Aufgabe 2.6

1 Grenzwert bestimmen

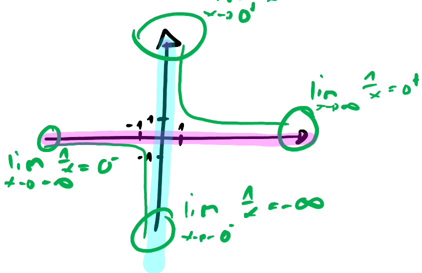
a) $\lim\limits_{x\to 2} f(x) = 0$ b) $\lim\limits_{x\to -3^+} f(x) = -2$ c) $\lim\limits_{x\to -3^-} f(x) = 2$ d) $\lim\limits_{x\to -3} f(x) =$ nicht def.

e) $\lim\limits_{x\to 0^+} f(x) = -1$ f) $\lim\limits_{x\to 0^-} f(x) =$ nicht definiert g) $\lim\limits_{x\to 0} f(x) =$ n.d.

h) $\lim\limits_{x\to \infty} f(x) = 1$ i) $\lim\limits_{x\to -\infty} f(x) = 0$

$\lim\limits_{x\to\infty} \frac{1}{x} = 0$

$\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$



3 Grenzwert bestimmen a) $x\to\infty$ b) $x\to -\infty$

$$g(x) = \frac{1}{2+(1/x)}$$

a) $\lim\limits_{x\to\infty} \left(\frac{1}{2+\underbrace{\frac{1}{x}}_{0}}\right) = \frac{1}{2}$

b) $\lim\limits_{x\to -\infty} \left(\frac{1}{2+\underbrace{\frac{1}{x}}_{0}}\right) = \frac{1}{2}$

5 Grenzwert

$$\lim_{x\to\infty} \frac{\sin 2x}{x} = \frac{2\cdot \boxed{\sin x}}{\boxed{x}} = 0 \; ?$$

strebt nach 0

$$\underbrace{\lim_{x\to\infty} \sin 2x}_{-1 \le L \le 1} \cdot \underbrace{\lim_{x\to\infty} \frac{1}{x}}_{0} = 0$$

7 Grenzwert rationaler Funktionen a) $x\to\infty$ b) $x\to -\infty$

$$f(x) = \frac{2x+3}{5x+7}$$

a) $\lim\limits_{x\to\infty} \left(\frac{2x+3}{5x+7}\right) = \frac{2}{5}$ b) $\lim\limits_{x\to -\infty} \left(\frac{2x+3}{5x+7}\right) = \frac{2}{5}$

9 Grenzwert rationaler Funktionen a) $x\to\infty$ b) $x\to -\infty$

$$h(x) = \frac{7x^3}{x^3-3x^2+6x}$$

a) $$\lim_{x\to\infty} \left(\frac{7x^3}{x^3-3x^2+6x}\right) = \frac{\frac{7x^3}{x^3}}{\frac{x^3}{x^3} - \frac{3x^2}{x^3} + \frac{6x}{x^3}} = \frac{7}{1 - \underbrace{\frac{3x^2}{x^3}}_{0} + \underbrace{\frac{6x}{x^3}}_{0}} = 7$$

b) $$\lim_{x\to -\infty} \left(\frac{7x^3}{x^3-3x^2+6x}\right) = \frac{\frac{7\cancel{x^3}}{\cancel{x^3}}}{1\,\cancel{\frac{x^3}{x^3}} - \underbrace{\frac{3x^2}{x^3}}_{0} + \underbrace{\frac{6x}{x^3}}_{0}} = 7$$

11 Grenzwert rationaler Funktionen a) $x\to\infty$ b) $x\to -\infty$

$$h(x) = \frac{-2x^3-2x+3}{3x^3+3x^2-5x}$$

a) $$\lim_{x\to\infty} \left(\frac{-2x^3-2x+3}{3x^3+3x^2-5x}\right) = \frac{\frac{-2x^3}{x^3} - \frac{2x}{x^3} + \frac{3}{x^3}}{\frac{3x^3}{x^3} + \frac{3x^2}{x^3} - \frac{5x}{x^3}} = \frac{-2 - \frac{2x}{x^3} + \frac{3}{x^3}}{3 + \frac{3x^2}{x^3} - \frac{5x}{x^3}} = -\frac{2}{3}$$

b) $$\lim_{x\to -\infty} \left(\frac{-2x^3-2x+3}{3x^3+3x^2-5x}\right) = \frac{\frac{-2x^3}{x^3} - \frac{2x}{x^3} + \frac{3}{x^3}}{\frac{3x^3}{x^3} + \frac{3x^2}{x^3} - \frac{5x}{x^3}} = \frac{-2 - \frac{2x}{x^3} + \frac{3}{x^3}}{3 + \frac{3x^2}{x^3} - \frac{5x}{x^3}} = -\frac{2}{3}$$

13 Grenzwert bestimmen

$$\lim_{x\to-\infty}\left(\frac{1-x^3}{x^2+7x}\right)^5 = \left(\frac{\frac{1}{x^3}-\frac{x^3}{x^3}}{\frac{x^2}{x^3}+\frac{7x}{x^3}}\right)^5 = \left(\frac{\frac{1}{x^3}-1}{\frac{x^2}{x^3}+\frac{7x}{x^3}}\right)^5 = \cancel{-1}\ \infty$$

5 → hoch 5 ist irrelevant!

→ 0

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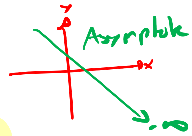
$$\lim_{x\to-\infty}\left(\frac{\sqrt[3]{x}-\sqrt[5]{x}}{\sqrt[3]{x}+\sqrt[5]{x}}\right) = \left(\frac{x^{\frac{1}{3}}-x^{\frac{1}{5}}}{x^{\frac{1}{3}}+x^{\frac{1}{5}}}\right) = \frac{\frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}}-\frac{x^{\frac{1}{5}}}{x^{\frac{1}{3}}}}{\frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}}+\frac{x^{\frac{1}{5}}}{x^{\frac{1}{3}}}} = \frac{1-\frac{x^{0.2}}{x^{0.3}}}{1+\frac{x^{0.2}}{x^{0.3}}} = 1 ✓$$

13 Klassenlösung



$$\lim_{x\to-\infty}\frac{1-x^3}{x^2+7x} = \infty$$

$$(1-x^3):(x^2+7x) = -x+7+\frac{-49x+1}{x^2+7x}$$

→ strebt nach 0

$-(-x^3-7x^2)$

$0+7x^2+1$

$-(7x^2+49x)$

$-49x+1$

$$\lim_{x\to-\infty}(-x+7) = \infty$$

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Wurzel ziehen erlaubt da 1 irrelevant ist

$$\lim_{x\to\infty}\frac{\sqrt{x^2+1}}{x+1} = \frac{x^{\frac{2}{2}}+1^{\frac{1}{2}}}{x+1} = 1$$

19 unendliche Grenzwerte

$$\lim_{x\to0^+}\left(\frac{1}{3x}\right) = \infty$$

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hier könnte -8 eingesetzt werden => 2*-8 = -16

$$\lim_{x\to-8^+}\left(\frac{2x}{x+8}\right) = -\infty$$

$$\underbrace{\lim_{x\to-8^+}\frac{1}{x+8}}_{-\infty}\cdot\underbrace{\lim_{x\to-8^+}2x}_{-16} = -\infty$$

da hier (-8) nicht eingesetzt werden darf (Division durch 0 ist nicht erlaubt), muss eine Zahl gefunden werden, welche in der Nähe liegt. Da (-8+) ist, wäre z.B. (-7.9) oder (-7.999999999) möglich.

23a

$$\lim_{x\to0^+}\frac{2}{3x^{\frac{1}{3}}} = \infty$$

23b

$$\lim_{x\to0^-}\frac{2}{3x^{\frac{1}{3}}} = -\infty$$

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$$\lim_{x\to(\pi/2)^-}(\tan x) = \infty$$

$$\tan x = \frac{\sin x}{\cos x}$$

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$$\lim\frac{1}{x^2-4}$$ Grenzwert bestimmen

a) $\lim_{x\to2^+}\frac{1}{x^2-4} = \infty$ b) $\lim_{x\to2^-}\frac{1}{x^2-4} = -\infty$ c) $\lim_{x\to-2^+}\frac{1}{x^2-4} = -\infty$ d) $\lim_{x\to-2^-}\frac{1}{x^2-4} = \infty$

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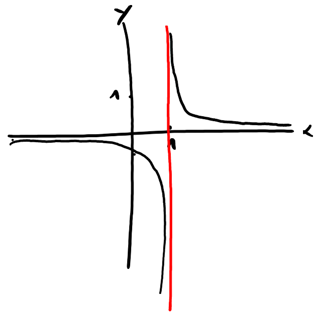
$$y = \frac{1}{x-1}$$



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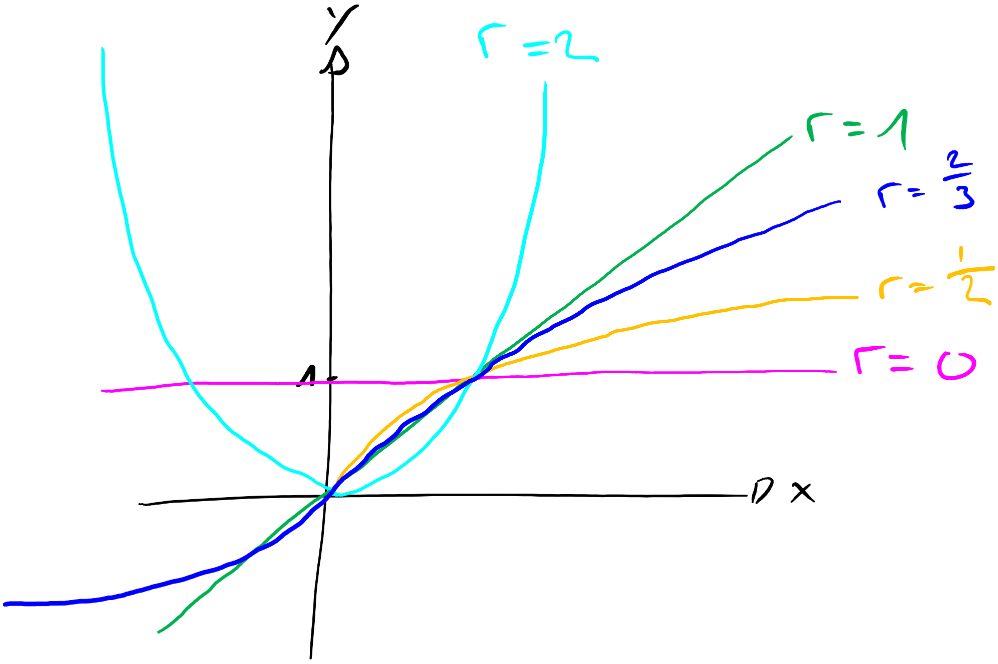
$$y = \frac{x+3}{x+2}$$



$y = x^r$

Potenzfunktion

$y = x^{\frac{1}{2}} = \sqrt{x}$

$y = x^{\frac{2}{3}} = \sqrt[3]{x^2}$

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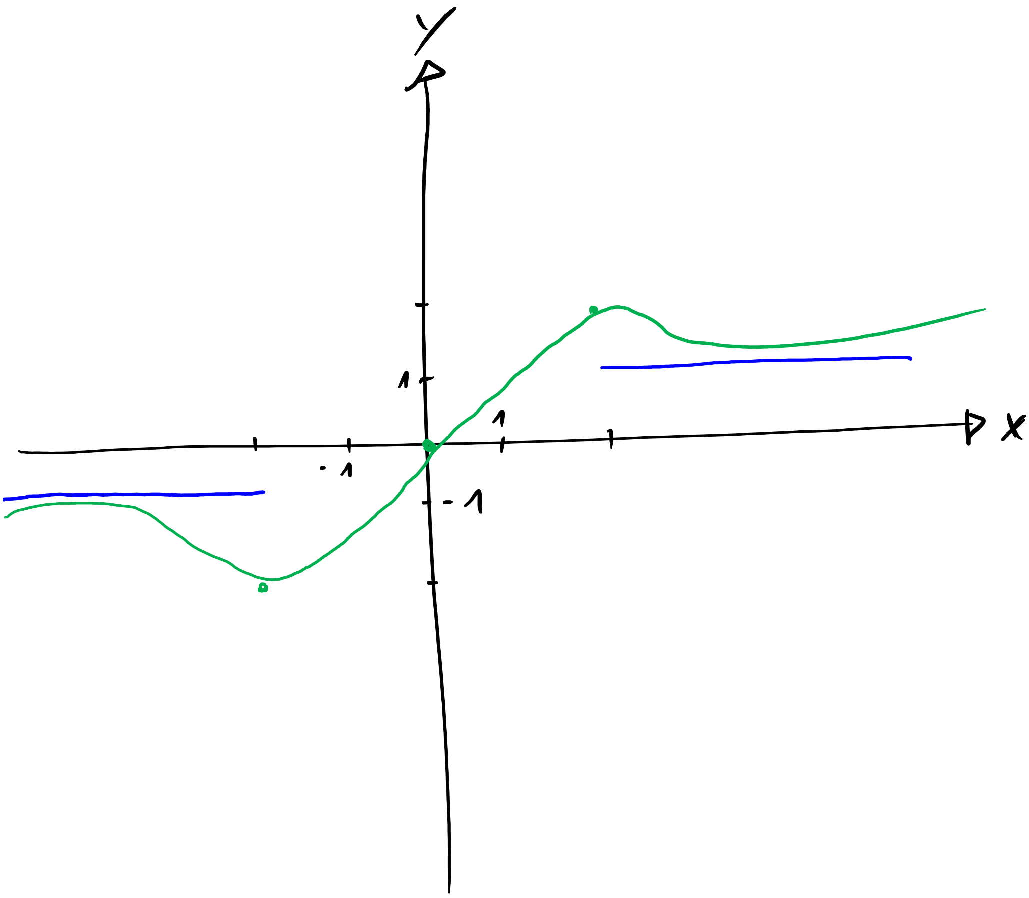

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$$\lim_{x \to \infty} \left(\sqrt{x^2+25} - \sqrt{x^2-1} \right)$$

$$= \frac{\left(\sqrt{x^2+25} - \sqrt{x^2-1}\right)\left(\sqrt{x^2+25} + \sqrt{x^2+1}\right)}{\sqrt{x^2+25} + \sqrt{x^2+1}}$$

$$= \frac{x^2+25 - x^2-1}{\sqrt{x^2+25} + \sqrt{x^2+1}} = \frac{26}{\sqrt{x^2+25} + \sqrt{x^2+1}} = 0$$

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$$\lim_{x \to -\infty} \left(2x + \sqrt{4x^2+3x-2} \right) = \frac{\left(2x + \sqrt{4x^2+3x-2}\right)\left(2x - \sqrt{4x^2+3x-2}\right)}{2x - \sqrt{4x^2+3x-2}}$$

$$= \frac{4x^2 - (4x^2+3x-2)}{2x - \sqrt{4x^2+3x-2}} = \frac{-3x+2}{2x - \sqrt{4x^2+3x-2}}$$

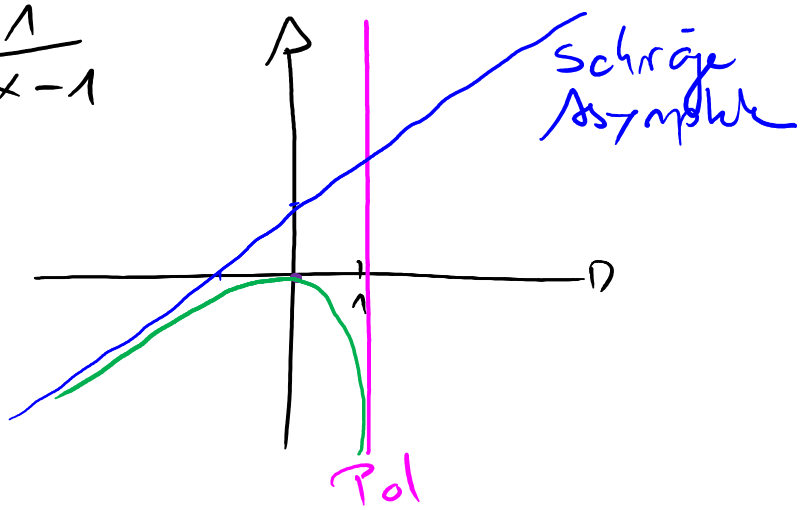
... durch x teilen

$$= \frac{-3 + \frac{2}{x}}{2 - \frac{\sqrt{4x^2+3x-2}}{x}} = \frac{-3 + \frac{2}{x}}{2 - \sqrt{\frac{4x^2+3x-2}{x^2}}} = \frac{-3 + \underbrace{\frac{2}{x}}_{\text{gegen } 0}}{2 - \sqrt{4 + \underbrace{\frac{3}{x}}_{\text{gegen } 0} - \underbrace{\frac{2}{x^2}}_{\text{gegen } 0}}}$$

$$= -\frac{3}{4}$$

(39) $y = \frac{x^2}{x-1}$ $\qquad \mathbb{D} = \mathbb{R} \setminus \{1\}$

$$x^2 : (x-1) = x+1+\frac{1}{x-1}$$
$$-(x^2-x)$$
$$0+x$$
$$-(x-1)$$
$$0+1$$



$x = 0$

(41)

$D = \mathbb{R} \setminus \{0\}$

$f(x) = \frac{1}{x} \cdot (x^2-1)$

wird dominieren

$f(x) = 0$ → schneidet bei ± 1

$$(x^2-1) : x = x - \frac{1}{x}$$
$$-x^2$$
$$-1$$

$$\underbrace{\lim_{x\to\infty} x}_{\infty} - \underbrace{\lim_{x\to\infty} \frac{1}{x}}_{0}$$