Vorlesung vom 20.02.2018

Kap. 4.7 (unbestimmtes Integral)

$$x^3 \underset{\text{aufleiten} \to \text{integrieren}}{\overset{\text{ableiten} \to \text{differenzieren}}{\rightleftarrows}} 3x^2$$

$$[x^4]' = 4x^3 \quad \to \text{ableiten}$$

$$\left[\frac{4x^4}{4}\right]' = 4x^4 \quad \to \text{aufleiten}$$

$$? \underset{\text{aufleiten}}{\rightleftarrows} x^5$$

$$? \to \frac{x^6}{6}$$

$$\left[\frac{x^6}{6}\right]' = \frac{1}{6}[x^6]' = \frac{1}{6} \cdot 6x^5 = x^5$$

eindeutig

$$\left.\begin{aligned} f_1(x) &= x^3 + 1 & f_1(x)' &= 3x^2 \\ f_2(x) &= x^3 - 2 & f_2(x)' &= 3x^2 \\ f_3(x) &= x^3 + 17 & f_3(x)' &= 3x^2 \end{aligned}\right\} \quad f(x) = x^3 + c \qquad f'(x) = 3x^2$$

$\Rightarrow$ Problemstelle: Beim Ableiten sind die Konstanten eindeutig & ist deshalb klar was damit „passiert"
Beim Aufleiten kann dies jedoch nicht eindeutig bestimmt werden

$$\boxed{F'(x) = f(x)} \longrightarrow \text{Stammfunktion}$$

$$F(x) = \int f(x)\, dx$$

dx → Integrationsvariable

$\int$ → Integralzeichen „Summe"

Bsp.:

$$\left.\begin{aligned} f(x) &= 3x^2y \\ f'(x) &= 3 \cdot 2x \cdot y = 6xy \end{aligned}\right\} \text{ableiten}$$

$$\left.F(x) = \int 2xy\, dx = 2y \int x\, dx = 2y\, \frac{x^2}{2} = x^2y + c\right\} \text{aufleiten}$$

$$[x^2y + c]' = 2xy + 0$$

Vorlesung vom 27.02.2018

Ableiten:

$[x^{17}]' = 17x^{16}$

① Exponent − 1
② Exponent → Faktor

Aufleiten:

$\int x^3 dx = \frac{x^4}{4} + c$

① Exponent + 1
② Exponent → Divisor

$[4x^{11}]' = 4 \cdot [x^{11}]' = 4 \cdot 11x^{10} = 44x^{10}$

Übungsblatt

2e) $\int (r+1)\, x^r dx = (r+1) \int x^r dx$

$= \cancel{(r+1)} \cdot \frac{x^{r+1}}{\cancel{r+1}}$

$= x^{r+1} + c$

2f) $\int x^{-0.5r} dx = \frac{x^{-0.5r+1}}{-0.5r+1} + c$ } Kontrolle mit ableiten

$\left[\frac{x^{-0.5r+1}}{-0.5+1}\right]' = -\frac{1}{0.5r+1} \left[x^{-0.5r+1}\right]' = -\frac{1}{0.5r+1} \cdot (-0.5r+1)\, x^{-0.5r}$

$= x^{-0.5r}$ ✓

$[3x^2 - 2x]' = [3x^2]' - [2x]'$

3a)

$\int (2x+1)\, dx = \int 2x\, dx + \int 1\, dx$

$= 2 \int x\, dx + \int dx$

$= \frac{2x^2}{x} + c_1 + x + c_2$

$= x^2 + x + c$

3d)

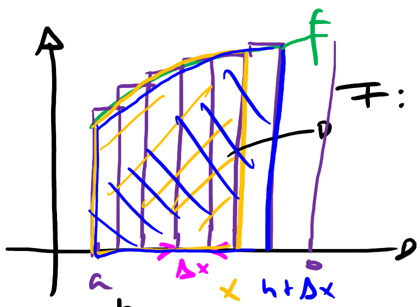
$$\int\left(\frac{1}{2}x^3-\frac{1}{4}x\right)dx=\frac{1}{2}\frac{x^4}{4}-\frac{1}{4}\frac{x^2}{2}+c=\frac{x^4}{8}-\frac{x^2}{8}+c$$

3f)

$$\int 2\sqrt{t}\cdot x\,dt=2x\int\sqrt{t}\,dt=2x\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+c=\frac{4x}{3}\cdot t^{\frac{3}{2}}+c$$

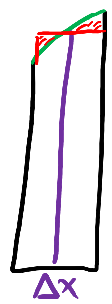
Integration

Kapitel 5.3 Riemannsche Summe = bestimmtes Integral



$$F:\ \lim_{n\to\infty}\sum f(c_k)\,\Delta x=\int_a^b f(x)\,dx$$

bestimmtes Integral



$$F(x)=\int_a^h f(x)\,dh$$

$$F(x+\Delta x)=\int_a^{x+\Delta x} f(x)\,dx$$

$$F(x+\Delta x)-F(h)=f(x)\,\Delta x\quad |:\Delta x$$

$$\frac{F(x+\Delta x)-F(h)}{\Delta x}=f(x)$$

$$\Rightarrow \lim_{\Delta x\to 0}\frac{F(x+\Delta x)-F(h)}{\Delta x}=F'(x)=f(x)$$

$F(x)$ ist Stammfunktion von $f(x)$

$$F(x)=\int_a^h f(x)\,dx+c$$

$$F(b)=\int_a^b f(x)\,dx+c$$

$$F(a)=\cancel{\int_a^a f(x)\,dx}+c$$

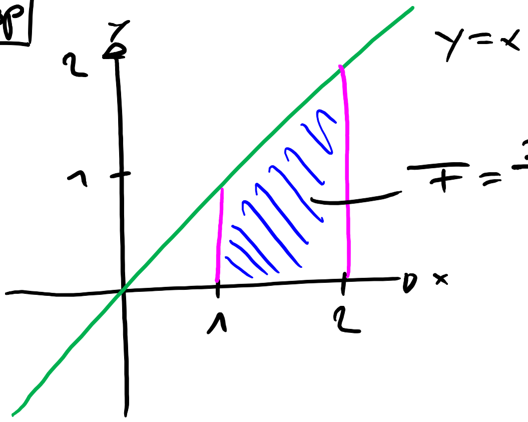
gäbe 0

$$F(b)=\int_a^b f(h)\,dh+F(a)$$

$$\int_a^b f(h)=F(b)-F(a)$$

Bsp

①



$F = \frac{3}{2}$

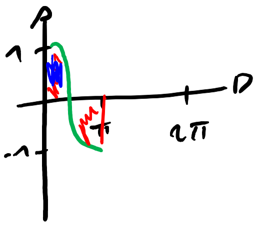
Stammfunktion

$$F: \int_1^2 x\,dx = \frac{x^2}{2}\Big|_1^2 = \frac{2^2}{2} - \frac{1^2}{2}$$

$$= \frac{4}{2} - \frac{1}{2} = \frac{3}{2} = 1{,}5$$

② 5.15 c)

$$\int_0^{\pi} \cos x \, dx = \underline{\underline{0}}$$



$$= \sin x \Big|_0^{\pi} = 0 - 0 = \underline{\underline{0}}$$

$$\int_0^{\frac{\pi}{2}} \cos x = \sin x \Big|_0^{\frac{\pi}{2}} = 1 - 0 = \underline{1}$$

5.17)

$$F = \left|\int_{-1}^{0} (x^3 - x^2 - 2x)\,dx\right| + \left|\int_{0}^{2} (x^3 - x^2 - 2x)\,dx\right|$$

① ②

$$① = \left(\frac{x^4}{4} - \frac{x^3}{3} - x^2\right)\Big|_{-1}^{0} = 0 - 0 - 0 - \left(\frac{1}{4} + \frac{1}{3} - 1\right) = \frac{5}{12}$$

$$② = \left(\frac{x^4}{4} - \frac{x^3}{3} - x^2\right)\Big|_{0}^{2} = \frac{16}{4} - \frac{8}{3} - 4 - 0$$

$$= \left|-\frac{8}{3}\right|$$

$$\frac{5}{12} + \frac{8}{3} = \underline{\underline{\frac{37}{12}}}$$

TR:

$\int_{\square}^{\square} \square \, d\alpha \rightarrow x$ als Platzhalter

Vorlesung vom 6.3.2018

Kapitel 5.5

Substitutionsregel hat zwei Bedingungen:

1) Integrand ist ein Produkt
2) ein Faktor ist ein Teil der innere Ableitung des Andern.

Bsp. 5.18:

$$\int (x^3+x)^5 (3x^2+1)\,dx$$

Substitution

$$u(x) = x^3 + x \qquad u' = \frac{du}{dx} = 3x^2+1$$

$$du = (3x^2+1)\,dx$$

$$= \int u^5\,du = \frac{u^6}{6} + c$$

Substitution „rückgängig"

$$= \frac{(x^3+x)^6}{6}$$

Bsp. 5.20:

$$\int x^2 \sin(x^3)\,dx$$

subst.

$$u = x^3 \qquad \frac{du}{dx} = 3x^2 \quad |\cdot dx$$

$$du = 3x^2\,dx \quad |:3$$

$$\frac{du}{3} = x^2\,dx$$

$$\frac{1}{3}\int \sin u \,\frac{du}{3} = -\frac{1}{3}\cdot \cos u + c$$

subst. rückgängig

$$= -\frac{1}{3}\cdot \cos(x^3) + c$$

Bsp. 5.21:

$$\int x\sqrt{2x+1}\,dx$$

subst.

$$\boxed{\begin{aligned} u &= 2x+1 \\ u-1 &= 2x \\ \frac{u-1}{2} &= x \end{aligned}} \qquad \frac{du}{dx} = 2 \qquad dx = \frac{du}{2}$$

$$\frac{1}{2}\int x\sqrt{u}\,du = \frac{1}{2}\int \left(\frac{u-1}{2}\right)\sqrt{u}\,du = \frac{1}{4}\int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du = \frac{1}{4}\left(\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right) + c$$

$$= \frac{1}{4}\left(\frac{2}{5}(2x+1)^{\frac{5}{2}} - \frac{2}{3}(2x+1)^{\frac{3}{2}}\right)$$

Bsp. 5.22

$$\int \frac{2z\,dz}{\sqrt[3]{z^2+1}} = \int 2z\,(z^2+1)^{-\frac{1}{3}}\,dz$$

subst.

$$u = z^2+1 \qquad \frac{du}{dz} = 2z$$

$$du = 2z\,dz$$

$$\int u^{-\frac{1}{3}}\,du = \frac{u^{\frac{2}{3}}}{\frac{2}{3}} + c = \frac{3}{2}(z^2+1)^{\frac{2}{3}} + c$$

rück. subst.

Kap. 5.6

Bsp. 5.24

$$\int_{-1}^{1} 3x^2\sqrt{x^3+1}\,dx$$

$u = x^3+1$ $\qquad u' = \frac{du}{dx} = 3x^2$

$\qquad du = 3x^2\,dx$

① $u(1) = 2$

$u(-1) = 0$

$$= \int_0^2 \sqrt{u}\,du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_0^2 = \frac{2}{3}\sqrt{8} - 0$$

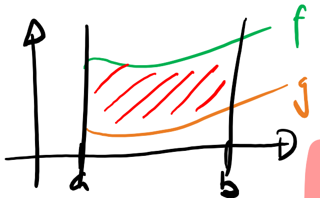
Alt.: $\frac{2}{3}\sqrt{2\cdot 4} = \frac{4}{3}\sqrt{2}$

② $$\int \sqrt{u}\,du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c$$

rück. subst.

$$= \frac{2}{3}(x^3+1)^{\frac{3}{2}}\Big|_{-1}^{1} = \frac{2}{3}\sqrt{8} - 0$$

$$\int_a^b f(x)\,dx =$$



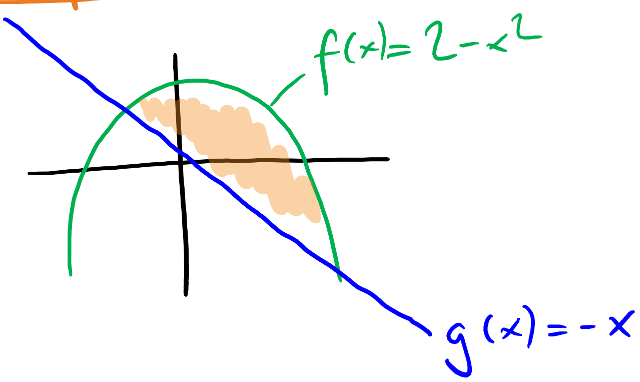
$$\Rightarrow \int_a^b f(x)\,dx - \int_a^b g(x)\,dx = \int_a^b (f(x) - g(x))\,dx$$

$\boxed{\text{!! f(x) muss oberhalb von g(x) sein !!}}$

$\Downarrow$

$f(x) \geq g(x)$

Bsp. 5.26:



$2 - x^2 = -x$

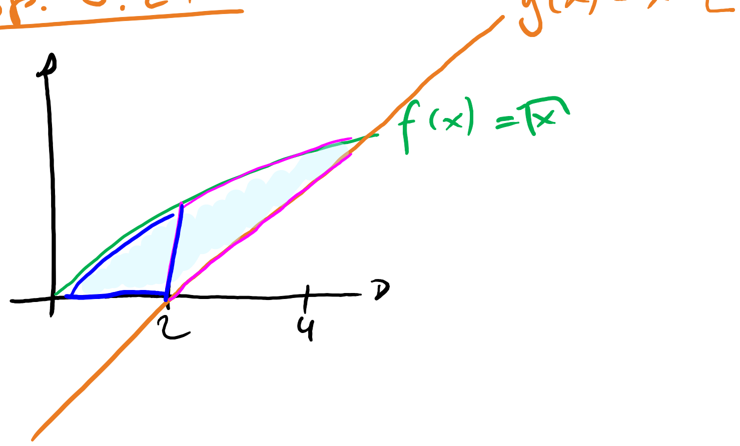
$x_1 = -1$

$x_2 = 2$

$$\int_{-1}^{2} (f(x) - g(x))\,dx = \int_{-1}^{2} (2 - x^2 + x)\,dx = \left(2x - \frac{x^3}{3} + \frac{x^2}{2}\right)\Bigg|_{-1}^{2}$$

$$= 4 - \frac{8}{3} + 2 - \left(-2 + \frac{1}{3} + \frac{1}{2}\right) = \frac{9}{2}$$

Bsp. 5.27

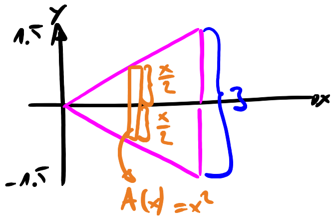


$$\int_2^4 \sqrt{x} - (x-2)\,dx + \int_0^2 \sqrt{x}\,dx = \frac{10}{3}$$

Vorlesung vom 13.03.2018

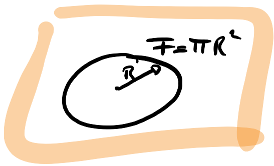
$$\int_a^b A(x)\,dx$$

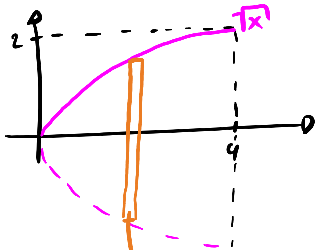
muss als Formel gegeben sein



$$\int_0^3 x^2\,dx = \frac{x^3}{3}\Big|_0^3 = \frac{27}{3} - 0 = 9\,[m^3]$$

Rotationskörper





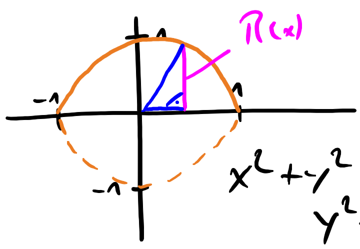
$A(x) = \pi R^2(x) = \pi(\sqrt{x})^2 = \pi x$

$$\int_4^0 \pi x\,dx = \frac{\pi x^2}{2}\Big|_0^4 = 8\pi\,[m^3]$$

Kugel

$$V = \frac{4\pi}{3} r^3$$

$$R(x) = \sqrt{1-x^2}$$



$$x^2 + y^2 = 1$$
$$y^2 = 1 - x^2$$
$$y = \pm\sqrt{1-x^2}$$

$$\int_{-1}^{1} \pi R^2(x)\,dx = \pi\int_{-1}^{1}(1-x^2)\,dx = 2\pi\int_0^1(1-x^2)\,dx = 2\pi\left(x - \frac{x^3}{3}\right)\Big|_0^1 = 2\pi\left(1-\frac{1}{3}\right) = \frac{4\pi}{3}$$

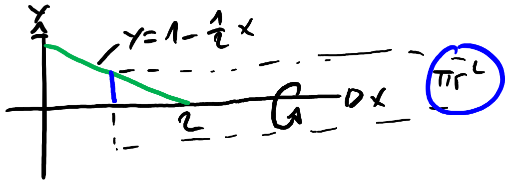
Vorlesung vom 27.3.18

6.1

$$V = \int_a^b \underline{A(x)}\, dx$$

Formel für Querschnittsfläche

Rotationskörper → Scheibenmethode



$$V = \int_0^2 \pi \left(1 - \frac{1}{2}x\right)^2 dx$$

$$= \pi \int_0^2 \left(1 - x + \frac{1}{4}x^2\right) dx = \pi\left(x - \frac{x^2}{2} + \frac{x^3}{4}\right)\Big|_0^2 = \pi\left(2 - 2 + \frac{8}{12}\right) = \pi \frac{2}{3}$$

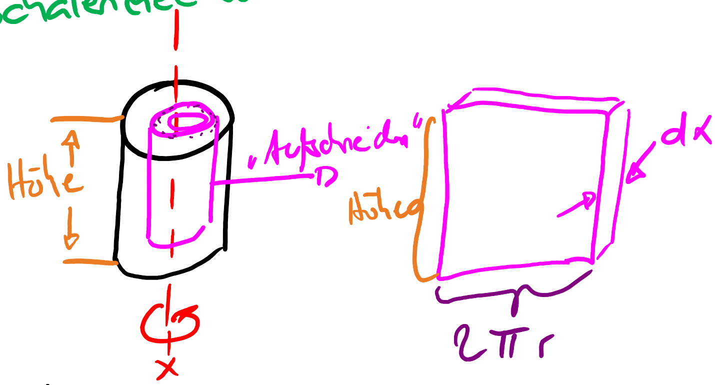
Ringmethode

$$V = \pi \int_a^b (R^2 - r^2)\, dx$$

6.2

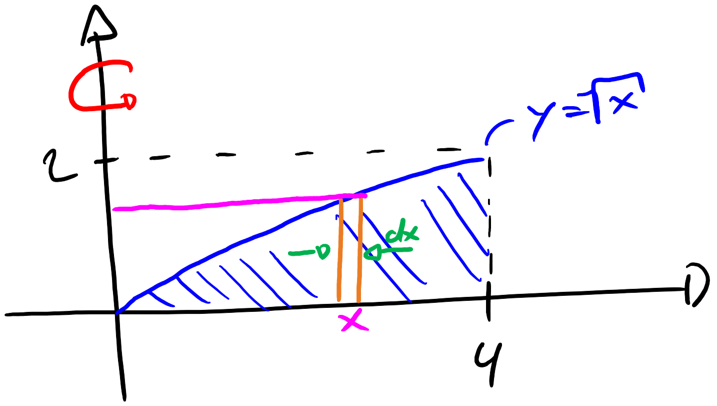
Kreisumfang: $2\pi r$

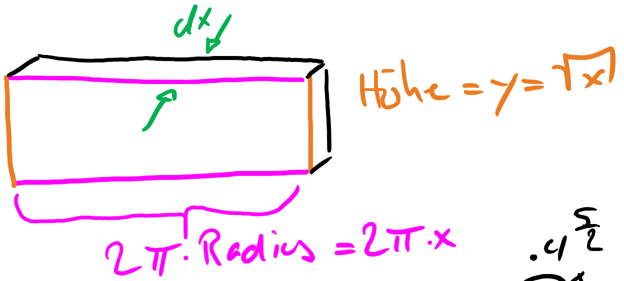
Schalenmethode



$$V = \int_a^b 2\pi \left(\text{Radius der Schale}\right) \cdot \left(\text{Höhe der Schale}\right) dx$$

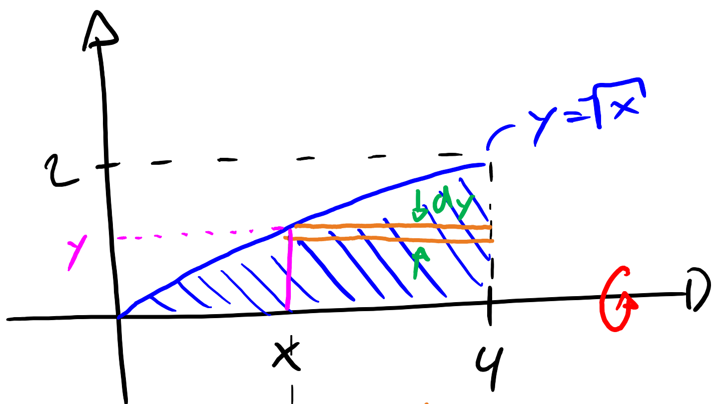
Rotation y-Achse





$$= \int_0^4 2\pi\, x^{\frac{3}{2}}\, dx = 2\pi \cdot \frac{x^{\frac{5}{2}}}{\frac{5}{2}} = \frac{4\pi}{5} \xrightarrow{\cdot 4^{\frac{5}{2}}} = \frac{128\pi}{5}$$

Rotation um x-Achse



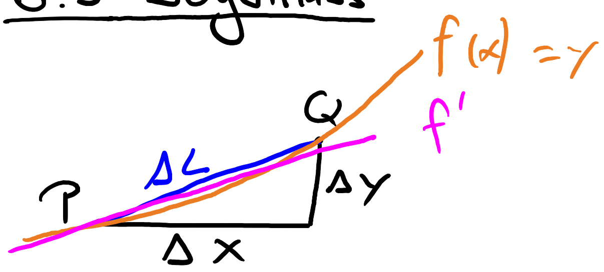
$$V = 2\pi \int_0^2 y \cdot (4 - y^2)\, dy$$

$$= 2\pi \int_0^2 (4y - y^3)\, dy$$

$$= 2\pi \left(2y^2 - \frac{y^4}{4}\right)\Big|_0^2$$

$$= 2\pi \left(2 \cdot 4 - \frac{2^4}{4}\right) = \underline{\underline{8\pi}}$$

6.3 Bogenmass



$$\Delta L^2 = \Delta x^2 + \Delta y^2$$

$$f' = y' \approx \frac{\Delta y}{\Delta x}$$

$$\Delta y \approx y' \cdot \Delta x \quad \Big\} \text{ Linearisierung}$$

$$\Delta L^2 = \Delta x^2 + (y')^2 \Delta x^2$$

$$\Delta L^2 = \left(1 + (y')^2\right) \Delta x^2$$

$$\boxed{\Delta L = \sqrt{(1 + (y')^2}\, \Delta x}$$

$$L = \int_a^b \sqrt{1 + (y')^2}\, dx$$ ⇒ wir brauchen einen gesamten Ausdruck in der Wurzel in Quadrat

Abb: 6.22

$$y = \frac{x^3}{12} + \frac{1}{x} \qquad x_1 = 1 \qquad x_2 = 4$$

$$y' = \frac{x^2}{4} - \frac{1}{x^2}$$

$$(y')^2 = \frac{x^4}{16} - 2\,\frac{\cancel{x^2}}{4} \cdot \frac{1}{\cancel{x^2}} + \frac{1}{x^4}$$

$$L = \int_1^4 \sqrt{1 + \frac{x^4}{16} - \frac{1}{2} + \frac{1}{x^4}}\, dx$$

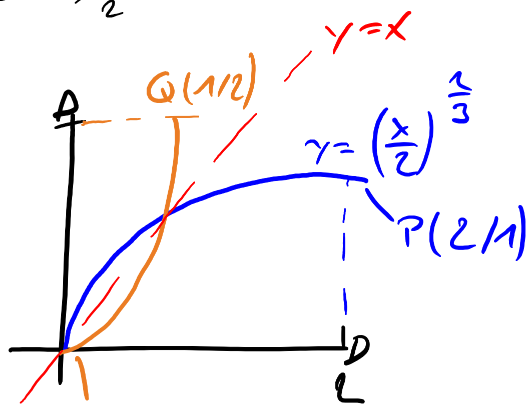
$$= \int_1^4 \sqrt{\frac{x^4}{16} + \frac{1}{2} + \frac{1}{x^4}} = \int_1^4 \sqrt{\left(\frac{x^2}{4} + \frac{1}{x^2}\right)^2}\, dx = \int_1^4 \left(\frac{x^2}{4} + \frac{1}{x^2}\right) dx$$

$$= \left(\frac{x^3}{12} - \frac{1}{x}\right)\Bigg|_1^4 = \frac{64}{12} - \frac{1}{4} - \left(\frac{1}{12} - 1\right) = \frac{64}{12} - \frac{3}{12} - \frac{1}{12} + \frac{12}{12} = \frac{72}{12} = \underline{\underline{6}}$$

Bsp.: 6.13

$$y=\left(\frac{x}{2}\right)^{\frac{2}{3}}=\sqrt[3]{\frac{x^2}{4}} \qquad x_1=0 \quad x_2=2$$

$$y'=\frac{2}{3}\left(\frac{x}{2}\right)^{-\frac{1}{3}}=\frac{2}{3}\cdot\frac{1}{\sqrt[3]{\frac{x}{2}}} \quad \text{↯} \to 0$$



$$y^{\frac{3}{2}}=\frac{x}{2} \Rightarrow x(y)=2y^{\frac{3}{2}}$$

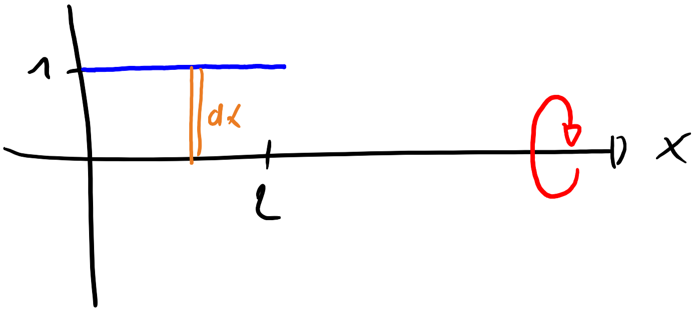
$$x'(y)=2\cdot\frac{3}{2}y^{\frac{1}{2}}=3\sqrt{y}$$

· blaue Funktion spiegeln

$$\int_0^1\sqrt{1+9y}\,dy \longleftarrow L=\int_c^d\sqrt{1+(x')^2}\,dy$$

$$=\int_0^1(1+9y)^{\frac{1}{2}}dy=\frac{1}{9}\cdot\frac{(1+9y)^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_0^1=\frac{2}{27}(1+9y)^{\frac{3}{2}}\Bigg|_0^1=\frac{2}{27}\left[10^{\frac{3}{2}}-1\right]=\frac{2}{27}(10\sqrt{10}-1)$$

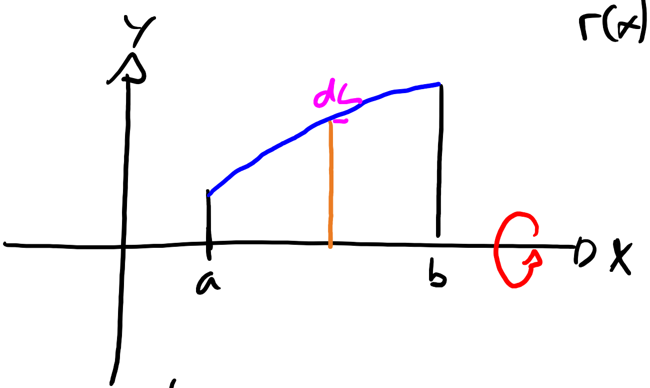
6.4 Rotationsfläche



$$2\pi\cdot 1\cdot h = \underline{\underline{4\pi}}$$

$\hookrightarrow_{D=0}$

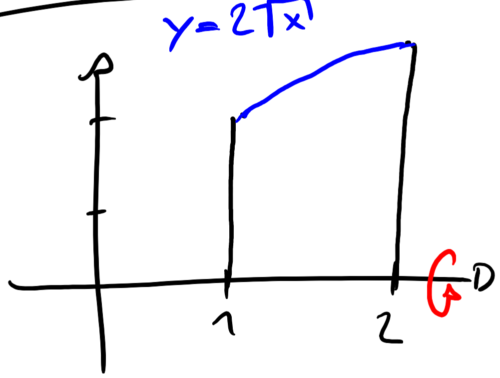
$$\int_0^2 2\pi\cdot 1\,dx = 2\pi x\Big|_0^2 = \underline{\underline{4\pi}}$$



$r(x) = f(x) = y$

$$O = \int_a^b 2\pi y \underbrace{\sqrt{1+(y')^2}}_{dL}\,dx$$

Bsp.: 6.15



$$y' = 2\,\frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{x}}$$

$$\sqrt{a}\cdot\sqrt{b} = \sqrt{ab}$$

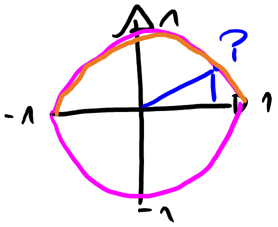
$$O = \int_1^2 2\pi\cdot 2\sqrt{x}\,\sqrt{1+\underbrace{\left(\frac{1}{\sqrt{x}}\right)^2}_{\frac{1}{x}}}\,dx = 4\pi\int_1^2 \sqrt{x+1}\,dx = 4\pi\int_1^2 (x+1)^{\frac{1}{2}}\,dx$$

O ↓ Oberfläche

$$= 4\pi\,\frac{(x+1)^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_1^2 = \frac{8\pi}{3}\,(x+1)^{\frac{3}{2}}\Bigg|_1^2 = \frac{8\pi}{3}\left(3^{\frac{3}{2}} - 2^{\frac{3}{2}}\right) = \frac{8\pi}{3}\left(3\sqrt{3} - 2\sqrt{2}\right)$$

Oberfläche-Kugel $= 4\pi r^2$

$r = 1$



$$1^2 = x^2 + y^2$$

$$y^2 = 1 - x^2$$

$$\boxed{y = \sqrt{1-x^2}}$$

$$y' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}}\cdot(-2x) = -\frac{x}{\sqrt{1-x^2}}$$

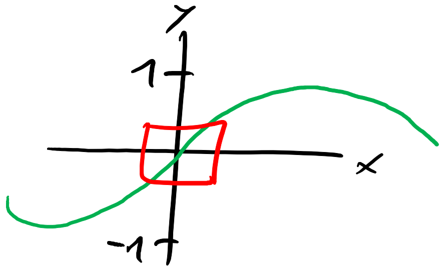
$$O = \int_{-1}^{1} 2\pi\sqrt{1-x^2}\,\underbrace{\sqrt{1+\frac{x^2}{1-x^2}}}_{\frac{1-\cancel{x^2}+\cancel{x^2}}{1-x^2}}\,dx$$

$$= \int_{-1}^{1} 2\pi\sqrt{\frac{1-x^2}{1-x^2}}\,dx = \int_{-1}^{1} 2\pi\,dx = 2\pi\Big|_{-1}^{1} = 2\pi\,(1-(-1)) = 4\pi$$

Kapitel 7 – Transzendente Funktionen

7.4 – l'hospital

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$



$$\lim_{x \to 0} \frac{f(x)}{g(x)} = \lim_{x \to 0} \frac{f'(x)}{g'(x)}$$

nicht Quotientenregel → Funktionen werden unabhängig geschaut!

Bsp.: 7.18

$$\lim_{x \to 0} \frac{1 - \cos x}{x + x^2} = \text{"}\frac{0}{0}\text{"}$$

$$\overset{\text{Abl.}}{=} \lim_{x \to 0} \frac{\sin x}{1 + 2x} = \frac{0}{1} = \underline{\underline{0}}$$

Bsp 7.20 b)

$$\lim_{x \to \infty} \frac{\ln x}{2\sqrt{x}} = \text{"}\frac{\infty}{\infty}\text{"}$$

$$\overset{\text{Abl.}}{=} \lim_{x \to x} \frac{\frac{1}{x}}{\cancel{2} \frac{1}{\cancel{2}\sqrt{x}}} = \lim_{x \to \infty} \frac{\frac{1}{x}}{x} = \frac{\sqrt{x}}{x} = \lim_{x \to \infty} \frac{1}{\sqrt{x}} = \underline{\underline{0}}$$

c)

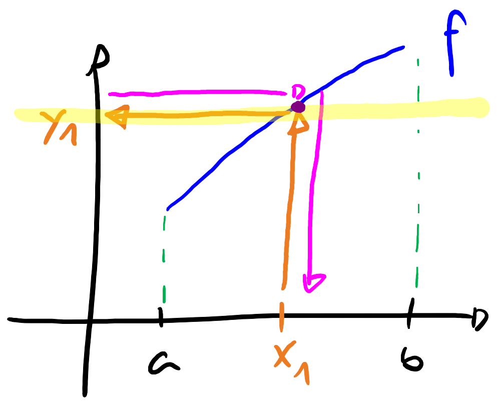
$$\lim_{x \to \infty} \frac{e^x}{x^2} = \text{"}\frac{\infty}{\infty}\text{"}$$

Mehrfaches Ableiten notwendig

$$\overset{\text{Abl.}}{=} \lim_{x \to \infty} \frac{e^x}{2x} = \text{"}\frac{\infty}{\infty}\text{"}$$

$$\overset{\text{Abl.}}{=} \lim_{x \to \infty} \frac{e^x}{2} = \text{"}\frac{\infty}{2}\text{"} = \underline{\underline{\infty}}$$

Kapitel 7.1 – inverse Funktionen



Funktion von x-Wert zu y-Wert

Umkehrfunktion von y-Wert zu x-Wert zurück → inverses

⚡ nicht jede Funktion hat ein inverses ⚡

! Wenn eine Funktion beim Horizontaltest nur einen Schnittpunkt hat, ist sie umkehrbar → inverses !

Bsp: $y = 2x + 3$

$P(1;5)$

$y - 3 = 2x$

$f^{-1}: x = \frac{y-3}{2}$

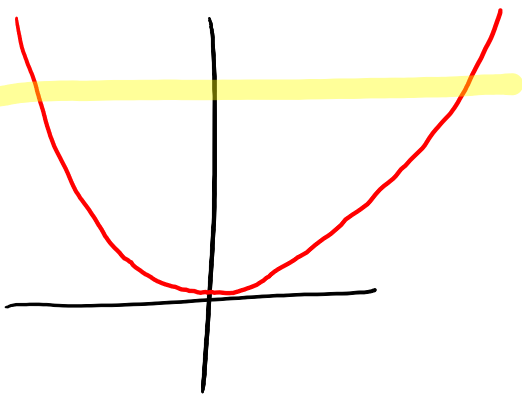
zu Wertetabelle auf S. 310

$\mathbb{D} = [0;2]$ → $\mathbb{D} = [3;7]$

$\mathbb{W} = [3;7]$ → $\mathbb{W} = [0;2]$

- Wertebereich wird zu Definitionsbereich & umgekehrt.

- X ist die freie Variable, deshalb muss bei f^{-1} die Variablen zum Einzeichnen getauscht werden



Durch das Einschränken wäre eine umkehrbar möglich (Bsp.: $\mathbb{R}_0^+$) obwohl die Funktion $y=x^2$ den Horizontaltest nicht bestehen würde

Taylorreihen

▷ unendlich viele Summanden

$$a_n = \frac{f^{(n)}(x_0)}{n!}$$

Bsp:

$$t_f(x) = \underbrace{0\cdot(x-1)^0}_{0} + 1\cdot(x-1)^1 - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \dots$$

$f(x) = \ln x$ $\quad$ $x_0 = 1$

$f^{(0)}(x) = \ln x$ $\quad$ $\ln 1 = 0$ $\quad$ $a_0 = \frac{0}{0!} = \underline{\underline{0}}$

$f'(x) = \frac{1}{x} = x^{-1}$ $\quad$ $\frac{1}{1} = 1$ $\quad$ $a_1 = \frac{1}{1!} = \underline{\underline{1}}$

$f''(x) = -x^{-2}$ $\quad$ $-\frac{1}{1^2} = -1$ $\quad$ $a_2 = -\frac{1}{2!}$

$f'''(x) = 2x^{-3}$ $\quad$ $\frac{2}{1^3} = 2$ $\quad$ $a_3 = \frac{2}{6} = \frac{1}{3}$

$f^{(4)}(x) = -6x^{-4}$ $\quad$ $-\frac{6}{1^4} = -6$ $\quad$ $a_4 = -\frac{6}{4!} = -\frac{6}{24} = -\frac{1}{4}$

allgemein

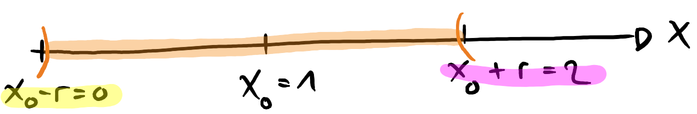
$$t_f(x) = \sum_{n=0}^{\infty} \frac{1}{n}(x-1)^n \cdot (-1)^{n+1}$$

Konvergenzbereich

$$r = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty}\left|\frac{\frac{1}{n}}{\frac{1}{n+1}}\right| = \lim_{n\to\infty}\left|\frac{n+1}{n}\right| = \underline{\underline{1}}$$

Fall $x=0$: $0 - 1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \dots = -\infty$

↳ 0 einsetzen $\quad$ ↳ divergent



Fall $x=2$: $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \approx 0.69$

↳ 2 einsetzen $\quad$ ↳ konvergent

⇒ Folgerung: 2 ist noch eingeschlossen & 0 nicht!

Approximation von Taylorreihe:

Summe in TR eingeben mit „$\sin \pi + \log_{□} □ \Rightarrow \sum_{□}^{□}$“

Restschätzung „zweitletzte Folie“

$$t_f(x) = P_n + R_n$$

$$\boxed{R^6 = \frac{f^{(7)}(X)}{7!} \cdot 1^7}$$

$$f^{(4)}(x) = -\frac{6}{x^4} = -6^{-4}$$

$$f^{(5)}(x) = 24x^{-5}$$

$$f^{(6)}(x) = -120x^{-6}$$

$$f^{(7)}(x) = 720x^{-7} = \frac{720}{x^7} = \frac{6!}{X^7}$$

$$X \in (x_0; x)$$

$$\in (1; 2)$$

$$\boxed{R_6 = \frac{6!}{X^7 \cdot 7!} = \frac{1}{X^7 \cdot 7} \approx \frac{1}{7} \approx 0.14}$$

Differentialgleichungen

- Die Lösung ist immer eine Funktion

Ansatz:

$\frac{dy}{dx} = 2x \quad | \cdot dx$

$\leftarrow y$

$\rightarrow x$

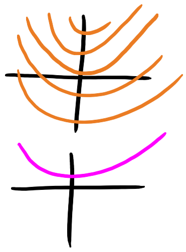
„alles mit y nach links & alles mit x nach rechts"

$\int dy = \int 2x \, dx$

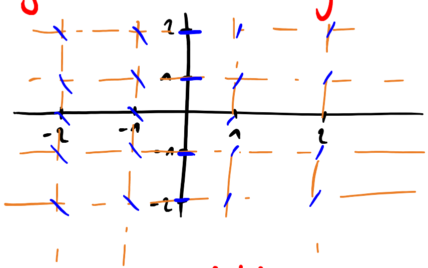
$y(x) = x^2 + c$ → „allgemeine Lösung"

$1 = 0^2 + c \Rightarrow c = 1$

$y(x) = x^2 + 1$ → „spezielle Lösung"



geometrische Lösung



⇒ Richtungsfeld

Aufgabe Richtungsfeld

$y' = y + 1$

Steigung = $y + 1$ ist unabhängig von x

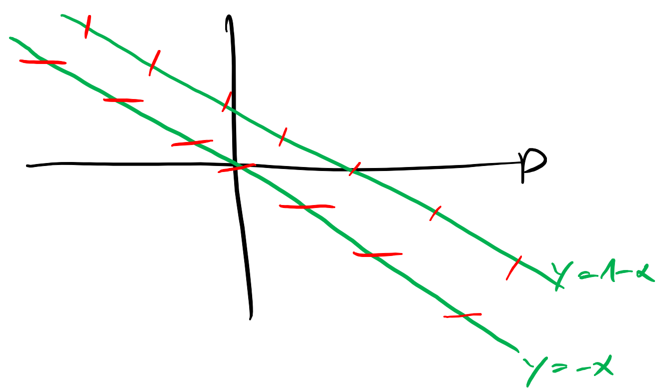
falls $y = 2$ → Steigung ist 3

$y' = x + y$

$x + y = \text{Konstante}$

$x + y = 0$

$y = -x$



$y' = -\frac{x}{y}$ $\qquad y \to 0 \quad y' \to \infty$

$x = 0 \quad x' = 0$

$y' = y^2 - x^2 \qquad y' = 0 \qquad y^2 - x^2 = 0$

$y^2 = x^2$

$y = \pm x$

$x = \pm y$

Probeprüfung 16

b)

Konstante

e

$e^0 = 1$

$x - 0.5y = 0 \qquad y = 2x$

Separierbare Differentialgleichung

$y' = x \cdot y$

$\frac{dy}{dx} = x \cdot y \quad | : y \cdot dx$

$\int \frac{dy}{y} = \int x \cdot dx$

$\ln y = \frac{x^2}{2} + c \quad | e^{\uparrow}$ „entlogarithmieren"

$e^{\ln y} = e^{\frac{x^2}{2} + c}$

$y(x) = e^{\frac{x^2}{2} + c} \longrightarrow$ Allg. Lösung

$= e^{\frac{x^2}{2}} \cdot \underbrace{e^c}_{\text{Konstante } K} \longrightarrow y(x) = K \cdot e^{\frac{x^2}{2}} \longrightarrow$ spezielle Lösung

Satz 1

$y' = x + y$

Subst.: $\underbrace{u = x + y}$

$y = u - x$

$y' = u' - 1$

$u' - 1 = u$

$$\downarrow$$

$$\frac{du}{dx} = u + 1 \qquad \Big| : (u+1)$$

$$\int \frac{du}{u+1} = \int dx$$

$$\ln(u+1) = x + c \qquad \Big| \; e^{\uparrow}$$

$$u + 1 = e^{x+c} = k \cdot e^{x}$$

Rücksubst.

$$x + y + 1 = k \cdot e^{x}$$

$$y = k \cdot e^{x} - x - 1 \rightarrow \text{Allg. Lösung}$$

$y' = \frac{y}{x}$

$u = \frac{y}{x}$

$y = u \cdot x$

$y' = u' \cdot x + u \cdot 1$

$y' = u$

$$u' \cdot x + u = u \qquad \Big| -u$$

$$u' = 0$$

$$\frac{du}{dx} = 0$$

$$\int du = c$$

$$u(x) = c$$

$$\frac{y}{x} = c$$

$$y(x) = c \cdot x \rightarrow \text{Allg. Lösung}$$

$$x \cdot y' - y = x \quad | : x$$

$$y' - \frac{y}{x} = 1 \Rightarrow y' = \frac{y}{x} + 1$$

Subst.: $u = \frac{y}{x}$

$$y = u \cdot x$$

$$y' = u' \cdot x + u \cdot 1$$

$$y' = u + 1$$

$$u' \cdot x + u = u + 1 \quad | - u$$

$$u' x = 1$$

$$u' = \frac{1}{x}$$

$$\frac{du}{dx} = \frac{1}{x} \quad | \cdot dx$$

$$\int du = \int \frac{dx}{x}$$

$$u(x) = \ln x + c \rightarrow \text{Alg. Lsg.}$$