

# Maintenance Scheduling System

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# Agenda

- ▶
- ▶ Introduction to Maintenance Scheduling
- ▶ Architecture of a Scheduling System
- ▶ Possible Contributions to Operation Research

# Strategic

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**Meta variables:**

$$s \in S \quad (1)$$

$$\tau \in [0, \infty] \quad (2)$$

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**Minimize:**

$$\begin{aligned} & \sum_{w \in W(\tau)} \sum_{p \in P(\tau)} \text{strategic\_value}_{wp}(\tau) \cdot \alpha_{wp}(\tau) \\ & + \sum_{p \in P(\tau)} \sum_{r \in R(\tau)} \text{strategic\_penalty} \cdot \epsilon_{pr}(\tau) \\ & + \sum_{p \in P(\tau)} \sum_{w1 \in W(\tau)} \sum_{w2 \in W(\tau)} \text{clustering\_value}_{w1, w2} \cdot \alpha_{w1p}(\tau) \cdot \alpha_{w2p}(\tau) \end{aligned} \quad (3)$$

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**Subject to:**

$$\sum_{w \in W(\tau)} \text{work\_order\_work}_{wr} \cdot \alpha_{wp}(\tau) \leq \text{resource}_{pr}(\tau, \beta(\tau)) + \epsilon_{pr}(\tau) \quad \forall p \in P(\tau) \quad \forall r \in R(\tau) \quad (4)$$

$$\sum_{w \in W(\tau)} \alpha_{wp}(\tau) = 1 \quad \forall p \in P(\tau) \quad (5)$$

$$\alpha_{wp}(\tau) = 0 \quad \forall (w, p) \in \text{exclude}(\tau) \quad (6)$$

$$\alpha_{wp}(\tau) = 1 \quad \forall (w, p) \in \text{include}(\tau) \quad (7)$$

$$\alpha_{wp}(\tau) \in \{0, 1\} \quad \forall w \in W(\tau) \quad \forall p \in P(\tau) \quad (8)$$

$$\epsilon_{pr}(\tau) \in \mathbb{R}^+ \quad \forall p \in P(\tau) \quad \forall r \in R(\tau) \quad (9)$$

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# Tactical

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## Meta variables:

$$s = S \quad (10)$$

$$\alpha(\tau) \quad (11)$$

$$\tau \in [0, \infty] \quad (12)$$


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## Minimize:

$$\sum_{o \in O(\tau, \alpha(\tau))} \sum_{d \in D(\tau)} \text{tactical\_value}_{do}(\tau) \cdot \beta_{do}(\tau) + \sum_{r \in R(\tau)} \sum_{d \in D(\tau)} \text{tactical\_penalty} \cdot \mu_{rd}(\tau) \quad (13)$$


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## Subject to:

$$\sum_{o \in O(\tau, \alpha(\tau))} \text{work}_o(\tau) \cdot \beta_{do}(\tau) \leq \text{tactical\_resource}_{dr}(\tau) + \mu_{rd}(\tau) \forall d \in D(\tau) \quad \forall r \in R(\tau) \quad (14)$$

$$\sum_{d=\text{latest\_finish}_o(\tau)}^{\text{latest\_finish}_o(\tau)} \sigma_{do}(\tau) = \text{duration}_o(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad (15)$$

$$\sum_{d^* \in D_{\Delta_{\text{duration}_o}(\tau)}(\tau)} \sigma_{d^*o}(\tau) = \text{duration}_o(\tau) \cdot \eta_{do}(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall d \in D(\tau) \quad (16)$$

$$\sum_{o \in O(\tau, \alpha(\tau))} \eta_{do}(\tau) = 1, \quad \forall d \in D(\tau) \quad (17)$$

$$\sum_{d \in D(\tau)} d \cdot \sigma_{do1}(\tau) + \Delta_o(\tau) = \sum_{d \in D(\tau)} d \cdot \sigma_{do2}(\tau) \quad \forall (o1, o2) \in \text{finish\_start}_{o1, o2} \quad (18)$$

$$\sum_{d \in D(\tau)} d \cdot \sigma_{do1}(\tau) = \sum_{d \in D(\tau)} d \cdot \sigma_{do2}(\tau) \quad \forall (o1, o2) \in \text{start\_start}_{o1, o2} \quad (19)$$

$$\beta_{do}(\tau) \leq \text{number}_o(\tau) \cdot \text{operating\_time}_o \quad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad (19)$$

$$\beta_{do}(\tau) \in \mathbb{R} \quad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad (20)$$

$$\mu_{rd}(\tau) \in \mathbb{R} \quad \forall r \in R(\tau) \quad \forall d \in D(\tau) \quad (21)$$

$$\sigma_{do}(\tau) \in \{0, 1\} \quad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad (22)$$

$$\eta_{do}(\tau) \in \{0, 1\} \quad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad (23)$$

$$\Delta_o(\tau) \in \{0, 1\} \quad \forall o \in O(\tau, \alpha(\tau)) \quad (24)$$


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# Supervisor

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**Meta variables:**

$$z \in Z \quad (25)$$

$$\alpha(\tau) \quad (26)$$

$$\theta(\tau) \quad (27)$$

$$\tau \in [0, \infty] \quad (28)$$

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**Maximize:**

$$\sum_{a \in A(\tau, \alpha(\tau))} \sum_{t \in T(\tau)} \text{supervisor\_value}_{at}(\tau, \lambda_t(\tau), \Lambda_t(\tau)) \cdot \gamma_{at}(\tau) \quad (29)$$

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**Subject to:**

$$\sum_{a \in A_o(\tau, \alpha(\tau))} \rho_a(\tau) = \text{work}_o(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad (30)$$

$$\sum_{t \in T(\tau)} \sum_{a \in A_o(\tau, \alpha(\tau))} \gamma_{at}(\tau) = \phi_o(\tau) \cdot \text{number}_o(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad (31)$$

$$\sum_{o \in O_w(\tau, \alpha(\tau))} \phi_o(\tau) = |O_w(\tau, \alpha(\tau))| \quad \forall w \in W(\tau, \alpha(\tau)) \quad (32)$$

$$\sum_{a \in A_o(\tau, \alpha(\tau))} \gamma_{at}(\tau) \leq 1 \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau) \quad (33)$$

$$\gamma_{at}(\tau) \leq \text{feasible}_{at}(\theta(\tau)) \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau) \quad (34)$$

$$\gamma_{at}(\tau) \in \{0, 1\} \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau) \quad (35)$$

$$\rho_a(\tau) \in [\text{lower\_activity\_work}_a(\tau), \text{work}_a(\tau)] \quad \forall a \in A(\tau, \alpha(\tau)) \quad (36)$$

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# Operational

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## Meta variables:

$$t \in T(\tau) \quad (37)$$

$$\alpha(\tau) \quad (38)$$

$$\gamma(\tau) \quad (39)$$

$$\tau \in [0, \infty] \quad (40)$$


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## Maximize:

$$\sum_{a \in A(\tau, \gamma_t(\tau))} \sum_{k \in K(\gamma(\tau))} \delta_{ak}(\tau) \quad (41)$$


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## Subject to:

$$\sum_{k \in K(\gamma(\tau))} \delta_{ak}(\tau) \cdot \pi_{ak}(\tau) = \text{activity\_work}_a(\tau, \rho(\tau)) \cdot \theta \quad (\tau) \forall a \in A(\tau, \gamma_t(\tau)) \quad (42)$$

$$\lambda_{a21}(\tau) \geq \Lambda_{a1\text{last}(a1)}(\tau) + \text{preparation}_{a1,a2} \quad \forall a1 \in A(\tau, \gamma_t(\tau)) \quad \forall a2 \in A(\tau, \gamma_t(\tau)) \quad (43)$$

$$\lambda_{ak}(\tau) \geq \Lambda_{ak-1}(\tau) - \text{constraint\_limit} \cdot (2 - \pi_{ak}(\tau) + \pi_{ak-1}(\tau)) \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad (44)$$

$$\delta_{ak}(\tau) = \Lambda_{ak}(\tau) - \lambda_{ak}(\tau) \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad (45)$$

$$\lambda_{ak}(\tau) \geq \text{event}_{ie} + \text{duration}_{ie} - \text{constraint\_limit} \cdot (1 - \omega_{akie}(\tau)) \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall i \in I(\tau) \quad \forall e \in E(\tau) \quad (46)$$

$$\Lambda_{ak}(\tau) \leq \text{event}_{ie} + \text{constraint\_limit} \cdot \omega_{akie}(\tau) \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall i \in I(\tau) \quad \forall e \in E(\tau) \quad (47)$$

$$\lambda_{a1}(\tau) \geq \text{time\_window\_start}_a(\beta(\tau)) \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad (48)$$

$$\Lambda_{a\text{last}(a)}(\tau) \leq \text{time\_window\_finish}_a(\beta(\tau)) \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad (49)$$

$$\pi_{ak}(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad (50)$$

$$\lambda_{ak}(\tau) \in [\text{availability\_start}(\tau), \text{availability\_finish}(\tau)] \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad (51)$$

$$\Lambda_{ak}(\tau) \in [\text{availability\_start}(\tau), \text{availability\_finish}(\tau)] \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad (52)$$

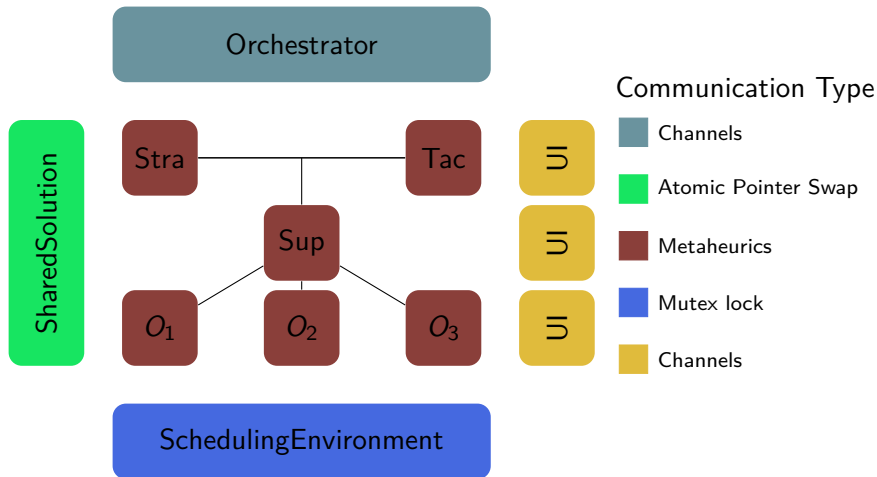
$$\delta_{ak}(\tau) \in [0, \text{work}_{a\_to\_o(a)}(\tau)] \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad (53)$$

$$\omega_{akie}(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall i \in I(\tau) \quad \forall e \in E(\tau) \quad (54)$$

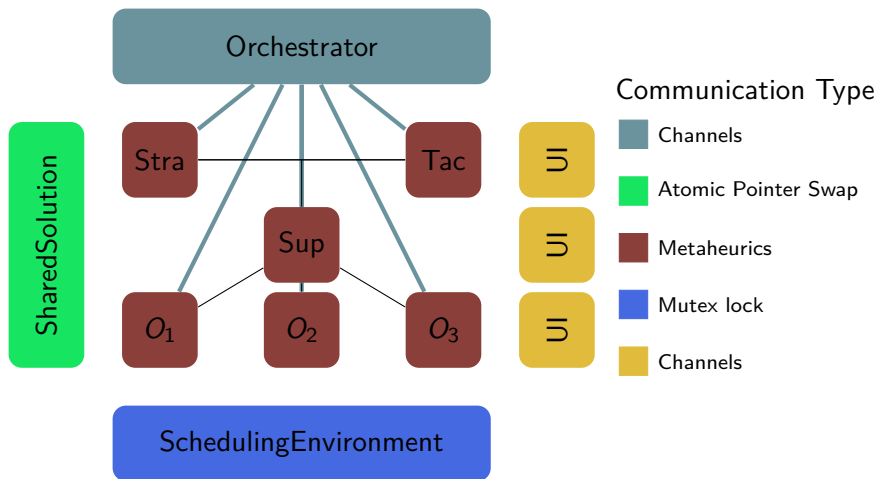
$$\theta_a(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad (55)$$


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# Architecture



# Architecture





## Possible contributions to Operation Research

1. Business
2. Technical
3. Academic

Business