1. a) $F'(P,\vec{v})$ = -8 b) $F'(P,\vec{v})$ = - 16

2. a) $F'_x(-1,2) = -14,\ F'_y(-1,2) = 4$
   b) $F'_x(1,3) = -2,\ F'_y(1,3) = -3$

3.. a) $F'_x(x,y) = \frac{x}{\sqrt{x^2-y^2}},\quad F'_y(x,y) = 6y - \frac{y}{\sqrt{x^2-y^2}}$

   b) $F'_x(x,y) = \frac{2\sqrt{y^2-x^2}+2\frac{x^2}{\sqrt{y^2-x^2}}}{y^2-x^2},\ F'_y(x,y) = \frac{2xy}{\sqrt{(y^2-x^2)^3}}$

   c) $F'_x(x,y,z) = \frac{5}{z}e^{\frac{x+y^2}{z}},\ F'_y(x,y,z) = \frac{10y}{z}e^{\frac{x+y^2}{z}},\ F'_z(x,y,z) = -\frac{5(x+y^2)}{z^2}e^{\frac{x+y^2}{z}}$

   d) $F'_s(r,s,t) = \left(rs + t^2 s^{\frac{1}{2}}\right)^{-\frac{2}{3}}\left(r + \frac{1}{2}t^2 s^{-\frac{1}{2}}\right) + rtln2$

   $F'_r(r,s,t) = (rs + t^2 s^{\frac{1}{2}})^{-\frac{2}{3}}s + stln2$

   $F'_t(r,s,t) = (rs + t^2 s^{\frac{1}{2}})^{-2/3}2ts^{1/2} + rsln2$

   e) $F'_x(x,y,z) = \frac{1}{cos^2\left(\frac{x+y^2}{z^3}\right)}\cdot\frac{1}{z^3}$, $F'_y(x,y,z) = \frac{1}{cos^2\left(\frac{x+y^2}{z^3}\right)}\cdot\frac{2y}{z^3},\ F'_z 2(x,y,z) = \frac{1}{cos^2\left(\frac{x+y^2}{z^3}\right)}\cdot\frac{-3(x+y^2)}{z^4}$

   f) $F'_u(u,v) = \frac{e^u}{e^u+e^v},\ F'_v(u,v) = \frac{e^v}{e^u+e^v}$

   g) $F'_u(u,v) = vu^{v-1},\quad F'_v(u,v) = u^v \ln(u)$

   h) $F'_x(x,y) = -3y(3-x)^{3y-1},\quad F'_y(x,y) = 3(3-x)^{3y}\ln(3-x)$

4. a) $F'(P,\vec{v})$ = -6+$\frac{\sqrt{3}}{2}$ b) $F'(P,\vec{v}) = \frac{2}{\sqrt{5}}$ c) $F'(P,\vec{v}) = -\frac{23}{\sqrt{50}}$

5. a) $F'_{max}(P) = \sqrt{2},\ F'_{min}(P) = -\sqrt{2}$

   b) $F'_{max}(P) = \sqrt{50},\ F'_{min}(P) = -\sqrt{50}$

   c) $F'_{max}(P) = \sqrt{241},\ F'_{min}(P) = -\sqrt{241}$

6. (Respuesta parcial)

   a) $F'_{max}(P) = 3,\ F'_{min}(P) = --3$

   b) $G'_{max}(P) = \sqrt{2}, G'_{min}(P) = -\sqrt{2},\ \vec{V}_{nula} = (\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2})$

   $\vec{V}_{nula} = (-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2})$

   c) $H'_{max}(P) = \sqrt{17}, H'_{min}(P) = -\sqrt{17},$

7. Debe nadar en la dirección dada por el vector $\left(\frac{4}{11},1\right)$. La profundidad no cambia en la dirección $\left(-1,\frac{4}{54}\right)$
1/

8. En la dirección dada por el vector $1/\sqrt{12}$ (2, 2, 2)

9. En la dirección dada por $\left(\sqrt{\frac{1}{2}},\sqrt{\frac{1}{2}},\ 0\right)$.

10. a. $J\bar{F}(1;2) = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$ b. $J\bar{F}(-1;0) = \begin{pmatrix} 1 & 2 \\ 1 & -1 \\ 3 & 0 \end{pmatrix}$ c. $J\bar{F}(0;0;0) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

11. a. $(\bar{g} \circ F)(x;y) = ((x^2 - 3xy)^{1/3} ; x^2 - 3xy)$ campo vectorial

$(F \circ \bar{g})(t) = t^{2/3} - 3t^{4/3}$ función escalar

b. $(g \circ F)(x;y) = 2^{-(x^2+xy)} \ln(x^2 + xy)$ campo escalar

c. $(\bar{g} \circ F)(x;y;z) = (x + yz ; 2^{x+yz})$ campo vectorial

$(F \circ \bar{g})$ no es posible

12. a.

$$J(\bar{f} \circ G)(x;y;z) = \begin{pmatrix} 9z(xz-y)^2 & -9(xz-y)^2 & 9x(xz-y)^2 \\ 0 & 0 & 0 \\ 2z\cos(2xz-2y) & -2\cos(2xz-2y) & 2x\cos(2xz-2y) \end{pmatrix}$$

.

$(G \circ \bar{f})'(t) = 9t^2\text{sen}(2t) + 6t^3\cos(2t)$

b. $\bar{\nabla}(G \circ \bar{F})(2;0) = (-32 ; 0)$

c. $J(\bar{F} \circ \bar{G})(-1;0) = \begin{pmatrix} 0 & -2 \\ 0 & -1 \end{pmatrix}$ $J(\bar{G} \circ \bar{F})(-2;0) = \begin{pmatrix} 0 & -8 \\ 0 & 0 \end{pmatrix}$

13. $\nabla H(P_0) = (25,\ 15)$

14. a. $div(\bar{F}) = 2x + 2,\ \ rot(\bar{F}) = (0,0,0)$

b. $div(\bar{F}) = yz + 2zy + 2yx^2z^2,\ \ rot(F) = (z^2 - 2zx^2y^2, xy, 2y^2xz^2 - xz)$

c. $div(\bar{F}) = -sen(xy + z)x - 5,\ \ rot(F) = (-sen(xy) + z, 0, -ysen(xy + z))$

15´. a. $f(x,y) = e^x(x-1) + y + sen(y) + C$

b. $f(x,y) = xsen(y) + x^2y + C$

c. $f(x,y) = \ln(x-1) + \frac{1}{3}y^3 + 2y - \frac{4}{3}$

d. $f(x,\ y) = y^2x + \frac{x^2}{2} + \frac{y^2}{2} - 35$

18. a. Si b. Si c.Si