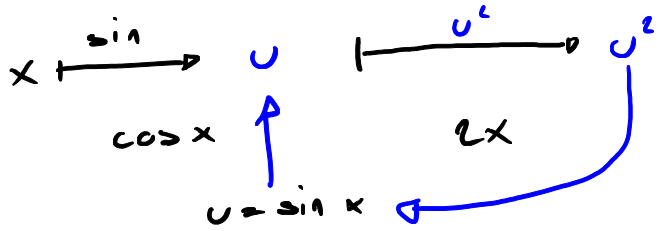
Kapitel 3.6

$y = \sin^2(x) = \underset{u}{\sin x} \cdot \underset{v}{\sin x}$

$y' = u'v + uv' = \cos x \cdot \sin x + \sin x \cdot \cos x = 2 \sin x \cos x$

$y = x^2$ $y = u^2$ $\quad y = \sin x$ $\quad u = \sin x$, $u' = \cos x$

$y' = 2x$ $\quad y' = \cos x$



Kettenregel

$f(g(x)) = (f \circ g)\, x$

$x \xrightarrow{g} g(x) \xrightarrow{f} f(g(x))$

g: innere Funktion; f: äußere Funktion

$f'(g(x)) = f'(x) \cdot g'(x)$

$\Rightarrow$ äußere Ableitung · innere Ableitung

Bsp:

$y = \sin(x^2 - 4)$

$x \xrightarrow{x^2-4} u \xrightarrow{\sin} \sin u$

innere Funktion: $y = x^2 - 4$, $y' = 2x$

äußere Funktion: $y = \sin u$, $y' = \cos u$

$y' = 2x \cdot \cos \underset{x^2-4}{u} = 2x \cdot \cos(x^2 - 4)$

Bsp:

1)

$y = u^2$ $\quad u = 3x^2 + 1$

$y' = 2u$ $\quad u' = 6x$

$y' = 2u \cdot 6x = 2(3x^2+1)6x = 12x(3x^2+1)$

4)

$y = \sin(\underbrace{-2x}_{u})$

$y = \sin x$ $\quad y' = \cos x$

$y = -2x$ $\quad y' = -2$

$y' = -2 \cdot \cos u = -2 \cdot \cos(-2x)$

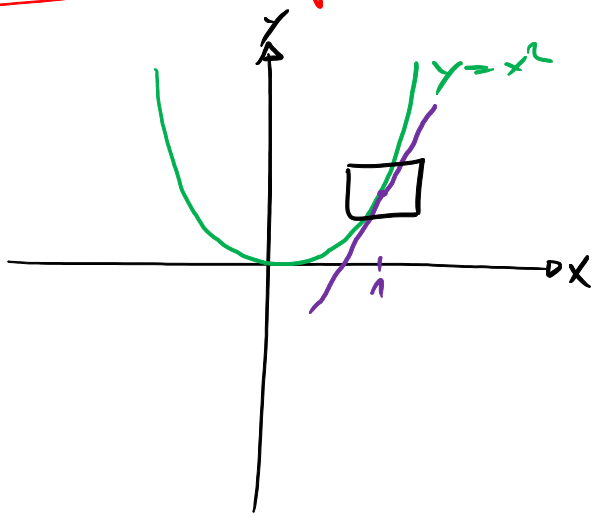
3) $\frac{\sin^2 x}{(x^3+1)^2} = \frac{u}{v}$ äußere Funktion $u' = 2\sin x \cos x$, $v' = 2(x^3+1)\cdot 3x^2$

$\left[\frac{u}{v}\right]' = \frac{u'v - uv'}{v^2}$

$= \frac{2\sin x \cos x (x^3+1)^2 - \sin^2 x \cdot 2(x^3+1)\cdot 3x^2}{(x^3+1)^4}$

$\frac{-2\sin x [\cos x (x^3+1) - \sin x \cdot 3x^2]}{(x^3+1)^3}$

3.9 Linearisierung



$y' = 2x$

$x = 1$

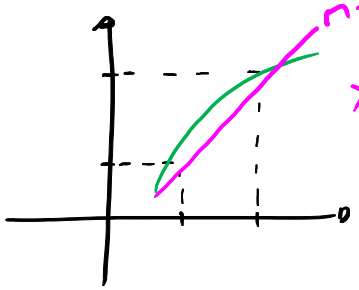
$y' = 2$

Stützpunkt

$x = a$

$y = f(a)$

$L(x) = f(a) + f'(a)\ (x-a)$



$m = \frac{y - y_1}{x - x_1}$

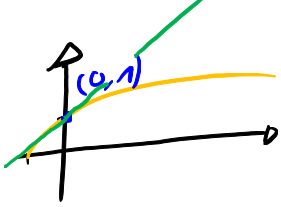
$y - y_1 = m(x - x_1) \quad | + y_1$

$y = y_1 + m(x - x_1)$

=

Bsp:

$y = \sqrt{x+1}$ $\quad y' = \frac{1}{2\sqrt{x+1}}$

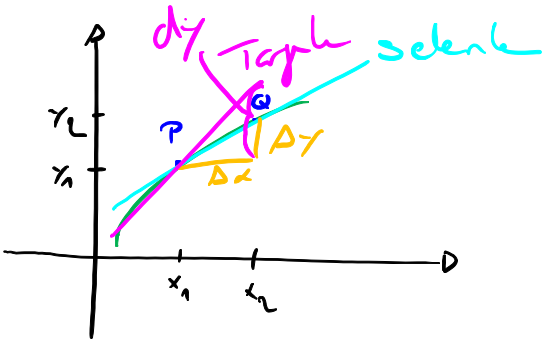


$a = 0$

$f(a) = f(0) = \sqrt{1} = 1$

$L(x) = 1 + \frac{1}{2\sqrt{0+1}} \cdot (x-0) = 1 + \frac{x}{2}$

Differentiale



$dy \simeq \Delta y$

$f \to f'(x) = \frac{dy}{dx}$

$m \simeq f'(x)$

$\frac{\Delta y}{\Delta x} \simeq f'(x) \Rightarrow \Delta y = dy = f'(x)\,dx$

Bsp:

$\sqrt{102}$

$\sqrt{100}$

$y = \sqrt{x}$

$x_1 = 100 \qquad x_2 = 102 \qquad \Delta x = dx = 2$

$y' = \frac{1}{2\sqrt{x}}$

$y' = \frac{1}{2\sqrt{100}} = \frac{1}{20}$

$dy = \frac{1}{20} \cdot dx = \frac{1}{20} \cdot 2 = \frac{1}{10}$

$\sqrt{102} = \sqrt{100} + \frac{1}{10} = 10.1$