2° Lista de Exercícios 11218813

(1)

a)

$$\begin{array}{c} L_1 \\ L_2 \\ L_3 \end{array} \left(\begin{array}{ccc|c} 0 & 1 & 2 & 0 \\ 1 & 1 & 3 & 2 \\ 2 & 1 & 5 & 5 \end{array}\right) \; L_3 \leftarrow 2L_2 - L_3 \sim \left(\begin{array}{ccc|c} 0 & 1 & 2 & 0 \\ 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & -1 \end{array}\right) \; L_3 \leftarrow L_3 - L_1$$

$$\sim \left(\begin{array}{ccc|c} 0 & 1 & 2 & 0 \\ 1 & 1 & 3 & 2 \\ 0 & 0 & -1 & -1 \end{array}\right) \; L_3 \leftarrow -L_3 \sim \left(\begin{array}{ccc|c} 0 & 1 & 2 & 0 \\ 1 & 1 & 3 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) \; L_2 \leftarrow L_2 - 3L_3$$

$$\sim \left(\begin{array}{ccc|c} 0 & 1 & 2 & 0 \\ 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right) \; L_1 \leftarrow L_1 - 2L_3 \sim \left(\begin{array}{ccc|c} 0 & 1 & 0 & -2 \\ 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right) \; L_2 \leftarrow L_2 - L_1$$

$$\sim \left(\begin{array}{ccc|c} 0 & 1 & 0 & -2 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

Temos que, $y = -2$, $x = 1$ e $z = 1$.

Portanto, possível e determinado.

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b)

$$\left(\begin{array}{ccc|c} 2 & 1 & 2 & 4 \\ 1 & 2 & 1 & 5 \\ 1 & 1 & 1 & 3 \end{array}\right) \leftarrow L_3 : L_2 - L_3 \sim \left(\begin{array}{ccc|c} 2 & 1 & 2 & 4 \\ 1 & 2 & 1 & 5 \\ 0 & 1 & 0 & 2 \end{array}\right) \leftarrow L_2 : L_2 - 2L_3$$

$$\sim \left(\begin{array}{ccc|c} 2 & 1 & 2 & 4 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \end{array}\right) L_1 \leftarrow L_1 - 2L_2 \sim \left(\begin{array}{ccc|c} 0 & 1 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \end{array}\right)$$

Se fizermos $L_1 \leftarrow L_1 - L_3$ ou $L_3 \leftarrow L_3 - L_1$ teremos uma linha nula na matriz, o que indica infinitas soluções.

Temos que $y = 2$, $x + z = 1 \Rightarrow z = 1 - x$, logo $(x, 2, 1 - x)$ então possível indeterminado.

c)

$$\left(\begin{array}{ccc|c} 2 & 1 & 2 & 8 \\ 1 & 2 & 1 & 5 \\ 1 & 1 & 1 & 5 \end{array}\right) L_2 \leftarrow L_2 - L_3 \sim \left(\begin{array}{ccc|c} 2 & 1 & 2 & 8 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 5 \end{array}\right) L_1 \leftarrow 2L_3 - L_1$$

$$\sim \left(\begin{array}{ccc|c} 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 \end{array}\right)$$

Chegamos a uma indeterminação $y = 2$ e $y = 0$.
Portanto, este sistema é impossível.

$\begin{matrix} 0 & 1 & 0 & | & 2 \end{matrix}$

Temos que $y = 2$, $x + z = 1 \Rightarrow z = 1 - x$, logo $(x, 2, 1-x)$ então possível indeterminado.

c)

$$\left(\begin{array}{ccc|c} 2 & 1 & 2 & 8 \\ 1 & 2 & 1 & 5 \\ 1 & 1 & 1 & 5 \end{array}\right) L_2 \leftarrow L_2 - L_3 \sim \left(\begin{array}{ccc|c} 2 & 1 & 2 & 8 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 5 \end{array}\right) L_1 \leftarrow 2L_3 - L_1$$

$$\sim \left(\begin{array}{ccc|c} 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 5 \end{array}\right)$$

Chegamos a uma indeterminação $y = 2$ e $y = 0$.
Portanto, este sistema é impossível.

2

$$AX = B \quad \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \left(\begin{array}{c|c} -3 & 0 \\ 3 & 3 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c|c} 1 & -1 & -3 & 0 \\ 1 & 2 & 3 & 3 \end{array}\right) L_1 \leftarrow 2L_1 + L_2$$

$$\sim \left(\begin{array}{cc|c|c} 3 & 0 & -3 & 3 \\ 1 & 2 & 3 & 3 \end{array}\right) \leftarrow L_1/3 \sim \left(\begin{array}{cc|c|c} 1 & 0 & -1 & 1 \\ 1 & 2 & 3 & 3 \end{array}\right) L_2 \leftarrow L_2 - L_1$$

$$\sim \left(\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 5 \end{array}\right)$$

Portanto, este sistema é impossível.

(2)

$$AX = B \quad \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \left(\begin{array}{c|c} -3 & 0 \\ 3 & 3 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c|c} 1 & -1 & -3 & 0 \\ 1 & 2 & 3 & 3 \end{array}\right) \; L_1 \leftarrow 2L_1 + L_2$$

$$\sim \left(\begin{array}{cc|c|c} 3 & 0 & -3 & 3 \\ 1 & 2 & 3 & 3 \end{array}\right) \leftarrow L_1/3 \sim \left(\begin{array}{cc|c|c} 1 & 0 & -1 & 1 \\ 1 & 2 & 3 & 3 \end{array}\right) \; L_2 \leftarrow L_2 - L_1$$

$$\sim \left(\begin{array}{cc|c|c} 1 & 0 & -1 & 1 \\ 0 & 2 & 4 & 2 \end{array}\right) \; L_2 \leftarrow L_2/2 \quad \sim \left(\begin{array}{cc|cc} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 1 \end{array}\right)_{//}$$

Portanto, $X = \begin{pmatrix} -1 & 1 \\ 2 & 1 \end{pmatrix}_{//}$