Game Theory

How do we incentivize good behavior?

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INPUT | OUTPUT

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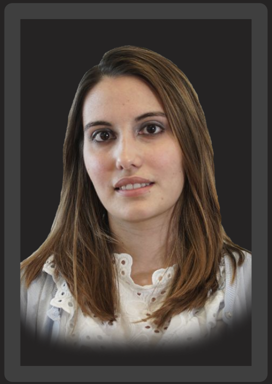

Aikaterini-Panagiota Stouka

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Desired Behavior

- A solid majority of stake should be delegated to 1000 stake pools of equal size.
- The stake pool operators should be online when needed.

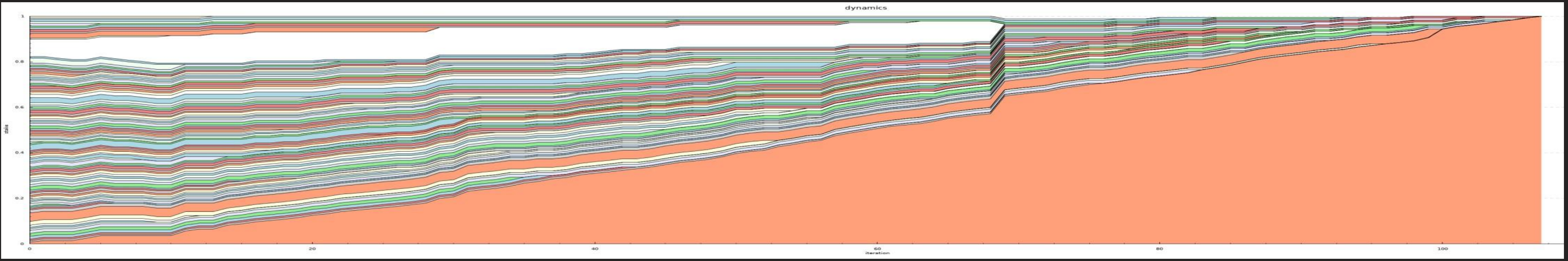
Basic Idea

The rewards pool from one epoch (five days) is distributed amongst stake pools and individual protocol participants according to their stake.

Problem with the Basic Idea

The basic idea is a good guideline, but too naive: The fewer pools there are, the lower total costs will be, the higher everybody's rewards will be.

So the system will tend towards a single dictatorial pool that everybody else delegates to.



Refinements

- Pool rewards are capped at 1‰ of available rewards.
- Pools are penalized for not being online when it is their turn.
- Pool leaders are compensated for their cost and effort by keeping a margin.
- Pool members are rewarded proportional to the stake they delegated to the pool.

INPUT | OUTPUT

Game Theory

For mathematicians, a game is a system where players can choose between strategies, and each player’s payoff is determined by his and everybody else’s strategy.

Example: Prisoner's Dilemma

Two players A and B, each having two strategies "stay silent" and "betray", with the following payoff-matrix:

	B stays silent.	B betrays.
A stays silent.	-1/-1	-3/0
A betrays.	0/-3	-2/-2

Nash Equilibrium

- One of the most important concepts in Game Theory.
- Named after John Forbes Nash Jr., Nobel Prize in Economics 1994.
- A Nash Equilibrium is a choice of strategy for each player with the property that no player has an incentive to unilaterally change his strategy.
- Any (reasonably well behaved) game has at least one Nash Equilibrium.
- Games played by rational players ”tend” to end up in a Nash Equilibrium.

Nash Equilibrium for Prisoner's Dilemma

	B stays silent.	B betrays.
A stays silent.	-1/-1	-3/0
A betrays.	0/-3	-2/-2

- If one player stays silent, he can always improve his payoff by betraying (provided the other player doesn't change his strategy).
- If both players betray, none has incentive to unilaterally change to staying silent.
- Therefore both players betraying is the (unique) Nash Equilibrium of the Prisoner's Dilemma.

The Staking Game

We consider the following “staking game”, where each player has the following strategies:

- Create a staking pool with a certain margin.
- Delegate his stake to one or more pools.

Nash Equilibria of the Staking Game

We prove: If reward distribution follows the rules explained earlier, then each Nash Equilibrium of this staking game has 1000 staking pools of the same size.

Thank you!