Aufgaben 5.4

①

$$\int_{-2}^{0}(2x+5)\,dx = \Big|(x^2+5x\Big|_{-2}^{0} = (0+0)-((-2)^2+(-10)) = 6$$

③

$$\int_{0}^{4}\left(3x-\frac{x^3}{4}\right)dx = \left|\frac{3x^2}{2}-\frac{x^4}{4}\cdot\frac{1}{4}\right|_0^4 = \frac{3x^2}{2}-\frac{x^4}{16}\Big|_0^4 = \left(\frac{3\cdot 16}{2}-\frac{4^4}{16}\right)-0 = \underline{\underline{8}}$$

④

$$\int_{0}^{1}(x^2+\sqrt{x})\,dx = \left|\frac{x^3}{3}+\frac{x^{1.5}}{1.5}\right|_0^1 = \left(\frac{1}{3}+\frac{1^{1.5}}{1.5}\right) = \underline{\underline{1}}$$

$2t \to t^2$

⑦

$$\int_{\frac{\pi}{2}}^{0}\left(\frac{1+\cos 2t}{2}\right)dt =$$

$\sin 2t = \cos(2t)\cdot 2$

$$\frac{1}{2}\int_{\frac{\pi}{2}}^{0}(1+\cos 2t)\,dt =$$

$\frac{1}{2}\cdot\sin 2t = \frac{1}{2}\cos(2t)\cdot 2$

$= \cos(2t)$

$$\frac{1}{2}\left(t+\frac{\sin 2t}{2}\right)$$

$\frac{1}{2}[\sin 2t]' = \cos 2t\cdot 2\cdot\frac{1}{2}$

$$= \frac{1}{2}\left(t+\frac{\sin 2t}{2}\right)\Bigg|_{\frac{\pi}{2}}^{0} = 0+0-\left(\frac{\pi}{4}+0\right) = \underline{\underline{-\frac{\pi}{4}}}$$

Flächeninhalt

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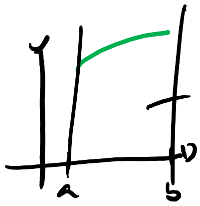
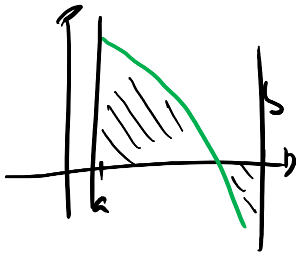


→ riemannsche Summe



$\Downarrow$

1) Hat es Nullstellen?

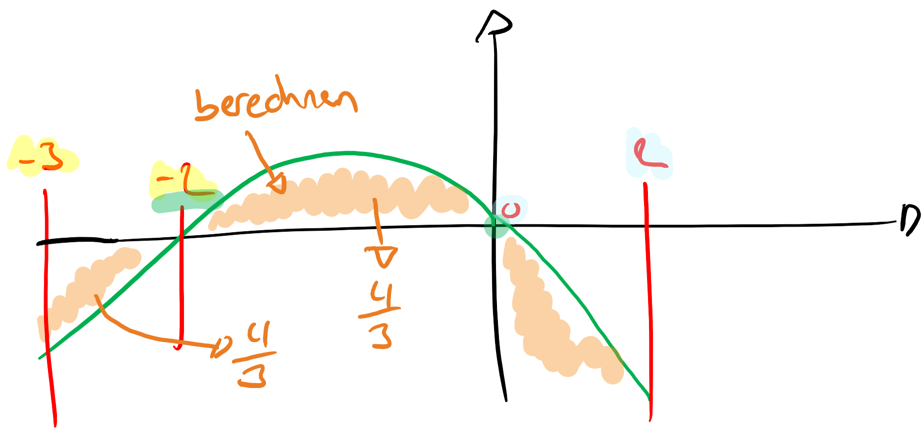
$f(x) = 0$

$-x^2 - 2x = 0$

$x(-x-2) = 0$

$x_1 = 0$

$x_2 = -2$



$$\int_{-3}^{-2} (-x^2-2x)\,dx = \left(-\frac{x^3}{3} - x^2\right)\Big|_{-3}^{-2}$$

$$= \frac{8}{3} - 4 - (\underbrace{9-9}_{0}) = -\frac{4}{3}$$

$$\int_{-2}^{0} \dots = \left(-\frac{x^3}{3} - x^2\right)\Big|_{-2}^{0} = 0 - \left(\frac{8}{3} - 4\right) = \frac{4}{3}$$

$$\int_{0}^{2} \dots = \left(-\frac{x^3}{3} - x^2\right)\Big|_{0}^{2} = -\frac{8}{3} - 4 - (0) = -\frac{20}{3}$$

$$F = \frac{4}{3} + \frac{4}{3} + \frac{20}{3} = \frac{28}{3}$$

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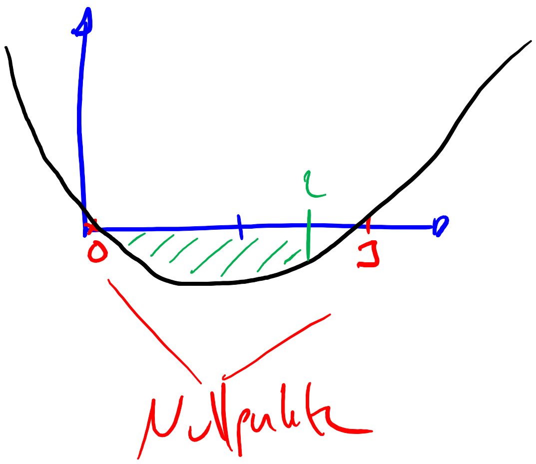
$$\int_0^{\pi} (1 + \cos x)\, dx = (x + \sin x)\Big|_0^{\pi} = \pi + 0 - (0+0) = \pi$$

2

$$\int_0^2 x(x-3)\, dx =$$

$$\int (x^2 - 3x)\, dx = \frac{x^3}{3} - \frac{3x^2}{2} \Big|_0^2 = \frac{0^3}{3} - \frac{3 \cdot 0^2}{2} - \frac{2^3}{3} - \frac{3 \cdot 2^2}{2} = -\frac{10}{3}$$



4

$$\int_0^1 (x^2 + x^{\frac{1}{2}})\, dx = \left(\frac{x^3}{3} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)\Bigg|_0^1 = \left(\frac{x^3}{3} + \frac{2}{3} x^{\frac{3}{2}}\right)\Bigg|_0^1 = \frac{1}{3} + \frac{2}{3} = 1$$

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$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sin x \, dx - \frac{1}{3}\pi = -\cos x \Big|_{\frac{\pi}{6}}^{\frac{5\pi}{6}} - \frac{\pi}{3} = \frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) - \frac{\pi}{3}$$

$$= \sqrt{3} - \frac{\pi}{3}$$