S T Q Q S S D   ___/___/___

Lista 9   Bruna Nogueira da Cruz, 11218813

①

a) $\begin{cases} x'' + 4x' - 5x = 6e^{t} \\ x(0) = 0, \; x'(0) = 7 \end{cases}$

Temos que a equação característica é: $\lambda^2 + 4\lambda + 5 = 0$ logo $\lambda_1 = 1$ e $\lambda_2 = -5$

Logo, $y_1 = e^{t}$, $y_2 = e^{-5t}$

Portanto,

$x_p(t) = t(a_1 e^{t}) \Rightarrow x_p'(t) = a_1 e^{t} + a_1 e^{t} t \Rightarrow x_p''(t) = a_1(2e^{t} + e^{t} t)$

Temos que $a_1(2e^{t} + e^{t} t) + 4(a_1 e^{t} + a_1 e^{t} t) - 5(a_1 e^{t} t) = 6e^{t} \Rightarrow x_p(t) = e^{t} t$

Por fim,

$x(t) = x_c(t) + x_p(t) = e^{t} t + c_1 e^{-5t} + c_2 e^{t} \Rightarrow x'(t) = e^{t} + e^{t} t - 5c_1 e^{-5t} + c_2 e^{t}$

$\Rightarrow x(0) = 0 \Rightarrow c_1 + c_2 = 0$ e $x'(0) = 7 \Rightarrow -5c_1 + c_2 + 1 = 7$

Então $\begin{cases} c_1 = -1 \\ c_2 = 1 \end{cases} \Rightarrow \boxed{x(t) = -e^{-5t} + e^{t}(t+1)}$

b) $\begin{cases} x'' - 2x' + 2x = 0 \\ x(0) = 1, \; x'(0) = 0 \end{cases}$

Temos a equação característica $\lambda^2 - 2\lambda + 2 = 0 \Rightarrow \lambda = 1 \pm i$

Logo

$z(t) = e^{(1+i)t} = e^{t} \cdot e^{it} = e^{t}\cos t + i e^{t} \operatorname{sen} t \Rightarrow y_1 = e^{t}\cos t$ e $y_2 = e^{t} \operatorname{sen} t$

Como $y = c_1 e^{t}\cos t + c_2 e^{t} \operatorname{sen} t$, então

$y(0) = e^{0}\cos(0) \cdot c_1 + e^{0}\operatorname{sen}(0) c_2 \Rightarrow c_1 = 1$

$y'(0) = e^{0}\cos(0) c_1 - e^{0}\operatorname{sen}(0) c_2 + e^{0}\cos(0) c_2 + e^{0}\operatorname{sen}(0) c_2 = 0 \Rightarrow c_2 = -1$

Portanto, $\boxed{y(t) = e^{t}(\cos t - \operatorname{sen} t)}$

c) $\begin{cases} x'' - 2x' + x = t \\ x(0) = 1, \; x'(0) = 1 \end{cases}$ Equação característica: $\lambda^2 - 2\lambda + 1 = 0 \Rightarrow \lambda_1 = 1$

Temos que

$\begin{cases} x_p(t) = a_1 + a_2 t \\ x_p'(t) = (a_1 + a_2 t)' = a_2 \\ x_p''(t) = 0 \end{cases} \Rightarrow \begin{cases} -2a_2 + a_1 + a_2 t = t \\ -2a_2 + a_1 = 0 \end{cases} \Rightarrow a_1 = 2$ e $a_2 = 1$

Logo, $x_p(t) = 2+t$ e $x(t) = c_1 e^t + c_2 e^t t + 2 + t$

Portanto

$$\begin{cases} x(0) = c_1 e^0 + c_2 e^0 0 + 2 \cdot 0 = 1 \Rightarrow c_1 = -1 \\ x'(0) = e^0 c_1 + e^0 c_2 + e^0 \cdot 0 + 1 = 1 \Rightarrow c_2 = 0 \end{cases}$$

Por fim, $x(t) = -e^t + 2 + t$

(2)

a) $h(t) = 12\cos t$

Temos que

$(e^{\lambda t})'' + 4(e^{\lambda t})' = 0 \Rightarrow \lambda^2 + 4\lambda = 0 \Rightarrow \lambda_1 = -4$ e $\lambda_2 = 0$

Assim

- $\lambda_1 = -4 \Rightarrow x_1(t) = c_1 e^{-4t} \Rightarrow x(t) = x_1(t) + x_2(t)$
- $\lambda_2 = 0 \Rightarrow x_2(t) = c_2 \Rightarrow x(t) = c_1 e^{-4t} + c_2$

Além disso, $x'' + 4x' = 12\cos t$

$$\Rightarrow \begin{cases} x_p'(t) = -a_1 \operatorname{sen}(t) + a_2 \cos(t) & \Rightarrow -a_1 \cos(t) - a_2 \operatorname{sen}(t) + 4(-a_1 \operatorname{sen}(t) + a_2 \cos(t)) = 12\cos(t) \\ x_p''(t) = -a_1 \cos(t) - a_2 \operatorname{sen}(t) & \Rightarrow a_1 = \frac{-12}{17},\ a_2 = \frac{48}{17} \end{cases}$$

Logo $x_p(t) = \frac{-12\cos(t)}{17} + \frac{48\operatorname{sen}(t)}{17}$

Portanto,

$x(t) = x_c(t) + x_p(t) = \frac{-12\cos(t)}{17} + \frac{48\operatorname{sen}(t)}{17} + c_1 e^{-4t} + c_2 \Rightarrow$ limitada

b) $h(t) = 12\cos 2t$

Por (a), $x(t) = c_1 e^{-4t} + c_2$

Além disso, $x'' + 4x' = 12\cos 2t$

$$\Rightarrow \begin{cases} x_p'(t) = -2a_1 \operatorname{sen}(2t) + 2a_2 \cos(2t) & \Rightarrow -4a_1 \cos(2t) - 4a_2 \operatorname{sen}(2t) + 4(-2a_1 \operatorname{sen}(2t) + 2a_2 \cos(2t)) \\ x_p''(t) = -4a_1 \cos(2t) - 4a_2 \operatorname{sen}(2t) & = 12\cos 2t \Rightarrow a_1 = -3/5,\ a_2 = -6/5 \end{cases}$$

Portanto,

$\frac{-3}{5}\cos(2t) + \frac{6}{5}\operatorname{sen}(2t) + c_1 e^{-4t} + c_2 \Rightarrow$ limitada