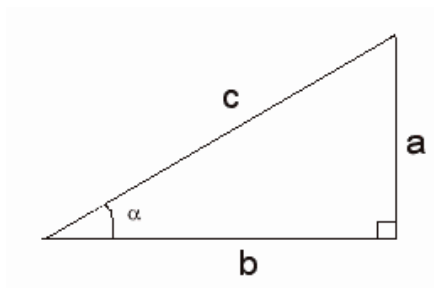# TRIGONOMETRIA

Razones trigonométricas elementales

$$\operatorname{sen}(\alpha) = \frac{\text{cateto opuesto}}{\text{hipotenusa}} = \frac{a}{c}$$

$$cos\,(\alpha) = \frac{\text{cateto contiguo}}{\text{hipotenusa}} = \frac{b}{c}$$

$$\operatorname{tg}(\alpha) = \frac{\text{cateto opuesto}}{\text{cateto contiguo}} = \frac{a}{b}$$

Fórmulas fundamentales

1) $\boxed{\operatorname{sen}^2\alpha + cos^{\,2}\alpha = 1}$
2) $\boxed{1 + \operatorname{tg}^2\alpha = \dfrac{1}{cos^{\,2}\alpha}}$
3) $\boxed{\operatorname{tg}\alpha = \dfrac{\operatorname{sen}\alpha}{cos\,\alpha}}$
4) $\boxed{\operatorname{cotg}\alpha = \dfrac{1}{\operatorname{tg}\alpha}}$
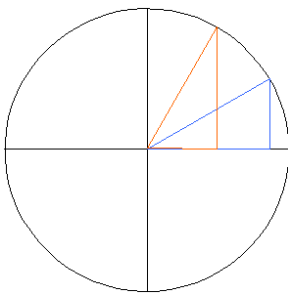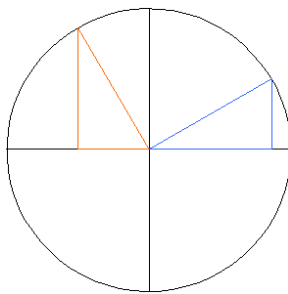
Razones trigonométricas de ángulos conocidos					
	0º	30º	45º	60º	90º
Seno	0	$\dfrac{1}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{\sqrt{3}}{2}$	1
Coseno	1	$\dfrac{\sqrt{3}}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{1}{2}$	0
Tangente	0	$\dfrac{\sqrt{3}}{3}$	1	$\sqrt{3}$	No existe
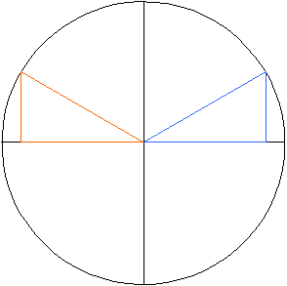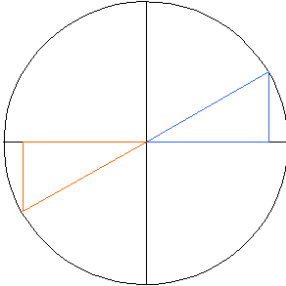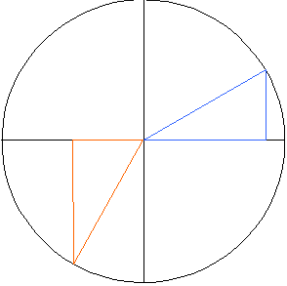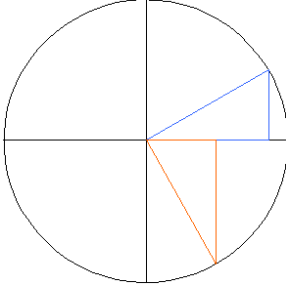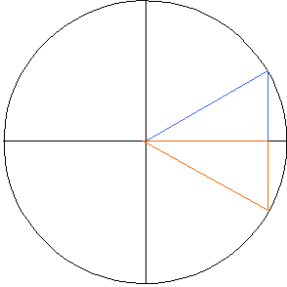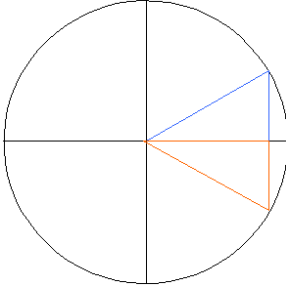
Signo según el cuadrante

Razones trigonométricas de la suma y de la diferencia	
Suma	**Diferencia**
$\operatorname{sen}(\alpha + \beta) = \operatorname{sen}\alpha\ cos\ \beta + cos\ \alpha\ \operatorname{sen}\beta$	$\operatorname{sen}(\alpha - \beta) = \operatorname{sen}\alpha\ cos\ \beta - cos\ \alpha\ \operatorname{sen}\beta$
$cos\ (\alpha + \beta) = cos\ \alpha\ cos\ \beta - \operatorname{sen}\alpha\ \operatorname{sen}\beta$	$cos\ (\alpha - \beta) = cos\ \alpha\ cos\ \beta + \operatorname{sen}\alpha\ \operatorname{sen}\beta$
$\operatorname{tg}(\alpha + \beta) = \dfrac{\operatorname{tg}\alpha\ +\ \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha\ \operatorname{tg}\beta}$	$\operatorname{tg}(\alpha - \beta) = \dfrac{\operatorname{tg}\alpha\ -\ \operatorname{tg}\beta}{1 + \operatorname{tg}\alpha\ \operatorname{tg}\beta}$

Razones trigonométricas del ángulo doble
$\operatorname{sen}(2\alpha) = 2\operatorname{sen}\alpha\ cos\ \alpha$
$cos\ (2\alpha) = cos\ ^2\alpha\ -\ \operatorname{sen}^2\alpha$
$\operatorname{tg}(2\alpha) = \dfrac{2\operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha}$

Razones trigonométricas del ángulo mitad
$\operatorname{sen}(\dfrac{\alpha}{2}) = \pm\sqrt{\dfrac{1 - cos\ \alpha}{2}}$
$cos\ (\dfrac{\alpha}{2}) = \pm\sqrt{\dfrac{1 + cos\ \alpha}{2}}$
$\operatorname{tg}(\dfrac{\alpha}{2}) = \pm\sqrt{\dfrac{1 - cos\ \alpha}{1 + cos\ \alpha}}$
El signo $\pm$ dependerá del cuadrante donde esté $\dfrac{\alpha}{2}$

Razones trigonométricas de $90 - \alpha$	Razones trigonométricas de $90 + \alpha$
$\operatorname{sen}(90 - \alpha) = cos\ \alpha$	$\operatorname{sen}(90 + \alpha) = cos\ \alpha$
$cos\ (90 - \alpha) = \operatorname{sen}\alpha$	$cos\ (90 + \alpha) = -\operatorname{sen}\alpha$
$\operatorname{tg}(90 - \alpha) = \dfrac{1}{\operatorname{tg}\alpha}$	$\operatorname{tg}(90 + \alpha) = -\dfrac{1}{\operatorname{tg}\alpha}$

Razones trigonométricas de $180 - \alpha$	Razones trigonométricas de $180 + \alpha$
$\operatorname{sen}(180 - \alpha) = \operatorname{sen} \alpha$	$\operatorname{sen}(180 + \alpha) = -\operatorname{sen} \alpha$
$cos\,(180 - \alpha) = -cos\,\alpha$	$cos\,(180 + \alpha) = -cos\,\alpha$
$\operatorname{tg}(180 - \alpha) = -\operatorname{tg} \alpha$	$\operatorname{tg}(180 + \alpha) = \operatorname{tg} \alpha$

Razones trigonométricas de $270 - \alpha$	Razones trigonométricas de $270 + \alpha$
$\operatorname{sen}(270 - \alpha) = -cos\,\alpha$	$\operatorname{sen}(270 + \alpha) = -cos\,\alpha$
$cos\,(270 - \alpha) = -\operatorname{sen} \alpha$	$cos\,(270 + \alpha) = \operatorname{sen} \alpha$
$\operatorname{tg}(270 - \alpha) = \dfrac{1}{\operatorname{tg} \alpha}$	$\operatorname{tg}(270 + \alpha) = -\dfrac{1}{\operatorname{tg} \alpha}$

Razones trigonométricas de $360 - \alpha$	Razones trigonométricas de $-\alpha$
$\operatorname{sen}(360 - \alpha) = -\operatorname{sen} \alpha$	$\operatorname{sen}(-\alpha) = -\operatorname{sen} \alpha$
$cos\,(360 - \alpha) = cos\,\alpha$	$cos\,(-\alpha) = cos\,\alpha$
$\operatorname{tg}(360 - \alpha) = -\operatorname{tg} \alpha$	$\operatorname{tg}(-\alpha) = -\operatorname{tg} \alpha$

Teorema del seno

$$\frac{a}{\operatorname{sen}\widehat{A}} = \frac{b}{\operatorname{sen}\widehat{B}} = \frac{c}{\operatorname{sen}\widehat{C}}$$
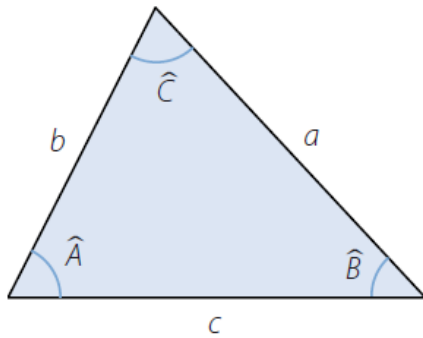
$$\widehat{A} + \widehat{B} + \widehat{C} = 180º$$

Teorema del coseno

$$a^2 = b^2 + c^2 - 2\,b\,c \cdot cos\,\widehat{A}$$

$$b^2 = a^2 + c^2 - 2\,a\,c \cdot cos\,\widehat{B}$$

$$c^2 = a^2 + b^2 - 2\,a\,b \cdot cos\,\widehat{C}$$

$$\widehat{A} + \widehat{B} + \widehat{C} = 180º$$