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**Calculation of Lie transformation .**

Two 6-vectors  and  of the canonic dynamic variables for ion and electron are correspondingly



Lie transformation **** is as follows (formulae (12) from [1]):



with Lie operator **** defined by “perturbed” Hamiltonian



Perturbed Hamiltonian  is a function of main parameters characterized the trajectories of the particles (only  for electron and only  for ion correspondingly):



Map  is defined as Lie transformation:



where matrix  is defined by unperturbed Hamiltonian :



This Hamiltonian gives the following equations of the motion:





and not necessary present the changing of the phase , because it does not affect the dynamics of the ion-electron scattering event.

As is known, the Lie transformation of an arbitrary function  of dynamic variables is characterized by the following property of similarity:



So, the Lie transformation  is described by two independent matrices  and  with the following nonzero entries:



It means that



In this expression the velocities  and the coordinate  are used.

Let’s input the following values:





and then (with ) Hamiltonians  and  are equal to



Let’s rearrange the components of the vectors  and , forming two new canonically conjugate vectors  and :

,

where the index  for variables  takes on values . Then



It means that in according with the definition of the Lie operator through the Poison brackets for each  one has



To receive the previous relation two additional 6-vectors were defined: “zero”-vector  and “unit”-vector .

So, for change  and  of 6-vectors  and  are as follows:



Therefore, recalling the expression for the Hamiltonian , we obtain



And, quite similarly, one finds that





Shift of the ion due to interaction with the electron is as follows:



or



where



Quite similarly



and





The changing of the electron parameters due to a collision with the ion can be found as



Further



and at last



References.

1. D.L. Bruhwiler, S.D. Webb. *New Algorithm for Dynamical Friction of Ions in a Magnetized Electron Beam.* AIP Conf. Proc. **1812**, 050006 (2017). <http://aip.scitation.org/doi/abs/10.1063/1.4975867>.
2. David Bruhwiler, Stephen Webb, Dan T. Abell. *A New Approach to Calculating Dynamical Friction for Magnetized Electron Cooling.* Presented at HSC Section Meeting, CERN (Hadron Synchrotron Collective effects), 24 April 2017, Geneva.