**Step 1: Origin**

Hamiltonian for 2-body magnetized collision ((1) from [1], slide #7 from [2]; further [1]-(1) and [2]-#7 for simplicity):



Let suppose that in external longitudinal magnetic field , which is described by vector-potential , electrons are magnetized in contrast to ions. Then ([1]-(2a) and [2]-#7):



In this formula  is a canonical momentum of electron ([1]-2c), [2]-#7):



where is an electron Larmor frequency.

Hamiltonian  describes the Coulomb collision between electron and ion ([1]-(2b) and [2]-#7):



Resulting equation of motion, in standard drift-kick symplectic form is as follows ([2]-#7):



and the transformation  is ([2]-#8)



Here ([2]-#8):



**Question:** why the transformation  includes the conservation only two components of momentum ? I think, it should be here .

In turn, the transformation  is ([2]-#9)



**Step 2: Transformation to action-angle variables**

Let’s input the coordinates  of the Larmor circle center



and move from set of canonic pairs of variables  for electrons to set of guiding center pairs . To do the required transition the following generating function is used ([1]-4 and [2]-#10):



For convenience, let’s rewrite the last expression as



then



Important relations follow from expressions for  and :



So, the transformation from guiding center to particle coordinates is as follows:





To find the reverse transformation let express  as



and then



Therefore, the electron parts of Hamiltonian transform to



so that Hamiltonian  transforms to



To find new expression for Hamiltonian  let’s firstly transform the denominator in its formula, using the values :



and then ([1]-(6b) and [2]-#10)



**Notices:** a) Wrong expression for variables  is given in [1] and second variable is not defined in [2] (slide #10);

b) some misprints in expression for  are given in [2] (slide #10), but the corresponding expression in [1]-(6b) is correct.

**Step 3: Removing fast dependence from Hamiltonian**

This removing requires:

1.  is a perturbation regarding ;
2. All electron trajectories stay at least one Larmor radius away from the ion;
3. For all time during interaction between ion and electron Larmor radius satisfies the following relation ([1]-(7), [2]-#11):



**Notice:** If this relation formally interprets as a restriction to the Larmor radius, then squaring it leads to a trivial condition. I think, something is wrong here.

Taking into account the above conditions let’s transform from pair variables  to . Result is as follows ([1]-(8),(9),(10) and [2]-#11):



**Notice:** I do not yet know how to make this transition.

Again, ion-electron interaction is described with transformation



where now ([2]-#12)



and ([1]-(11a),(11b),(11c) and [2]-#13)



**Notices:** a) There are misprints in formulae (11a) and (11b) in [1];

b) I think, it is necessary to add the expression for  to formulae (11).

**Step 4 (Another approach): analytical map for scattering of ion with magnetized electron**

Technic of Lie operators allows describe the interaction of ions with magnetized electrons analytically.

The so-called Magnus expansion gives the following factored map ( is a total time of the interaction):



The unperturbed “drift” map  is defined by unperturbed Hamiltonian 



Actual interaction is described by Hamiltonian :



where



and this expression for  means that unperturbed solutions for coordinates and momenta are “inserted” into the partial Hamiltonian .

Details of calculation of  are presented in Appendix A. result is as follows:

**Notice:** Unfortunately, I have not completed my “investigations” with the latest materials in [1] (part “Analytic…”) and [2] (slides #17-#21).

**Appendix A.** Calculation of Lie operator .

This operator equals



with “perturbed” Hamiltonian



where



i.e. the following values are defined:



and



Map  is defined as Lie operator:



with



Pairs of canonic variables  for  are



Then





In this expression velocities  and coordinate  are used.

Hamiltonian  describes only the collision of an ion with an electron and this Hamiltonian "works" is preceded by the stage when the momenta of the colliding particles are conserved, and the coordinates change in accordance with the equations of motion in the drift space:



For this reason, dependence  on time is as follows:



Let’s input the following values:



Then



and further





At last,



and then



So (),



and



References

1. D.L. Bruhwiler, S.D. Webb. *New Algorithm for Dynamical Friction of Ions in a Magnetized Electron Beam.* AIP Conf. Proc. **1812**, 050006 (2017). <http://aip.scitation.org/doi/abs/10.1063/1.4975867>.
2. David Bruhwiler, Stephen Webb, Dan T. Abell. *A New Approach to Calculating Dynamical Friction for Magnetized Electron Cooling.* Presented at HSC Section Meeting, CERN (Hadron Synchrotron Collective effects), 24 April 2017, Geneva.