Taylorreihen

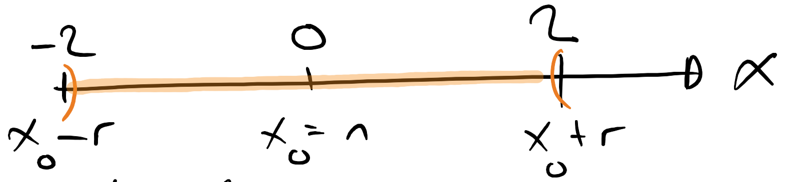
1b)

$$\sum_{n=0}^{\infty} \frac{x^n}{2^n} = \sum_{n=0}^{\infty} \frac{1}{2^n} \cdot x^n$$

$$r = \lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty} \left|\frac{\frac{1}{2^n}}{\frac{1}{2^{n+1}}}\right| = \lim_{n\to\infty} \left|\frac{2^{n+1}}{2^n}\right| = \underline{\underline{2}}$$

$$a_n = \frac{1}{2^n}$$

$$a_{n+1} = \frac{1}{2^{n+1}}$$



$\boxed{-2 < x < 2}$

Fall $x = 2$ $\quad \frac{2^0}{2^0} + \frac{2^1}{2^1} + \frac{2^2}{2^2} + \ldots \rightarrow$ (bestimmt) divergent gegen ∞

Fall $x = -2$ $\quad \underbrace{\frac{(-2)^0}{2^0}}_{1} + \underbrace{\frac{(-2)^1}{2^1}}_{-1} + \underbrace{\frac{(-2)^2}{2^2}}_{+1} + \underbrace{\frac{(-2)^3}{2^3}}_{-1} + \ldots \rightarrow$ (unbestimmt) divergent

---

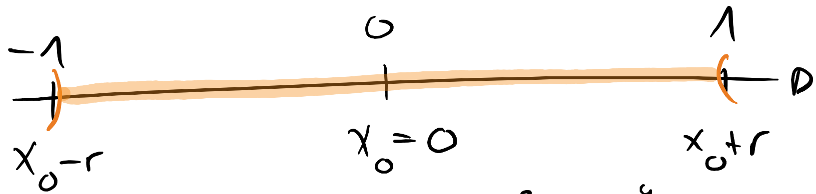
1a) $P(x) = x + \frac{x^2}{2^2} + \frac{x^3}{3^2} + \frac{x^4}{4^2} + \ldots$

$$a_n = \frac{1}{n^2}$$

$$\sum_{n=1}^{\infty} \frac{x^n}{n^2}$$

$$a_{n+1} = \frac{1}{(n+1)^2}$$

$$r = \lim_{n\to\infty} \left|\frac{\frac{1}{n^2}}{\frac{1}{(n+1)^2}}\right| = \lim_{n\to\infty} \left|\frac{(n+1)^2}{n^2}\right| = 1$$



Fall $x = 1$ $\quad \left.\begin{array}{l} 1 + \frac{1^2}{2^2} + \frac{1^3}{3^2} + \frac{1^4}{4^2} + \ldots \\ 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \ldots \end{array}\right\}$ Konvergent

Fall $x = -1$

$$\left.\begin{aligned} &-1 + \frac{1^2}{2^2} - \frac{1^3}{3^2} + \frac{1^4}{4^2} - \dots \\ &-1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} - \dots \end{aligned}\right\} \text{konvergent}$$

Aufgabe 2

$x_0 = 0$

$$a_n = \frac{f^{(n)}(x_0)}{n!}$$

$f(x) = \cos(0) = 1$

$f^{(1)}(x) = -\sin(0) = 0$

$f^{(2)}(x) = -\cos(0) = -1$

$f^{(3)}(x) = \sin(0) = 0$

$f^{(4)}(x) = \cos(0) = 1$

$f^{(5)}(x) = -\sin(0) = 0$

$f^{(6)}(x) = -\cos(0) = -1$

$a_0 = \frac{1}{0!} = 1$

$a_1 = 0$

$a_2 = \frac{-1}{2!} = -\frac{1}{2}$

$a_3 = 0$

$a_4 = \frac{1}{4!} = \frac{1}{24}$

$a_5 = 0$

$a_6 = \frac{-1}{6!} = \frac{-1}{720}$

$$t_f(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \dots$$

$$\boxed{\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot x^{2n}} = (x^2)^n$$

Substitution

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot t^n$$

$$r = \lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty} \left| - \frac{(2n+2)!}{(2n)!} \right|$$

$$= \lim_{n\to\infty} \left| - \frac{(2n+2)(2n+1)\cancel{(2n)!}}{\cancel{(2n)!}} \right| = \underline{\underline{\infty}}$$

$$f(x) = \frac{1}{1-x} \qquad x_0 = -1$$

$$\frac{f^{(n)}}{}$$

$$f(x) = \frac{1}{1-x} = \frac{1}{2} \qquad a_0 = \frac{\frac{1}{2}}{0!} = \frac{1}{2}$$

$$f'(x) = \cdot \quad (1-x)^{-2} = \frac{1}{4} \qquad a_1 = \frac{\frac{1}{4}}{1!} = \frac{1}{4}$$

$$f''(x) = (-2)\,(1-x)^{-3} = -\frac{2}{8} \qquad a_2 = \frac{-\frac{1}{4}}{2!} = -\frac{1}{8}$$

$$f'''(x) - 6(1-x)^{-4} = \frac{6}{16} \qquad a_3 = \frac{\frac{3}{8}}{3!} = \frac{1}{16}$$

$$\vdots \qquad \vdots$$

$$t_f(x) = \frac{1}{2}(x+1)^0 + \frac{1}{4}(x+1)^1 - \frac{1}{8}(x+1)^2 + \frac{1}{16}(x+1)^3 + \dots$$

$$= \sum_{n=0}^{\infty} \frac{1}{2^{n+1}} \quad (x+1)^n$$