

Números complejos

1. i. a. $3 - 6i$ b. $1 + \frac{5}{2}i$ c. $\frac{9}{2} + 4i$ d. $2 + i$

3. a. Las longitudes de sus diagonales son: 2 y $\sqrt{20}$.

4. a. $S = \{1, 2\}$

b. $S = \{-2, 1 + \sqrt{3}i, 1 - \sqrt{3}i\}$

c. $S = \left\{ \sqrt[4]{2}e^{\frac{5\pi}{12}i}, \sqrt[4]{2}e^{\frac{11\pi}{12}i}, \sqrt[4]{2}e^{\frac{17\pi}{12}i}, \sqrt[4]{2}e^{\frac{23\pi}{12}i} \right\}$

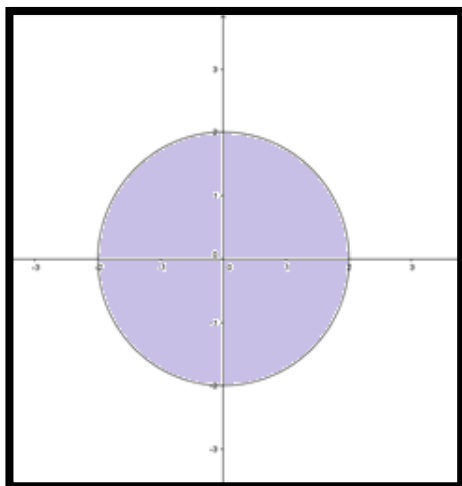
d. $S = \left\{ \sqrt[12]{2}e^{\frac{\pi}{24}i}, \sqrt[12]{2}e^{\frac{3\pi}{8}i}, \sqrt[12]{2}e^{\frac{17\pi}{24}i}, \sqrt[12]{2}e^{\frac{25\pi}{24}i}, \sqrt[12]{2}e^{\frac{11\pi}{8}i}, \sqrt[12]{2}e^{\frac{41\pi}{24}i} \right\}$

e. $S = \{3, 3i, -3, -3i\}$

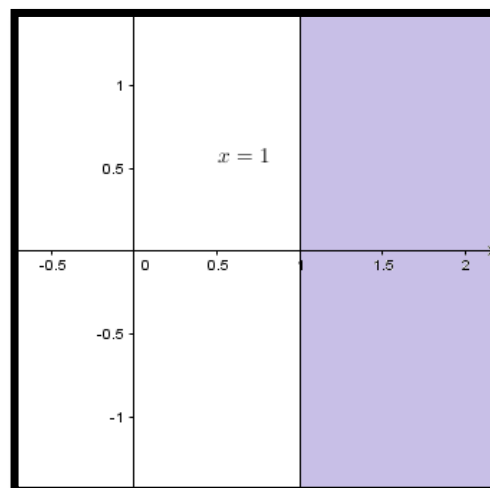
f. $S = \{-2, 2, 1 + 2i, 1 - 2i\}$

5.

a.

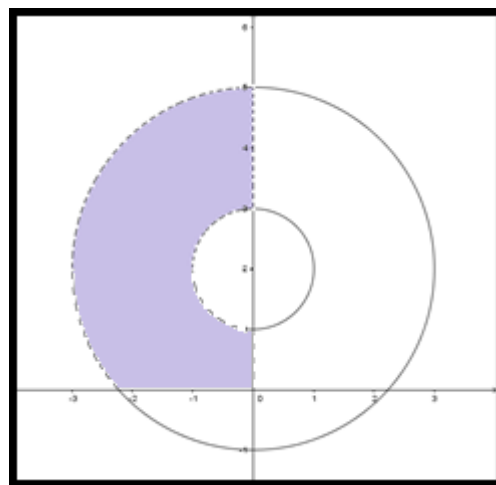
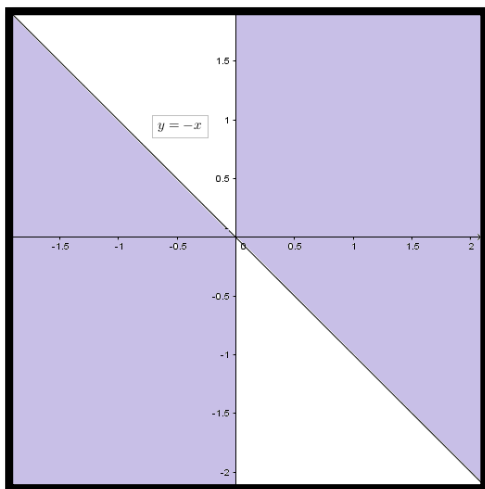


b.



c.

d.



6.

- a. $R = \{z \in \mathbb{C} : \operatorname{Re}(z) \leq 3, \operatorname{Im}(z) \leq 2\}$
- b. $R = \{z \in \mathbb{C} : \operatorname{Re}(z) \leq \operatorname{Im}(z), \operatorname{Re}(z) \geq 0, \operatorname{Im}(z) \leq 4\}$
- c. $R = \{z \in \mathbb{C} : \frac{\pi}{4} \leq \arg(z) \leq \frac{7}{4}\pi \wedge |z| \leq 2\}$

Polinomios

8. i. $p(x) + r(x) = -x^3 + 6x - 2$ ii. $q(x) - r(x) = x^3 + x^2 - 5x + 3$ iii. $p(x)q(x) = x^3 + x$ iv. $p(x) + 2r(x) - q(x) = -2x^3 - x^2 + 11x - 5$

9. i. a) $\sigma(p_1) = \{-1, 0, 1\}$ b) En $\mathbb{Q}[t], \mathbb{R}[t]$ y $\mathbb{C}[t]$: $p_1(t) = t(t-1)(t+1)$

ii. a) $\sigma(p_2) = \{0(\text{doble}), 1\}$ b) En $\mathbb{Q}[t], \mathbb{R}[t]$ y $\mathbb{C}[t]$: $p_2(t) = -t^2(t-1)$

iii. a) $\sigma(p_3) = \{-\sqrt{2}, \sqrt{2}, \sqrt{2}i, -\sqrt{2}i\}$

b) En $\mathbb{Q}[t]$: $p_3(t) = (t^2 - 2)(t^2 + 2)$

En $\mathbb{R}[t]$: $p_3(t) = (t - \sqrt{2})(t + \sqrt{2})(t^2 + 2)$

En $\mathbb{C}[t]$: $p_3(t) = (t - \sqrt{2})(t + \sqrt{2})(t - \sqrt{2}i)(t + \sqrt{2}i)$

iv. a) $\sigma(p_4) = \{-3, -2, 0, 2, 3\}$ b) En $\mathbb{Q}[t], \mathbb{R}[t]$ y $\mathbb{C}[t]$: $p_4(t) = t(t-2)(t+2)(t-3)(t+3)$

v. a) $\sigma(p_1 + p_2) = \{0, 1\}$ b) En $\mathbb{Q}[t], \mathbb{R}[t]$ y $\mathbb{C}[t]$: $(p_1 + p_2)(t) = t(t-1)$

vi. a) $\sigma(p_1 p_2) = \{0(\text{triple}), 1(\text{doble}), -1\}$ b) En $\mathbb{Q}[t], \mathbb{R}[t]$ y $\mathbb{C}[t]$: $(p_1 p_2)(t) = -t^3(t-1)^2(t+1)$

10. a. $a = \frac{1}{5}, \sigma(p) = \{2, -1\}$

b. $a = -\frac{253}{6}, b = \frac{223}{3}$

c. $a = -16, b = 12$.

11. a. $c(w) = w, r(w) = 25w^2 - 1$

b. $c(u) = 0, r(u) = u^3 - 25u$

c. $c(t) = t^4 - t^2 + t + 1, r(t) = -t^2 + 2t - 1$

d. $c(t) = t^6 + t^4 + t^2 + 1, r(t) = 0$.

e. $c(x) = 3x + 15, r(x) = 0$

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a. $p(t) = \frac{1}{9}(t+3)(t+1)(t-1)(t-3)$

b. $q(t) = \frac{1}{48}(t+3)^2(t+2)t^2(t-2)^2(t-3)$

c. $r(t) = \frac{1}{4}(t+2)(t+1)^2(t-1)^2(t-2)$

d. $s(t) = -\frac{1}{9}(t+2)^2(t+1)t^2(t-2)^2$

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i. En $R[t]$ y $C[t]$: $p(t) = (t-1)^2(t-2)(t-3)$

ii. En $R[t]$: $p(t) = (t^2 - 2t + 5)(t-1)(t-2)$ En $C[t]$: $p(t) = (t-(1+2i))(t-(1-2i))(t-1)(t-2)$

iii. En $R[t]$ y $C[t]$: $p(t) = t(t-1)(t+1)(t-2)(t+2)$

iv. En $R[t]$ y $C[t]$: $p(t) = t(t-1)(t-2)(t-3)(t-4)$

v. En $R[t]$: $p(t) = -2.(t^2 + 4)(t-3)$ En $C[t]$: $p(t) = -2.(t-2i)(t+2i)(t-3)$

vi. En $R[t]$ y $C[t]$: $p(t) = -3t^3(t+2)^2(t-2)^2$