Derivación de campos vectoriales. Regla de la cadena. Derivación implícita.

1. a. $J\bar{F}(1;2)=\begin{pmatrix}2 & 1\\ 4 & 2\end{pmatrix}$ b. $J\bar{F}(-1;0)=\begin{pmatrix}1 & 2\\ 1 & -1\\ 3 & 0\end{pmatrix}$ c. $J\bar{F}(0;0;0)=\begin{pmatrix}0 & 1 & 0\\ 1 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$

2. a. $(\bar{g}\circ F)(x;y) = ((x^2-3xy)^{1/3} ; x^2-3xy)$ campo vectorial
$(F\circ\bar{g})(t) = t^{2/3}-3t^{4/3}$ función escalar

b. $(g\circ F)(x;y) = 2^{-(x^2+xy)}\ln(x^2+xy)$ campo escalar

c. $(\bar{g}\circ F)(x;y;z) = (x+yz ; 2^{x+yz})$ campo vectorial
$(F\circ\bar{g})$ no es posible

3. a. $H(x;y) = x^2 + y - 2xy$ b. $H'_x(1;1) = 0$ $H'_y(1;1) = -1$

4. a. $z'(0) = 2e^3$ b. $\dfrac{dz}{dt} = -(\text{sen}t)\,\text{tg}(\sqrt{t}) + \dfrac{\cos t}{2\sqrt{t}\cos^2(\sqrt{t})}$

5. A. $\dfrac{\partial z}{\partial u}=\dfrac{e^{2u}-e^{-2u}}{e^{2u}+e^{-2u}}$ $\dfrac{\partial z}{\partial v}=\dfrac{1}{v}$ b- $dz = 2x\,dx$

6. a. $J(\bar{f}\circ G)(x;y;z)=\begin{pmatrix}9z(xz-y)^2 & -9(xz-y)^2 & 9x(xz-y)^2\\ 0 & 0 & 0\\ 2z\cos(2xz-2y) & -2\cos(2xz-2y) & 2x\cos(2xz-2y)\end{pmatrix}$

$(G\circ\bar{f})'(t) = 9t^2\text{sen}(2t) + 6t^3\cos(2t)$

b. $\nabla(G\circ\bar{F})(2;0) = (-32 ; 0)$

c. $J(\bar{F}\circ\bar{G})(-1;0)=\begin{pmatrix}0 & -2\\ 0 & -1\end{pmatrix}$ $J(\bar{G}\circ\bar{F})(-2;0)=\begin{pmatrix}0 & -8\\ 0 & 0\end{pmatrix}$

7. $\dfrac{dV}{dt} = -5890{,}49\,\dfrac{cm^3}{h}$

8. $\nabla G(1,-1)=(2,-1)$

9. a. $\nabla H(P_0) = (25,\ 15)$ b. $\dfrac{105\sqrt{13}}{13}$

10. $\dfrac{9}{2\sqrt{20}}$

11. - a. $J(\bar{F}\circ\bar{G})(1;0)=\begin{pmatrix}10 & 22\\ 2e+2 & 2e+5\\ 6 & 9\end{pmatrix}$ b. $\dfrac{-2e^3+5}{\sqrt{29}}$

13. a. Recta normal: $\bar{X} = (2,1)\lambda + (1;0)$, $\lambda\in R$ Recta tangente: $\bar{X} = (-1;2)\lambda + (1,0)$, $\lambda\in R$

b. Recta normal: $X = \alpha(0,-3)+(0,1)$ *Recta tangente* $X = \alpha(-3,0)+(0,1)\ \alpha\in R$

c. Recta normal: $\bar{X}=\lambda(1;8)+(-1;1)$, $\lambda\in R$ Recta tangente: $\bar{X}=\lambda(-8;1)+(-1;1)$, $\lambda\in R$

14. a. Recta normal: $\bar{X} = \lambda(1;0;0)$, $\lambda\in R$ Plano tangente: $x = 3$

b. Recta normal: $\bar{X} = \lambda(1\,;\,1\,;\,-\sqrt{2}) + (1\,;\,1;\,\sqrt{2})\ ,\ \lambda \in R$ Plano tangente: $x + y - \sqrt{2}z = 0$

c. Recta normal: $\bar{X} = \lambda(4\,;\,0\,;\,0) + (2\,;\,0\,;\,8)\quad \lambda \in R$ Plano tangente: $x = 2$

15. A. $(2\sqrt{2}, 0, 2\sqrt{2})$ b. $\vec{n} = (2, 0, 0)$ c. $\vec{n} = (-2, 0, 1)$

16. $\vec{v} = (-495, 50, 76)$

17. b. 1) $z_x'(1;1) = -5/3$ $z_y'(1;1) = -1/2$ 2) $z_x'(1;1) = 0$ $z_y'(1;1) = 1/2$

18. $dz(0;0) = -1/2\,\Delta x - 3/2\,\Delta y$

19. a. $y_x' = \dfrac{y\cos x - e^x}{z - \operatorname{sen} x}$ $y_z' = -\dfrac{y}{z - \operatorname{sen} x}$

b. $y_x' = \dfrac{y}{y - x}$ $y_z' = \dfrac{z}{y - x}$

20. b. $F'_{\max}(2, 3) = \frac{2\sqrt{2}}{5}$ c. $z(2.01\,;\,3.02) \cong 1.012$

21. $h'(1) = 13/2$

22. $G(2.95\,;\,3.01) \cong -2.045$

23. $\vec{\nabla} z\left(\frac{2}{3}, -\frac{1}{3}\right) = (-1/2, 0)$