

# Maintenance Scheduling System

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# Agenda

- ▶ Introduction to Maintenance Scheduling
- ▶ Architecture of a Scheduling System
- ▶ Possible Contributions to Operation Research

## Mathematical Notation: Sets

$$a \in A_{b,c}^m(t, x, y)$$

- ▶ a: set element
- ▶ A: set itself
- ▶ b: set element from set B
- ▶ c: set element from set C
- ▶ m: model formulation m
- ▶ t: time
- ▶ x: value of decision variable from a different model
- ▶ y: value of decision variable from a different model

## Mathematical Notation: Parameters

*name\_of\_parameter*<sub>a,b</sub>(*t*, *x*, *y*)

- ▶ parameters are functions of set elements and input parameters
- ▶ a: set element from A
- ▶ b: set element from B
- ▶ t: time
- ▶ x: value of decision variable from another model
- ▶ y: value of decision variable from another model

## Mathematical Notation: Variables

$$x_{a,b}^m(t)$$

- ▶ variables are functions of set elements, specified model, and time
- ▶ x: decision variable
- ▶ a: set element from A
- ▶ b: set element from B
- ▶ m: specifying the model
- ▶ t: time
- ▶ Notice: decision variables cannot depend on other decision variables as it would make them belong to the same model.

# Strategic

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**Meta variables:**

$$s \in S \quad (1)$$

$$\beta(\tau) \quad (2)$$

$$\tau \in [0, \infty] \quad (3)$$

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**Maximize:**

$$\begin{aligned} & \sum_{w \in W(\tau)} \sum_{p \in P(\tau)} \text{strategic\_value}_{wp}(\tau) \cdot \alpha_{wp}(\tau) \\ & - \sum_{p \in P(\tau)} \sum_{r \in R(\tau)} \text{strategic\_penalty} \cdot \epsilon_{pr}(\tau) \\ & + \sum_{p \in P(\tau)} \sum_{w1 \in W(\tau)} \sum_{w2 \in W(\tau)} \text{clustering\_value}_{w1, w2} \cdot \alpha_{w1p}(\tau) \cdot \alpha_{w2p}(\tau) \end{aligned} \quad (4)$$

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**Subject to:**

$$\sum_{w \in W(\tau)} \text{work\_order\_work}_{wr} \cdot \alpha_{wp}(\tau) \leq \text{resource}_{pr}(\tau, \beta(\tau)) + \epsilon_{pr}(\tau) \quad \forall p \in P(\tau) \quad \forall r \in R(\tau) \quad (5)$$

$$\sum_{w \in W(\tau)} \alpha_{wp}(\tau) = 1 \quad \forall p \in P(\tau) \quad (6)$$

$$\alpha_{wp}(\tau) = 0 \quad \forall (w, p) \in \text{exclude}(\tau) \quad (7)$$

$$\alpha_{wp}(\tau) = 1 \quad \forall (w, p) \in \text{include}(\tau) \quad (8)$$

$$\alpha_{wp}(\tau) \in \{0, 1\} \quad \forall w \in W(\tau) \quad \forall p \in P(\tau) \quad (9)$$

$$\epsilon_{pr}(\tau) \in \mathbb{R}^+ \quad \forall p \in P(\tau) \quad \forall r \in R(\tau) \quad (10)$$

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# Tactical

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**Meta variables:**

$$s \in S \quad (11)$$

$$\alpha(\tau) \quad (12)$$

$$\tau \in [0, \infty] \quad (13)$$

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**Minimize:**

$$\sum_{o \in O(\tau, \alpha(\tau))} \sum_{d \in D(\tau)} \text{tactical\_value}_{do}(\tau) \cdot \beta_{do}(\tau) + \sum_{r \in R(\tau)} \sum_{d \in D(\tau)} \text{tactical\_penalty} \cdot \mu_{rd}(\tau) \quad (14)$$

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**Subject to:**

$$\sum_{o \in O(\tau, \alpha(\tau))} \text{work}_o(\tau) \cdot \beta_{do}(\tau) \leq \text{tactical\_resource}_{dr}(\tau) + \mu_{rd}(\tau) \forall d \in D(\tau) \quad \forall r \in R(\tau) \quad (15)$$

$$\sum_{d=\text{earliest\_start}_o(\tau)}^{\text{latest\_finish}_o(\tau)} \sigma_{do}(\tau) = \text{duration}_o(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad (16)$$

$$\sum_{d^* \in D_{\text{duration}_o(\tau)}(\tau)} \sigma_{d^*o}(\tau) = \text{duration}_o(\tau) \cdot \eta_{do}(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall d \in D(\tau) \quad (17)$$

$$\sum_{o \in O(\tau, \alpha(\tau))} \eta_{do}(\tau) = 1, \quad \forall d \in D(\tau) \quad (18)$$
$$\sum_{d \in D(\tau)} d \cdot \sigma_{do1}(\tau) + \Delta_o(\tau) = \sum_{d \in D(\tau)} d \cdot \sigma_{do2}(\tau) \quad \forall (o1, o2) \in \text{finish\_start}_{o1, o2} \quad (19)$$

$$\sum_{d \in D(\tau)} d \cdot \sigma_{do1}(\tau) = \sum_{d \in D(\tau)} d \cdot \sigma_{do2}(\tau) \quad \forall (o1, o2) \in \text{start\_start}_{o1, o2} \quad (20)$$

$$\beta_{do}(\tau) \leq \text{number}_o(\tau) \cdot \text{operating\_time}_o \quad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad (21)$$

$$\beta_{do}(\tau) \in \mathbb{R} \quad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad (22)$$

$$\mu_{rd}(\tau) \in \mathbb{R} \quad \forall r \in R(\tau) \quad \forall d \in D(\tau) \quad (23)$$

$$\sigma_{do}(\tau) \in \{0, 1\} \quad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad (24)$$

$$\eta_{do}(\tau) \in \{0, 1\} \quad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad (25)$$

$$\Delta_o(\tau) \in \{0, 1\} \quad \forall o \in O(\tau, \alpha(\tau)) \quad (26)$$

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# Supervisor

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**Meta variables:**

$$z \in Z \quad (26)$$

$$\alpha(\tau) \quad (27)$$

$$\theta(\tau) \quad (28)$$

$$\tau \in [0, \infty] \quad (29)$$

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**Maximize:**

$$\sum_{a \in A(\tau, \alpha(\tau))} \sum_{t \in T(\tau)} \text{supervisor\_value}_{at}(\tau, \lambda_t(\tau), \Lambda_t(\tau)) \cdot \gamma_{at}(\tau) \quad (30)$$

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**Subject to:**

$$\sum_{a \in A_o(\tau, \alpha(\tau))} \rho_a(\tau) = \text{work}_o(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad (31)$$

$$\sum_{t \in T(\tau)} \sum_{a \in A_o(\tau, \alpha(\tau))} \gamma_{at}(\tau) = \phi_o(\tau) \cdot \text{number}_o(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad (32)$$

$$\sum_{o \in O_w(\tau, \alpha(\tau))} \phi_o(\tau) = |O_w(\tau, \alpha(\tau))| \quad \forall w \in W(\tau, \alpha(\tau)) \quad (33)$$

$$\sum_{a \in A_o(\tau, \alpha(\tau))} \gamma_{at}(\tau) \leq 1 \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau) \quad (34)$$

$$\gamma_{at}(\tau) \leq \text{feasible}_{at}(\theta(\tau)) \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau) \quad (35)$$

$$\gamma_{at}(\tau) \in \{0, 1\} \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau) \quad (36)$$

$$\rho_a(\tau) \in [\text{lower\_activity\_work}_a(\tau), \text{work}_a(\tau)] \quad \forall a \in A(\tau, \alpha(\tau)) \quad (37)$$

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# Operational

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**Meta variables:**

$$t \in T(\tau) \quad (38)$$

$$\alpha(\tau) \quad (39)$$

$$\gamma(\tau) \quad (40)$$

$$\tau \in [0, \infty] \quad (41)$$

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**Maximize:**

$$\sum_{a \in A(\tau, \gamma_t(\tau))} \sum_{k \in K(\gamma(\tau))} \delta_{ak}(\tau) \quad (42)$$

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**Subject to:**

$$\sum_{k \in K(\gamma(\tau))} \delta_{ak}(\tau) \cdot \pi_{ak}(\tau) = \text{activity\_work}_a(\tau, \rho(\tau)) \cdot \theta \quad (\tau) \forall a \in A(\tau, \gamma_t(\tau)) \quad (43)$$

$$\lambda_{a21}(\tau) \geq \Lambda_{a1, \text{last}(a1)}(\tau) + \text{preparation}_{a1, a2} \quad \forall a1 \in A(\tau, \gamma_t(\tau)) \quad \forall a2 \in A(\tau, \gamma_t(\tau)) \quad (44)$$

$$\lambda_{ak}(\tau) \geq \Lambda_{ak-1}(\tau) - \text{constraint\_limit} \cdot (2 - \pi_{ak}(\tau) + \pi_{ak-1}(\tau)) \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad (45)$$

$$\delta_{ak}(\tau) = \Lambda_{ak}(\tau) - \lambda_{ak}(\tau) \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad (46)$$

$$\lambda_{ak}(\tau) \geq \text{event}_{ie} + \text{duration}_{ie} - \text{constraint\_limit} \cdot (1 - \omega_{akie}(\tau)) \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall i \in I(\tau) \quad \forall e \in E(\tau) \quad (47)$$

$$\Lambda_{ak}(\tau) \leq \text{event}_{ie} + \text{constraint\_limit} \cdot \omega_{akie}(\tau) \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall i \in I(\tau) \quad \forall e \in E(\tau) \quad (48)$$

$$\lambda_{a1}(\tau) \geq \text{time\_window\_start}_a(\beta(\tau)) \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad (49)$$

$$\Lambda_{a, \text{last}(a)}(\tau) \leq \text{time\_window\_finish}_a(\beta(\tau)) \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad (50)$$

$$\pi_{ak}(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad (51)$$

$$\lambda_{ak}(\tau) \in [\text{availability\_start}(\tau), \text{availability\_finish}(\tau)] \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad (52)$$

$$\Lambda_{ak}(\tau) \in [\text{availability\_start}(\tau), \text{availability\_finish}(\tau)] \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad (53)$$

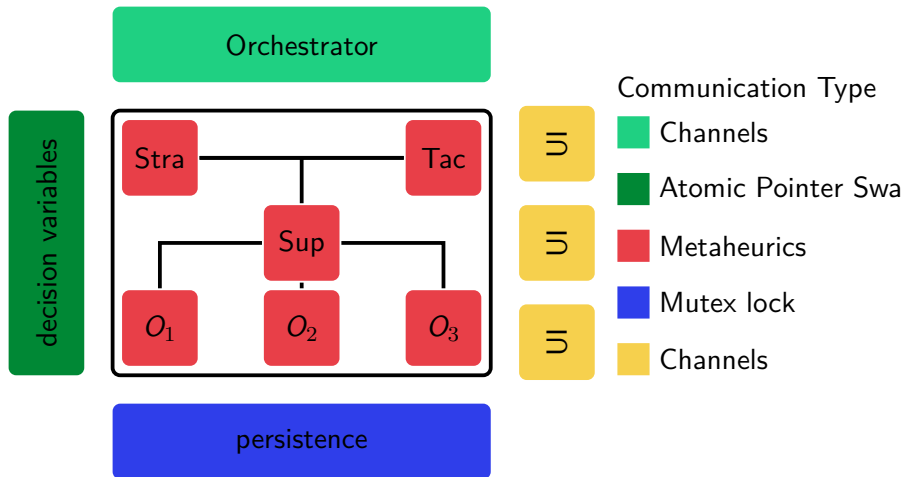
$$\delta_{ak}(\tau) \in [0, \text{work}_{a, \text{to\_o}(a)}(\tau)] \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad (54)$$

$$\omega_{akie}(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall i \in I(\tau) \quad \forall e \in E(\tau) \quad (55)$$

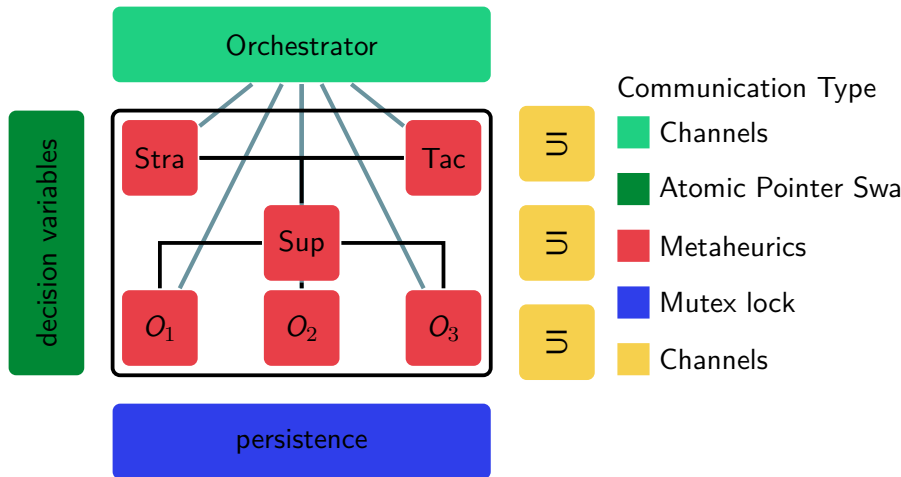
$$\theta_a(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad (56)$$

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# Architecture

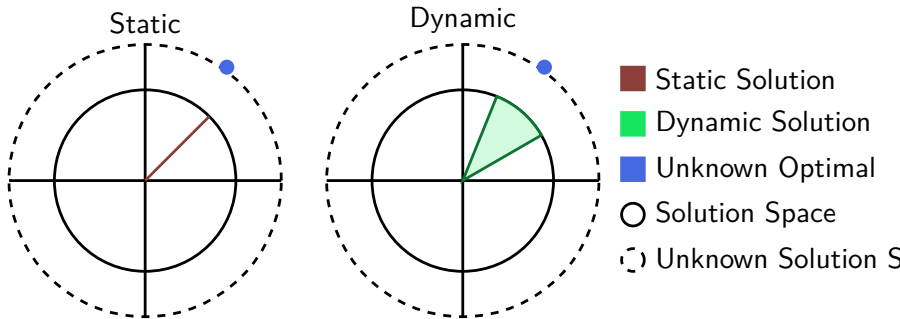


# Architecture



1. Reactive Versus Static Constraints
2. Dynamic
3. Business
4. Technical
5. Academic

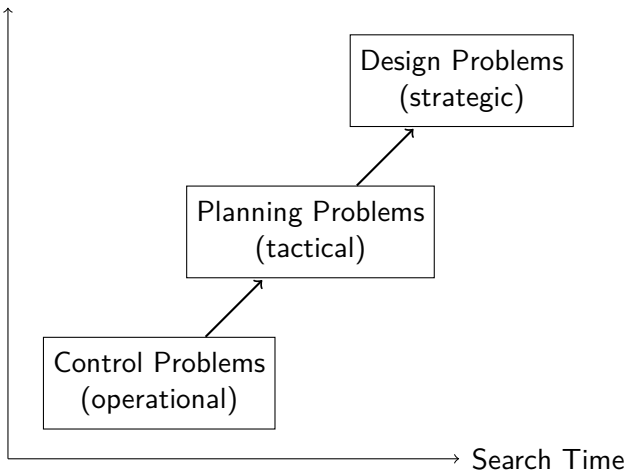
## Dynamic versus Static Models



- ▶ Mathematical models guide direction but does not provide direct solutions.
- ▶ Static solutions are rarely fully executable.
- ▶ Dynamic models are less constrained and ensure a contained optimal solution.
- ▶ Remember: The real optimal solution is ever knowable at  $\text{time} = 0$

# Dynamic

Quality of Solutions



Business