

1. $F'(P, \vec{v}) = -26$

3. a) $F'_x(-1, 2) = -14, F'_y(-1, 2) = 4$

b) $F'_x(1, 3) = -2, F'_y(1, 3) = -3$

4. a) $F'_x(x, y) = \frac{x}{\sqrt{x^2 - y^2}}, F'_y(x, y) = 6y - \frac{y}{\sqrt{x^2 - y^2}}$

b) $F'_x(x, y) = \frac{2\sqrt{y^2 - x^2} + 2\frac{x^2}{\sqrt{y^2 - x^2}}}{y^2 - x^2}$

$F'_y(x, y) = \frac{2xy}{\sqrt{(y^2 - x^2)^3}}$

c) $F'_x(x, y, z) = \frac{5}{z} e^{\frac{x+y^2}{z}}, F'_y(x, y, z) = \frac{10y}{z} e^{\frac{x+y^2}{z}}, F'_z(x, y, z) = -\frac{5(x+y^2)}{z^2} e^{\frac{x+y^2}{z}}$

d) $F'_u(u, v) = \frac{e^u}{e^{u+e^v}}, F'_v(u, v) = \frac{e^v}{e^{u+e^v}}$

e) $F'_u(u, v) = v u^{v-1}, F'_v(u, v) = u^v \ln(u)$

f) $F'_x(x, y) = -3y(3-x)^{3y-1}, F'_y(x, y) = 3(3-x)^{3y} \ln(3-x)$

g) $F'_x(x, y) = \sin y e^{x \sin y}, F'_y(x, y) = x \cos y e^{x \sin y}$

h) $F'_s(s, u, v) = -u^3 \sin\left(\frac{s}{v}\right) \cdot \frac{1}{v}, F'_u(s, u, v) = 3u^2 \cos\left(\frac{s}{v}\right), F'_v(s, u, v) = \frac{su^3}{v^2} \sin\left(\frac{s}{v}\right)$

i) $F'_x(x, y) = \frac{2}{2x+y}, F'_y(x, y) = \frac{1}{2x+y}$

6. a) $\nabla F(1, -1) = \left(-\frac{19}{6}, 0\right)$

b) $\nabla F(-1, 2) = (-12, 9)$

c) $\nabla F(1, 1, 1) = (2, 2, 3)$

8. a) $F'(P, \vec{v}) = -6 + \frac{\sqrt{3}}{2}$

b) $F'(P, \vec{v}) = \frac{2}{\sqrt{5}}$

c) $F'(P, \vec{v}) = -\frac{23}{\sqrt{50}}$

9. (Respuesta parcial) a) $F'_{\max}(P) = \sqrt{2}, F'_{\min}(P) = -\sqrt{2}$

b) $F'_{\max}(P) = \sqrt{50}, F'_{\min}(P) = -\sqrt{50}$

c) $F'_{\max}(P) = \sqrt{241}, F'_{\min}(P) = -\sqrt{241}$

d) $F'_{\max}(P) = 3, F'_{\min}(P) = -3$

e) $G'_{\max}(P) = \sqrt{2}, G'_{\min}(P) = -\sqrt{2}, \vec{V}_{nula} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ o $\vec{V}_{nula} = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

f) $H'_{\max}(P) = \sqrt{17}, H'_{\min}(P) = -\sqrt{17}$

10. Comenzará a rodar en la dirección $\left(\frac{-80}{\sqrt{11584}}, \frac{-72}{\sqrt{11584}}\right)$ - Debe caminar en la dirección $\left(\frac{80}{\sqrt{11584}}, \frac{72}{\sqrt{11584}}\right)$

11. En la dirección dada por $\left(\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}}, 0\right)$.

12. Debe nadar en la dirección dada por el vector $\left(\frac{4}{11}, 1\right)$. La profundidad no cambia en la dirección $\left(-1, \frac{4}{54}\right)$

13. $\left(\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}}\right)$

14.

a)

$$F''_{xx}(x, y) = \frac{(-x^2 + y^2)}{(x^2 - y^2)^2} \quad F''_{xy}(x, y) = \frac{2xy}{(x^2 - y^2)^2},$$

$$F''_{yy}(x, y) = \frac{(x^2 - y^2)}{(x^2 - y^2)^2}$$

b)

$$F''_{xx}(x, y) = 9y(y - 1)(3x)^{y-2}$$

$$F''_{xy}(x, y) = 3(3 - x)^{y-1} + 3y(3x)^{y-1}\ln(3x)$$

$$F''_{yy}(x, y) = \ln^2(3x) \cdot (3x)^{y-1}$$

c)

$$F''_{xx}(x, y) = -\frac{y^2}{x^2}, \quad F''_{xy}(x, y) = \frac{2y}{x}, \quad F''_{yy}(x, y) = 2\ln(xy) + 3$$

$$d) \quad F''_{xx}(x, y, z) = 0 = F''_{xy}(x, y, z) = F''_{xz}(x, y, z)$$

$$F''_{yy}(x, y, z) = -6yz, \quad F''_{yz}(x, y, z) = -3y^2, \quad F''_{zz}(x, y, z) = 16$$