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Lista de Transformações

① Seja $T: V \to W$ uma função, em que V e W são espaços vetoriais de dimensão finita com escalares reais.

V, F, V, V, F, F, V, F

② a)

$T(1,0,0) = (1,2)$ $T(0,1,1) = (-1,-2)$ $T(0,1,-1) = (5,-4)$

Temos que,

$$\left[\begin{array}{ccc|c|c} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 1 & -1 & -2 \\ 0 & 1 & -1 & 5 & -4 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c|c} 1 & 0 & 0 & 1 & 2 \\ 0 & 2 & 0 & 4 & -6 \\ 0 & 1 & -1 & 5 & -4 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c|c} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 1 & -3 & 1 \end{array}\right] \Rightarrow T(x,y,z) = (x+2y-3z,\ 2x-3y+z)$$

b) Seja matriz de $T \Rightarrow (S,T) = T(x,y,z)$ Temos que,

$$\begin{bmatrix} S \\ \end{bmatrix} = \begin{bmatrix} 1 & 2 & -3 \\ \end{bmatrix} \cdot \begin{bmatrix} x \\ \end{bmatrix}$$

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$$\Rightarrow \left[\begin{array}{ccc|c|c} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 1 & -3 & 1 \end{array}\right] \Rightarrow T(x,y,z) = (x+2y-3z,\ 2x-3y+z)$$

b) Seja matriz de $T \Rightarrow (s,t) = T(x,y,z)$ Temos que,

$$\begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} 1 & 2 & -3 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

③ Seja $B = \{e_{11}, e_{12}, e_{21}, e_{22}\}$ com

$e_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $e_{12} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $e_{21} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ $e_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$$T(A) = A^t$$

a) Seja $M_2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Sabendo que $T(U+V) = T(U) + T(V)$, temos que

$$U = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \quad V = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \Rightarrow U+V = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix}$$

Logo,

$$T(U+V) = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix} = T(U) + T(V)$$

Sabemos que $\alpha T(U) = T(\alpha U)$ $\alpha \in \mathbb{R}$

$$U = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad e \quad \alpha U = \begin{bmatrix} \alpha a & \alpha b \\ \alpha c & \alpha d \end{bmatrix}$$

Logo,

$$T(\alpha U) = \begin{bmatrix} \alpha a & \alpha b \\ \alpha c & \alpha d \end{bmatrix} \quad e \quad \alpha T(U) = \alpha \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \alpha a & \alpha b \\ \alpha c & \alpha d \end{bmatrix} = T(\alpha U),$$

Portanto T é transformação linear.

b) Seja $A = \begin{bmatrix} a & b \end{bmatrix} \Rightarrow A = \begin{bmatrix} a \end{bmatrix}$ ... A^T ... A^{-1} $\begin{bmatrix} a \end{bmatrix}$

$\begin{bmatrix} c & d \end{bmatrix} \quad \begin{bmatrix} \alpha c & \alpha d \end{bmatrix}$

Logo,

$$T(\alpha v) = \begin{bmatrix} \alpha a & \alpha b \\ \alpha c & \alpha d \end{bmatrix} \quad e \quad \alpha T(v) = \alpha \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \alpha a & \alpha b \\ \alpha c & \alpha d \end{bmatrix} = T(\alpha v),$$

Portanto, T é transformação linear.

b) Seja $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow A' = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$, temos $A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \Rightarrow A^{T'} = \begin{bmatrix} a \\ c \\ b \\ d \end{bmatrix}$

Logo,

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Portanto,

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$