1. Firstly, I’ll rewrite all formulas on the slides #20,21 in presentation for IOTA/FAST Workshop [1] and I’ll enumerate them:

 (1)

 (2)

 (3)

 (4)

 (5)

 (6)

Let

 (7)

then

 (8)

1. I start from the following expression ( is, on my understanding, the value of electron Larmor radius; this is a transformed expression (1)):

. (1a)

I do not quite understand its origin and therefore I have a few questions:

1. The dimension of the left part is , but the dimension of the right part is . **How it can be?**
2. The value  is the distance from the origin of coordinates (this is a position of ion with charge ) to the electron, the center of whose Larmor circle has the coordinates . So, the value  is a simply the potential of interaction between these particles. Thus, the right-hand side of this expression is the difference the potential energies of the particles at the zero instant of time and at time . Therefore, the right part must be equal to changing of kinetic energy of electron (the changes in the parameters of the transverse motion of the electron are neglected), i.e. left side is



Now the dimensions of both parts of the equation correspond to each other, but, unfortunately, the equation differs from (1).

So, the correct equation is as follows:

 (1b)

1. Let’s neglect by the ion velocity  in the expression (2) for the relative velocity . Then  and for this reason instead expression (1b) one has

 (1c)

**This is the correct equation replacing the expression (1).**

1. Let’s analyze the expression (3). It has the form of the continuity equation, but in this case on the right side there must be a divergence of the flow vector:

 (9)

and dimension of this expression is . If  is exactly the flow, then the dimension of the right-hand side will be  and then left side has a dimension  like . Thus, the left-hand side of expression (3) is derivative of the electric field over time and the right-hand side is the stream vector and therefore

 (10)

Nevertheless, equation (5) looks as if, into expression for the flow  instead of speed  was used the change in velocity . **Why?** So, corrected equation (5) (using expression (1c) for value of ) is as follows:

 (5a)

1. After the integration (it is also assumed that the electron velocity remains constant during the integration time) one obtains exactly expression (6):

 (6)

1. Next question is regarding expression (7): **Why?**
2. Let's find the time-averaged force  acting on the ions in a unit volume from the side of the found electric field:

 (11)

Once again: it is necessary understand that the above expression is the average longitudinal frictional force acting on ions that are in a unit volume of the beam. Substitution the expression (7) into (6) and last into (11) gives



or



i.e. it is received the final expression (8).

Let's summarize the implicit assumptions and unclear questions during the obtaining of the final expression (8):

1. The differing dimension of the right and left parts of the original equation (1).
2. The lost velocity  in the denominator of the right-hand side of equation (1) (compare equations (1) and (1c)).
3. The physical meaning of each of the sides of equation (3).
4. Why the flow vector  from equation (3) is determined by the increment  of the electron velocity, and not by the velocity  itself.
5. Why is the longitudinal coordinate  of the electron in equation (7) determined by the longitudinal ion velocity , and not by the electron velocity ? Ultimately the replacement of the value  by  in this equation will dramatically change the result (8).
6. The above scheme does not allow us to find the transverse components of the frictional force.

Reference.

[1]. David Bruhwiler, Stephen Webb, Dan T. Abel, Yury Eidelman. *A new approach to calculating dynamic friction for magnetized electron coolers.* Fermilab Workshop on Megawatt Ring & IOTA/FAST Collaboration Meeting. 9 May 2018, Batavia.