

## Aufgaben 4.4

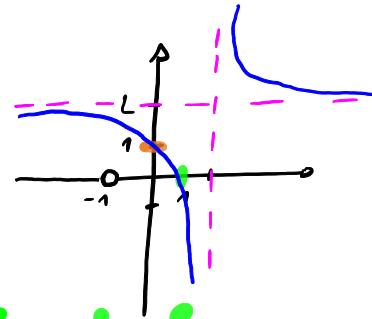
26)  $y = \frac{2x^2 + x - 1}{x^2 - 1}$

1. Nullstellen
2. Definitionsbereich

$y(0) = 1$

$y(x) = 0 \quad 2x^2 + x - 1 = 0$

$x_1 = \frac{1}{2} \quad x_2 = -1 \quad \text{L} \rightarrow \text{MODE:5}$



$D = \mathbb{R} \setminus \{-1\}$  (P)

$$\begin{aligned} \frac{(2x^2 + x - 1) : (x+1)}{-(2x^2 + 2x)} &= 2x - 1 \\ \frac{x-1}{-(x-1)} &= 0 \end{aligned} \quad y = \frac{2x^2 + x - 1}{x^2 - 1} = \frac{(x+1)(2x-1)}{(x+1)(x-1)}$$

horizontale Asymptote

$$\begin{aligned} \frac{(2x^2 + x - 1) : (x^2 - 1)}{-(2x^2 - 2)} &= 2 + \frac{x+1}{x^2 - 1} \\ x+1 & \end{aligned}$$

Zähler = mögliche Schnittpunkte mit x

Nenner = möglicher Pol

28)

$$y = -\frac{x^2 - 2}{x^2 - 1} \quad D = \mathbb{R} \setminus \{-1, 1\}$$

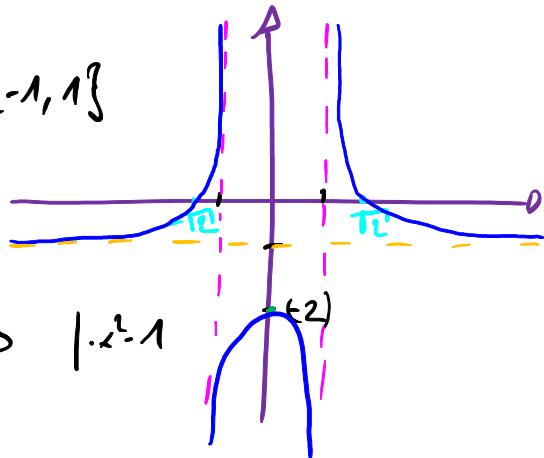
$$y(0) = -2$$

$$y(x) = 0 \quad -\frac{x^2 - 2}{x^2 - 1} = 0 \quad | \cdot x^2 - 1$$

$$-(x^2 - 2) = 0$$

$$x^2 = 2$$

$$x_{1,2} = \pm\sqrt{2}$$



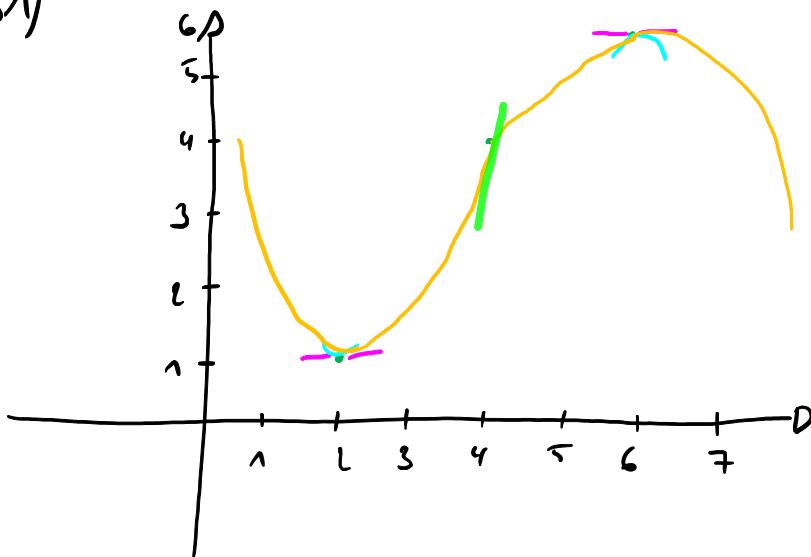
$$\lim_{x \rightarrow \pm\infty} y = -1 \quad \text{horizontal Asymptote}$$

$$y' = -\frac{2x(x^2 - 1) - (x^2 - 2)2x}{(x^2 - 1)^2} = \frac{2x^3 - 2x - 2x^3 + 4x}{(x^2 - 1)^2}$$

$$y' = -\frac{2x}{(x^2 - 1)^2}$$

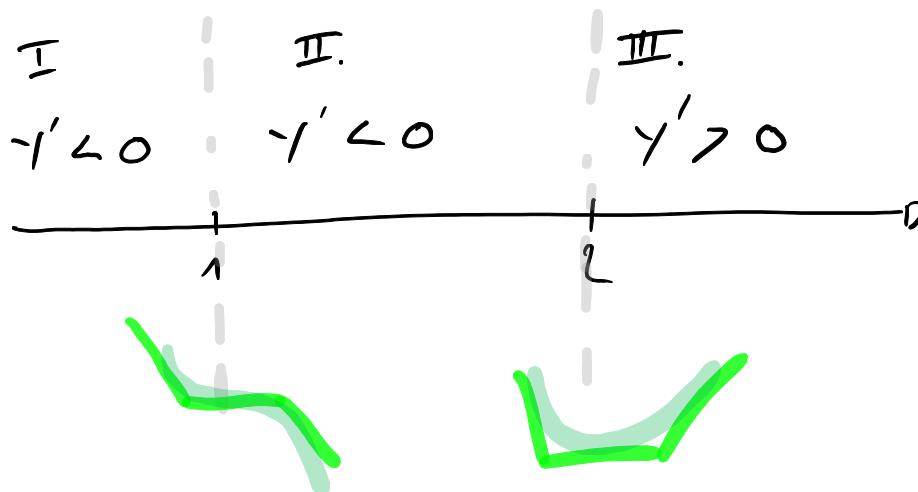
$$\boxed{y'(x) = 0} \quad \Rightarrow x = 0$$

31)



33)

$$\boxed{y' = 0} \quad x_1 = 1 \quad x_2 = 2$$



35)

$$y'' = (x+1)(x-2)$$

$$\boxed{y'' = 0}$$

$$x = -1 \quad x = 2$$

I.

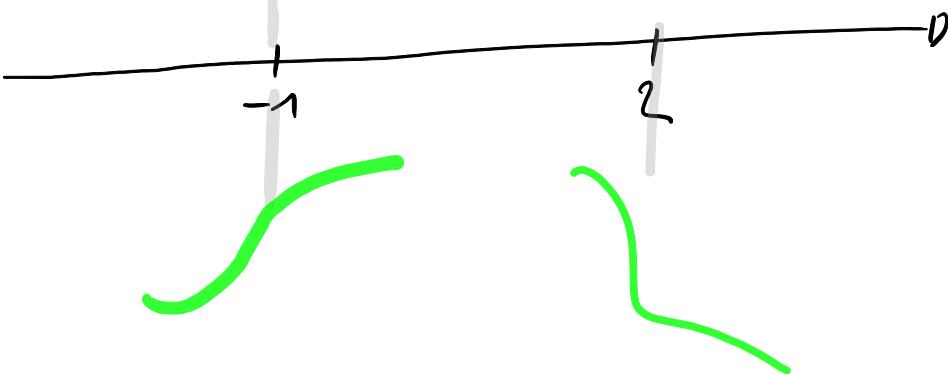
$$y'' > 0$$

II.

$$y'' < 0$$

III.

$$y'' > 0$$



## Aufgaben 4.5

1)  $F(a, b) = a \cdot b = 100$    
 $b = \frac{100}{a}$

$$U(a, b) = 2a + 2b$$

$$U(a) = 2a + \frac{200}{a}$$

$$U'(a) = 0$$

$$U'(a) = 2 - \frac{200}{a^2} = 0$$

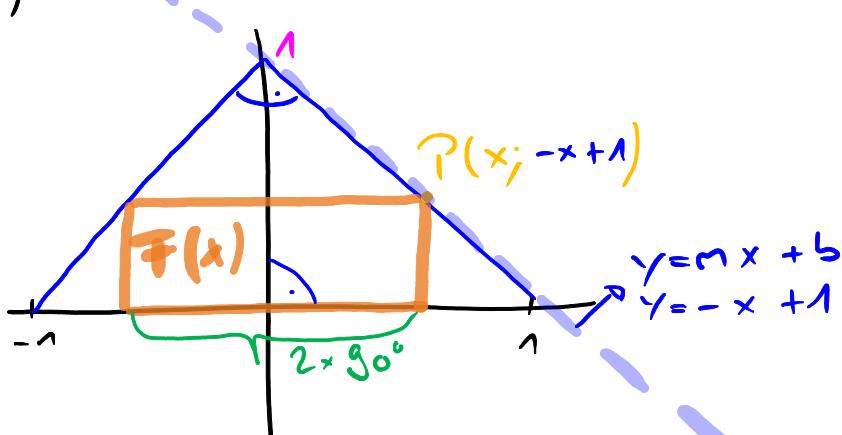
$$2a^2 - 200 = 0$$

$$a^2 = 100$$

$$a = \pm 10 \quad a > 0$$

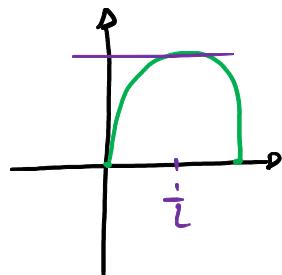
$$b = 10$$

2)



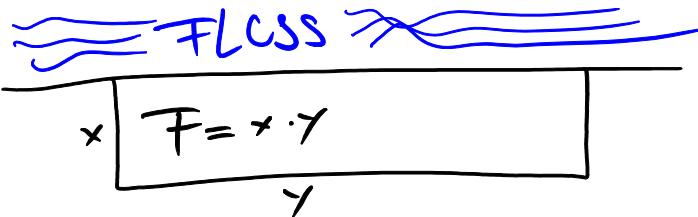
$$\begin{aligned}f(x) &= 2x \cdot (-x + 1) = -2x^2 + 2x \\&= 2x(-x + 1)\end{aligned}$$

$$\begin{aligned}f'(x) &= 0 = -4x + 2 \\4x &= 2 \\x &= \frac{1}{2}\end{aligned}$$



$$f\left(\frac{1}{2}\right) = 1 \cdot \frac{1}{2} = \underline{\underline{\frac{1}{2}}}$$

3)



$$U(x, y) = 2x + y = 800$$

$$y = 800 - 2x$$

$$F(x) = x(800 - 2x) = 800x - 2x^2$$

$$x_1 = 0$$

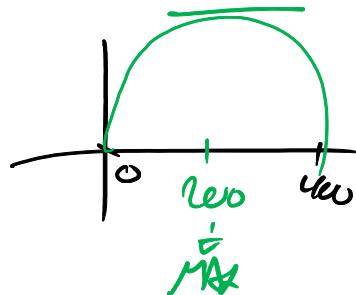
$$x_2 = 400$$

$$F'(x) = 800 - 4x = 0$$

$$x = 200$$

$$y = 400$$

$$F = 200 \cdot 400 = 80000$$



14)

$$f(x) = x^2 + \frac{a}{x}$$

d)  
 $f'(x) = 0 \quad \text{und} \quad f''(x) > 0$

$\downarrow$

$f'(2) = 0 \quad \& \quad f''(2) > 0$

$$f'(x) = 2x - \frac{a}{x^2}$$

$$2 \cdot 2 - \frac{a}{2^2} = 0$$

$$4 - \frac{a}{4} = 0$$

$$a = 16$$

b)  $f'(x) = 2 + \frac{2a}{x^3} = 0$

$$f''(1) = 0 \quad 2 + \frac{2 \cdot a}{1^3} = 0$$

$$2 + 2a = 0$$

$$a = -1$$