**Step 1: Origin**

Hamiltonian for 2-body magnetized collision ((1) from [1], slide #7 from [2]; further [1]-(1) and [2]-#7 for simplicity):



Let suppose that in external longitudinal magnetic field , which is described by vector-potential , electrons are magnetized in contrast to ions. Then ([1]-(2a) and [2]-#7):



In this formula  is a canonical momentum of electron ([1]-2c), [2]-#7):



where is an electron Larmor frequency.

Hamiltonian  describes the Coulomb collision between electron and ion ([1]-(2b) and [2]-#7):



Resulting equation of motion, in standard drift-kick symplectic form is as follows ([2]-#7):



and the transformation  is ([2]-#8)



Here ([2]-#8):



**Question:** why the transformation  includes the conservation only two components of momentum ? I think, it should be here .

In turn, the transformation  is ([2]-#9)



**Step 2: Transformation to action-angle variables**

Let’s input the coordinates  of the Larmor circle center



and move from set of canonic pairs of variables  for electrons to set of guiding center pairs . To do the required transition the following generating function is used ([1]-4 and [2]-#10):



For convenience, let’s rewrite the last expression as



then



Important relations follow from expressions for  and :



So, the transformation from guiding center to particle coordinates is as follows:





To find the reverse transformation let express  as



and then



Therefore, the electron parts of Hamiltonian transform to



so that Hamiltonian  transforms to



To find new expression for Hamiltonian  let’s firstly transform the denominator in its formula, using the values :



and then ([1]-(6b) and [2]-#10)



**Notices:** a) Wrong expression for variables  is given in [1] and second variable is not defined in [2] (slide #10);

b) some misprints in expression for  are given in [2] (slide #10), but the corresponding expression in [1]-(6b) is correct.

**Step 3: Removing fast dependence from Hamiltonian**

This removing requires:

1.  is a perturbation regarding ;
2. All electron trajectories stay at least one Larmor radius away from the ion;
3. For all time during interaction between ion and electron Larmor radius satisfies the following relation ([1]-(7), [2]-#11):



**Notice:** If this relation formally interprets as a restriction to the Larmor radius, then squaring it leads to a trivial condition. I think, something is wrong here.

Considering the above conditions let’s transform from pair variables  to . Result is as follows ([1]-(8),(9),(10) and [2]-#11):



**Notice:** I do not yet know how to make this transition.

Again, ion-electron interaction is described with transformation



where now ([2]-#12)



and ([1]-(11a),(11b),(11c) and [2]-#13)



**Notices:** a) There are misprints in formulae (11a) and (11b) in [1];

b) I think, it is necessary to add the expression for  to formulae (11).

**Step 4 (Another approach): analytical description of the scattering of ion with magnetized electron**

Technic of Lie operators allows describe the interaction of ions with magnetized electrons analytically.

The so-called Magnus expansion gives the following factored map ( is a total time of the interaction):



The unperturbed “drift” map  is defined by unperturbed Hamiltonian 



This Hamiltonian gives the following equations of the motion:





and not necessary present the changing of the phase , because it does not affect the dynamics of the ion-electron scattering event.

It is convenient further to use two 6-vectors  and  of the canonic dynamic variables for ion and electron are correspondingly



Actual interaction is described by “perturbed” Hamiltonian :



where Hamiltonian  is a function of main parameters characterized the trajectories of the particles (only  for electron and only  for ion correspondingly):



As is known, the Lie transformation of an arbitrary function  of dynamic variables is characterized by the following property of similarity:



So, the Lie transformation  is described by two independent matrices  and  with the following nonzero entries:



It means that



In this expression the velocities  and the coordinate  are used.

Let’s input the following values:





and then (with ) Hamiltonians  and  are equal to



Let’s define two 6-vector for canonical conjugate pairs  of the coordinates and momenta of ion and electron as

,

where the index  for variables  takes on values . Then



It means that in according with the definition of the Lie operator through the Poison brackets for each  one has



To receive the previous relation two additional 6-vectors were defined: “zero”-vector  and “unit”-vector .

So, for change  and  of 6-vectors  and  are as follows:



Therefore, recalling the expression for the Hamiltonian , we obtain



And



And, quite similarly, one finds that





The changing of the electron parameters due to a collision with the ion can be found as



References

1. D.L. Bruhwiler, S.D. Webb. *New Algorithm for Dynamical Friction of Ions in a Magnetized Electron Beam.* AIP Conf. Proc. **1812**, 050006 (2017). <http://aip.scitation.org/doi/abs/10.1063/1.4975867>.
2. David Bruhwiler, Stephen Webb, Dan T. Abell. *A New Approach to Calculating Dynamical Friction for Magnetized Electron Cooling.* Presented at HSC Section Meeting, CERN (Hadron Synchrotron Collective effects), 24 April 2017, Geneva.