Números complejos

1. i. a. $3-6i$ b. $1+\frac{5}{2}i$ c. $\frac{9}{2}+4i$ d. $3+\frac{3}{2}i$

4. a. Al sumar $2-i$, el número real z se traslada dos unidades hacia la derecha y una unidad hacia abajo. Al multiplicarlo por -3i el módulo de z se triplica y se produce una rotación de $\frac{3}{2}\pi$.

   b. Rotación de intensidad $\frac{\pi}{4}$.

   c. Rotación de intensidad $\frac{3}{2}\pi$ y traslación de dos unidades hacia la izquierda.

5. a. $z_1=\sqrt[6]{2}\left(\cos\frac{\pi}{12}+i\operatorname{sen}\frac{\pi}{12}\right)$, $z_2=\sqrt[6]{2}\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)$, $z_3=\sqrt[6]{2}\left(\cos\frac{17}{12}\pi+i\operatorname{sen}\frac{17}{12}\pi\right)$

   b. $z_1=2,\ z_2=-2,\ z_3=2i,\ z_4=-2i$

   c. $z_1=-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i,\ z_2=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$

   d.

   $z_1=\cos\frac{\pi}{10}+i\operatorname{sen}\frac{\pi}{10},\ z_2=i,\ z_3=\cos\frac{9}{10}\pi+i\operatorname{sen}\frac{9}{10}\pi,\ z_4=\cos\frac{13}{10}\pi+i\operatorname{sen}\frac{13}{10}\pi,\ z_5=\cos\frac{17}{10}\pi+i\operatorname{sen}\frac{17}{10}\pi$

   e. $[(1+2i)+(3-4i)](2-i)^2=2(2-i)^3$

   f. $(1+3i)/(3-i)=i$

   g. $(1+i)^{14}=128e^{\frac{7}{2}\pi i}$

   h. $i^{84}=1$

   i. $(-i)^{35}=i$

6. a. $S=\{1,2\}$

   b. $S=\{-2,\ 1+\sqrt{3}i,\ 1-\sqrt{3}i\}$

   c. $S=\left\{\sqrt[4]{2}e^{\frac{5}{12}\pi i},\ \sqrt[4]{2}e^{\frac{11}{12}\pi i},\ \sqrt[4]{2}e^{\frac{17}{12}\pi i},\ \sqrt[4]{2}e^{\frac{23}{12}\pi i}\right\}$

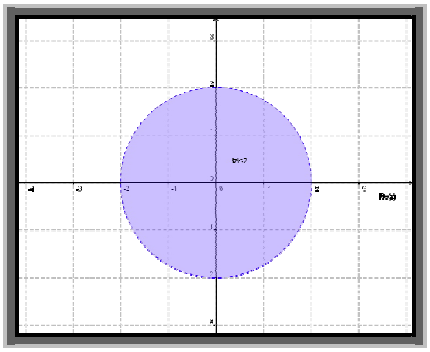
   d. $S=\left\{\sqrt[12]{2}e^{\frac{\pi}{24}i},\ \sqrt[12]{2}e^{\frac{3}{8}\pi i},\ \sqrt[12]{2}e^{\frac{17}{24}\pi i},\ \sqrt[12]{2}e^{\frac{25}{24}\pi i},\ \sqrt[12]{2}e^{\frac{11}{8}\pi i},\ \sqrt[12]{2}e^{\frac{41}{24}\pi i}\right\}$

   e. $S=\{3,3i,\ -3,-3i\}$

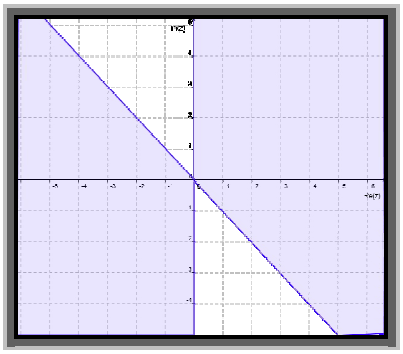
   f. $S=\{-2,2,\ 1+2i,\ 1-2i\}$

7.

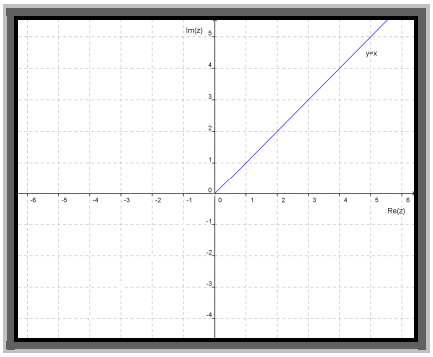
a.



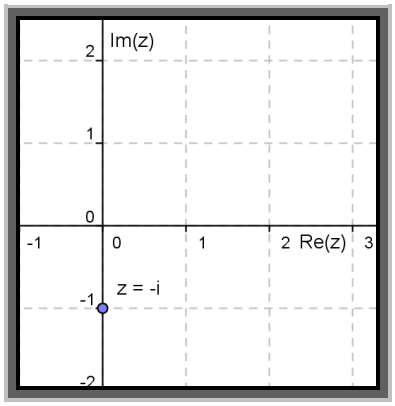
b.



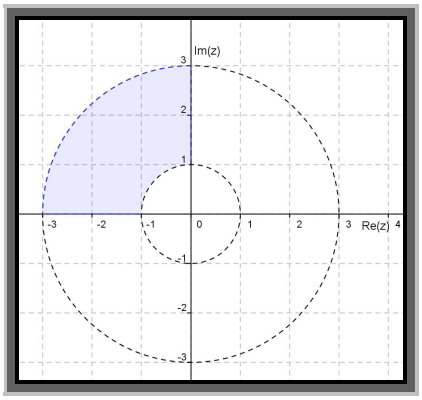
c.



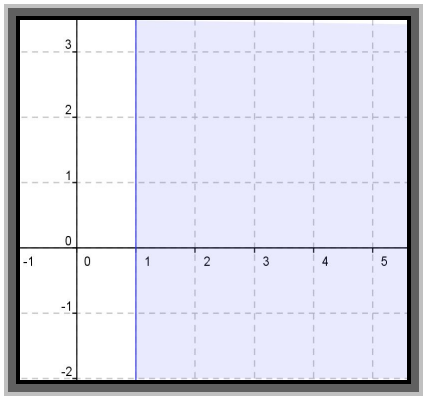
d.



e.



f.



8. a. $A = \{z \in C \ / \ Re(z) \leq 3 \ \wedge Im(z) \leq 2\}$

b. $A = \left\{z \in C / \ Im(z) \geq Re(z), \ \frac{\pi}{4} \leq arg(z) \leq \frac{\pi}{2}, Im(z) \leq 4\right\}$

c. $A = \left\{z \in C \ / |z| \leq 2 \ , \ \frac{\pi}{4} \leq arg(z) \leq \frac{7}{4}\pi\right\}$

Polinomios

9. i. $p(x) + r(x) = -x^3 + 6x - 2$ ii, $q(x) - r(x) = x^3 + x^2 - 5x + 3$ iii. $p(x)q(x) = x^3 + x$ iv. $p(x)+2r(x)-q(x)= -2x^3 - x^2+11x-5$

10. i. a) $\sigma(p_1) = \{-1, 0 , 1\}$ b) En Q[t], R[t] y C[t]: $p_1(t) = t(t-1)(t+1)$

ii. a) $\sigma(p_2) = \{0(doble), 1\}$ b) En Q[t], R[t] y C[t]: $p2(t) = -t^2(t-1)$

iii. a) $\sigma(p_3) = \{-\sqrt{2}, \ \sqrt{2}, \ \sqrt{2}i, \ -\sqrt{2}i\}$

b) En Q[t]: $p_3(t) = (t^2 - 2)(t^2 + 2)$

En R[t]: $p_3(t) = (t-\sqrt{2})(t+\sqrt{2})(t^2 + 2)$

En C[t]: $p_3(t) = (t-\sqrt{2})(t+\sqrt{2})(t-\sqrt{2}i)(t+\sqrt{2}i)$

iv. a) $\sigma(p_4) = \{-3, -2, \ 0, 2, 3\}$ b) En Q[t], R[t] y C[t]: $p_4(t)= t(t-2)(t+2)(t-3)(t+3)$

v. a) $\sigma(p_1 + p_2) = \{0, 1\}$ b) En Q[t], R[t] y C[t]: $(p_1 + p_2)(t) = t(t-1)$

vi. a) $\sigma(p_1p_2) = \{0(triple), 1(doble), -1\}$ b) En Q[t], R[t] y C[t]: $(p_1p_2)(t) = -t^3(t-1)^2(t+1)$

11. a. $a = \frac{1}{5}$, $\sigma(p) = \{2, -1\}$

b. $a = -\frac{253}{6}$, $b = \frac{223}{3}$

c. a = -16, b = 12.

12. a. $c(w) = w$, $r(w) = 25w^2 - 1$

b. $c(u) = 0$, $r(u) = u^3 - 25u$

c. $c(t) = t^4 - t^2 + t + 1$, $r(t) = -t^2 + 2t - 1$

d. $c(t) = t^6 + t^4 + t^2 + 1$, $r(t) = 0$.

e. $c(x) = 3x + 15$, $r(x) = 0$

13. a. $p(t) = \frac{1}{9}(t+3)(t+1)(t-1)(t-3)$

b. $q(t) = \frac{1}{48}(t+3)^2(t+2)t^2(t-2)^2(t-3)$

c. $r(t) = \frac{1}{4}(t+2)(t+1)^2(t-1)^2(t-2)$

d. $s(t) = -\frac{1}{9}(t+2)^2(t+1)t^2(t-2)^2$

14. i. En R[t] y C[t]: $p(t) = (t-1)^2(t-2)(t-3)$

ii. En R[t]: $p(t) = (t^2 - 2t + 5)(t-1)(t-2)$ En C[t]: $p(t) = (t-(1+2i))(t-(1-2i))(t-1)(t-2)$

iii. En R[t] y C[t]: $p(t) = t(t-1)(t+1)(t-2)(t+2)$

iv. En R[t] y C[t]: $p(t) = t(t-1)(t-2)(t-3)(t-4)$

v. En R[t]: $p(t) = -2.(t^2+4)(t-3)$ En C[t]: $p(t) = -2.(t-2i)(t+2i)(t-3)$

vi. En R[t] y C[t]: $p(t) = -3t^3(t+2)^2(t-2)^2$