1. a. Es transformación lineal b. No es TL c. Es TL d. Es TL

2. a. T(x, y) = (3x – 2y, -x + 4y) b. $T(x, y) = \left(-\frac{x+y}{3}, \frac{2x+2y}{3}\right)$

c. No existe una TL que cumpla lo pedido. d. T(x, y, z) = (8x + 9y + 6z, -5x-9y – 5z)
e. T(x, y, z) = (z, x – z, 0).

3. a. i. $(0, 0) \in$ Nu T ii. $(2, 3) \in$ Nu T iii. $(3, -2) \notin$ Nu T iv. $\left(1, \frac{1}{3}\right) \notin$ Nu T

b. i. $(3, -6) \in$ Im T ii. $(2, 3) \notin$ Im T iii. $(1, -2) \in$ Im T iv. $(4, -3) \notin$ Im T

4. a. Nu T = {(0 0)} (no existe base del núcleo) Im T = gen{(1 0 0) (-2 -5 0)}
b. Nu T = gen {(0 1 1)} Im T = gen {(-2 0 1) (1 -1 0)}
c. Nu T = gen {(3 0 1 0)(2 1 0 0)} Im T = gen {(3 0 1) (0 1 0)}

5. a. T(x, y) = (-x, 3x, 4x) b. T(x, y, z) = (-x + 2z - y, 0, 0) c. No existe

6. Verdadero

7. a. $M(T) = \begin{pmatrix} \frac{1}{3} & \frac{2}{5} \\ -1 & 5 \end{pmatrix}$ b. $M(T) = \begin{pmatrix} 1 & 1 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{4} \\ 0 & 1 & -1 \end{pmatrix}$ c. $M(T) = \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ -4 & 0 \end{pmatrix}$

8. a. T(1 -5 3) = (7 13 21), T(0 0 0) = (0 0 0), T(1 -1 1) = (3 3 5)
b. Im T = R^3, Nu T = {(0 0 0)}
c. T(x, y, z) = (2x – y, x – 3y – z, -3y + 2z)

9.
a. $\sigma(T) = \{4, 2\}$ B = {(1, 1)(-1, 1)}
b. $\sigma(T) = \{4\}$ B = {(1, 0)}
c. $\sigma(T) = \{-1, 1, 3\}$ B = {(1, 0, 0), (-3/2, 1, -1), (2, 1, -2)}
d. $\sigma(T) = \{1, 2, 3\}$ B = {(1, 0, 0), (0, 1, 2), (1, 0, 1)}

10.
a. k = - 2 b $\sigma(T) = \{1, -3\}$

11.
a. $\sigma(A) = \{1, -1\}$ $P = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$ b. $\sigma(A) = \{-1, 3, 2\}$ $P = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

c. $\sigma(A) = \{0, 6(\text{doble})\}$. $P = \begin{pmatrix} 1 & -1 & 2 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$

d. $\sigma(A) = \{0, -\frac{3+\sqrt{41}}{2}, \frac{\sqrt{41}-3}{2}\}$ $P = \begin{pmatrix} 4 & \frac{3+\sqrt{41}}{4} & \frac{3-\sqrt{41}}{4} \\ 1 & 1 & 1 \\ 0 & -\frac{3+\sqrt{41}}{2} & \frac{-3+\sqrt{41}}{2} \end{pmatrix}$