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(1)

$$A = \begin{pmatrix} 0 & 0 & 0 \\ -4 & 1 & -2 \\ 2 & 0 & 2 \end{pmatrix}$$

a) $A = \begin{pmatrix} 0 & 0 & 0 \\ -4 & 1 & -2 \\ 2 & 0 & 2 \end{pmatrix} \Rightarrow A - \lambda I = \begin{pmatrix} -\lambda & 0 & 0 \\ -4 & 1-\lambda & -2 \\ 2 & 0 & 2-\lambda \end{pmatrix} \Rightarrow \det(A - \lambda I) = \lambda(1-\lambda)(2-\lambda) = 0$

Portanto, seus autovalores são 0, 1 e 2.

b)

• $\lambda = 0$

$A - \lambda I = A - 0 \cdot I = A \cdot v_0 = \begin{pmatrix} 0 & 0 & 0 \\ -4 & 1 & -2 \\ 2 & 0 & 2 \end{pmatrix} \cdot v_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Logo, $\begin{cases} 0x + 0y + 0z = 0 \\ -4x + 1y + (-2)z = 0 \\ 2x + 0y + 2z = 0 \end{cases} \Rightarrow x = -z, \ y = 2z$ Portanto, $v_0 = (1, 2, -1)$

• $\lambda = 1$

$(A - I) \cdot v_1 = \begin{pmatrix} -1 & 0 & 0 \\ -4 & 0 & -2 \\ 2 & 0 & 1 \end{pmatrix} v_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Logo $\begin{cases} -x = 0 \\ -4x + 0y - 2z = 0 \\ 2x + 0y + z = 0 \end{cases} \Rightarrow x = 0, \ z = 0, \ \forall y$ Portanto, $v_1 = (0, 1, 0)$

• $\lambda = 2$

$(A - 2I) \, v_2 = \begin{pmatrix} -2 & 0 & 0 \\ -4 & -1 & -2 \\ 2 & 0 & 0 \end{pmatrix} \cdot v_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Logo $\begin{cases} x = 0 \\ -4x - y - 2z = 0 \\ y = -2z \end{cases} \Rightarrow x = 0, \ y = -2z$ Portanto, $v_2 = (0, -2, 1)$

c) Para formar uma base, o determinante da base B deve ser diferente de 0, logo

$$B = \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 0 \\ 2 & 1 & -2 & 2 & 1 \\ -1 & 0 & 1 & -1 & 0 \end{array}\right] \Rightarrow \det(B) = -1 \Rightarrow \text{base (L.I)}$$

d) Como $[T]_C = [T]_B \cdot M$, então

$$\begin{pmatrix} 0 & 0 & 0 \\ -4 & 1 & -2 \\ -2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 1 & 0 & 1 \end{pmatrix}$$

Logo,

$$[T]_B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \\ -2 & 0 & -2 \end{pmatrix}$$ //

e) $B = PAP^{-1}$, logo

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ -1 & 0 & -1 \end{pmatrix} \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & -1 & 0 & -1 \end{array}\right)$$

Então,

$$P^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ -1 & 0 & -1 \end{pmatrix}$$

Portanto,

$$B = P \cdot AP^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ -1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ -4 & 1 & -2 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ -1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$ //