Vorlesung vom 20.02.2018

Kap. 4.7 (unbestimmtes Integral)

$$\begin{tikzcd} x^3 \arrow[r, bend left, "\text{ableiten}"] & 3x^2 \arrow[l, bend left, "\text{aufleiten}"] \end{tikzcd}$$

ableiten → differenzieren

aufleiten → integrieren

$$\left[x^4\right]' = 4x^3$$ → ableiten

$$\left[\frac{4x^4}{4}\right]' = 4x^4$$ → aufleiten

$$\begin{tikzcd} ? \arrow[r, bend left] \arrow[d] & x^5 \arrow[l, bend left, "\text{aufleiten}"] \\ \frac{x^6}{6} & \end{tikzcd}$$

$$\left[\frac{x^6}{6}\right]' = \frac{1}{6}\left[x^6\right]' = \frac{1}{6}\cdot 6x^5 = x^5$$

eindeutig

$$f_1(x) = x^3 + 1 \qquad f_1(x)' = 3x^2$$

$$f_2(x) = x^3 - 2 \qquad f_2(x)' = 3x^2$$

$$f_3(x) = x^3 + 17 \qquad f_3(x)' = 3x^2$$

$$f(x) = x^3 + c \qquad f'(x) = 3x^2$$

⇒ Problematik: Beim Ableiten sind die Konstanten eindeutig & ist deshalb klar was damit „passiert"
Beim Aufleiten kann dies jedoch nicht eindeutig bestimmt werden

$$\boxed{F'(x) = f(x)}$$ → Stammfunktion

$$F(x) = \int f(x)\, dx$$

dx → Integrationsvariable

$\int$ → Integralzeichen „Summe"

Bsp.:

$$f(x) = 3x^2y$$
$$f'(x) = 3\cdot 2x\cdot y = 6xy$$
ableiten

$$F(x) = \int 2xy\, dx = 2y\int x\, dx = 2y\,\frac{x^2}{2} = x^2y + c$$
aufleiten

$$\left[x^2y + c\right]' = 2xy + 0$$

Vorlesung vom 27.02.2018

Ableiten:

$$[x^{17}]' = 17x^{16}$$

① Exponent -1
② Exponent $\to$ Faktor

Aufleiten:

$$\int x^3 dx = \frac{x^4}{4} + c$$

① Exponent $+1$
② Exponent $\to$ Divisor

$$[4x^{11}]' = 4 \cdot [x^{11}]' = 4 \cdot 11 x^{10} = 44x^{10}$$

Übungsblatt

2e) $$\int (r+1) x^r dx = (r+1) \int x^r dx$$
$$= \cancel{(r+1)} \cdot \frac{x^{r+1}}{\cancel{r+1}}$$
$$= x^{r+1} + c$$

2f) $$\int x^{-0.5r} dx = \frac{x^{-0.5r+1}}{-0.5r+1} + c$$

} Kontrolle mit ableiten

$$\left[\frac{x^{-0.5r+1}}{-0.5+1}\right]' = -\frac{1}{0.5r+1}\left[x^{-0.5r+1}\right]' = -\frac{1}{0.5r+1} \cdot (-0.5r+1)\, x^{-0.5r}$$
$$= x^{-0.5r} \checkmark$$

$$[3x^2 - 2x]' = [3x^2]' - [2x]'$$

3a)
$$\int (2x+1) dx = \int 2x\, dx + \int 1\, dx$$
$$= 2 \int x\, dx + \int dx$$
$$= \frac{2x^2}{x} + c_1 + x + c_2$$
$$= x^2 + x + c$$

3d)

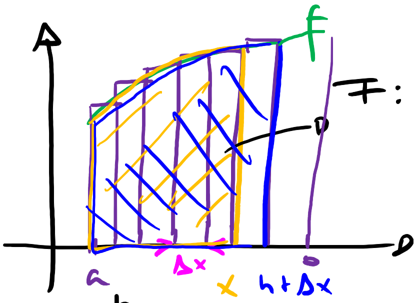
$$\int\left(\frac{1}{2}x^3 - \frac{1}{4}x\right)dx = \frac{1}{2}\frac{x^4}{4} - \frac{1}{4}\frac{x^2}{2} + c = \frac{x^4}{8} - \frac{x^2}{8} + c$$

3f)

$$\int 2\sqrt{t}\cdot x\,dt = 2x\int\sqrt{t}\,dt = 2x\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{4x}{3}\cdot t^{\frac{3}{2}} + c$$

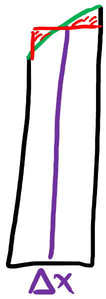
Integration

Kapitel 5.3 Riemannsche Summe = bestimmtes Integral



$$F:\ \lim_{n\to\infty}\sum f(c_k)\,\Delta x = \int_a^b f(x)\,dx$$

bestimmtes Integral



$$F(x) = \int_a^h f(x)\,dh$$

$$F(x+\Delta x) = \int_a^{x+\Delta x} f(x)\,dx$$

$$F(x+\Delta x) - F(h) = f(x)\,\Delta x \quad \Big|:\Delta x$$

$$\frac{F(x+\Delta x) - F(h)}{\Delta x} = f(x)$$

$$\Rightarrow \lim_{\Delta x\to 0}\frac{F(x+\Delta x) - F(h)}{\Delta x} = F'(x) = f(x)$$

$F(x)$ ist Stammfunktion von $f(x)$

$$F(x) = \int_a^h f(x)\,dx + c$$

$$F(b) = \int_a^b f(x)\,dx + c$$

$$F(a) = \cancel{\int_a^a f(x)\,dx} + c$$

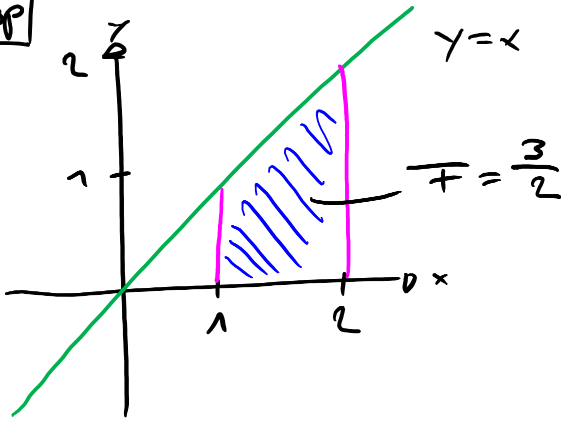
gäbe 0

$$F(b) = \int_a^b f(h)\,dh + F(a)$$

$$\int_a^b f(h) = F(b) - F(a)$$

Bsp

1



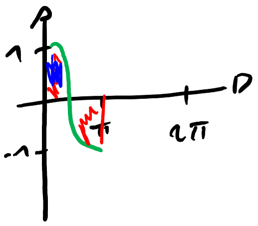
Stammfunktion

$$F: \int_1^2 x\,dx = \frac{x^2}{2}\Big|_1^2 = \frac{2^2}{2} - \frac{1^2}{2}$$

$$= \frac{4}{2} - \frac{1}{2} = \frac{3}{2} = 1{,}5$$

2 5.15c)

$$\int_0^{\pi} \cos x \; dx = \underline{\underline{0}}$$



$$= \sin x \Big|_0^{\pi} = 0 - 0 = \underline{\underline{0}}$$

$$\int_0^{\frac{\pi}{2}} \cos x = \sin x \Big|_0^{\frac{\pi}{2}} = 1 - 0 = \underline{1}$$

5.17)

$$F = \left| \int_{-1}^{0} (x^3 - x^2 - 2x)\,dx \right| + \left| \int_{2}^{0} (x^3 - x^2 - 2x)\,dx \right|$$

(1) under the first integrand, (2) under the second.

$$1 = \left(\frac{x^4}{4} - \frac{x^3}{3} - x^2\right)\Big|_{-1}^{0} = 0 - 0 - 0 - \left(\frac{1}{4} + \frac{1}{3} - 1\right) = \frac{5}{12}$$

$$2 = \left(\frac{x^4}{4} - \frac{x^3}{3} - x^2\right)\Big|_0^2 = \frac{16}{4} - \frac{8}{3} - 4 - 0$$

$$= \left| -\frac{8}{3} \right|$$

$$\frac{5}{12} + \frac{8}{3} = \underline{\underline{\frac{37}{12}}}$$