# 1 Fenics

## 1.1 Math stuff

### 1.1.1 Variational Form of the Poisson equation

$$D\nabla^2 u = f$$
$$D\nabla^2 uv = fv$$
$$D\int_\Omega \nabla^2 u dx = \int_\Omega fv dx$$

**left hand side** By adding $\int_\Omega \nabla u \nabla v dx$ and using Green's first identity the *lhs* yields

$$D\int_\Omega \nabla^2 u dx = D\left(\int_{\partial\Omega} v(\nabla u \cdot \vec{n}) ds - \int_\Omega \nabla u \cdot \nabla v dx\right) \tag{1.1}$$

**right hand side** ...

### 1.1.2 Boundary Conditions

Since the test function must vanish on $\partial\Omega$, only terms concerning $\nabla u \cdot \vec{n}$ must be substituted into 1.1. In order to specify different BCs on different parts of the boundary, the boundary integral in 1.1 can be constructed as a sum

$$\int_{\partial\Omega} v(\nabla u \cdot \vec{n}) ds = \sum \int_{\partial\Omega} v(\nabla u \cdot \vec{n}) ds_i$$

over the pieces $s_i$ of the boundary[1]. The above admits both Neumann and nonlinear boundary conditions of the form

$$\nabla u \cdot \vec{n} = const$$
$$\nabla u \cdot \vec{n} = q(u, \vec{p})$$

where $q$ depends on $u$ and possible a parameter vector $\vec{p}$.

[1]It is my understanding that this piecewise construction is possible as result of the FEM approach since the solution $u$ is taken from the $H^1$ Sobolev space, which allows for discontinues derivatives.