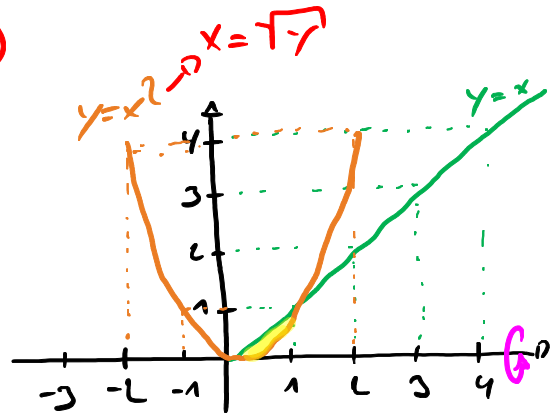
Kapitel 6.2

(26)



Schalenmethode:

$$V = \int_a^b 2\pi \,(\text{Radius Schale}) \cdot (\text{Höhe Schale})\, dx$$

Ringmethode:

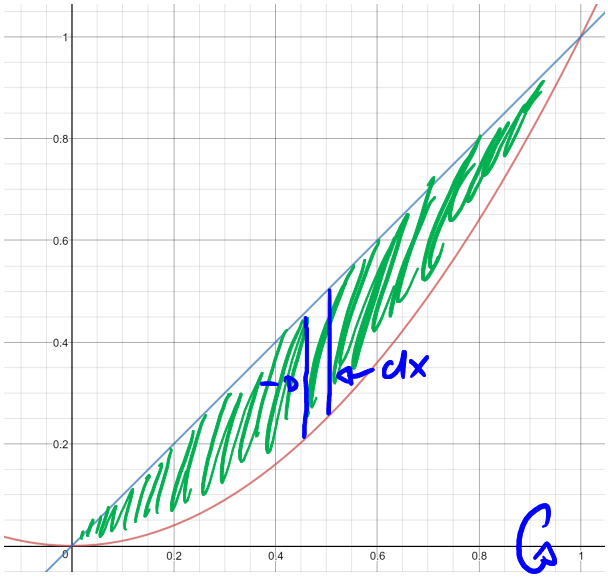
$$V = \pi \int_a^b (R^2 - r^2)\, dx$$

Rotation x-Achse

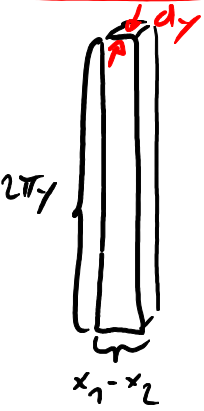
Ringmethode

$$V = \pi \int_a^b (R^2 - r^2)\, dx = \pi \int_0^1 (x^2 - x^4)\, dx$$

$$V = \pi \left(\frac{x^3}{3} - \frac{x^5}{5} \right) \Big|_0^1 = \pi \left(\frac{1}{3} - \frac{1}{5} \right) = \frac{2\pi}{15}$$



Schalenmethode

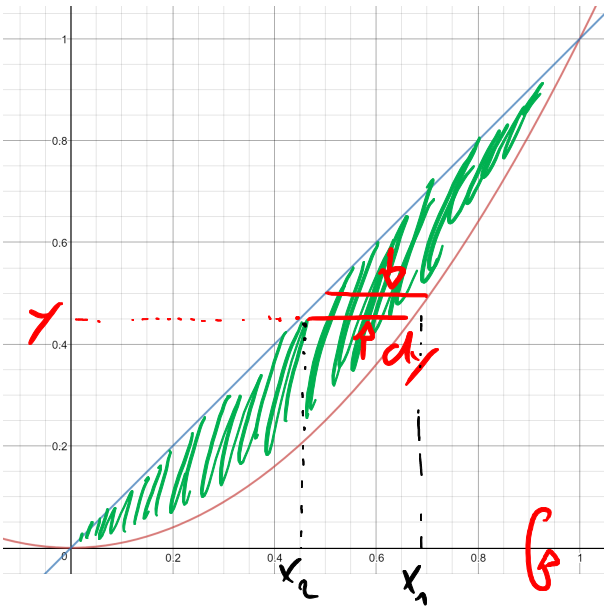


$$V = \int_0^1 2\pi y \left(\sqrt{y} - y \right) dy$$

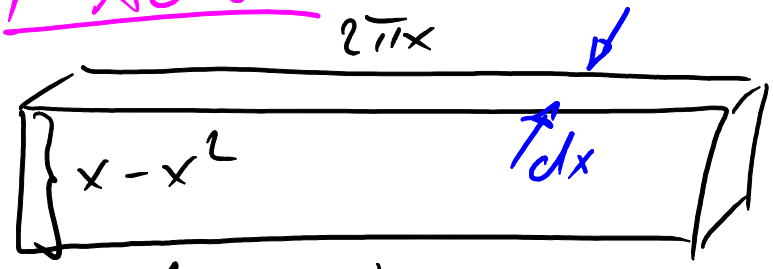
$$= 2\pi \int_0^1 \left(y^{\frac{3}{2}} - y^2 \right) dy$$

$$= 2\pi \left(\frac{y^{\frac{5}{2}}}{\frac{5}{2}} - \frac{y^3}{3} \right) \Big|_0^1 = 2\pi \left(\frac{2}{5} - \frac{1}{3} \right)$$

$$= \frac{2\pi}{15}$$



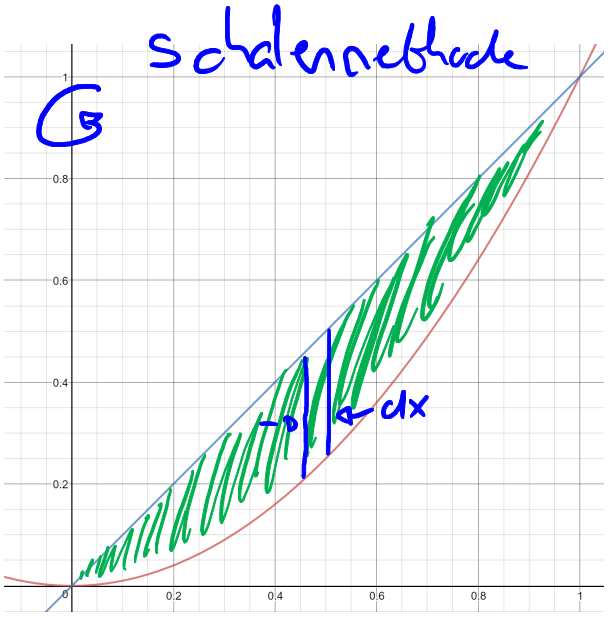
Y-Achse



$$V = 2\pi \int_0^1 (x^2 - x^3)\,dx$$

$$= 2\pi \left(\frac{x^3}{3} - \frac{x^4}{4} \right) \Big|_0^1 = 2\pi \left(\frac{1}{3} - \frac{1}{4} \right)$$

$$= 2\pi \, \frac{4-3}{12} = \frac{\pi}{6}$$



$$V = \int_0^1 \pi (R^2 - r^2)\,dy$$

$$= \pi \int_0^1 (y - y^2)\,dy$$

$$= \pi \left(\frac{y^2}{2} - \frac{y^3}{3} \right) \Big|_0^1 = \pi \left(\frac{1}{2} - \frac{1}{3} \right) = \pi \, \frac{3-2}{6}$$

$$= \frac{\pi}{6}$$