Aufgaben 4.7 „Aufleiten"

① a) $2x$ $F(x) = x^2 + K$ b) x^2 $F(x) = \frac{1}{3}x^3 + K$ c) $x^2 - 2x + 1$
$F(x) = \frac{1}{3}x^3 - x^2 + x + K$

② a) $-3x^{-4}$ $F(x) = x^{-3} + K$ b) x^{-4} $F(x) = \frac{1}{-3}x^{-3} + K$ c) $x^{-4} + 2x + 3$
$F(x) = \frac{1}{-3}x^{-3} + x^2 + 3x + K$

③ a) $\frac{1}{x^2} = x^{-2}$ $F(x) = -x^{-1} + K$ b) $\frac{5}{x^2} = 5x^{-2}$ $F(x) = -5x^{-1} + K$ c) $2 - \frac{5}{x^2} = 2 - 5x^{-2}$
$F(x) =$

④ a) $\frac{3}{2}\sqrt{x} = \frac{3}{2}x^{0.5}$ $F(x) = \frac{1.5}{1.5}x^{1.5} + K$ b) $\frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-0.5}$ $F(x) = \frac{1}{1.5}x^{1.5} + K$ c) $\sqrt{x} + \frac{1}{\sqrt{x}} = x^{0.5} + x^{-0.5} = 2x^{0.5}$
$F(x) = \frac{2}{1.5}x^{1.5} + K$

⑤ a) $\frac{2}{3}x^{-\frac{1}{3}}$ $F(x) = x^{\frac{2}{3}} + K$ b) $\frac{1}{3}x^{-\frac{2}{3}}$ $F(x) = x^{\frac{1}{3}} + K$ c) $-\frac{1}{3}x^{-\frac{4}{3}}$ $F(x) = x^{-\frac{1}{3}} + K$

⑥ a) $\pi \sin \pi x$ $F(x) = -\cos \pi x^2 + K$ b) $3 \sin x$ $F(x) = \frac{3}{2} - \cos x^2 + K$ c) $\sin \pi x - 3 \sin 3x$
$F(x) = \frac{1}{2} - \cos \pi x^2 - 3 \cdot \cos \frac{3}{2}x^2 + K$

⑧ $\int (x+1)\,dx$ $F(x) = \frac{1}{2}x^2 + x + K$

⑨ $\int (3t^2 + \frac{5}{2})\,dt$ $F(x) = t^3 +$ _____

⑩ $\int (2x^3 - 5x + 7)\,dx$ $F(x) = \frac{1}{2}x^4 - \frac{5}{2}x^2 + 7x + K$

⑪ $\int (\frac{1}{x^2} - x^2 - \frac{1}{3})\,dx$ $F(x) = \ln|x^2| - \frac{1}{3}x^3 - \frac{1}{3}x + K$

⑫ $\int x^{-\frac{1}{3}}\,dx$ $F(x) = \frac{1}{\frac{2}{3}}x^{\frac{2}{3}} + K$

⑬ $\int (\sqrt{x} + 2\sqrt[3]{x})\,dx$ $F(x) = \frac{1}{1.5}x^{1.5} + \frac{3}{4}x^{\frac{4}{3}} + K$

⑭ $\int (8y - \frac{2}{y\sqrt{y}})\,dy$ $F(x) = 4y^2 -$ _____

(15)

$\int 2x(1-x^{-3})$ $\quad F(x) = x^2\left(x + \frac{1}{2}x^{-2}\right) + k$

(16)

$\int \frac{t\sqrt{t} - \sqrt{t}}{t^2}\,dt$ $\quad F(x) = \frac{\frac{1}{2}t^2 \cdot \frac{1}{1.5}t^{1.5} - \frac{1}{1.5}t^{1.5}}{\frac{1}{3}t^3}$

(17)

$\int (-2\cos t)\,dt$ $\quad F(x) = -1\sin t^2$

(18)

$\int 7\sin\frac{\theta}{3}\,d\theta$ $\quad F(x) =$ _____

um $\frac{1}{3}$ zu kompensieren

$\int 7\sin\frac{\theta}{3}\,d\theta = 7\int \sin\frac{\theta}{3}\,d\theta = 7\left(-\cos\frac{\theta}{3}\right)\cdot 3 + c$

$= -21\cos\frac{\theta}{3} + c$

Kontrolle:

$\left[-21\cos\frac{\theta}{3} + c\right]' = -21\left[\cos\frac{\theta}{3}\right]' = -21\left(-\sin\frac{\theta}{3}\right)\cdot\frac{1}{3} = 7\sin\frac{\theta}{3}$

(19)

a) falsch ✓ b) ~~nicht~~ falsch c) falsch ✓

(27)

$\left[\frac{(7x-2)^4}{28} + c\right]' = \left[\frac{(7x-2)^4}{28}\right]' = \frac{1}{28}\left[(7x-2)^4\right] = \frac{1}{28}\cdot \underbrace{4(7x-2)^3}_{\text{äussere Abl.}}\cdot \underbrace{7}_{\text{innere}}$

$= (7x-2)^3$ ⇒ korrekt

(28)

$\left[-\frac{1}{x+1} + c\right]' = \left[-\frac{1}{x+1}\right]' = \left[(-1)(x+1)^{-1}\right]' = (-1)\cdot(x+1)^{-2}\cdot(-1) = (x+1)^{-2} = \frac{1}{(x+1)^2}$

⇒ korrekt

Musterlösung

6a)

$$\int -\pi \sin \pi x \, dx = -\pi \int \sin \pi x \, dx = -\pi \cdot \frac{1}{\pi} (-\cos \pi x) + c$$

innere Abl. vergessen

$$\frac{1}{\pi} \cdot [\pi \cos \pi x + c]' = \pi \cdot (-\sin \pi x) \cdot \pi \cdot \frac{1}{\pi}$$

Korrekte Lösung

$\cos \pi x + c$

① Ableitung „abschließen"
② Lösung ableiten & vergleichen
③ fehlende Lösung ergänzen