

# Multi-model Maintenance Scheduling

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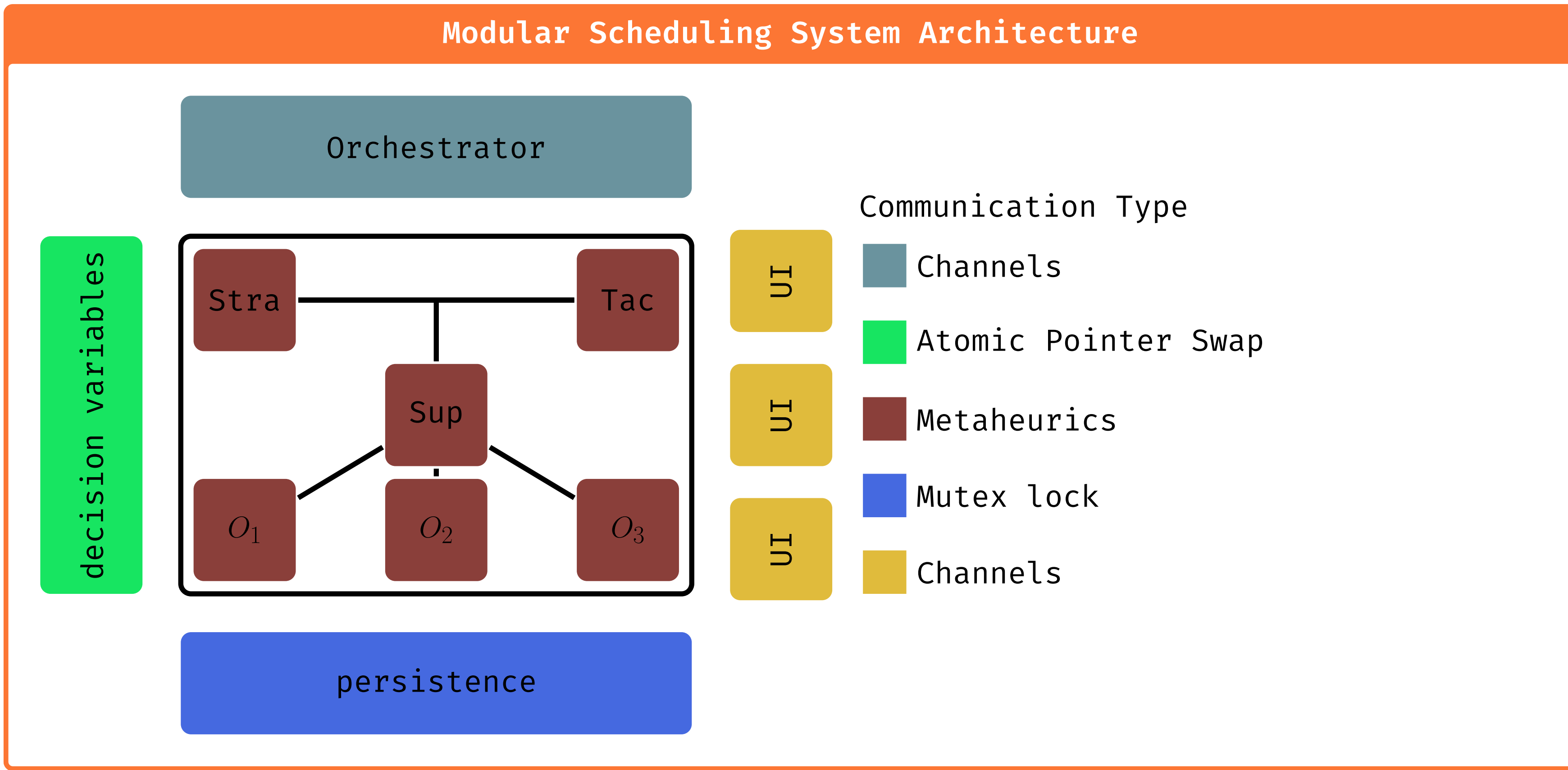
## Introduction

Current Operation Research methods have proven difficult to implement in operational settings. The poster presents a methodology to decompose a large-scale decision process into a series of modules that each represents the decisions taken by each individual stakeholder making up the scheduling process.

## Research Questions

1. How to implement a scheduling system that can coordinate in real-time?
2. How to coordinate multiple stakeholders in real-time that has different mathematical model requirements?
3. How to synchronize state across a high number of metaheuristics spread across different CPU threads?
4. How to intergrate metaheuristics into the workflow of a working scheduler?
5. Can you coordinate metaheuristics based on different mathematical models in real-time?
6. Which modern software architecture should be used to create scalably metaheuristic based scheduling systems
7. Which of the latest techniques in modern software development can be utilized to integrate metaheuristics directly into a business' IT infrastructure
8. How to create modular algorithm components that can solve well defined decision problems while also integrating into a larger decision making process
- 9.

## Solution Method



## Algorithm: Actor based Large Neighborhood Search

**Algorithm 1** Actor-based Large Neighborhood Search

```
1: Input  $Q$  = message queue
2: Input  $P$  = problem instance
3: Input  $X$  = initial schedule
4: Input  $S$  = SharedSolution
5: repeat
6:    $X^t = \text{clone}(X)$ 
7:   while  $Q.\text{has\_message}()$  do
8:      $P.\text{update}(S, m)$ 
9:      $X^t.\text{destruct}(S, m)$ 
10:  end while
11:   $X^t.\text{repair}(S)$ 
12:  if  $\text{accept}(X^t, X)$  then
13:     $X.\text{update}(X^t)$ 
14:  end if
15:  if  $c(X^t) < c(X)$  then
16:     $X.\text{update}(X^t)$ 
17:     $S.\text{atomic\_pointer\_swap}(X)$ 
18:  end if
19:   $Q.\text{push}(m)$ 
20: until
```

## Future Work

- Extend the approach to international supply chains.
- Incorporate real-time data analytics for dynamic decision-making.
- Explore applications in other sectors like healthcare and transportation.

## Conclusion

## Methodology

Strategic	
<b>Meta variables:</b>	
$s \in S$	(1)
$\beta(\tau)$	(2)
$\tau \in [0, \infty]$	(3)
<b>Minimize:</b>	
$\sum_{w \in W(\tau)} \sum_{p \in P(\tau)} \text{strategic\_value}_{wp}(\tau) \cdot \alpha_{wp}(\tau) + \sum_{p \in P(\tau)} \sum_{r \in R(\tau)} \text{strategic\_penalty} \cdot \epsilon_{pr}(\tau) + \sum_{p \in P(\tau)} \sum_{w \in W(\tau)} \sum_{u \in W(\tau)} \text{clustering\_value}_{w1, w2} \cdot \alpha_{w1p}(\tau) \cdot \alpha_{w2p}(\tau)$	(4)
<b>Subject to:</b>	
$\sum_{w \in W(\tau)} \text{work\_order\_work}_{wp} \cdot \alpha_{wp}(\tau) \leq \text{resource}_{pr}(\tau, \beta(\tau)) + \epsilon_{pr}(\tau) \quad \forall p \in P(\tau) \quad \forall r \in R(\tau)$	(5)
$\sum_{w \in W(\tau)} \alpha_{wp}(\tau) = 1 \quad \forall p \in P(\tau)$	(6)
$\alpha_{wp}(\tau) = 0 \quad \forall (w, p) \in \text{exclude}(\tau)$	(7)
$\alpha_{wp}(\tau) = 1 \quad \forall (w, p) \in \text{include}(\tau)$	(8)
$\alpha_{wp}(\tau) \in \{0, 1\} \quad \forall w \in W(\tau) \quad \forall p \in P(\tau)$	(9)
$\epsilon_{pr}(\tau) \in \mathbb{R}^+ \quad \forall p \in P(\tau) \quad \forall r \in R(\tau)$	(10)

Tactical	
<b>Meta variables:</b>	
$s \in S$	(11)
$\alpha(\tau)$	(12)
$\tau \in [0, \infty]$	(13)
<b>Minimize:</b>	
$\sum_{o \in O(\tau, \alpha(\tau))} \sum_{d \in D(\tau)} \text{tactical\_value}_{do}(\tau) \cdot \beta_{do}(\tau) + \sum_{r \in R(\tau)} \sum_{d \in D(\tau)} \text{tactical\_penalty} \cdot \mu_{rd}(\tau)$	(14)
<b>Subject to:</b>	
$\sum_{o \in O(\tau, \alpha(\tau))} \text{work}_o(\tau) \cdot \beta_{do}(\tau) \leq \text{tactical\_resource}_{dr}(\tau) + \mu_{rd}(\tau) \quad \forall d \in D(\tau) \quad \forall r \in R(\tau)$	(15)
$\text{latest\_finish}_o(\tau) \cdot \sigma_{do}(\tau) = \text{duration}_o(\tau) \quad \forall o \in O(\tau, \alpha(\tau))$	(16)
$\text{d=earliest\_start}_o(\tau) \cdot \sigma_{rd}(\tau) = \text{duration}_o(\tau) \cdot \eta_{do}(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall d \in D(\tau)$	(17)
$\sum_{o \in O(\tau, \alpha(\tau))} \eta_{do}(\tau) = 1, \quad \forall d \in D(\tau)$	(18)
$\sum_{d \in D(\tau)} d \cdot \sigma_{do}(\tau) + \Delta_o(\tau) = \sum_{d \in D(\tau)} d \cdot \sigma_{dd}(\tau) \quad \forall (o1, o2) \in \text{finish\_start}_{o1, o2}$	(19)
$\sum_{d \in D(\tau)} d \cdot \sigma_{dd}(\tau) = \sum_{d \in D(\tau)} d \cdot \sigma_{dd}(\tau) \quad \forall (o1, o2) \in \text{start\_start}_{o1, o2}$	(20)
$\beta_{do}(\tau) \leq \text{number}_o(\tau) \cdot \text{operating\_time}_o \quad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau))$	(21)
$\beta_{do}(\tau) \in \mathbb{R} \quad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau))$	(22)
$\mu_{rd}(\tau) \in \mathbb{R} \quad \forall r \in R(\tau) \quad \forall d \in D(\tau)$	(23)
$\sigma_{do}(\tau) \in \{0, 1\} \quad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau))$	(24)
$\eta_{do}(\tau) \in \{0, 1\} \quad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau))$	(25)
$\Delta_o(\tau) \in \{0, 1\} \quad \forall o \in O(\tau, \alpha(\tau))$	(26)

Supervisor	
<b>Meta variables:</b>	
$z \in Z$	(26)
$\alpha(\tau)$	(27)
$\theta(\tau)$	(28)
$\tau \in [0, \infty]$	(29)
<b>Maximize:</b>	
$\sum_{a \in A(\tau, \gamma(\tau))} \sum_{t \in T(\tau)} \text{supervisor\_value}_{at}(\tau, \lambda_t(\tau), \Lambda_t(\tau)) \cdot \gamma_{at}(\tau)$	(30)
<b>Subject to:</b>	
$\sum_{a \in A(\tau, \gamma(\tau))} \rho_{at}(\tau) = \text{work}_a(\tau) \quad \forall o \in O(\tau, \alpha(\tau))$	(31)
$\sum_{t \in T(\tau)} \sum_{a \in A(\tau, \gamma(\tau))} \gamma_{at}(\tau) = \phi_o(\tau) \cdot \text{number}_o(\tau) \quad \forall o \in O(\tau, \alpha(\tau))$	(32)
$\sum_{o \in O(\tau, \alpha(\tau))} \phi_o(\tau) =  O_w(\tau, \alpha(\tau))  \quad \forall w \in W(\tau, \alpha(\tau))$	(33)
$\sum_{a \in A(\tau, \gamma(\tau))} \gamma_{at}(\tau) \leq 1 \quad \forall a \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau)$	(34)
$\gamma_{at}(\tau) \leq \text{feasible}_{at}(\theta(\tau)) \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau)$	(35)
$\gamma_{at}(\tau) \in \{0, 1\} \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau)$	(36)
$\rho_{at}(\tau) \in [\text{lower\_activity\_work}_a(\tau), \text{work}_a(\tau)] \quad \forall a \in A(\tau, \gamma(\tau))$	(37)

Operational	
<b>Meta variables:</b>	
$t \in T(\tau)$	(38)
$\alpha(\tau)$	(39)
$\gamma(\tau)$	(40)
$\tau \in [0, \infty]$	(41)
<b>Maximize:</b>	
$\sum_{a \in A(\tau, \gamma(\tau))} \sum_{k \in K(\gamma(\tau))} \delta_{ak}(\tau)$	(42)
<b>Subject to:</b>	
$\sum_{k \in K(\gamma(\tau))} \delta_{ak}(\tau) \cdot \pi_{ak}(\tau) = \text{activity\_work}_{ka}(\tau, \rho(\tau)) \cdot \theta \quad (\tau) \quad \forall a \in A(\tau, \gamma(\tau))$	(43)
$\lambda_{a2}(\tau) \geq \lambda_{a1}(\tau) + \text{preparation}_{a1, a2} \quad \forall a1 \in A(\tau, \gamma(\tau)) \quad \forall a2 \in A(\tau, \gamma(\tau))$	(44)
$\lambda_{ak}(\tau) \geq \lambda_{a, k-1}(\tau) - \text{constraint\_limit} \cdot (2 - \pi_{ak}(\tau) + \pi_{a, k-1}(\tau)) \quad \forall a \in A(\tau, \gamma(\tau)) \quad \forall k \in K(\gamma(\tau))$	(45)
$\delta_{ak}(\tau) = \lambda_{ak}(\tau) - \lambda_{ak}(\tau) \quad \forall a \in A(\tau, \gamma(\tau)) \quad \forall k \in K(\gamma(\tau))$	(46)
$\lambda_{ak}(\tau) \geq \text{event}_{ak} + \text{duration}_{ak} - \text{constraint\_limit} \cdot (1 - \omega_{ake}(\tau)) \quad \forall a \in A(\tau, \gamma(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall e \in E(\tau)$	(47)
$\lambda_{ak}(\tau) \leq \text{event}_{ak} + \text{constraint\_limit} \cdot \omega_{ake}(\tau) \quad \forall a \in A(\tau, \gamma(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall e \in E(\tau)$	(48)
$\lambda_{a1}(\tau) \geq \text{time\_window\_start}_a(\tau) \quad \forall a \in A(\tau, \gamma(\tau))$	(49)
$\lambda_{a1}(\tau) \leq \text{time\_window\_finish}_a(\tau) \quad \forall a \in A(\tau, \gamma(\tau))$	(50)
$\pi_{ak}(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma(\tau)) \quad \forall k \in K(\gamma(\tau))$	(51)
$\lambda_{ak}(\tau) \in [\text{availability\_start}(\tau), \text{availability\_finish}(\tau)] \quad \forall a \in A(\tau, \gamma(\tau)) \quad \forall k \in K(\gamma(\tau))$	(52)
$\lambda_{ak}(\tau) \in [\text{availability\_start}(\tau), \text{availability\_finish}(\tau)] \quad \forall a \in A(\tau, \gamma(\tau)) \quad \forall k \in K(\gamma(\tau))$	(53)
$\delta_{ak}(\tau) \in [0, \text{work}_{ka\_to\_a}(\tau)] \quad \forall a \in A(\tau, \gamma(\tau)) \quad \forall k \in K(\gamma(\tau))$	(54)
$\omega_{ake}(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall e \in E(\tau)$	(55)
$\theta_a(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma(\tau))$	(56)

## Results



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