Maintenance Scheduling System

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Agenda

- Introduction to Maintenance Scheduling
- Architecture of a Scheduling System
- Possible Contributions to Operation Research

Mathematical Notation: Sets

$a \in A^m_{b,c}(t, x, y)$

- a: set element
- A: set itself
- b: set element from set B
- c: set element from set C
- m: model formulation m
- t: time
- x: value of decision variable from a different model
- y: value of decision variable from a different model

Mathematical Notation: Parameters

$name\_of\_parameter_{a,b}(t, x, y)$

- parameters are functions of set elements and input parameters
- a: set element from A
- b: set element from B
- t: time
- x: value of decision variable from another model
- y: value of decision variable from another model

Mathematical Notation: Variables

$x^m_{a,b}(t)$

- variables are functions of set elements, specified model, and time
- x: decision variable
- a: set element from A
- b: set element from B
- m: specifying the model
- t: time
- Notice: decision variables cannot depend on other decision variables as it would make them belong to the same model.

Strategic

Meta variables:

$$s \in S \tag{1}$$

$$\beta(\tau) \tag{2}$$

$$\tau \in [0, \infty] \tag{3}$$

Maximize:

$$\sum_{w \in W(\tau)} \sum_{p \in P(\tau)} \mathit{strategic\_value}_{wp}(\tau) \cdot \alpha_{wp}(\tau) - \sum_{p \in P(\tau)} \sum_{r \in R(\tau)} \mathit{strategic\_penalty} \cdot \epsilon_{pr}(\tau) + \sum_{p \in P(\tau)} \sum_{w1 \in W(\tau)} \sum_{w2 \in W(\tau)} \mathit{clustering\_value}_{w1,w2} \cdot \alpha_{w1p}(\tau) \cdot \alpha_{w2p}(\tau) \tag{4}$$

Subject to:

$$\sum_{w \in W(\tau)} \mathit{work\_order\_work}_{wr} \cdot \alpha_{wp}(\tau) \leq \mathit{resource}_{pr}(\tau, \beta(\tau)) + \epsilon_{pr}(\tau) \quad \forall p \in P(\tau) \quad \forall r \in R(\tau) \tag{5}$$

$$\sum_{w \in W(\tau)} \alpha_{wp}(\tau) = 1 \quad \forall p \in P(\tau) \tag{6}$$

$$\alpha_{wp}(\tau) = 0 \quad \forall (w, p) \in \mathit{exclude}(\tau) \tag{7}$$

$$\alpha_{wp}(\tau) = 1 \quad \forall (w, p) \in \mathit{include}(\tau) \tag{8}$$

$$\alpha_{wp}(\tau) \in \{0, 1\} \quad \forall w \in W(\tau) \quad \forall p \in P(\tau) \tag{9}$$

$$\epsilon_{pr}(\tau) \in \mathbb{R}^{+} \quad \forall p \in P(\tau) \quad \forall r \in R(\tau) \tag{10}$$

Tactical

Meta variables:

$$s \in S \tag{11}$$

$$\alpha(\tau) \tag{12}$$

$$\tau \in [0, \infty] \tag{13}$$

Minimize:

$$\sum_{o \in O(\tau,\alpha(\tau))} \sum_{d \in D(\tau)} tactical\_value_{do}(\tau) \cdot \beta_{do}(\tau) + \sum_{r \in R(\tau)} \sum_{d \in D(\tau)} tactical\_penalty \cdot \mu_{rd}(\tau) \tag{14}$$

Subject to:

$$\sum_{o \in O(\tau,\alpha(\tau))} work_o(\tau) \cdot \beta_{do}(\tau) \leq tactical\_resource_{dr}(\tau) + \mu_{rd}(\tau) \forall d \in D(\tau) \quad \forall r \in R(\tau) \tag{15}$$

$$\sum_{d=earliest\_start_o(\tau)}^{latest\_finish_o(\tau)} \sigma_{do}(\tau) = duration_o(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{16}$$

$$\sum_{d^* \in D_{duration_o(\tau)}(\tau)} \sigma_{d^*o}(\tau) = duration_o(\tau) \cdot \eta_{do}(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall d \in D(\tau) \tag{17}$$

$$\sum_{o \in O(\tau,\alpha(\tau))} \eta_{do}(\tau) = 1, \quad \forall d \in D(\tau)$$

$$\sum_{d \in D(\tau)} d \cdot \sigma_{do1}(\tau) + \Delta_o(\tau) = \sum_{d \in D(\tau)} d \cdot \sigma_{do2}(\tau) \quad \forall (o1, o2) \in finish\_start_{o1,o2} \tag{18}$$

$$\sum_{d \in D(\tau)} d \cdot \sigma_{do1}(\tau) = \sum_{d \in D(\tau)} d \cdot \sigma_{do2}(\tau) \quad \forall (o1, o2) \in start\_start_{o1,o2} \tag{19}$$

$$\beta_{do}(\tau) \leq number_o(\tau) \cdot operating\_time_o \quad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{20}$$

$$\beta_{do}(\tau) \in \mathbb{R} \qquad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{21}$$

$$\mu_{rd}(\tau) \in \mathbb{R} \qquad \forall r \in R(\tau) \quad \forall d \in D(\tau) \tag{22}$$

$$\sigma_{do}(\tau) \in \{0, 1\} \qquad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{23}$$

$$\eta_{do}(\tau) \in \{0, 1\} \qquad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{24}$$

$$\Delta_o(\tau) \in \{0, 1\} \quad \forall o \in O(\tau, \alpha(\tau)) \tag{25}$$

Supervisor

Meta variables:

$$z \in Z \tag{26}$$

$$\alpha(\tau) \tag{27}$$

$$\theta(\tau) \tag{28}$$

$$\tau \in [0, \infty] \tag{29}$$

Maximize:

$$\sum_{a \in A(\tau,\alpha(\tau))} \sum_{t \in T(\tau)} \mathit{supervisor\_value}_{at}(\tau, \lambda_t(\tau), \Lambda_t(\tau)) \cdot \gamma_{at}(\tau) \tag{30}$$

Subject to:

$$\sum_{a \in A_o(\tau,\alpha(\tau))} \rho_a(\tau) = \mathit{work}_o(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{31}$$

$$\sum_{t \in T(\tau)} \sum_{a \in A_o(\tau,\alpha(\tau))} \gamma_{at}(\tau) = \phi_o(\tau) \cdot \mathit{number}_o(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{32}$$

$$\sum_{o \in O_w(\tau,\alpha(\tau))} \phi_o(\tau) = |O_w(\tau, \alpha(\tau))| \quad \forall w \in W(\tau, \alpha(\tau)) \tag{33}$$

$$\sum_{a \in A_o(\tau,\alpha(\tau))} \gamma_{at}(\tau) \leq 1 \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau) \tag{34}$$

$$\gamma_{at}(\tau) \leq \mathit{feasible}_{at}(\theta(\tau)) \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau) \tag{35}$$

$$\gamma_{at}(\tau) \in \{0, 1\} \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau) \tag{36}$$

$$\rho_a(\tau) \in [\mathit{lower\_activity\_work}_a(\tau), \mathit{work}_a(\tau)] \quad \forall a \in A(\tau, \alpha(\tau)) \tag{37}$$

Operational

Meta variables:

$$t \in T(\tau) \tag{38}$$

$$\alpha(\tau) \tag{39}$$

$$\gamma(\tau) \tag{40}$$

$$\tau \in [0, \infty] \tag{41}$$

Maximize:

$$\sum_{a \in A(\tau, \gamma_t(\tau))} \sum_{k \in K(\gamma(\tau))} \delta_{ak}(\tau) \tag{42}$$

Subject to:

$$\sum_{k \in K(\gamma(\tau))} \delta_{ak}(\tau) \cdot \pi_{ak}(\tau) = activity\_work_a(\tau, \rho(\tau)) \cdot \theta \quad (\tau) \forall a \in A(\tau, \gamma_t(\tau)) \tag{43}$$

$$\lambda_{a21}(\tau) \geq \Lambda_{a1 last(a1)}(\tau) + preparation_{a1,a2} \quad \forall a1 \in A(\tau, \gamma_t(\tau)) \quad \forall a2 \in A(\tau, \gamma_t(\tau)) \tag{44}$$

$$\lambda_{ak}(\tau) \geq \Lambda_{ak-1}(\tau) - constraint\_limit \cdot (2 - \pi_{ak}(\tau) + \pi_{ak-1}(\tau))$$
$$\forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{45}$$

$$\delta_{ak}(\tau) = \Lambda_{ak}(\tau) - \lambda_{ak}(\tau) \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{46}$$

$$\lambda_{ak}(\tau) \geq event_{ie} + duration_{ie} - constraint\_limit \cdot (1 - \omega_{akie}(\tau))$$
$$\forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall i \in I(\tau) \quad \forall e \in E(\tau) \tag{47}$$

$$\Lambda_{ak}(\tau) \leq event_{ie} + constraint\_limit \cdot \omega_{akie}(\tau)$$
$$\forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall i \in I(\tau) \quad \forall e \in E(\tau) \tag{48}$$

$$\lambda_{a1}(\tau) \geq time\_window\_start_a(\beta(\tau)) \quad \forall a \in A(\tau, \gamma_t(\tau)) \tag{49}$$

$$\Lambda_{alast(a)}(\tau) \leq time\_window\_finish_a(\beta(\tau)) \quad \forall a \in A(\tau, \gamma_t(\tau)) \tag{50}$$

$$\pi_{ak}(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{51}$$

$$\lambda_{ak}(\tau) \in [availability\_start(\tau), availability\_finish(\tau)]$$
$$\forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{52}$$

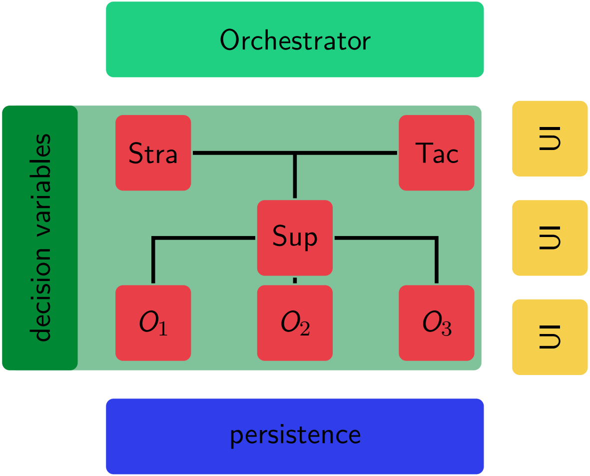
$$\Lambda_{ak}(\tau) \in [availability\_start(\tau), availability\_finish(\tau)]$$
$$\forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{53}$$

$$\delta_{ak}(\tau) \in [0, work_{a\_to\_o(a)}(\tau)] \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{54}$$

$$\omega_{akie}(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall i \in I(\tau) \quad \forall e \in E(\tau) \tag{55}$$

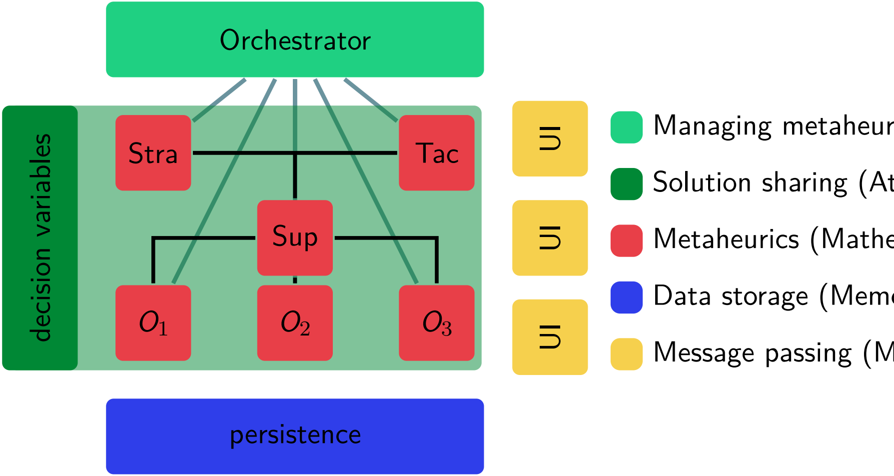
$$\theta_a(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma_t(\tau)) \tag{56}$$

Architecture



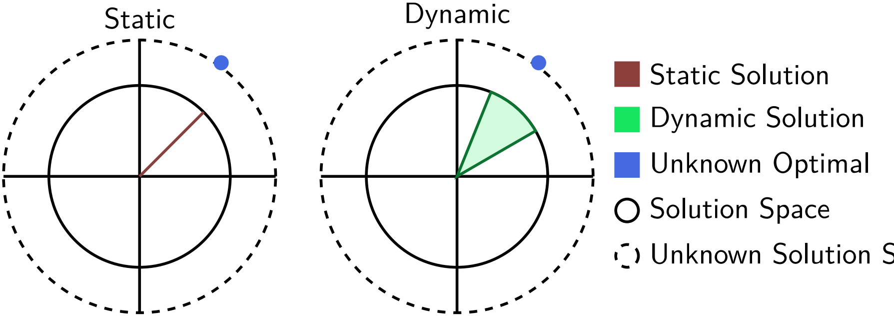
- Managing metaheur
- Solution sharing (At
- Metaheurics (Mathe
- Data storage (Mem
- Message passing (M

Architecture



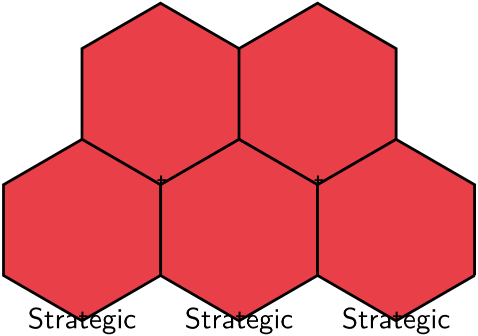
1. Reactive Versus Static Constraints
2. Dynamic
3. Business
4. Technical
5. Academic

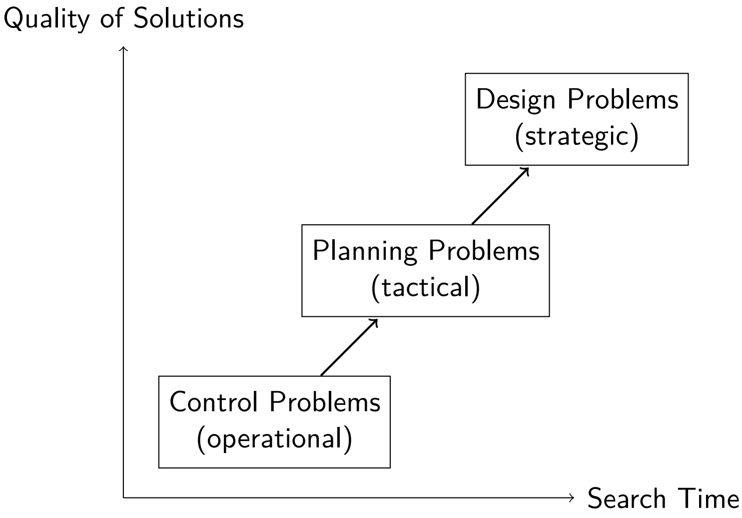
Dynamic versus Static Models



- Mathematical models guide direction but does not provide direct solutions.
- Static solutions are rarely fully executable.
- Dynamic models are less constrained and ensure a contained optimal solution.
- Remember: The real optimal solution is ever knowable at time $= 0$

Uncertainty, Complexity, and Value




Quality of Solutions
Design Problems
(strategic)
Planning Problems
(tactical)
Control Problems
(operational)
Search Time

Business