5.6 Aufgaben

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a)

$$\int_0^{2\pi} \frac{\cos t}{\sqrt{4+3\sin t}}\,dt$$

$u = 4 + 3\sin t \qquad \frac{du}{dt} = 3\cdot\cos t \qquad \frac{du}{3} = \cos t\,dt$

aufleiten

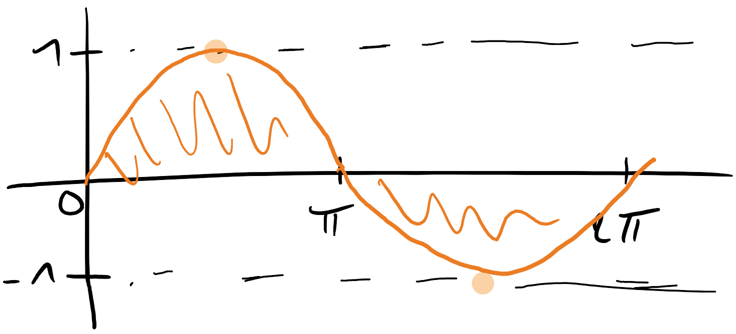
$$\frac{1}{3}\int \frac{du}{\sqrt{u}} = \frac{1}{3}\int u^{-\frac{1}{2}}\,du = \frac{1}{3}\,\frac{u^{\frac{1}{2}}}{\frac{1}{2}} = \frac{2}{3}\sqrt{u} + c$$

a)

$u(2\pi) = 4 + 0$

$u(0) = 4 + 0$

$$= \frac{2}{3}\sqrt{u}\,\Big|_4^4 = 0$$



b)

$u(\pi) = 4 + 0$

$u(-\pi) = 4 + 0$

$$= \frac{2}{3}\sqrt{u}\,\Big|_4^4 = 0$$

8

– Gibt es Nullstellen in Integrationsintervall? $[-2; 2]$

$$x \cdot \sqrt{4-x^2} = 0$$

$x_1 = 0$

$x_2 = 2$

$x_3 = -2$

$$\int_{-2}^{0} x\sqrt{4-x^2}\, dx$$

$u = 4 - x^2$

$\frac{du}{dx} = -2x$

$\frac{du}{-2} = x\, dx$

$= -\frac{1}{2} \int_{-2}^{0} \sqrt{u}\, du$

$= -\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} \cdot u^{\frac{3}{2}} = -\frac{1}{3} \cdot u^{\frac{3}{2}} + c$

$u(-2) = 0$

$u(0) = 4$

$= -\frac{1}{3} u^{\frac{3}{2}} \Big|_0^4 = -\frac{1}{3} \cdot 8 = -\frac{8}{3}$

Fläche $= 2 \cdot \left|-\frac{8}{3}\right| = \frac{16}{3}$

(14)

Gegeben: $f(x) = x^4 - 2x^2$ &
$g(x) = 2x^2$

Fläche zwischen f & g (und ihre Schnittpunkte)

1) Schnittpunkte (Integrationsgrenzen)

$x^4 - 2x^2 = 2x^2$ → beide Gleichungen gleichsetzen

$x^4 = 4x^2$

$x^4 - 4x^2 = 0$

$x^2(x^2 - 4) = 0$

$x_1 = 0$ } Nullstelle

$x_2 = 2$
$x_3 = (-2)$ } Integrationsgrenzen

2) Integral bestimmen

$$\int_{-2}^{0} (x^4 - 2x^2 - 2x^2)\,dx = \int_{-2}^{0} (x^4 - 4x^2)\,dx$$

$$= \left(\frac{x^5}{5} - \frac{4x^3}{3}\right)\Big|_{-2}^{0} = 0 - 0 - \left(-\frac{32}{5} + \frac{32}{3}\right) = \frac{96}{15} - \frac{160}{15} = -\frac{64}{15}$$

$$\text{Fläche}_{-2-0} = \frac{64}{15}$$

$$\left(\frac{x^5}{5} - \frac{4x^3}{3}\right)\Big|_{0}^{2} = \frac{32}{5} - \frac{32}{3} = -\frac{64}{15}$$

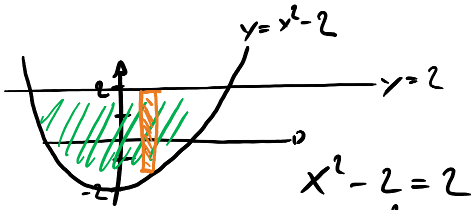
$$\text{Fläche}_{0-2} = \frac{64}{15}$$

$$\text{Fläche}_{total} = \frac{128}{15}$$

(19)

$y = x^2 - 2$

$y = 2$



$x^2 - 2 = 2$

$x^2 = 4$

$x = \pm 2$

$$\int_{-2}^{2} 2 - (x^2 - 2)\,dx = \int_{-2}^{2} (-x^2 + 4)\,dx$$

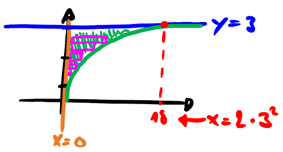
$$= -\frac{x^3}{3} + 4x\Big|_{-2}^{2} = 2\left(-\frac{x^3}{3} + 4x\right)\Big|_{0}^{2} = 2\left(-\frac{8}{3} + 8\right) = 2 \cdot \frac{16}{3} = \frac{32}{3}$$

(21)

$x = 2y^2$

$x = 0$

$y = 3$



$$\boxed{3 \cdot 18} - \int_{0}^{18} \sqrt{\frac{x}{2}}\,dx \quad \text{Variante 1}$$

Integration nach y (Variante 2)

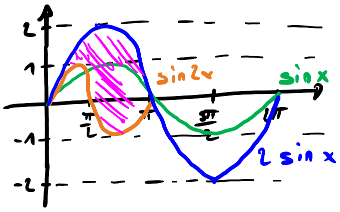
$$\int_{0}^{3} 2y^2\,dy = 2\int_{0}^{3} \frac{y^3}{3} = \frac{2y^3}{3}\Big|_{0}^{3} = 18$$

(25)

$y = 2\sin x$

$y = \sin 2x$

$0 \leq x \leq \pi$



$2\sin x = \sin 2x$

$2\sin x = 2\sin x \cos x \quad |:2\sin x$

$1 = \cos x \quad \Rightarrow x = 0$

$[\cos 2x]' = -\sin 2x \cdot 2$

⇒ Keine Schnittpunkte

$$\int_{0}^{\pi} (2\sin x - \sin 2x)\,dx = -2\cos x + \frac{1}{2}\cos x\Big|_{0}^{\pi}$$

$$= -2(-1) + \frac{1}{2} \cdot 1 - \left(-2 + \frac{1}{2} \cdot 1\right) = \underline{4}$$