Kapitel 3.6

Ableitung $\frac{dy}{dx} = \underbrace{f'(g(x)) g'(x)}_{(f \circ g)'x}$, wenn $y = f(v)$ $v = g(x)$

Aufgabe 1

$y = 6v - 9$ $\quad y' = 6$ $\quad 12x^3 \cdot 6 = 12x^3$

$v = (1/2)x^4$ $\quad v' = 2x^3$

Aufgabe 2

$y = \sin v$ $\quad y' = \cos v$ $\quad 3 \cdot \cos(3x+1)$

$v = 3x + 1$ $\quad v' = 3$

Aufgabe 3

$y = (2x+1)^5$

$y = v^5$ $\quad y' = 5v^4$ $\quad 2 \cdot 5v^4 = 10 \cdot (2x+1)^4$

$v = 2x + 1$ $\quad v' = 2$

Aufgabe 4

$\left[\frac{u}{v}\right]' = \frac{u'v - uv'}{v^2}$

$y = (1 - \frac{x}{7})^{-7}$

$y = v^{-7}$ $\quad y' = -7v^{-8}$

$v = 1 - \frac{x}{7}$ $\quad v' = \frac{1 \cdot 7 - x \cdot 0}{49} = \frac{1}{7}$

$-7 \cdot \frac{1}{7}(1 - \frac{x}{7})^{-8} = -(1 - \frac{x}{7})^{-8}$

$-7v^{-8} \cdot (-\frac{1}{7}) = (1 - \frac{x}{7})^{-8}$

Aufgabe 5

$p = \sqrt{3 - t}$

$p = \sqrt{v}$ $\quad p' = \frac{1}{2\sqrt{v}}$ $\quad \frac{1}{2\sqrt{3-t}}$

$v = 3 - t$ $\quad v' = 1$

Aufgabe 6

$s = \frac{4}{3\pi} \sin 3t + \frac{4}{6\pi} \cos 5t$

$s = \frac{4}{3\pi} \sin(u) + \frac{4}{6\pi} \cos(v)$ $\quad s' =$

Aufgabe 8

$y = x^2 \sin^4 x + x \cos^{-2} x$

$y = \sin^4(x)$ $\quad y' = 4\cos(x) + \sin^3(x)$

$v = \cos^{-2}(x) = \frac{1}{\cos^2(x)}$ $\quad v' = \frac{2\sin(x)}{\cos^3(x)}$

$y_1 = x^2 \sin^4 x$

$y_1' = 2x \sin^4 x + x^2 \cdot 4\sin^3 x \cdot \cos x$

$y_2 = \frac{x}{\cos^2 x} = x \cos^{-2} x$

$y_2' = \frac{1 \cdot \cos^2 x - x \, 2\cos x \cdot (-\sin x)}{\cos^4 x}$ } Quotientenregel

$y_2' = 1 \cdot \cos^{-2} x + x(-2)\cos^{-3} x \cdot (-\sin x)$ } Produktregel

Aufgabe 9

$y = \frac{1}{21}(3x-2)^7 + \left(4 - \frac{1}{2x}\right)^{-1}$

$y = \frac{1}{21}(u_1)^7 + u_2^{-1}$ $\quad y' = 7u^6 + -u^{-2}$

$u_1 = 3x - 2$ $\quad u_1' = 3$

$u_2 = \left(4 - \frac{1}{2x}\right)$ $\quad u_2' = \frac{1}{4x}$

$y' =$

Aufgabe 14

$\frac{u'v - uv'}{v^2}$

$y = \left(1 + \frac{1}{x}\right)^3$

$y = u^3$ $\quad y' = 3u^2$ $\quad y' = 6u$

$u = 1 + \frac{1}{x}$ $\quad u' = \frac{0 \cdot x - 1 \cdot 1}{x^2} = -\frac{1}{x^2}$ $\quad u' = \frac{0 \cdot x^2 - 1 \cdot 2x}{x^4} = \frac{2x}{x^4} = \frac{2}{x^3}$

$y' = 6\left(\frac{2}{x^3}\right)$

$y = \left(1 + \frac{1}{x}\right)^3$

$y' = 3\left(1 + \frac{1}{x}\right)^2 \cdot \left(-\frac{1}{x^2}\right)$

$= 3\left(1 + \frac{2}{x} + \frac{1}{x^2}\right)\left(-\frac{1}{x^2}\right)$

$= -\frac{3}{x^2} - \frac{6}{x^3} - \frac{3}{x^4}$

$y' = \frac{6}{x^3} + \frac{12}{x^4} + \frac{12}{x^5}$

Tsep.:

$u(x) = 6x - 9$ $\quad u' = 6$

$y(u) = \frac{1}{2}u^4$ $\quad y' = 2u^3$

$y(u(x)) = \frac{1}{2}(6x-9)^4$

$y' = 12(6x-9)^3$