Berg Michael, Brunner Pascal Gruppe 2

Aufgabe 1

1) a) geg: $y^{(4)} + 1.1 y'' - 0.1 y' - 0.3 y = \sin(x) + 5$ $\quad y(0) = y''(0) = y'''(0) = 0 \quad y'(0) = 2$

$x_0 = 0 \qquad h = 0.1$

1) Auflösen nach der höchsten Ableitung:

$y^{(4)} = \sin(x) + 5 - 1.1 y'' + 0.1 y' + 0.3 y$

2) Hilfsfunktionen einführen

$z_1(x) = y(x)$

$z_2(x) = y'(x)$

$z_3(x) = y''(x)$

$z_4(x) = y'''(x)$

3) Hilfsfunktion ableiten & in grösste Hilfsfunktion einsetzen

$z_1'(x) = y' = z_2$

$z_2'(x) = y'' = z_3$

$z_3'(x) = y''' = z_4$

$z_4'(x) = y^{(4)} \Rightarrow z_4' = \sin(x) + 5 - 1.1 z_4 + 0.4 z_3 + 0.3 z_1$

4) DGL in vektorielle Form schreiben

$$z' = \begin{pmatrix} z_1' \\ z_2' \\ z_3' \\ z_4' \end{pmatrix} = \begin{pmatrix} z_2 \\ z_3 \\ z_4 \\ \sin(x) + 5 - 1.1 z_4 + 0.4 z_3 + 0.3 z_1 \end{pmatrix} = f(x, z), \qquad z(0) = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix}$$

Euler: $x_{i+1} = x_i + h \qquad z_{i+1} = z_i + h \cdot f(x_i, z_i)$

$$z_1 = z_0 + 0.1 \begin{pmatrix} z_2 \\ z_3 \\ z_4 \\ \sin(x_0) + 5 - 1.1 z_4 + 0.4 z_3 + 0.3 z_1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix} + 0.1 \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 + 5 \cancel{- 1.1 \cdot 0} \cancel{+ 0.4 \cdot 0} \cancel{+ 0.3 \cdot 0} \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0.2 \\ 0 \\ 0 \\ 0.5 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 0.2 \\ 2 \\ 0 \\ 0.5 \end{pmatrix}}} = z_1$$

Runge-Kutta

$$k_1 = f(x_0, z_0) = f\left(0, \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ 0 \\ 0 \\ \cancel{0} + 5 \cancel{- 1.1 \cdot 0} \cancel{+ 0.4 \cdot 0} \cancel{+ 0.3 \cdot 0} \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 5 \end{pmatrix}$$

$$k_2 = f\left(x_0 + \frac{h}{2}, z_0 + \frac{h}{2} \cdot k_1\right) = f\left(0.05, \begin{pmatrix} 0.1 \\ 2 \\ 0 \\ 0.25 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ 0 \\ 0.25 \\ 4.8050 \end{pmatrix}$$

$$k_3 = f\left(x_0 + \frac{h}{2}, z_0 + \frac{h}{2} \cdot k_2\right) = \begin{pmatrix} 2 \\ 0.25 \\ 4.75 \\ 4.77 \end{pmatrix}$$

$$k_4 = f(x_0 + h, z_0 + h \cdot k_3) = \begin{pmatrix} 2.025 \\ 0.475 \\ 0.477 \\ 5.122 \end{pmatrix}$$

$$z_1 = z_0 + h \cdot \frac{1}{6} (k_1 + k_2 + k_3 + k_4) = \begin{pmatrix} 0.2 \\ 2.02 \\ 0.17 \\ 0.48 \end{pmatrix}$$

b) geg: $x^2 \cdot y'' + x \cdot y' + y(x^2 - n^2) = 0$ $y(1) = y'(1) = 2$

$x_0 = 1$ $h = 0.1$ $n^2 = 1$

1) $y'' = -\frac{y'}{x} - y + \frac{n^2 y}{x^2}$

2) $z_1(x) = y(x)$
$z_2(x) = y'(x)$

3) $z_1'(x) = y' = z_2$
$z_2'(x) = y'' = -\frac{y'}{x} - y + \frac{n^2 y}{x^2}$
$= \frac{z_2}{x} - z_1 + \frac{n^2 z_1}{x^2}$

4) $z' = \begin{pmatrix} z_1' \\ z_2' \end{pmatrix} = \begin{pmatrix} z_2 \\ -\frac{z_2}{x} - z_1 + \frac{n^2 \cdot z_1}{x^2} \end{pmatrix} = f(x, z)$, $z(1) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

Euler: $x_{i+1} = x_i + h$ $z_{i+1} = z_i + h \cdot f(x_i, z_i)$

$z_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix} + 0.1 \cdot f\left(x_i, \begin{pmatrix} 2 \\ 2 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ 2 \end{pmatrix} + 0.1 \begin{pmatrix} 2 \\ -\frac{2}{1} - 2 + \frac{1 \cdot 2}{1} \end{pmatrix} = \begin{pmatrix} 2.2 \\ 1.8 \end{pmatrix}$

Runge-Kutta

$k_1 = f(x_0, z_0) = f\left(1, \begin{pmatrix} 2 \\ 2 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ -\frac{2}{1} - 2 + \frac{1 \cdot 2}{1} \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$k_2 = f\left(x_0 + \frac{h}{2}, z_0 + \frac{h}{2} \cdot k_1\right) = \begin{pmatrix} -2.1 \\ 2.2 \end{pmatrix}$

$k_3 = f\left(x_0 + \frac{h}{2}, z_0 + \frac{h}{2} \cdot k_2\right) = \begin{pmatrix} -1.85 \\ 1.976 \end{pmatrix}$

$k_4 = f(x_0 + h, z_0 + h \cdot k_3) = \begin{pmatrix} -1.802 \\ 1.9 \end{pmatrix}$

$z_1 = z_0 + h \cdot \frac{1}{6}(k_1 + k_2 + k_3 + k_4) = \begin{pmatrix} 1.996 \\ 2.009 \end{pmatrix}$