Lista de Exercícios 1

(1)

$\{(u-v)(u-v) = u^2 - uv - uv + v^2 = u^2 - 2uv + v^2\}$

a) $\|u+v\|^2 - \|u-v\|^2 = 4u \cdot v$

$(u+v)^2 - (u-v)^2$ // Pois $\|x\|^2 = x \cdot x$ (norma de um vetor)

$u^2 + 2uv + v^2 - (u^2 - 2uv + v^2)$

$\cancel{u^2} + 2uv + \cancel{v^2} - \cancel{u^2} + 2uv - \cancel{v^2}$

$+2uv + 2uv = 4u \cdot v$ //

b) u, v são ortogonais $\Leftrightarrow \|u+v\| = \|u-v\|$

Seja $u = (u_1, u_2, u_3)$ e $v = (v_1, v_2, v_3)$

$\|u+v\| = \|u-v\|$

$\|u+v\|^2 = \|u-v\|^2$

$\Leftrightarrow (u+v)^2 = (u-v)^2$

$\Leftrightarrow u^2 + 2uv + v^2 = u^2 - 2uv + v^2$

$\Rightarrow u^2 - u^2 + 2uv + 2uv + v^2 - v^2 = 0$

// Pois $u \perp v \Rightarrow u \cdot v = 0$

$u^2 + 2uv + v^2 - (u^2 - 2uv + v^2)$

$u^2 + 2uv + v^2 - u^2 + 2uv - v^2$

$+2uv + 2uv = 4u \cdot v$ //

b) u, v são ortogonais $\Leftrightarrow \|u+v\| = \|u-v\|$.

Seja $u = (u_1, u_2, u_3)$ e $v = (v_1, v_2, v_3)$

$\|u+v\| = \|u-v\|$

$\|u+v\|^2 = \|u-v\|^2$

$\Leftrightarrow (u+v)^2 = (u-v)^2$

$\Leftrightarrow u^2 + 2uv + v^2 = u^2 - 2uv + v^2$

$\Leftrightarrow u^2 - u^2 + 2uv + 2uv + v^2 - v^2 = 0$

$0 + 2\cancel{uv}^{0} + 2\cancel{uv}^{0} + 0 = 0$ // Por $u \perp v \Rightarrow u \cdot v = 0$

$\therefore 0 = 0$ ✓

c) u, v são ortogonais $\Leftrightarrow \|u+v\|^2 = \|u\|^2 + \|v\|^2$

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$\|u+v\|^2 = \|u-v\|^2$

$\iff (u+v)^2 = (u-v)^2$

$\iff u^2 + 2uv + v^2 = u^2 - 2uv + v^2$

$\iff u^2 - u^2 + 2uv + 2uv + v^2 - v^2 = 0$

$0 + \cancelto{0}{2uv} + \cancelto{0}{2uv} + 0 = 0$ // Por $u \perp v \Rightarrow u \cdot v = 0$

$\therefore\ 0 = 0$ ✓

c) u, v são ortogonais $\iff \|u+v\|^2 = \|u\|^2 + \|v\|^2$

$\|u+v\|^2 = \|u\|^2 + \|v\|^2$

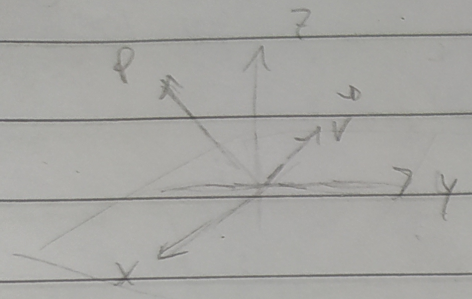
$(u+v)^2 = \|u\|^2 + \|v\|^2$

$u^2 + \cancelto{0}{2uv} + v^2 = \|u\|^2 + \|v\|^2$ // Por $u \perp v \Rightarrow u \cdot v = 0$

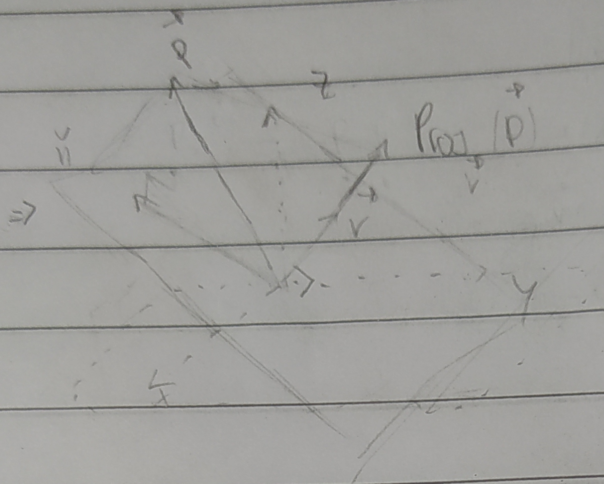
$u^2 + v^2 = \|u\|^2 + \|v\|^2$

$\|u\|^2 + \|v\|^2 = \|u\|^2 + \|v\|^2$ ✓ // Norma de um vetor: $u \cdot u = \|u\|^2$

a) Temos que



$\Rightarrow$



$$\mathrm{Proj}_{\vec{v}}\,\vec{P} = \frac{\vec{P}\cdot\vec{v}}{\|\vec{v}\|^2}\cdot\vec{v} \Rightarrow \mathrm{Proj}_{\vec{v}}\,\vec{P} = \frac{(2,-1,3)(1,1,1)}{\|(1,1,1)\|^2}\cdot\vec{v} = \frac{2-1+3}{1^2+1^2+1^2}\cdot\vec{v} = \frac{4}{3}\vec{v} \;//$$

Logo,

$$\vec{P} - \mathrm{Proj}_{\vec{v}}\,\vec{P} = (2,-1,3) - \left(\frac{4}{3}, \frac{4}{3}, \frac{4}{3}\right) = \left(\frac{2}{3}, \frac{-7}{3}, \frac{5}{3}\right) = D \;//$$

b)

Seja r uma reta que conecta P e D (sombra de P em π)

$r: \dots \Rightarrow r: (2,-1,3) + \lambda\,[P-D] \Rightarrow r: (2,-1,3) + \lambda\left(\frac{4}{3}, \frac{4}{3}, \frac{4}{3}\right)$ ...

$$\text{Proj}_{\vec{v}}\vec{P} = \frac{\vec{P}\cdot\vec{v}}{\|\vec{v}\|^2}\cdot\vec{v} \Rightarrow \text{Proj}_{\vec{v}}\vec{P} = \frac{(2,-1,3)(1,1,1)}{\|(1,1,1)\|^2}\cdot\vec{v} = \frac{2-1+3}{1^2+1^2+1^2}\cdot\vec{v} = \frac{4}{3}\vec{v}$$ //

Logo,

$$\vec{P} - \text{Proj}_{\vec{v}}\vec{P} = (2,-1,3) - \left(\frac{4}{3}, \frac{4}{3}, \frac{4}{3}\right) = \left(\frac{2}{3}, -\frac{7}{3}, \frac{5}{3}\right) = D$$ //

b)

Seja r uma reta que conecta P e D (sombra de P em π)

$r: P + \lambda\vec{u} \Rightarrow r: (2,-1,3) + \lambda[P-D] \Rightarrow r: (2,-1,3) + \lambda\left(\frac{4}{3}, \frac{4}{3}, \frac{4}{3}\right)$ equação da reta //

Seja $Q = (x,y,z)$ um ponto do plano $\mathcal{E}$, $\vec{QO}$ é um vetor em $\mathcal{E}$

$\vec{QO}\cdot\vec{n} = (x-0, y-0, z-0)\cdot(1,0,1) = 0 \Rightarrow x + 0\cdot y + z = 0 \Rightarrow x + z = 0$ equação do plano //

Temos que o ponto que está em r e $\mathcal{E}$ é dado por

$$2 + \lambda\frac{4}{3} + 3 + \lambda\frac{4}{3} = 0 \Rightarrow 5 + 2\lambda\frac{4}{3} = 0 \Rightarrow \lambda = -\frac{15}{8}$$ //

Aplicando em r: $(2,-1,3) - \frac{15}{8}\left(\frac{4}{3}, \frac{4}{3}, \frac{4}{3}\right) = \boxed{\left(-\frac{1}{2}, -\frac{7}{2}, \frac{1}{2}\right)}$ //

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