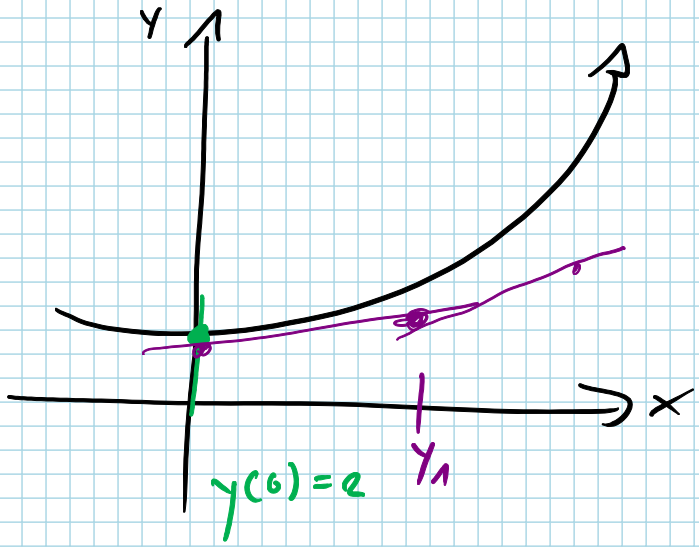


Euler

$$y' = x^2 \cdot y \approx \frac{y_1 - y_0}{\underbrace{x_1 - x_0}_{h}}$$

$$\Rightarrow y_1 \approx h \cdot f(x_0, y_0) + y_0$$

Aufg 2 a) geg: $y' = \frac{x^2}{y} = f(x,y)$

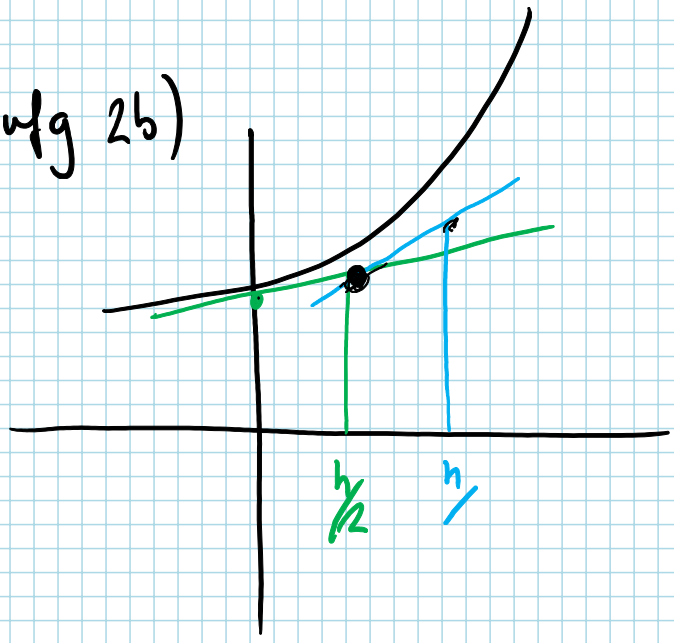
$x_1 = 0.7$ $\quad y_1 = y_0 + h f(x_0, y_0) = 2 + 0.7 \cdot (0,2) = 2$

$x_2 = 1.4$ $\quad y_2 = 2 + h f(0,7, 2) = 2.1715$

$x_3 = 2.1$ $\quad y_3 = 2.1715 + h \cdot f(1.4, 2.1715) = \underline{2.8033}$

$$err_{abs} = |y(x_3) - y_3| = \underline{0.386}$$

Aufg 2b)



Mittelpt-Verfahren

$$y_{i+1} = y_i + h \cdot f(x_{i+\frac{1}{2}}, y_{i+\frac{1}{2}})$$

$$f = \frac{x^2}{y} = y'$$

$x_0 = 0$: $y_0 = 2$

$x_{1/2} = 0.35$: $y_{1/2} = 2 + 0.35 f(x_0, y_0) = 2$

$x_1 = 0.7$ $y_1 = 2 + 0.7 \cdot f(0.35, 2) = 2.0429$

$x_{3/2} = 1.05$ $y_{3/2} = 2.0429 + 0.35 f(0.7, 2.0429) = 2.1268$

$x_2 = 1.4$ $y_2 = 2.0429 + 0.7 \cdot f(1.05, 2.1268) = 2.4058$

$x_{5/4} = 1.75$ $y_{5/4} = 2.4058 + 0.35 f(1.4, 2.4058) = 2.6309$

$x_3 = 2.1$ $y_3 = 2.4058 + 0.7 \cdot f(1.75, 2.6309) = 3.2025$

$$err_{abs} = |y(x_i) - y_3| = 0.0128$$

Aufg 2c) Modif. Euler

$x_0 = 0$ $y_0 = 2$

$\tilde{x}_1 = 0.7$ $\tilde{y}_1 = 2$

$\tilde{x}_2 = 1.4$ $\tilde{y}_2 = \tilde{y}_1 + 0.7 \cdot f(0.7, 2) = 2.1715$

$k_1 = 0$

$k_2 = \frac{2.1715 - 2}{0.7} = 0.245 \Rightarrow y_1 = 2 + 0.7 \cdot \frac{0.245}{2}$

$x_1 = 0.7$ $y_1 = 2.0858$

$\tilde{x}_2 = 1.4$ $\tilde{y}_2 = 2.0858 + 0.7 \cdot f(0.7, 2.0585) = 2.2524$

$\tilde{x}_3 = 2.1$ $\tilde{y}_3 = 2.2524 + 0.7 f(1.4, 2.2524) = 2.8615$

$k_1 = \frac{2.2524 - 2.0858}{0.7} = 0.238$ $k_2 = \frac{2.8615 - 2.2524}{0.7} = 0.870$

$\Rightarrow y_2 = 2.0858 + 0.7 \cdot \frac{0.238 + 0.87}{2} = 2.4736$

$\tilde{x}_3 = 2.1 \quad \tilde{y}_3 = 2.4736 + 0.7 f(1.4, 2.4736) = 3.0283$

$\tilde{x}_4 = 2.8 \quad \tilde{y}_4 = 3.0283 + 0.7 f(2.1, 3.0283) = 4.0477$

$k_1 = \frac{3.0283 - 2.4736}{0.7} = 0.792 \qquad k_2 = \frac{4.0477 - 3.0283}{0.7} = 1.456$

$\Rightarrow y_3 = 2.4736 + 0.7 \frac{0.792 + 1.456}{2} = 3.2604$

$err_{abs} : \left| y(x_3) - y_3 \right| = 0.071$