Multi-actor Maintenance Scheduling

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Contents

Chapter 1

Introduction

Maintenance scheduling is in its nature a multi actor process. Many stakeholders have to coordinate in both time and space to allow for an efficient and effective execution. This thesis will propose a generalized multi-agent scheduling system and it will argue that for the field of maintenance scheduling to more forward similar approaches will have to be adopted. Other approaches may be very different but they will share many of the aspects.

This Ph.D. will present a generalized dynamic multi-model approach to maintenance scheduling which will be model after a practical maintenace handbook Palmer [2019]. This book written by the experienced practitioner Richard D. Palmer will be guiding throughout the thesis, so it serves as the main source of validation, and invalidation throughout the thesis, as we explore the academic maintenance scheduling literture and also, and more importantly, it will also be the source which above all else will us us through the process of creating a generalize model setup for maintenance scheduling.

1.1 The General Maintenance Scheduling Process

This section will provide an overview of the maintenance scheduling process in the most abstracted way possible. It will be important to understand this setup throughly as most industries that perform maintanance of a considerable scale follow this process. Many industries are of course unique and deviate from general framework in specific work but the fundamentals are usually quite similar.

This figure shows

Type	Symbol	Description

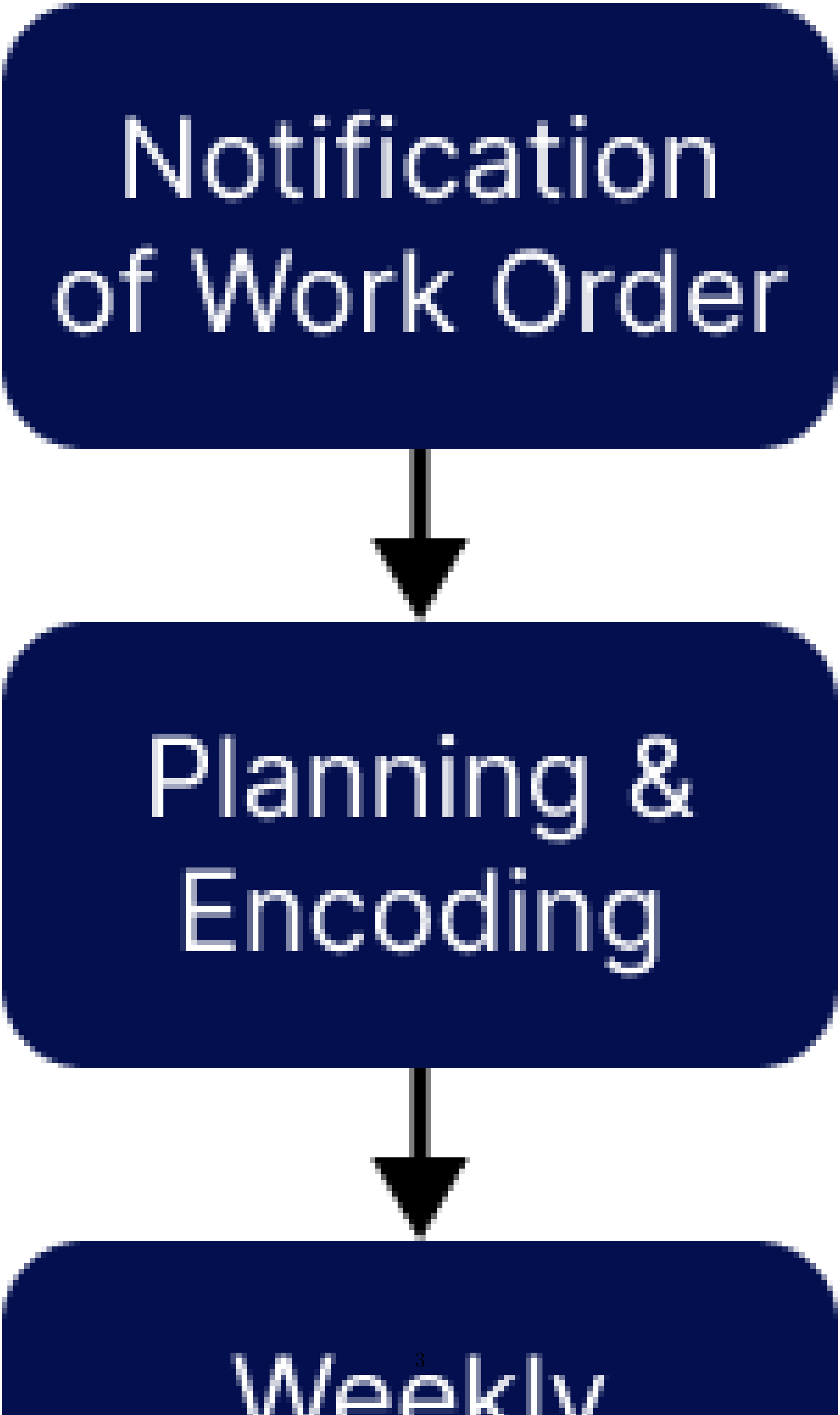
Notification of Work Order
Planning & Encoding
Weekly

Table 1.1: Summary of Sets, Parameters, and Variables

Type	Symbol	Description
Set	S	Set of suppliers
Parameter	d_i	Demand of customer i (units)
Variable	x_{ij}	Quantity shipped from supplier i to customer j

Table 1.2: Example of a Simple Table

Type	Symbol	Description
Set	S	Set of suppliers
Parameter	d_i	Demand of customer i (units)
Variable	x_{ij}	Quantity shipped from supplier i to customer j

Chapter 2

Modelling the Generalized Setup

To model the maintenace process in its entirety we will need mathematical modelling tools that are powerful enough to describe the maintenance scheduling process. As the process requires multiple different actors we need a system that enables us to model each of them and specific

To effectively model a system that contains multiple actors also The system will be described in accordance with the 1.1 Palmer [2019].

The maintenance scheduling problem is NP-hard and real-time optimal solutions will never be a feasible approach unless we use a multi-model setup where each model enriches the overall solution in the way that it is most capable of.

2.1 Parameter Table

Element	**Set**	**Dependent Variables**	**Description**
w	$W(\tau)$	τ	The set of all work orders
p	$P(\tau)$	τ	The set of all weekly periods
r	$R(\tau)$	τ	The set of all resources
o	$O(\tau,)$	τ	The set of all operations
d	$D(\tau)$	τ	The set of all days
a	$A(\tau,)$	τ	The set of all activities
t	$T(\tau)$	τ	The set of all technicians
k	$K(\gamma(\tau))$	τ	The set of all technician work segments
i	$I(\tau)$	τ	The set of all time instances
e	$E(\tau)$	τ	The set of all technician events

Table 2.1: Sets used in the model setup

Parameter	set	var	**Description**
$strategic\_value_{wp}(\tau)$	set	var	DESCRIPTION
$strategic\_penalty$	set	var	DESCRIPTION
$clustering\_value_{w1,w2}$	set	var	DESCRIPTION
$resource_{pr}(\tau, \beta(\tau))$	set	var	DESCRIPTION
$work\_order\_work_{wr}$	set	var	DESCRIPTION
$include(\tau)$	set	var	DESCRIPTION
$exclude(\tau)$	set	var	DESCRIPTION
$tactical\_value_{do}(\tau)$	set	var	DESCRIPTION
$tactical\_penalty$	set	var	DESCRIPTION
$work(\tau)$	set	var	DESCRIPTION
$tactical\_resource_{dr}(\tau)$	set	var	DESCRIPTION
$start\_start_{o1,o2}$	set	var	DESCRIPTION
$finish\_start_{o1,o2}$	set	var	DESCRIPTION
$earliest\_start_o(\tau)$	set	var	DESCRIPTION
$latest\_finish_o(\tau)$	set	var	DESCRIPTION
$number_o(\tau)$	set	var	DESCRIPTION
$operating\_time_o$	set	var	DESCRIPTION
$duration_o(\tau)$	set	var	DESCRIPTION
$supervisor\_value_{at}(\tau, \lambda_t(\tau), \Lambda_t(\tau))$	set	var	DESCRIPTION
$feasible_{at}(\theta(\tau))$	set	var	DESCRIPTION
$work\_order\_to\_operations_w$	set	var	DESCRIPTION
$operations\_in\_work\_order_w$	set	var	DESCRIPTION
$activities\_for\_operation_o$	set	var	DESCRIPTION
$lower\_activity\_work_a(\tau)$	set	var	DESCRIPTION
$activity\_work_a(\tau, \rho(\tau))$	set	var	DESCRIPTION
$preparation_{a1,a2}$	set	var	DESCRIPTION
$event_{ie}$	set	var	DESCRIPTION
$duration_{ie}$	set	var	DESCRIPTION
$constraint\_limit$	set	var	DESCRIPTION
$time\_window\_start_a(\beta(\tau))$	set	var	DESCRIPTION
$time\_window\_finish_a(\beta(\tau))$	set	var	DESCRIPTION
$availability\_start(\tau)$	set	var	DESCRIPTION
$availability\_finish(\tau)$	set	var	DESCRIPTION

Table 2.2: Parameters used in the model setup

Variable	Set Selectors	Dependent Variable	Description
$\alpha(\tau)$	w, p	τ	DESCRIPTION
$\epsilon_{pr}(\tau)$	p, r	τ	DESCRIPTION
$\beta(\tau)$	d, o	τ	DESCRIPTION
$\mu_{rd}(\tau)$	r, d	τ	DESCRIPTION
$\sigma(\tau)$	d, o	τ	DESCRIPTION
$\eta_{do}(\tau)$	d, o	τ	DESCRIPTION
$\Delta_o(\tau)$	o	τ	DESCRIPTION
$\gamma(\tau)$	a, t	τ	DESCRIPTION
$\phi_o(\tau)$	o	τ	DESCRIPTION
$\rho(\tau)$	a	τ	DESCRIPTION
$\delta_{ak}(\tau)$	a, k	τ	DESCRIPTION
$\pi(\tau)$	a, k	τ	DESCRIPTION
$\lambda(\tau)$	a, k	τ	DESCRIPTION
$\Lambda(\tau)$	a, k	τ	DESCRIPTION
$\omega_{akie}(\tau)$	a, k, i, e	τ	DESCRIPTION
$\theta(\tau)$	a	τ	DESCRIPTION
τ	None	None	DESCRIPTION

Table 2.3: Variables used in the model setup

Meta variables:

$$s \in S \tag{2.1}$$

$$\tau \in [0, \infty] \tag{2.2}$$

Minimize:

$$\begin{aligned}
&\sum_{w \in W(\tau)} \sum_{p \in P(\tau)} strategic_value_{wp}(\tau) \cdot \alpha_{wp}(\tau) \\
&+ \sum_{p \in P(\tau)} \sum_{r \in R(\tau)} strategic_penalty \cdot \epsilon_{pr}(\tau) \\
&+ \sum_{p \in P(\tau)} \sum_{w1 \in W(\tau)} \sum_{w2 \in W(\tau)} \quad clustering_value_{w1,w2} \cdot \alpha_{w1p}(\tau) \cdot \alpha_{w2p}(\tau)
\end{aligned} \tag{2.3}$$

Subject to:

$$\sum_{w \in W(\tau)} work\_order\_work_{wr} \cdot \alpha_{wp}(\tau) \leq \ resource_{pr}(\tau, \beta(\tau)) + \epsilon_{pr}(\tau) \quad \forall p \in P(\tau) \quad \forall r \in R(\tau) \tag{2.4}$$

$$\sum_{w \in W(\tau)} \alpha_{wp}(\tau) = 1 \quad \forall p \in P(\tau) \tag{2.5}$$

$$\alpha_{wp}(\tau) = 0 \quad \forall (w, p) \in exclude(\tau) \tag{2.6}$$

$$\alpha_{wp}(\tau) = 1 \quad \forall (w, p) \in include(\tau) \tag{2.7}$$

$$\alpha_{wp}(\tau) \in \{0, 1\} \quad \forall w \in W(\tau) \quad \forall p \in P(\tau) \tag{2.8}$$

$$\epsilon_{pr}(\tau) \in \mathbb{R}^+ \quad \forall p \in P(\tau) \quad \forall r \in R(\tau) \tag{2.9}$$

Meta variables:

$$s = S \tag{2.10}$$

$$\alpha(\tau) \tag{2.11}$$

$$\tau \in [0, \infty] \tag{2.12}$$

Minimize:

$$\sum_{o \in O(\tau, \alpha(\tau))} \sum_{d \in D(\tau)} tactical\_value_{do}(\tau) \cdot \beta_{do}(\tau) + \sum_{r \in R(\tau)} \sum_{d \in D(\tau)} tactical\_penalty \cdot \mu_{rd}(\tau) \tag{2.13}$$

Subject to:

$$\sum_{o \in O(\tau, \alpha(\tau))} work_o(\tau) \cdot \beta_{do}(\tau) \leq tactical\_resource_{dr}(\tau) + \mu_{rd}(\tau) \forall d \in D(\tau) \quad \forall r \in R(\tau) \tag{2.14}$$

$$\sum_{d = earliest\_start_o(\tau)}^{latest\_finish_o(\tau)} \sigma_{do}(\tau) = duration_o(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{2.15}$$

$$\sum_{d^* \in D_{duration_o(\tau)}(\tau)} \sigma_{d^*o}(\tau) = duration_o(\tau) \cdot \eta_{do}(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall d \in D(\tau) \tag{2.16}$$

$$\sum_{o \in O(\tau, \alpha(\tau))} \eta_{do}(\tau) = 1, \quad \forall d \in D(\tau)$$

$$\sum_{d \in D(\tau)} d \cdot \sigma_{do1}(\tau) + \Delta_o(\tau) = \sum_{d \in D(\tau)} d \cdot \sigma_{do2}(\tau) \quad \forall (o1, o2) \in finish\_start_{o1,o2} \tag{2.17}$$

$$\sum_{d \in D(\tau)} d \cdot \sigma_{do1}(\tau) = \sum_{d \in D(\tau)} d \cdot \sigma_{do2}(\tau) \quad \forall (o1, o2) \in start\_start_{o1,o2} \tag{2.18}$$

$$\beta_{do}(\tau) \leq number_o(\tau) \cdot operating\_time_o \quad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{2.19}$$

$$\beta_{do}(\tau) \in \mathbb{R} \qquad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{2.20}$$

$$\mu_{rd}(\tau) \in \mathbb{R} \qquad \forall r \in R(\tau) \quad \forall d \in D(\tau) \tag{2.21}$$

$$\sigma_{do}(\tau) \in \{0, 1\} \qquad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{2.22}$$

$$\eta_{do}(\tau) \in \{0, 1\} \qquad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{2.23}$$

$$\Delta_o(\tau) \in \{0, 1\} \quad \forall o \in O(\tau, \alpha(\tau)) \tag{2.24}$$

Meta variables:

$$z \in Z \tag{2.25}$$

$$\alpha(\tau) \tag{2.26}$$

$$\theta(\tau) \tag{2.27}$$

$$\tau \in [0, \infty] \tag{2.28}$$

Maximize:

$$\sum_{a \in A(\tau, \alpha(\tau))} \sum_{t \in T(\tau)} supervisor\_value_{at}(\tau, \lambda_t(\tau), \Lambda_t(\tau)) \cdot \gamma_{at}(\tau) \tag{2.29}$$

Subject to:

$$\sum_{a \in A_o(\tau, \alpha(\tau))} \rho_a(\tau) = work_o(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{2.30}$$

$$\sum_{t \in T(\tau)} \sum_{a \in A_o(\tau, \alpha(\tau))} \gamma_{at}(\tau) = \phi_o(\tau) \cdot number_o(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{2.31}$$

$$\sum_{o \in O_w(\tau, \alpha(\tau))} \phi_o(\tau) = |O_w(\tau, \alpha(\tau))| \quad \forall w \in W(\tau, \alpha(\tau)) \tag{2.32}$$

$$\sum_{a \in A_o(\tau, \alpha(\tau))} \gamma_{at}(\tau) \leq 1 \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau) \tag{2.33}$$

$$\gamma_{at}(\tau) \leq feasible_{at}(\theta(\tau)) \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau) \tag{2.34}$$

$$\gamma_{at}(\tau) \in \{0, 1\} \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau) \tag{2.35}$$

$$\rho_a(\tau) \in [lower\_activity\_work_a(\tau), work_a(\tau)] \quad \forall a \in A(\tau, \alpha(\tau)) \tag{2.36}$$

Meta variables:

$$t \in T(\tau) \tag{2.37}$$

$$\alpha(\tau) \tag{2.38}$$

$$\gamma(\tau) \tag{2.39}$$

$$\tau \in [0, \infty] \tag{2.40}$$

Maximize:

$$\sum_{a \in A(\tau, \gamma_t(\tau))} \sum_{k \in K(\gamma(\tau))} \delta_{ak}(\tau) \tag{2.41}$$

Subject to:

$$\sum_{k \in K(\gamma(\tau))} \delta_{ak}(\tau) \cdot \pi_{ak}(\tau) = activity\_work_a(\tau, \rho(\tau)) \cdot \theta \quad (\tau) \forall a \in A(\tau, \gamma_t(\tau)) \tag{2.42}$$

$$\lambda_{a21}(\tau) \geq \Lambda_{a1last(a1)}(\tau) + preparation_{a1,a2} \quad \forall a1 \in A(\tau, \gamma_t(\tau)) \quad \forall a2 \in A(\tau, \gamma_t(\tau)) \tag{2.43}$$

$$\lambda_{ak}(\tau) \geq \Lambda_{ak-1}(\tau) - constraint\_limit \cdot (2 - \pi_{ak}(\tau) + \pi_{ak-1}(\tau))$$
$$\forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{2.44}$$

$$\delta_{ak}(\tau) = \Lambda_{ak}(\tau) - \lambda_{ak}(\tau) \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{2.45}$$

$$\lambda_{ak}(\tau) \geq event_{ie} + duration_{ie} - constraint\_limit \cdot (1 - \omega_{akie}(\tau))$$
$$\forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall i \in I(\tau) \quad \forall e \in E(\tau) \tag{2.46}$$

$$\Lambda_{ak}(\tau) \leq event_{ie} + constraint\_limit \cdot \omega_{akie}(\tau)$$
$$\forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall i \in I(\tau) \quad \forall e \in E(\tau) \tag{2.47}$$

$$\lambda_{a1}(\tau) \geq time\_window\_start_a(\beta(\tau)) \quad \forall a \in A(\tau, \gamma_t(\tau)) \tag{2.48}$$

$$\Lambda_{alast(a)}(\tau) \leq time\_window\_finish_a(\beta(\tau)) \quad \forall a \in A(\tau, \gamma_t(\tau)) \tag{2.49}$$

$$\pi_{ak}(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{2.50}$$

$$\lambda_{ak}(\tau) \in [availability\_start(\tau), availability\_finish(\tau)]$$
$$\forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{2.51}$$

$$\Lambda_{ak}(\tau) \in [availability\_start(\tau), availability\_finish(\tau)]$$
$$\forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{2.52}$$

$$\delta_{ak}(\tau) \in [0, work_{a\_to\_o(a)}(\tau)] \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{2.53}$$

$$\omega_{akie}(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall i \in I(\tau) \quad \forall e \in E(\tau) \tag{2.54}$$

$$\theta_a(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma_t(\tau)) \tag{2.55}$$

possible sections * All Stochasticity will be handled by user interaction. * In maintenance scheduling, data is assumed to be correct. Yes that must be the prevailing idea here.

Bibliography

Richard D. Palmer. *Maintenance Planning and Scheduling Handbook, 4th Edition*. McGraw Hill, 4th edition edition, September 2019.