1. a. Es transformación lineal b. No es TL c. Es TL d. Es TL e. Es TL

2. a. T(x, y) = (3x – 2y, -x + 4y) b. $T(x, y) = \left(-\frac{x+y}{3}, \frac{2x+2y}{3}\right)$

c. No existe una TL que cumpla lo pedido. d. T(x, y, z) = (8x + 9y + 6z, -5x-9y – 5z)
e. T(x, y, z) = (z, x – z, 0).

3. a. T(x;y) = (-x; y) b. T(x;y) = (-y; -x) c. T(x;y) = (x + 3y; y)

4. a. i. (0, 0) ∈ Nu T ii. (2, 3) ∈ Nu T iii. (3, -2) ∉ Nu T iv. $\left(1, \frac{1}{3}\right) \notin$ Nu T

b. i. (3, -6) ∈ Im T ii. (2, 3) ∉ Im T iii. (1, -2) ∈ Im T iv. (4, -3) ∉ Im T

5. a. i. $\begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix} \in$ Nu T b. $\begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix} \notin Nu\ T$ c. $\begin{pmatrix} 1 & -2 \\ 0 & \frac{1}{2} \end{pmatrix} \in$ Nu T

b. i.(1,2) ∈ Im T ii. (0, 1) ∉ Im T iii. (-4, -8) ∈ Im T

6. a. Nu T = {(0 0)} (no existe base del núcleo) Im T = gen{(1 0 0) (-2 -5 0)}
b. Nu T = gen {(0 1 1)} Im T = gen {(-2 0 1) (1 -1 0)}
c. Nu T = gen {(3 0 1 0)(2 1 0 0)} Im T = gen {(3 0 1) (0 1 0)}

7. a. T(x, y) = (-x, 3x, 4x) b. T(x, y, z) = (-x + 2z - y, 0,0) c. No existe

8. Verdadero

9. a. $M(T) = \begin{pmatrix} \frac{1}{3} & \frac{2}{5} \\ -1 & 5 \end{pmatrix}$ b. $M(T) = \begin{pmatrix} 1 & 1 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{4} \\ 0 & 1 & -1 \end{pmatrix}$ c. $M(T) = \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ -4 & 0 \end{pmatrix}$

10.
a. T(1 -5 3) = (7 13 21), T(0 0 0) = (0 0 0), T(1 -1 1) = (3 3 5)
b. Im T = R^3, Nu T = {(0 0 0)}
c. T(x, y, z) = (2x – y, x – 3y – z, -3y + 2z)

13. a. $M_{BB'} = \begin{pmatrix} 4 & -3 \\ 4 & -2 \end{pmatrix}$ b. $M_{BB'} = \begin{pmatrix} 1 & 1 & -2 \\ 2 & -2 & 0 \end{pmatrix}$ c. $M_{BB'} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{pmatrix}$

14.

$T(v_1 + v_4) = 2w_1 - w_2 + 3w_3$, $T(-v_2 + 2v_3) = 4w_1 + w_2 + 9w_3$,
$T(v_1 + v_2 - v_3) = -2w_1 - w_2 - 5w_3$

15.

a

. $C(B, B') = \begin{pmatrix} \frac{7}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$ b. $\lfloor v \rfloor_{B'} = \begin{pmatrix} \frac{29}{2} \\ \frac{5}{2} \end{pmatrix}$ c. $\lfloor v' \rfloor_B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

16.

$M_{BE} = \begin{pmatrix} 4 & 0 \\ 0 & -1 \end{pmatrix}$

17.

a. $\sigma(A) = \{4,2\}$ $B = \{(1,1), (-1,1)\}$

b. $\sigma(A) = \{4\}$ $S_4 = gen\{(1,0)\}$

c. $\sigma(A) = \{3, -1, 1\}$ $B = \{(-2,-1,2), (3,-2,2), (1,0,0)\}$

d. $\sigma(A) = \{1, 2, 3\}$ $B = \{(1,0,0), (0,1,2), (1,0,1)\}$

18.

a. k = - 2 b $\sigma(T) = \{1, -3\}$.

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19.

a. $\sigma(A) = \{1, -1\}$ P = $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$ b. $\sigma(A) = \{-1, 3, 2\}$ P = $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

c. $\sigma(A) = \{0, 6(\text{doble})\}$. P = $\begin{pmatrix} 1 & -1 & 2 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$

d. $\sigma(A) = \{0, -\frac{3+\sqrt{41}}{2}, \frac{\sqrt{41}-3}{2}\}$ P= $\begin{pmatrix} 4 & \frac{3+\sqrt{41}}{4} & \frac{3-\sqrt{41}}{4} \\ 1 & 1 & 1 \\ 0 & -\frac{3+\sqrt{41}}{2} & \frac{-3+\sqrt{41}}{2} \end{pmatrix}$