

Measures of classification performance

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Some images in these slides are from (or adapted from):

A. Geron, Hands-on Machine Learning with Scikit-Learn, Keras and TensorFlow, O'Reilly, 2020

The MNIST dataset

70000 images

Each image: 28x28
features

Each feature can take
values in $[0, 255]$



A binary classification problem

We want to build a “5-detector” capable of distinguishing between just two classes,

- 5 (C_1 or “positives”, P)
- not-5 (C_0 or “negatives”, N).

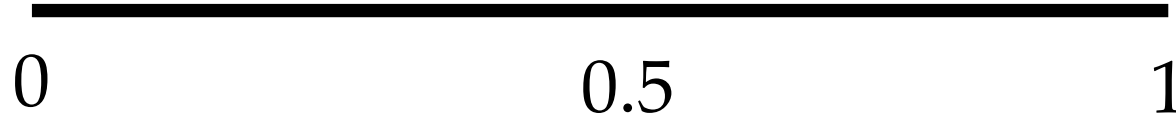
Let us assume we have built our classifier.

How can I evaluate its performance on the test set?

Remember: we are always interested in generalization, i.e. how well it will perform on unseen data (that is the test set)

Confusion matrix

Classifier output: probability of being a 1



		predicted	
actual		TN	FP <i>Type 1 error (false alarm)</i>
		FN <i>Type 2 error (miss)</i>	TP

Confusion Matrix

actual	predicted	
	TN	FP
FN	TP	

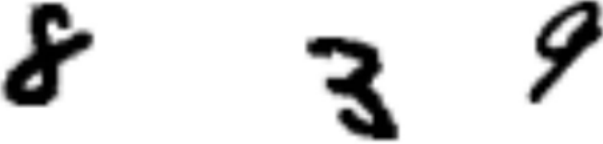



		Predicted	
		Negative	Positive
Actual	Negative		
	Positive		

Diagram illustrating a Confusion Matrix for handwritten digit classification. The matrix is divided into four quadrants based on Actual (rows) and Predicted (columns) outcomes.

- TN (True Negative):** Top-left quadrant, labeled "Negative" for both Actual and Predicted. Contains handwritten digits 8, 3, and 9.
- FP (False Positive):** Top-right quadrant, labeled "Negative" for Actual and "Positive" for Predicted. Contains handwritten digit 6.
- FN (False Negative):** Bottom-left quadrant, labeled "Positive" for Actual and "Negative" for Predicted. Contains handwritten digits 7 and 2.
- TP (True Positive):** Bottom-right quadrant, labeled "Positive" for both Actual and Predicted. Contains handwritten digits 5, 5, and 5.

Measures

actual \ predicted		
	TN	FP
	FN	TP

Accuracy: percentage of correct predictions

$$\frac{TP + TN}{TP + TN + FP + FN}$$

Precision: the accuracy of the correct prediction (*how precise am I? what is the % of correct, out of all those that I predict as P?*)

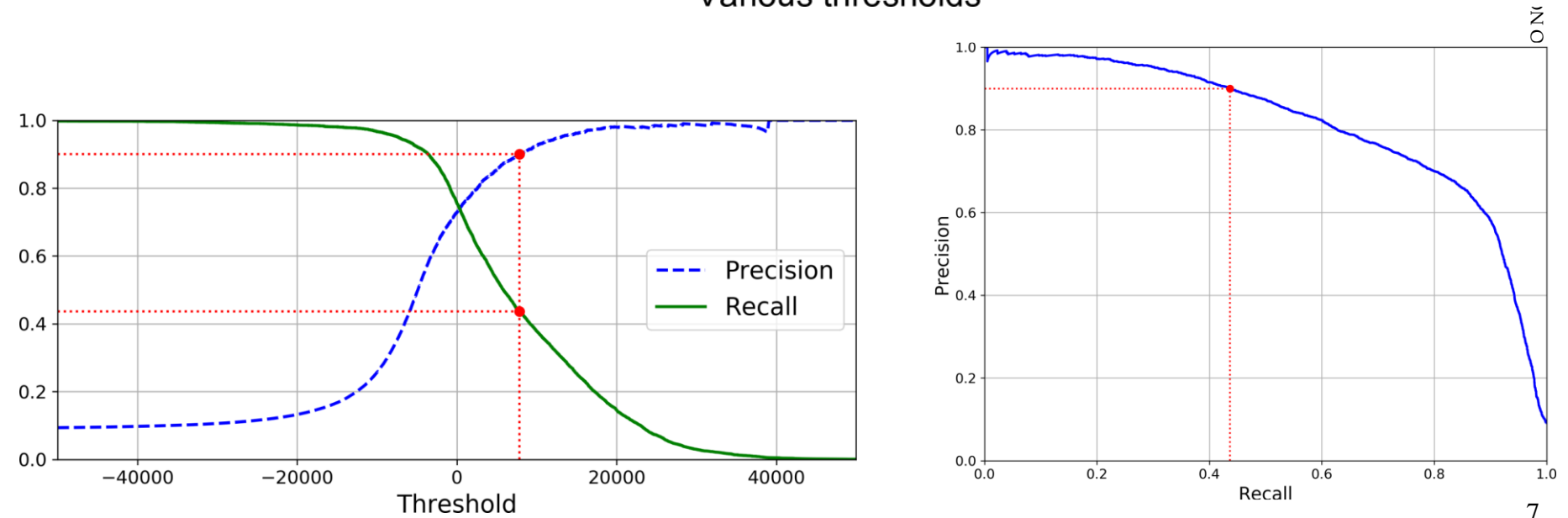
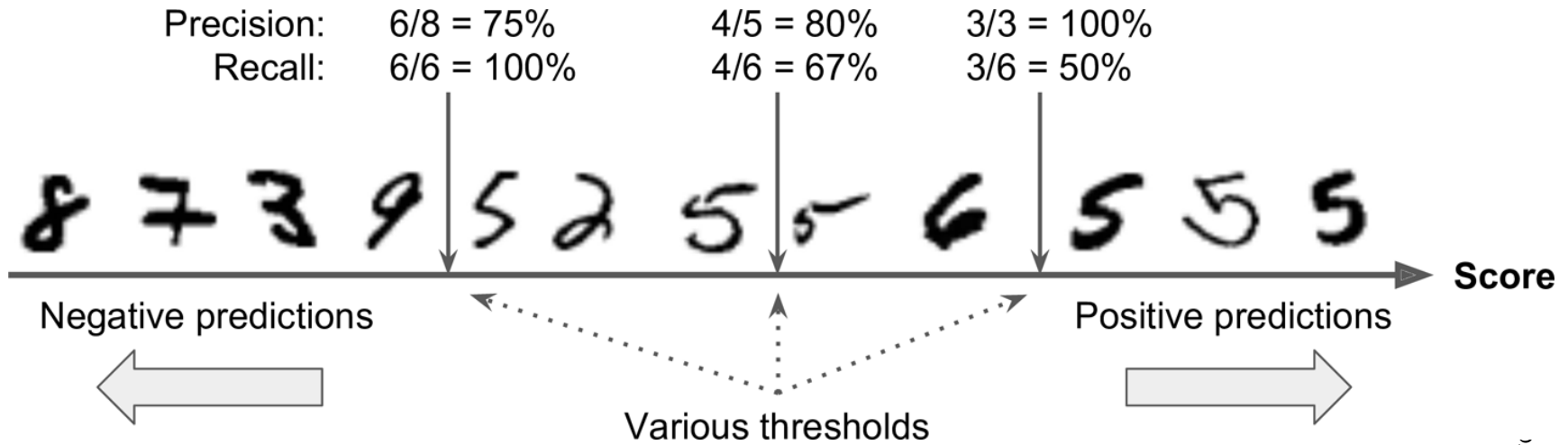
$$\frac{TP}{TP + FP}$$

Recall: percentage of positive instances that are correctly predicted (*how much do I “cover” the P? what is the % of correct, out of all those that are P?*)

$$\frac{TP}{P} = \frac{TP}{TP + FN}$$

Unfortunately: increasing precision reduces recall, and vice versa...

Precision/recall trade-off



F_1 measure

$$\text{HM}(x_1, \dots, x_n) = \frac{n}{\frac{1}{x_1} + \dots + \frac{1}{x_n}}$$

The harmonic mean of precision and recall:

$$F_1 = \frac{2}{\frac{1}{\text{precision}} + \frac{1}{\text{recall}}} = 2 \times \frac{\text{precision} \times \text{recall}}{\text{precision} + \text{recall}} = \frac{TP}{TP + \frac{FN + FP}{2}}$$

It is higher when precision and recall are both high.

These considerations help us to pick a threshold.
Can I use these ideas to compare the performance of classifiers?

The receiver operating characteristic (ROC) curve

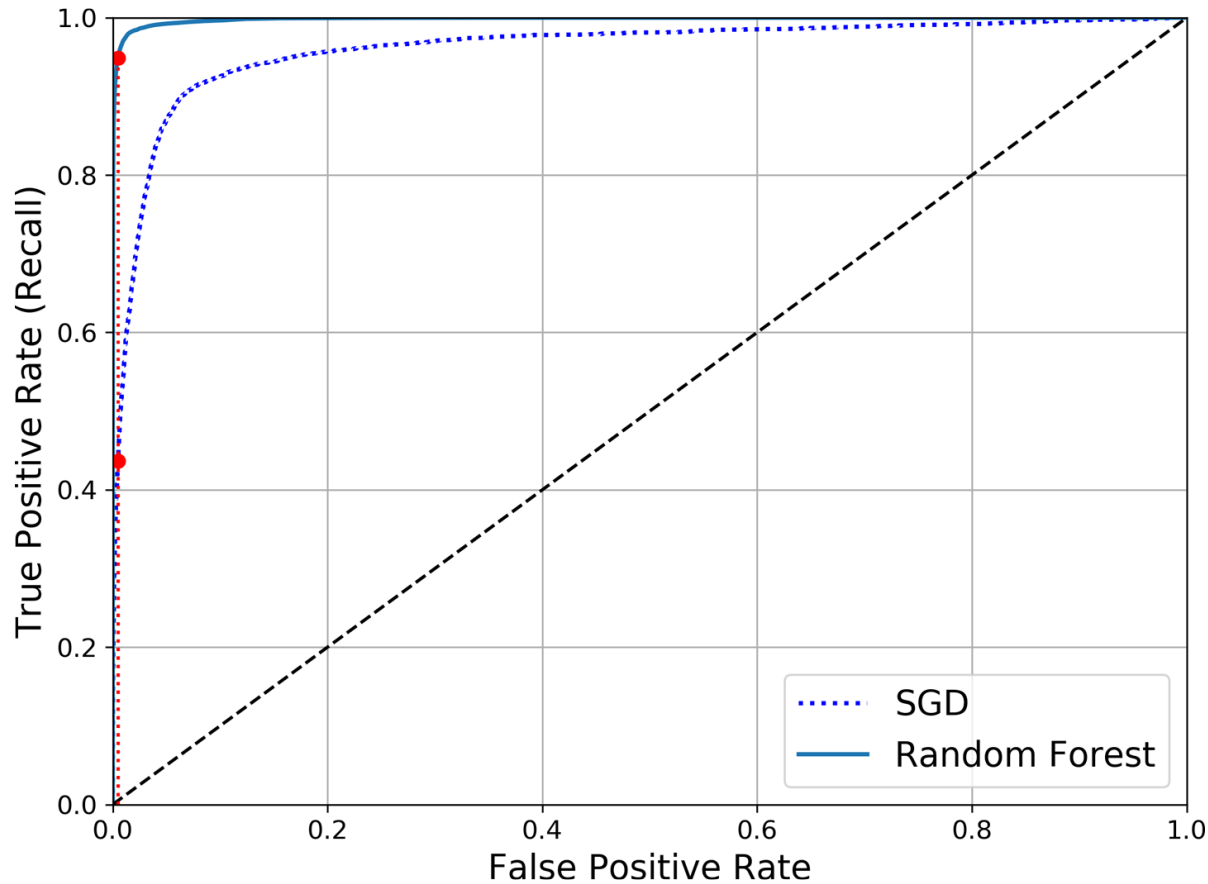
actual	predicted	
	TN	FP
	FN	TP

True positive rate (= recall, sensitivity): % of positive instances that are correctly predicted

$$\frac{TP}{P} = \frac{TP}{TP + FN}$$

False positive rate: the % of negative instances that are incorrectly predicted.

$$1 - \text{specificity} = 1 - \frac{TN}{N} = \frac{FP}{FP + TN}$$



One way to compare classifiers is to measure the *area under the curve* (AUC).

A purely random classifier will have a ROC AUC equal to 0.5.