Números complejos

1. i. a. $3-6i$ b. $1+\frac{5}{2}i$ c. $\frac{9}{2}+4i$ d. $3+\frac{3}{2}i$

3. a. $z_1=\sqrt[6]{2}\left(\cos\frac{\pi}{12}+i\,sen\frac{\pi}{12}\right)$, $z_2=\sqrt[6]{2}\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)$, $z_3=\sqrt[6]{2}\left(\cos\frac{17}{12}\pi+i\,sen\frac{17}{12}\pi\right)$

b. $z_1=2,\ z_2=-2,\ z_3=2i,\ z_4=-2i$

c. $z_1=-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i,\ z_2=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$

d.

$z_1=\cos\frac{\pi}{10}+i\,sen\frac{\pi}{10},\ z_2=i,\ z_3=\cos\frac{9}{10}\pi+i\,sen\frac{9}{10}\pi,\ z_4=\cos\frac{13}{10}\pi+i\,sen\frac{13}{10}\pi,\ z_5=\cos\frac{17}{10}\pi+i\,sen\frac{17}{10}\pi$

e. $[(1+2i)+(3-4i)](2-i)^2=2(2-i)^3$

f. $(1+3i)/(3-i)=i$

g. $(1+i)^{14}=128e^{\frac{7}{2}\pi i}$

h. $i^{84}=1$

i. $(-i)^{35}=i$

j. $z_1=-2i,\quad z_2=2i$

4. a. $S=\{1,2\}$

b. $S=\left\{-2,\ 1+\sqrt{3}i,\ 1-\sqrt{3}i\right\}$

c. $S=\left\{\sqrt[4]{2}e^{\frac{5}{12}\pi i},\ \sqrt[4]{2}e^{\frac{11}{12}\pi i},\ \sqrt[4]{2}e^{\frac{17}{12}\pi i},\ \sqrt[4]{2}e^{\frac{23}{12}\pi i}\right\}$

d. $S=\left\{\sqrt[12]{2}e^{\frac{\pi}{24}i},\ \sqrt[12]{2}e^{\frac{3}{8}\pi i},\ \sqrt[12]{2}e^{\frac{17}{24}\pi i},\ \sqrt[12]{2}e^{\frac{25}{24}\pi i},\ \sqrt[12]{2}e^{\frac{11}{8}\pi i},\ \sqrt[12]{2}e^{\frac{41}{24}\pi i}\right\}$

e. $S=\{3,3i,\ -3,-3i\}$

f. $S=\{-2,2,\ 1+2i,\ 1-2i\}$

Polinomios

5. i. $p(x)+r(x)=-x^3+6x-2$ ii, $q(x)-r(x)=x^3+x^2-5x+3$ iii. $p(x)q(x)=x^3+x$ iv. $p(x)+2r(x)-q(x)=-2x^3-x^2+11x-5$

6. i. a) $\sigma(p_1)=\{-1,0,1\}$ b) En Q[t], R[t] y C[t]: $p_1(t)=t(t-1)(t+1)$

ii. a) $\sigma(p_2)=\{0(doble),1\}$ b) En Q[t], R[t] y C[t]: $p2(t)=-t^2(t-1)$

iii. a) $\sigma(p_3)=\left\{-\sqrt{2},\ \sqrt{2},\ \sqrt{2}i,\ -\sqrt{2}i\right\}$

b) En Q[t]: $p_3(t)=(t^2-2)(t^2+2)$

En R[t]: $p_3(t)=(t-\sqrt{2})(t+\sqrt{2})(t^2+2)$

En C[t]: $p_3(t)=(t-\sqrt{2})(t+\sqrt{2})(t-\sqrt{2}i)(t+\sqrt{2}i)$

iv. a) $\sigma(p_4)=\{-3,-2,\ 0,2,3\}$ b) En Q[t], R[t] y C[t]: $p_4(t)=t(t-2)(t+2)(t-3)(t+3)$

v. a) $\sigma(p_1 + p_2) = \{0, 1\}$ b) En Q[t], R[t] y C[t]: $(p_1 + p_2)(t) = t(t-1)$

vi. a) $\sigma(p_1 p_2) = \{0(\text{triple}), 1(\text{doble}), -1\}$ b) En Q[t], R[t] y C[t]: $(p_1 p_2)(t) = -t^3(t-1)^2(t+1)$

7. a. $a = \frac{1}{5}$, $\sigma(p) = \{2, -1\}$

b. $a = -\frac{253}{6}$, $b = \frac{223}{3}$

c. a = -16, b = 12.

8. a. $c(w) = w$, $r(w) = 25w^2 - 1$

b. $c(u) = 0$, $r(u) = u^3 - 25u$

c. $c(t) = t^4 - t^2 + t + 1$, $r(t) = -t^2 + 2t - 1$

d. $c(t) = t^6 + t^4 + t^2 + 1$, $r(t) = 0$.

e. $c(x) = 3x + 15$, $r(x) = 0$

9. a. $p(t) = \frac{1}{9}(t+3)(t+1)(t-1)(t-3)$

b. $q(t) = \frac{1}{48}(t+3)^2(t+2)t^2(t-2)^2(t-3)$

c. $r(t) = \frac{1}{4}(t+2)(t+1)^2(t-1)^2(t-2)$

d. $s(t) = -\frac{1}{9}(t+2)^2(t+1)t^2(t-2)^2$

10. i.

En R[t] y C[t]: $p(t) = (t-1)^2(t-2)(t-3)$

ii. En R[t]: $p(t) = (t^2 - 2t + 5)(t-1)(t-2)$ En C[t]: $p(t) = (t-(1+2i))(t-(1-2i))(t-1)(t-2)$

iii. En R[t] y C[t]: $p(t) = t(t-1)(t+1)(t-2)(t+2)$

iv. En R[t] y C[t]: $p(t) = t(t-1)(t-2)(t-3)(t-4)$

v. En R[t]: $p(t) = -2.(t^2+4)(t-3)$ En C[t]: $p(t) = -2.(t-2i)(t+2i)(t-3)$

vi. En R[t] y C[t]: $p(t) = -3t^3(t+2)^2(t-2)^2$