Multi-agent Maintenance Scheduling: The Making of a Science

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Chapter 1

Introduction

Maintenance scheduling is in its nature a multi actor process. Many stakeholders have to coordinate in both time and space to allow for an efficient and effective execution. This thesis will propose a generalized multi-agent scheduling system and it will argue that for the field of maintenance scheduling to more forward similar approaches will have to be adopted. Other approaches may be very different but they will share many of the aspects.

This Ph.D. will present a generalized dynamic multi-model approach to maintenance scheduling which will be model after a practical maintenace handbook Palmer [2019]. This book written by the experienced practitioner Richard D. Palmer will be guiding throughout the thesis, so it serves as the main source of validation, and invalidation throughout the thesis, as we explore the academic maintenance scheduling literture and also, and more importantly, it will also be the source which above all else will us us through the process of creating a generalize model setup for maintenance scheduling.

1.1 The General Maintenance Scheduling Process

This section will provide an overview of the maintenance scheduling process in the most abstracted way possible. It will be important to understand this setup throughly as most industries that perform maintanance of a considerable scale follow this process. Many industries are of course unique and deviate from general framework in specific work but the fundamentals are usually quite similar.

This figure shows

Chapter 2

Modelling the Generalized Setup

To model the maintenace process in its entirety we will need mathematical modelling tools that are powerful enough to describe the maintenance scheduling process. As the process requires multiple different actors we need a system that enables us to model each of them and specific

To effectively model a system that contains multiple actors also The system will be described in accordance with the **??** Palmer [2019].

The maintenance scheduling problem is NP-hard and real-time optimal solutions will never be a feasible approach unless we use a multi-model setup where each model enriches the overall solution in the way that it is most capable of.

2.1 Parameter Table

Element	Set	Dependent Variables	Description
w	$W(\tau)$	τ	The set of all work orders
p	$P(\tau)$	τ	The set of all weekly periods
r	$R(\tau)$	τ	The set of all resources
o	$O(\tau,)$	τ	The set of all operations
d	$D(\tau)$	τ	The set of all days
a	$A(\tau,)$	τ	The set of all activities
t	$T(\tau)$	τ	The set of all technicians
k	$K(\gamma(\tau))$	τ	The set of all technician work segments
i	$I(\tau)$	τ	The set of all time instances
e	$E(\tau)$	τ	The set of all technician events

Table 2.1: Sets used in the model setup

2.2 The Strategic Model

The Strategic Model have multiple different purposes.

- Schedule Work Order out across the weekly periods
- Prioritize all the different released work orders
- Respect the available weekly hours available for each trait

The Strategic model is responsible for grouping work orders into weekly or biweekly periods depending on which kind of maintenance setup that one is running. This kind of model closely resembles a variant of the multi-compartment multi-knapsack problem.

Meta variables:

$s \in S$

$\tau \in [0, \infty]$

Minimize:

$$\sum_{w \in W(\tau)} \sum_{p \in P(\tau)} strategic\_value_{wp}(\tau) \cdot \alpha_{wp}(\tau) \tag{2.1}$$

$$+ \sum_{p \in P(\tau)} \sum_{r \in R(\tau)} strategic\_penalty \cdot \epsilon_{pr}(\tau) \tag{2.2}$$

$$+ \sum_{p \in P(\tau)} \sum_{w1 \in W(\tau)} \sum_{w2 \in W(\tau)} clustering\_value_{w1,w2} \cdot \alpha_{w1p}(\tau) \cdot \alpha_{w2p}(\tau) \tag{2.3}$$

Subject to:

$$\sum_{w \in W(\tau)} work\_order\_work_{wr} \cdot \alpha_{wp}(\tau) \leq resource_{pr}(\tau) + \epsilon_{pr}(\tau) \quad \forall p \in P(\tau) \quad \forall r \in R(\tau) \tag{2.4}$$

$$\sum_{w \in W(\tau)} \alpha_{wp}(\tau) = 1 \quad \forall p \in P(\tau) \tag{2.5}$$

$$\alpha_{wp}(\tau) = 0 \quad \forall (w,p) \in exclude(\tau) \tag{2.6}$$

$$\alpha_{wp}(\tau) = 1 \quad \forall (w,p) \in include(\tau) \tag{2.7}$$

$$\alpha_{wp}(\tau) \in \{0,1\} \quad \forall w \in W(\tau) \quad \forall p \in P(\tau) \tag{2.8}$$

$$\epsilon_{pr}(\tau) \in \mathbb{R}^+ \quad \forall p \in P(\tau) \quad \forall r \in R(\tau) \tag{2.9}$$

$$\tau \in [0, \infty] \tag{2.10}$$

Parameter	set	var	**Description**
$strategic\_value_{wp}(\tau)$	set	var	DESCRIPTION
$strategic\_penalty$	set	var	DESCRIPTION
$clustering\_value_{w1,w2}$	set	var	DESCRIPTION
$resource_{pr}(\tau)$	set	var	DESCRIPTION
$work\_order\_work_{wr}$	set	var	DESCRIPTION
$include(\tau)$	set	var	DESCRIPTION
$exclude(\tau)$	set	var	DESCRIPTION
$tactical\_value_{do}(\tau)$	set	var	DESCRIPTION
$tactical\_penalty$	set	var	DESCRIPTION
$work(\tau)$	set	var	DESCRIPTION
$tactical\_resource_{dr}(\tau)$	set	var	DESCRIPTION
$start\_start_{o1,o2}$	set	var	DESCRIPTION
$finish\_start_{o1,o2}$	set	var	DESCRIPTION
$earliest\_start_{o}(\tau)$	set	var	DESCRIPTION
$latest\_finish_{o}(\tau)$	set	var	DESCRIPTION
$number_{o}(\tau)$	set	var	DESCRIPTION
$operating\_time_{o}$	set	var	DESCRIPTION
$duration_{o}(\tau)$	set	var	DESCRIPTION
$supervisor\_value_{at}(\tau, \lambda_t(\tau), \Lambda_t(\tau))$	set	var	DESCRIPTION
$feasible_{at}(\theta(\tau))$	set	var	DESCRIPTION
$work\_order\_to\_operations_{w}$	set	var	DESCRIPTION
$operations\_in\_work\_order_{w}$	set	var	DESCRIPTION
$activities\_for\_operation_{o}$	set	var	DESCRIPTION
$lower\_activity\_work_{a}(\tau)$	set	var	DESCRIPTION
$activity\_work_{a}(\tau, \rho(\tau))$	set	var	DESCRIPTION
$preparation_{a1,a2}$	set	var	DESCRIPTION
$event_{ie}$	set	var	DESCRIPTION
$duration_{ie}$	set	var	DESCRIPTION
$constraint\_limit$	set	var	DESCRIPTION
$time\_window\_start_{a}(\beta(\tau))$	set	var	DESCRIPTION
$time\_window\_finish_{a}(\beta(\tau))$	set	var	DESCRIPTION
$availability\_start(\tau)$	set	var	DESCRIPTION
$availability\_finish(\tau)$	set	var	DESCRIPTION

Table 2.2: Parameters used in the model setup

Variable	Set Selectors	Dependent Variable	Description
$\alpha(\tau)$	w, p	τ	DESCRIPTION
$\epsilon_{pr}(\tau)$	p, r	τ	DESCRIPTION
$\beta(\tau)$	d, o	τ	DESCRIPTION
$\mu_{rd}(\tau)$	r, d	τ	DESCRIPTION
$\sigma(\tau)$	d, o	τ	DESCRIPTION
$\eta_{do}(\tau)$	d, o	τ	DESCRIPTION
$\Delta_o(\tau)$	o	τ	DESCRIPTION
$\gamma(\tau)$	a, t	τ	DESCRIPTION
$\phi_o(\tau)$	o	τ	DESCRIPTION
$\rho(\tau)$	a	τ	DESCRIPTION
$\delta_{ak}(\tau)$	a, k	τ	DESCRIPTION
$\pi(\tau)$	a, k	τ	DESCRIPTION
$\lambda(\tau)$	a, k	τ	DESCRIPTION
$\Lambda(\tau)$	a, k	τ	DESCRIPTION
$\omega_{akie}(\tau)$	a, k, i, e	τ	DESCRIPTION
$\theta(\tau)$	a	τ	DESCRIPTION
τ	None	None	DESCRIPTION

Table 2.3: Variables used in the model setup

2.3 The Tactical Model

- Respect precedence constraints

- Respect daily resource requirements for each trait

- Penalize exceeded daily capacity

After the strategic model has optimized its schedule the tactical agent will continue scheduling the output at a more detailed level. This means that now the tactical agent will schedule out on each of the days of the work orders scheduled by the strategic agent.

The tactical model is responsible for providing an initial suggestion for a weekly schedule, below we see the model for the tactical agent.

Meta variables:

$s = S$

$\tau \in [0, \infty]$

$\alpha(\tau)$

Minimize:

$$\sum_{o \in O(\tau,\alpha(\tau))} \sum_{d \in D(\tau)} tactical\_value_{do}(\tau)!!!!!!!!!!!!! \cdot \beta_{do}(\tau)$$

$$+ \sum_{r \in R(\tau)} \sum_{d \in D(\tau)} tactical\_penalty \cdot \mu_{rd}(\tau) \quad (2.11)$$

Subject to:

$$\sum_{o \in O(\tau,\alpha(\tau))} work_o(\tau) \cdot \beta_{do}(\tau)$$

$$\leq tactical\_resource_{dr}(\tau) + \mu_{rd}(\tau)$$

$$\forall d \in D(\tau) \quad \forall r \in R(\tau) \quad (2.12)$$

$$\sum_{d=earliest\_start_o(\tau)}^{latest\_finish_o(\tau)} \sigma_{do}(\tau) = duration_o(\tau)$$

$$\forall o \in O(\tau, \alpha(\tau)) \quad (2.13)$$

$$\sum_{d^* \in D_{duration_o(\tau)}(\tau)} \sigma_{d^*o}(\tau)$$

$$= duration_o(\tau) \cdot \eta_{do}(\tau)$$

$$\forall o \in O(\tau, \alpha(\tau)) \quad \forall d \in D(\tau) \quad (2.14)$$

$$\sum_{o \in O(\tau,\alpha(\tau))} \eta_{do}(\tau) = 1,$$

$$\forall d \in D(\tau)$$

$$\sum_{d \in D(\tau)} d \cdot \sigma_{do1}(\tau) + \Delta_o(\tau) = \sum_{d \in D(\tau)} d \cdot \sigma_{do2}(\tau)$$

$$\forall (o1, o2) \in finish\_start_{o1,o2} \quad (2.15)$$

$$\sum_{d \in D(\tau)} d \cdot \sigma_{do1}(\tau) = \sum_{d \in D(\tau)} d \cdot \sigma_{do2}(\tau)$$

$$\forall (o1, o2) \in start\_start_{o1,o2} \quad (2.16)$$

$$\beta_{do}(\tau) \leq number_o(\tau) \cdot operating\_time_o$$

$$\forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad (2.17)$$

$$\beta_{do}(\tau) \in \mathbb{R}$$

$$\forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad (2.18)$$

$$\mu_{rd}(\tau) \in \mathbb{R}$$

$$\forall r \in R(\tau) \quad \forall d \in D(\tau) \quad (2.19)$$

$$\sigma_{do}(\tau) \in \{0, 1\}$$

$$\forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad (2.20)$$

$$\eta_{do}(\tau) \in \{0, 1\}$$

$$\forall d \in D(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \quad (2.21)$$

$$\Delta_o(\tau) \in \{0, 1\} \quad (2.22)$$

$$\forall o \in O(\tau, \alpha(\tau)) \quad (2.23)$$

$$\tau \in [0, \infty] \quad (2.24)$$

2.4 The Supervisor Model

The maintenance supervisor is considered the most central person in a maintenance scheduling system. All the work of the planner and scheduler should be considered a service for the supervisor.

The supervisor has multiple different responsibilities among them are:

- Assigning work orders
- Creating a daily schedule
- Keeping the schedule up-to-date

2.5 The Operational Model

Meta variables:

$\tau \in [0, \infty]$

$z \in Z$

$\alpha(\tau)$

$\theta(\tau)$

Maximize:

$$\sum_{a \in A(\tau,\alpha(\tau))} \sum_{t \in T(\tau)} supervisor\_value_{at}(\tau, \lambda_t(\tau), \Lambda_t(\tau)) \cdot \gamma_{at}(\tau) \tag{2.25}$$

Subject to:

$$\sum_{a \in A_o(\tau,\alpha(\tau))} \rho_a(\tau) = work_o(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{2.26}$$

$$\sum_{t \in T(\tau)} \sum_{a \in A_o(\tau,\alpha(\tau))} \gamma_{at}(\tau) = \phi_o(\tau) \cdot number_o(\tau) \quad \forall o \in O(\tau, \alpha(\tau)) \tag{2.27}$$

$$\sum_{o \in O_w(\tau,\alpha(\tau))} \phi_o(\tau) =!!!!|O_w(\tau, \alpha(\tau))| \quad \forall w \in W(\tau, \alpha(\tau)) \tag{2.28}$$

$$\sum_{a \in A_o(\tau,\alpha(\tau))} \gamma_{at}(\tau) \leq 1 \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau) \tag{2.29}$$

$$\gamma_{at}(\tau) \leq feasible_{at}(\theta(\tau)) \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau) \tag{2.30}$$

$$\gamma_{at}(\tau) \in \{0, 1\} \quad \forall o \in O(\tau, \alpha(\tau)) \quad \forall t \in T(\tau) \tag{2.31}$$

$$\rho_a(\tau) \in [lower\_activity\_work_a(\tau), work_a(\tau)] \quad \forall a \in A(\tau, \alpha(\tau)) \tag{2.32}$$

$$\tau \in [0, \infty] \tag{2.33}$$

In the supervisor model shown in **??** the set O and W comes from the tactical algorithm and value v and the information of whether or not the operation can be assigned to a specific operational model comes from the operational model itself and is captured in the.

Can this be done? What should the Supervisor have here? He should have what is necessary to handle the.

Here the o is a single operation and o2 is another operation. It is crucial to understand here that the main decision variable, x defines an ordering of the operations that a single operational agent will do the operations in.

The $\lambda_{ak}(\tau)$ is the start time of job i in segment k and $\Lambda_{ak}(\tau)$ is the finish time of job i in segment k. $\delta_{ak}(\tau)ak$ is the processing time of each segment.

Meta variables:

$\tau \in [0, \infty]$

$t \in T(\tau)$

$\alpha(\tau)$

$\gamma(\tau)$

$$\sum_{a \in A(\tau,\gamma_t(\tau))} \sum_{k \in K(\gamma(\tau))} \delta_{ak}(\tau) \tag{2.34}$$

Subject to:

$$\sum_{k \in K(\gamma(\tau))} \delta_{ak}(\tau) \cdot \pi_{ak}(\tau) = activity\_work_a(\tau, \rho(\tau)) \cdot \theta(\tau) \quad \forall a \in A(\tau, \gamma_t(\tau)) \tag{2.35}$$

$$\lambda_{a21}(\tau) \geq \Lambda_{a1last(a1)}(\tau) + preparation_{a1,a2} \quad \forall a1 \in A(\tau, \gamma_t(\tau)) \quad \forall a2 \in A(\tau, \gamma_t(\tau)) \tag{2.36}$$

$$\lambda_{ak}(\tau) \geq \Lambda_{ak-1}(\tau) - constraint\_limit \cdot (2 - \pi_{ak}(\tau) + \pi_{ak-1}(\tau)) \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{2.37}$$

$$\delta_{ak}(\tau) = \Lambda_{ak}(\tau) - \lambda_{ak}(\tau) \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{2.38}$$

$$\lambda_{ak}(\tau) \geq event_{ie} + duration_{ie} - constraint\_limit \cdot (1 - \omega_{akie}(\tau)) \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall i \in I(\tau) \quad \forall e \in E(\tau) \tag{2.39}$$

$$\Lambda_{ak}(\tau) \leq event_{ie} + constraint\_limit \cdot \omega_{akie}(\tau) \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall i \in I(\tau) \quad \forall e \in E(\tau) \tag{2.40}$$

$$\lambda_{a1}(\tau) \geq time\_window\_start_a(\beta(\tau)) \quad \forall a \in A(\tau, \gamma_t(\tau)) \tag{2.41}$$

$$\Lambda_{alast(a)}(\tau) \leq time\_window\_finish_a(\beta(\tau)) \quad \forall a \in A(\tau, \gamma_t(\tau)) \tag{2.42}$$

$$\pi_{ak}(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{2.43}$$

$$\lambda_{ak}(\tau) \in [availability\_start(\tau), availability\_finish(\tau)] \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{2.44}$$

$$\Lambda_{ak}(\tau) \in [availability\_start(\tau), availability\_finish(\tau)] \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{2.45}$$

$$\delta_{ak}(\tau) \in [0, work_{a\_to\_o(a)}(\tau)] \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \tag{2.46}$$

$$\omega_{akie}(\tau) \in \{0, 1\} \quad \forall a \in A(\tau, \gamma_t(\tau)) \quad \forall k \in K(\gamma(\tau)) \quad \forall i \in I(\tau) \quad \forall e \in E(\tau) \tag{2.47}$$

$\theta_a(\tau) \in \{0, 1\}$

possible sections * All Stochasticity will be handled by user interaction. * In maintenance scheduling, data is assumed to be correct. Yes that must be the prevailing idea here.

Maximize:

Bibliography

Richard D. Palmer. *Maintenance Planning and Scheduling Handbook, 4th Edition.* McGraw Hill, 4th edition edition, September 2019.