

1. a. Es transformación lineal b. No es TL c. Es TL d. Es TL e. No es TL f. es TL

2. a. $T(x, y) = (3x - 2y, -x + 4y)$ b. $T(x, y) = \left(-\frac{x+y}{3}, \frac{2x+2y}{3}\right)$

c. No existe una TL que cumpla lo pedido. d. $T(x, y, z) = (8x + 9y + 6z, -5x - 9y - 5z)$

e. $T(x, y, z) = (z, x - z, 0)$.

3. a. i. $(0, 0) \in \text{Nu } T$ ii. $(2, 3) \in \text{Nu } T$ iii. $(3, -2) \notin \text{Nu } T$ iv. $\left(1, \frac{1}{3}\right) \notin \text{Nu } T$
b. i. $(3, -6) \in \text{Im } T$ ii. $(2, 3) \notin \text{Im } T$ iii. $(1, -2) \in \text{Im } T$ iv. $(4, -3) \notin \text{Im } T$

4. a. $\text{Nu } T = \{(0, 0)\}$ (no existe base del núcleo) $\text{Im } T = \text{gen}\{(1, 0, 0), (-2, -5, 0)\}$

b. $\text{Nu } T = \text{gen}\{(0, 1, 1)\}$ $\text{Im } T = \text{gen}\{(-2, 0, 1), (1, -1, 0)\}$

c. $\text{Nu } T = \text{gen}\{(3, 0, 1), (2, 1, 0), (0, 0, 0)\}$ $\text{Im } T = \text{gen}\{(3, 0, 1), (0, 1, 0)\}$

d. $\text{Nu } T = \text{gen}\left\{\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}\right\}$ $\text{Im } T = \text{gen}\left\{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right\}$

e. $\text{Nu } T = \text{gen}\left\{\begin{pmatrix} -12 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 3 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}\right\}$, $\text{Im } T = \text{gen}\left\{\begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}, (0, 1)\right\}$

5. a. $T(x, y) = (-x, 3x, 4x)$ b. $T(x, y, z) = (-x + 2z, -y, 0, 0)$ c. No existe

d. Una TL que cumple lo pedido es $T\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = (-a_{11}, 3a_{11} + a_{12})$. No es única.

6. Verdadero

7. a. $M(T) = \begin{pmatrix} \frac{1}{3} & \frac{2}{5} \\ -1 & 5 \end{pmatrix}$ b. $M(T) = \begin{pmatrix} 1 & 1 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{4} \\ 0 & 0 & -1 \end{pmatrix}$ c. $M(T) = \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ -4 & 0 \end{pmatrix}$

8. a. $T(1, -5, 3) = (7, 13, 21)$, $T(0, 0, 0) = (0, 0, 0)$, $T(1, -1, 1) = (3, 3, 5)$

b. $\text{Im } T = \mathbb{R}^3$, $\text{Nu } T = \{(0, 0, 0)\}$

c. $T(x, y, z) = (2x - y, x - 3y - z, -3y + 2z)$

9.

a. $M_{B'}(T) = \begin{pmatrix} 4 & -3 \\ 4 & -2 \end{pmatrix}$ b. $M_{BB'}(T) = \begin{pmatrix} 1 & 1 & -2 \\ 2 & -2 & 0 \end{pmatrix}$ c. $M_{BB'}(T) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 2 & -4 \end{pmatrix}$

10.

- a. $T(-1 \ 1 \ 0) = (4 \ 6 \ 2 \ 2)$, $T(2 \ 4 \ 0) = (0 \ 4 \ -4 \ -4)$, $T(-1 \ 1 \ 1) = (6 \ 8 \ 2 \ 4)$
 b. $\text{Nu } T = \{(0 \ 0 \ 0)\}$ $\text{Im } T = \text{gen}\{(1 \ 1 \ 0 \ 1), (1 \ 4 \ 2 \ 0), (-4 \ -5 \ -1 \ -3)\}$

11.

- a. $\sigma(T) = \{4, 2\}$ $B = \{(1, 1), (-1, 1)\}$
 b. $\sigma(T) = \{4\}$ $B = \{(1, 0)\}$
 c. $\sigma(T) = \{-1, 1, 3\}$ $B = \{(1, 0, 0), (-3/2, 1, -1), (2, 1, -2)\}$
 d. $\sigma(T) = \{1, 2, 3\}$ $B = \{(1, 0, 0), (0, 1, 2), (1, 0, 1)\}$

12.

- a. $k = -2$ b. $\sigma(T) = \{1, -3\}$

13.

- $B = \{(1, 4/3, -1), (0, 1, 0), (0, 1, -1)\}$ b. $\text{Im } T = \mathbb{R}^3$

14.

- a. $\sigma(A) = \{1, -1\}$ $P = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$ b. $\sigma(A) = \{-1, 3, 2\}$ $P = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

- c. $\sigma(A) = \{0, 6(\text{doble})\}$. $P = \begin{pmatrix} 1 & -1 & 2 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$

- d. $\sigma(A) = \{0, -\frac{3+\sqrt{41}}{2}, \frac{\sqrt{41}-3}{2}\}$ $P = \begin{pmatrix} 4 & \frac{3+\sqrt{41}}{4} & \frac{3-\sqrt{41}}{4} \\ 1 & 1 & 1 \\ 0 & -\frac{3+\sqrt{41}}{2} & \frac{-3+\sqrt{41}}{2} \end{pmatrix}$