Maintenance Scheduling System

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Agenda

- Introduction to Maintenance Scheduling
- Architecture of a Scheduling System
- Possible Contributions to Operation Research

Mathematical Notation: Sets

$a \in A^m_{b,c}(t, x, y)$

- a: set element
- A: set itself
- b: set element from set B
- c: set element from set C
- m: model formulation m
- t: time
- x: value of decision variable from a different model
- y: value of decision variable from a different model

Mathematical Notation: Parameters

$name\_of\_parameter_{a,b}(t, x, y)$

- parameters are functions of set elements and input parameters
- a: set element from A
- b: set element from B
- t: time
- x: value of decision variable from another model
- y: value of decision variable from another model

Mathematical Notation: Variables

$x^m_{a,b}(t)$

- variables are functions of set elements, specified model, and time
- x: decision variable
- a: set element from A
- b: set element from B
- m: specifying the model
- t: time
- Notice: decision variables cannot depend on other decision variables as it would make them belong to the same model.

Strategic

Meta variables:

$$s \in S \tag{1}$$

$$\beta(\tau) \tag{2}$$

$$\tau \in [0, \infty] \tag{3}$$

Minimize:

$$\begin{aligned}
&+ \sum_{w \in W(\tau)} \sum_{p \in P(\tau)} \mathit{strategic_urgency}_{wp}(\tau) \cdot \alpha_{wp}(\tau) \\
&+ \sum_{p \in P(\tau)} \sum_{r \in R(\tau)} \mathit{strategic_resource_penalty} \cdot \epsilon_{pr}(\tau) \\
&- \sum_{p \in P(\tau)} \sum_{w1 \in W(\tau)} \sum_{w2 \in W(\tau)} \mathit{clustering_value}_{w1,w2} \cdot \alpha_{w1p}(\tau) \cdot \alpha_{w2p}(\tau)
\end{aligned} \tag{4}$$

Subject to:

$$\sum_{w \in W(\tau)} \mathit{work\_order\_workload}_{wr} \cdot \alpha_{wp}(\tau) \leq \mathit{resource}_{pr}(\tau, \beta(\tau)) + \epsilon_{pr}(\tau)$$

$$\forall p \in P(\tau) \quad \forall r \in R(\tau) \tag{5}$$

$$\sum_{w \in W(\tau)} \alpha_{wp}(\tau) = 1 \quad \forall p \in P(\tau) \tag{6}$$

$$\alpha_{wp}(\tau) = 0, \quad \mathit{if} \quad \mathit{exclude}_{wp}(\tau) \quad \forall w \in W(\tau) \quad \forall p \in P(\tau) \tag{7}$$

$$\alpha_{wp}(\tau) = 1, \quad \mathit{if} \quad \mathit{include}_{wp}(\tau) \quad \forall w \in W(\tau) \quad \forall p \in P(\tau) \tag{8}$$

$$\alpha_{wp}(\tau) \in \{0, 1\} \quad \forall w \in W(\tau) \quad \forall p \in P(\tau) \tag{9}$$

$$\epsilon_{pr}(\tau) \in \mathbb{R}^{+} \quad \forall p \in P(\tau) \quad \forall r \in R(\tau) \tag{10}$$

Tactical

Meta variables:

$$s \in S \tag{11}$$

$$\alpha(\tau) \tag{12}$$

$$\tau \in [0, \infty] \tag{13}$$

Minimize:

$$\sum_{o \in O(\tau,\alpha(\tau))} \sum_{d \in D(\tau)} tactical\_value_{do}(\tau) \cdot \beta_{do}(\tau) + \sum_{r \in R(\tau)} \sum_{d \in D(\tau)} tactical\_penalty \cdot \mu_{rd}(\tau) \tag{14}$$

Subject to:

$$\sum_{o \in O(\tau,\alpha(\tau))} work_o(\tau) \cdot \beta_{do}(\tau) \leq tactical\_resource_{dr}(\tau) + \mu_{rd}(\tau) \forall d \in D(\tau) \quad \forall r \in R(\tau) \tag{15}$$

$$\sum_{d = earliest\_start_o(\tau)}^{latest\_finish_o(\tau)} \sigma_{do}(\tau) = duration_o(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{16}$$

$$\sum_{d^* \in D_{duration_o(\tau)}(\tau)} \sigma_{d^* o}(\tau) = duration_o(\tau) \cdot \eta_{do}(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall d \in D(\tau) \tag{17}$$

$$\sum_{o \in O(\tau,\alpha(\tau))} \eta_{do}(\tau) = 1, \quad \forall d \in D(\tau)$$

$$\sum_{d \in D(\tau)} d \cdot \sigma_{do1}(\tau) + \Delta_o(\tau) = \sum_{d \in D(\tau)} d \cdot \sigma_{do2}(\tau) \quad \forall (o1, o2) \in finish\_start_{o1,o2} \tag{18}$$

$$\sum_{d \in D(\tau)} d \cdot \sigma_{do1}(\tau) = \sum_{d \in D(\tau)} d \cdot \sigma_{do2}(\tau) \quad \forall (o1, o2) \in start\_start_{o1,o2} \tag{19}$$

$$\beta_{do}(\tau) \leq number_o(\tau) \cdot operating\_time_o \quad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{20}$$

$$\beta_{do}(\tau) \in \mathbb{R} \qquad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{21}$$

$$\mu_{rd}(\tau) \in \mathbb{R} \qquad \forall r \in R(\tau) \quad \forall d \in D(\tau) \tag{22}$$

$$\sigma_{do}(\tau) \in \{0, 1\} \qquad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{23}$$

$$\eta_{do}(\tau) \in \{0, 1\} \qquad \forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{24}$$

$$\Delta_o(\tau) \in \{0, 1\} \quad \forall o \in O(\tau, \alpha(\tau)) \tag{25}$$

$$\tag{26}$$

Supervisor

Meta variables:

$$z \in Z \tag{27}$$
$$\alpha(\tau) \tag{28}$$
$$\theta(\tau) \tag{29}$$
$$\tau \in [0, \infty] \tag{30}$$

Maximize:

$$\sum_{a \in A(\tau,\alpha(\tau))} \sum_{t \in T(\tau)} \mathit{supervisor\_value}_{at}(\tau, \lambda_t(\tau), \Lambda_t(\tau)) \cdot \gamma_{at}(\tau) \tag{31}$$

Subject to:

$$\sum_{a \in A_o(\tau,\alpha(\tau))} \rho_a(\tau) = \mathit{work}_o(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{32}$$
$$\sum_{t \in T(\tau)} \sum_{a \in A_o(\tau,\alpha(\tau))} \gamma_{at}(\tau) = \phi_o(\tau) \cdot \mathit{number}_o(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{33}$$
$$\sum_{o \in O_w(\tau,\alpha(\tau))} \phi_o(\tau) = |O_w(\tau, \alpha(\tau))| \quad \forall w \in W(\tau, \alpha(\tau)) \tag{34}$$
$$\sum_{a \in A_o(\tau,\alpha(\tau))} \gamma_{at}(\tau) \leq 1 \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau) \tag{35}$$
$$\gamma_{at}(\tau) \leq \mathit{feasible}_{at}(\theta(\tau)) \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau) \tag{36}$$
$$\gamma_{at}(\tau) \in \{0, 1\} \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau) \tag{37}$$
$$\rho_a(\tau) \in [\mathit{lower\_activity\_work}_a(\tau), \mathit{work}_a(\tau)] \quad \forall a \in A(\tau, \alpha(\tau)) \tag{38}$$

(39)

Operational

Meta variables:

$$t \in T(\tau) \tag{40}$$

$$\alpha(\tau) \tag{41}$$

$$\gamma(\tau) \tag{42}$$

$$\tau \in [0, \infty] \tag{43}$$

Maximize:

$$\sum_{a \in A(\tau, \gamma_t(\tau))} \sum_{k \in K(\gamma(\tau))} \delta_{ak}(\tau) \tag{44}$$

Subject to:

$$\sum_{k \in K(\gamma(\tau))} \delta_{ak}(\tau) \cdot \pi_{ak}(\tau) = activity\_work_a(\tau, \rho(\tau)) \cdot \theta_a(\tau) \quad \forall a \in A(\tau, \gamma_t(\tau)) \tag{45}$$

$$\lambda_{a21}(\tau) \geq \Lambda_{a1last(a1)}(\tau) + preparation_{a1,a2} \quad \forall a1 \in A(\tau, \gamma_t(\tau)) \quad \forall a2 \in A(\tau, \gamma_t(\tau)) \tag{46}$$

$$\lambda_{ak}(\tau) \geq \Lambda_{ak-1}(\tau) - constraint\_limit \cdot (2 - \pi_{ak}(\tau) + \pi_{ak-1}(\tau))$$
$$\forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{47}$$

$$\delta_{ak}(\tau) = \Lambda_{ak}(\tau) - \lambda_{ak}(\tau) \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{48}$$

$$\lambda_{ak}(\tau) \geq event_{ie} + duration_{ie} - constraint\_limit \cdot (1 - \omega_{akie}(\tau))$$
$$\forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall i \in I(\tau) \quad \forall e \in E(\tau) \tag{49}$$

$$\Lambda_{ak}(\tau) \leq event_{ie} + constraint\_limit \cdot \omega_{akie}(\tau)$$
$$\forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall i \in I(\tau) \quad \forall e \in E(\tau) \tag{50}$$

$$\lambda_{a1}(\tau) \geq time\_window\_start_a(\beta(\tau)) \quad \forall a \in A(\tau, \gamma_t(\tau)) \tag{51}$$

$$\Lambda_{alast(a)}(\tau) \leq time\_window\_finish_a(\beta(\tau)) \quad \forall a \in A(\tau, \gamma_t(\tau)) \tag{52}$$

$$\pi_{ak}(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{53}$$

$$\lambda_{ak}(\tau) \in [availability\_start(\tau), availability\_finish(\tau)]$$
$$\forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{54}$$

$$\Lambda_{ak}(\tau) \in [availability\_start(\tau), availability\_finish(\tau)]$$
$$\forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{55}$$

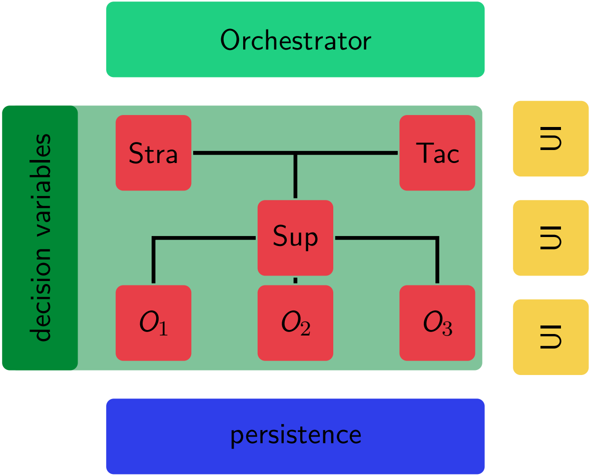
$$\delta_{ak}(\tau) \in [0, work_{a\_to\_o(a)}(\tau)] \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{56}$$

$$\omega_{akie}(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall i \in I(\tau) \quad \forall e \in E(\tau) \tag{57}$$

$$\theta_a(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma_t(\tau)) \tag{58}$$

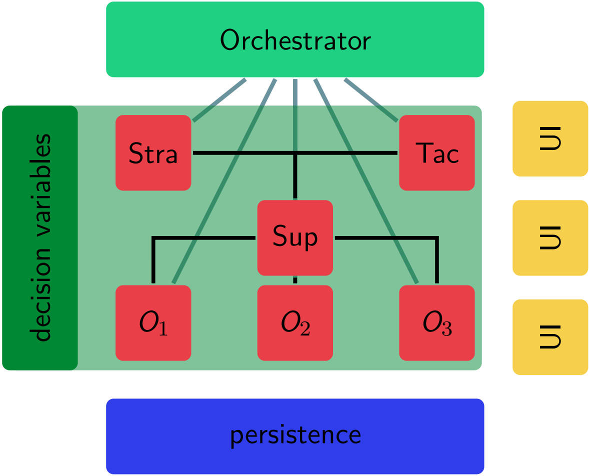
(59)

Architecture



- Managing metahe
- Solution sharing (
- Actor-based Opti
- Data storage (Me
- Message passing (

Architecture



- Managing metahe
- Solution sharing (
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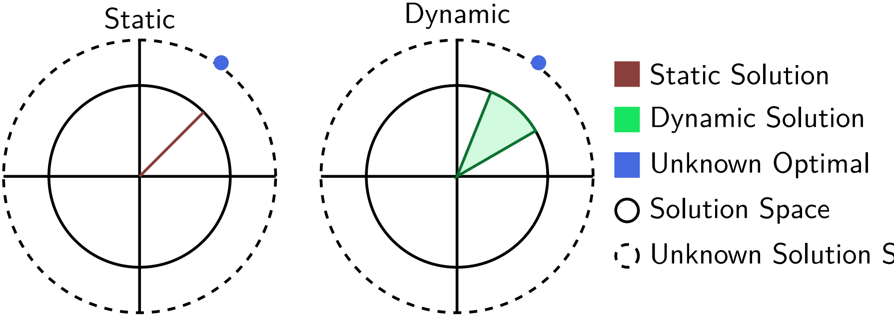
- Message passing (

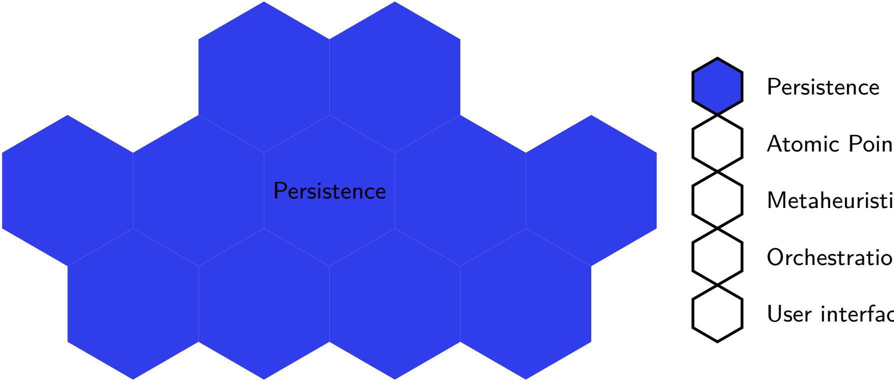
1. Reactive Versus Static Constraints
2. Dynamic
3. Business
4. Technical
5. Academic

Dynamic versus Static Models



- Mathematical models guide direction but does not provide direct solutions.
- Static solutions are rarely fully executable.
- Dynamic models are less constrained and ensure a contained optimal solution.
- Remember: The real optimal solution is ever knowable at time $= 0$

Model setup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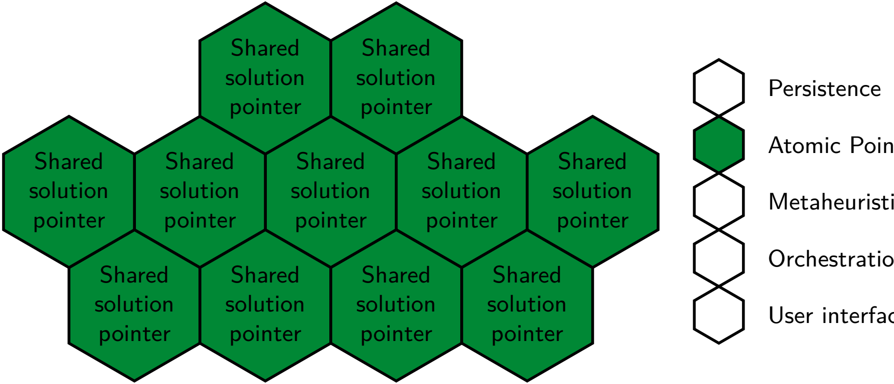
Model setup


Shared solution pointer
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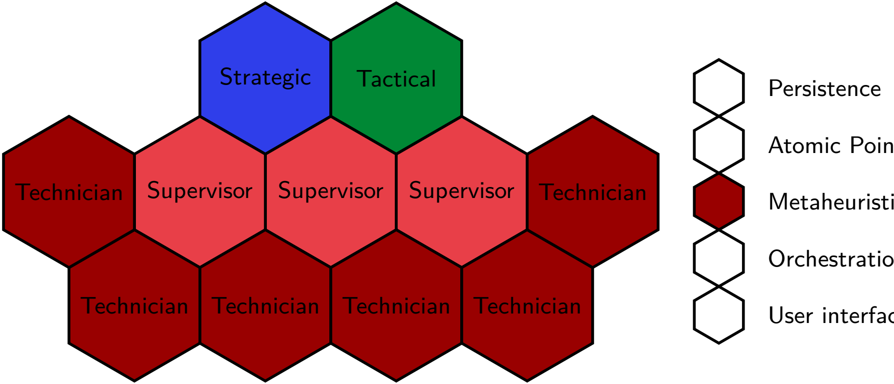

Strategic
Tactical
Technician
Supervisor
Supervisor
Supervisor
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Persistence
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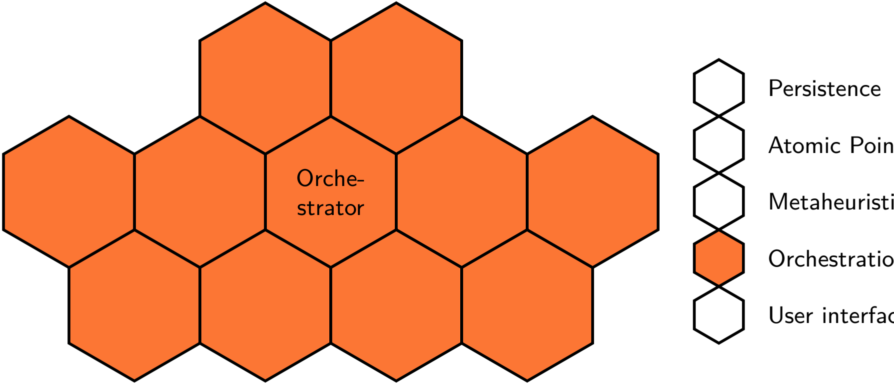
Model setup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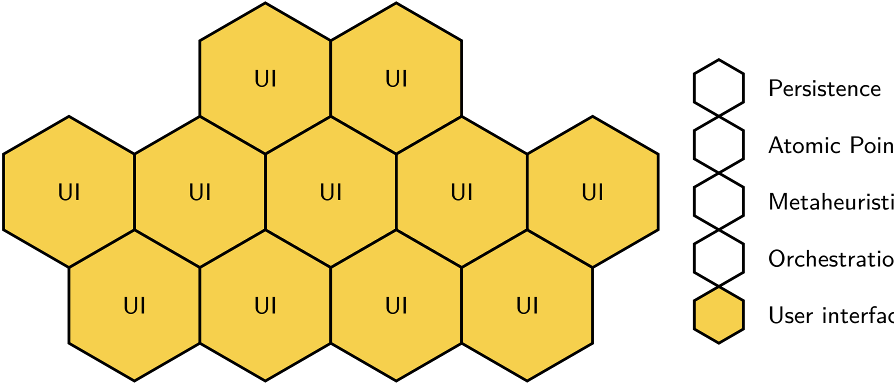
Model setup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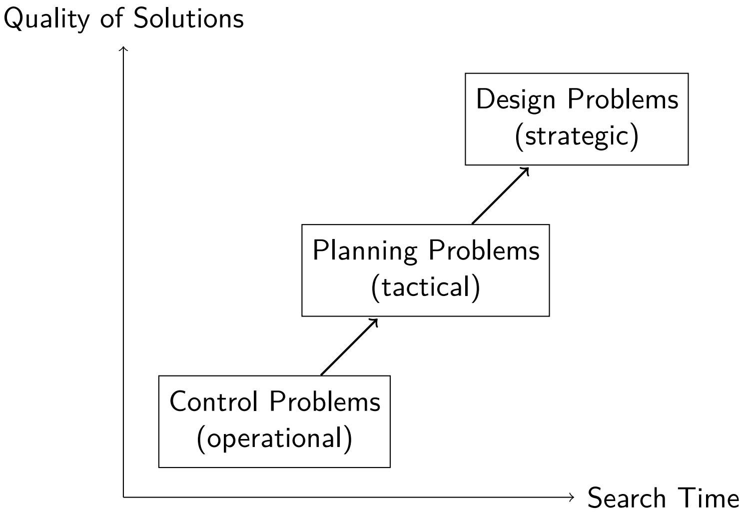
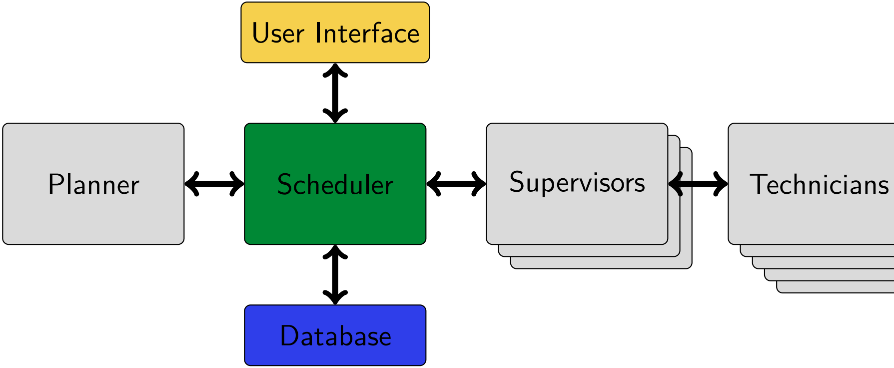
The Story of Zaabalawi

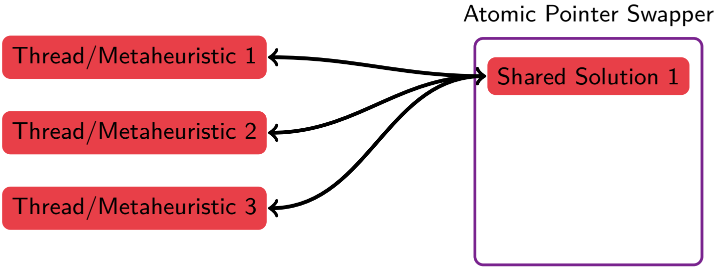


Quality of Solutions
Design Problems
(strategic)
Planning Problems
(tactical)
Control Problems
(operational)
Search Time

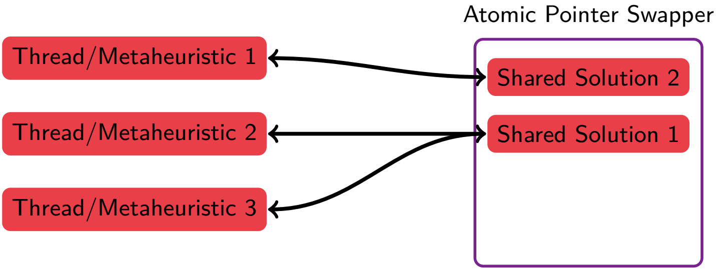
Business


User Interface
Planner
Scheduler
Supervisors
Technicians
Database

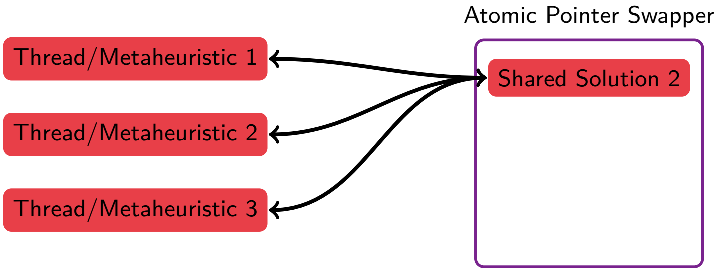
Atomic Pointer Swapping



Thread one finds a better solution and swaps it in



Thread two and three loads the new Shared Solution at the top of their optimization loop



Shared Solution 1 is dropped from memory when is it no longer referenced by any threads