Beg Michael, Brunner Pascal – Gruppe 2

Das Integral

gegeben: $$I(a) = 2\int_1^a x \cdot \ln(x^2)\,dx$$

liegt in Abhängigkeit der oberen Intervallgrenze a als Wertetabelle vor

mit Stützpunkte

	0	1	2	3
a	$e-\frac{1}{2}$	$e-\frac{1}{4}$	$e+\frac{1}{4}$	$e+\frac{1}{2}$
$I(a)$	3.9203	5.9169	11.3611	14.8550

$\Rightarrow n = 3$

a) Ges: $I(a)$ für $a = e$ mittels Lagrange-Interpolation interpolieren

$$P_n(x) = \sum_{i=0}^{n} l_i(x) \cdot y_i \qquad l_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{x - x_j}{x_i - x_j} \quad (i = 0,\dots,n)$$

$(j \neq 0)$

$$l_0(e) = \frac{[e-(e-\frac{1}{4})]}{[(e-\frac{1}{2})-(e-\frac{1}{4})]} * \frac{[e-(e+\frac{1}{4})]}{[(e-\frac{1}{2})-(e+\frac{1}{4})]} * \frac{[e-(e+\frac{1}{2})]}{[(e-\frac{1}{2})-(e+\frac{1}{2})]} = \frac{\frac{1}{4}\cdot(-\frac{1}{4})(-\frac{1}{2})}{(-\frac{1}{4})\cdot(-\frac{3}{4})\cdot(-1)} = \frac{1/32}{-3/16} = -\frac{1}{6}$$

$$l_1(e) = \frac{[e-(e-\frac{1}{2})]}{[(e-\frac{1}{4})-(e-\frac{1}{2})]} * \frac{[e-(e+\frac{1}{4})]}{[(e-\frac{1}{4})-(e+\frac{1}{4})]} * \frac{[e-(e+\frac{1}{2})]}{[(e-\frac{1}{4})-(e+\frac{1}{2})]} = \frac{\frac{1}{2}\cdot(-\frac{1}{4})(-\frac{1}{2})}{\frac{1}{4}\cdot(-\frac{1}{2})\cdot(-\frac{3}{4})} = \frac{1/16}{3/32} = \frac{2}{3}$$

$$l_2(e) = \frac{[e-(e-\frac{1}{2})]}{[(e+\frac{1}{4})-(e-\frac{1}{2})]} * \frac{[e-(e-\frac{1}{4})]}{[(e+\frac{1}{4})-(e-\frac{1}{4})]} * \frac{[e-(e+\frac{1}{2})]}{[(e+\frac{1}{4})-(e+\frac{1}{2})]} = \frac{\frac{1}{2}\cdot\frac{1}{4}(-\frac{1}{2})}{\frac{3}{4}\cdot\frac{1}{2}\cdot(-\frac{1}{4})} = \frac{-1/16}{-3/32} = \frac{2}{3}$$

$$l_3(e) = \frac{[e-(e-\frac{1}{2})]}{[(e+\frac{1}{2})-(e-\frac{1}{2})]} * \frac{[e-(e-\frac{1}{4})]}{[(e+\frac{1}{2})-(e-\frac{1}{4})]} * \frac{[e-(e+\frac{1}{4})]}{[(e+\frac{1}{2})-(e+\frac{1}{4})]} = \frac{\frac{1}{2}\cdot\frac{1}{4}(-\frac{1}{4})}{1\cdot\frac{3}{4}\cdot\frac{1}{4}} = \frac{-1/32}{3/16} = -\frac{1}{6}$$

$$P_3(e) = l_0(e) * 3.9203 + l_1(e)\cdot 5.9169 + l_2(e)\cdot 11.3611 + l_3(e)\cdot 14.855$$
$$\underline{\underline{= 8.3895}}$$

b) $I(e) = 2\cdot\int_1^e x\cdot\ln(x^2)\,dx \rightarrow$ MATLAB: `2.*int(x.*log(x^2),1,exp(1))` $= 8.3891$

$I(e)$ = exakter Wert für $a = e$

Abs. Err: $|I(e) - P_3(e)| = 4.4139\cdot 10^{-4}$

Rel. Err: $\frac{|I(e) - P_3(e)|}{|I(e)|} = 5.291\cdot 10^{-5}$

c) Ausgerechnet in MATLAB aus Serie 3 $= I_{roms}$

Abs. Err: $|I(e) - I_{roms}| = 3.926 \cdot 10^{-4}$

Rel. Err: $\dfrac{|I(e) - I_{roms}|}{|I(e)|} = 4.673 \cdot 10^{-5}$

$\Rightarrow$ Romberg-Extrapolation ist besser

Aufg 2

gegeben:

Höhe über Meer [m]	0	1250	2500	3750	5000	10000
Atmosphärendruck [hPa]	1013	***NaN***	747	***NaN***	540	226

gesucht: Schätzung für NaN-Werte mittels Aitken-Neville

x	x_{i-j}	x_{i-j+1}	$\dots$	x_i
y	y_{i-j}	y_{i-j+1}	$\dots$	y_i

$$p_{i0} = y_i$$

$$p_{ij} = \frac{(x_i - x)p_{i-1,j-1} + (x - x_{i-j})p_{i,j-1}}{x_i - x_{i-j}}$$

Für 1250 zu schätzen in $x = 1250$ einsetzen

x	y			
0	$P_{00} = 1013$			
2500	$P_{10} = 747$	$P_{11} = \dfrac{(2500-1250)\cdot 1013 + (1250-0)\cdot 747}{2500-0} = 880$		
5000	$P_{20} = 540$	$P_{21} = 850.5$	$P_{22} = 872.625$	
10'000	$P_{30} = 226$	$P_{31} = 775.5$	$P_{32} = 863$	$\underline{\underline{P_{33} = 871.42}}$

$x = 3750$

x	y			
0	$P_{00} = 1013$			
2500	$P_{10} = 747$	$P_{11} = 614$		
5000	$P_{20} = 540$	$P_{21} = 643.5$	$P_{22} = 636.1$	
10'000	$P_{30} = 226$	$P_{31} = 618.5$	$P_{32} = 639.3$	$\underline{\underline{P_{33} = 637.3}}$