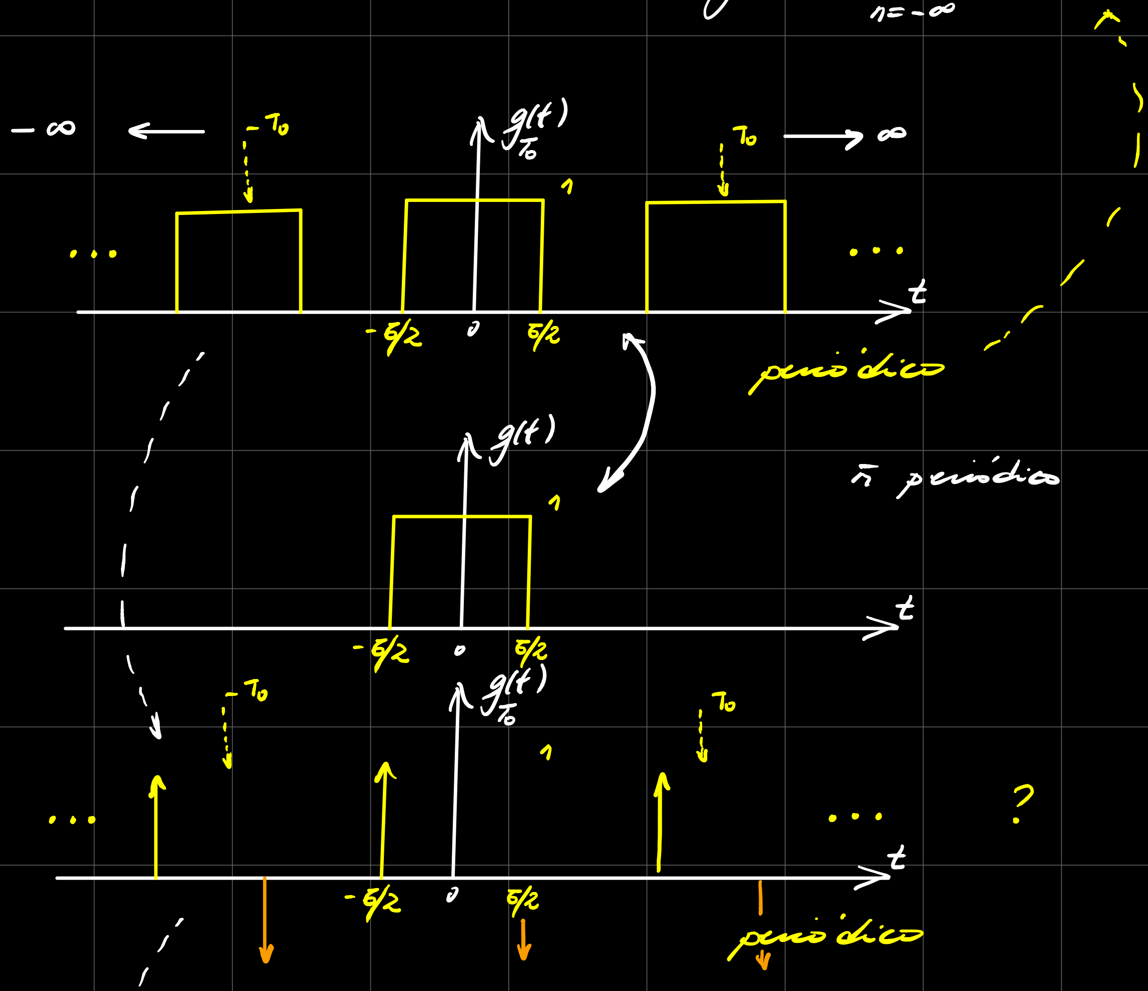
TRANSFORMADA DE FOURIER

Série de Fourier

$$D_n = \frac{1}{T_0}\int_{T_0} g(t)\, e^{-jn\omega_0 t}\, dt$$

$$g(t) = \sum_{n=-\infty}^{\infty} D_n\, e^{jn\omega_0 t}$$



$$\underset{\text{isolado}}{g(t)} = \lim_{T_0 \to \infty} \underset{\text{periódico}}{g_{T_0}(t)}$$

Série de $T_0 = 1s$, $T_0 = 10s$, $T_0 = 100s$, $T_0 \to \infty$

Fourier

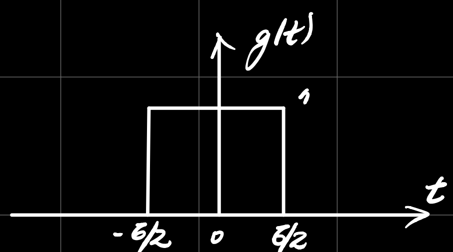
$$D_n = \underbrace{\frac{1}{T_0}\int_{T_0} g(t)\cdot e^{-jn\omega_0 t}\,dt}_{G(\omega)}$$

$T_0 \to \infty$

$$G(\omega) = \int_{-\infty}^{+\infty} g(t)\, e^{-j\omega t}\,dt$$

$$D_n = \frac{G(n\omega_0)}{T_0}$$

série transformada



$$D_n = \frac{1}{T_0}\int_{-\tau/2}^{+\tau/2} 1\cdot e^{-jn\omega_0 t}\,dt = \frac{1}{T_0}\left.\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right|_{-\tau/2}^{+\tau/2}$$

$$= \frac{1}{T_0}\cdot\left[\frac{-e^{-jn\omega_0\tau/2} + e^{+jn\omega_0\tau/2}}{+jn\omega_0}\right] = \frac{2j\,\mathrm{sen}\, n\omega_0\tau/2}{jn\omega_0\cdot T_0}$$

discreto!

$$D_n = \frac{2\,\tau/2}{T_0}\,\frac{\mathrm{sen}\, n\omega_0\tau/2}{n\omega_0\,\tau/2} = \frac{\tau}{T_0}\cdot\frac{\mathrm{sen}\,(n\omega_0\tau/2)}{n\omega_0\tau/2}$$

$$G(\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\,dt = \int_{-\tau/2}^{+\tau/2} e^{-j\omega t}\,dt =$$

$$= \left.\frac{e^{-j\omega t}}{-j\omega}\right|_{-\tau/2}^{+\tau/2} = \frac{-e^{-j\omega\tau/2} + e^{+j\omega\tau/2}}{+j\omega} = \frac{2j\,\mathrm{sen}\,\omega\tau/2}{+j\omega}$$

$$= \frac{2\,\tau/2\;\mathrm{sen}\,\omega\tau/2}{\omega\,\tau/2} = \tau\cdot\frac{\mathrm{sen}\,\omega\tau/2}{\omega\tau/2}$$

n...

contínuo!

$$\frac{\tau}{T_0}\cdot\frac{\text{sen}\,(n\omega_0\tau/2)}{n\omega_0\tau/2} \leftrightarrow \tau\cdot\frac{\text{sen}\,\omega\tau/2}{\omega\tau/2}$$

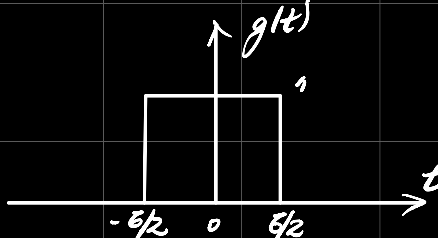
D_n $\longleftrightarrow$ $G(\omega)$

Série de Fourier — Transformada de Fourier

$$D_n = \frac{G(n\omega_0)}{T_0}$$

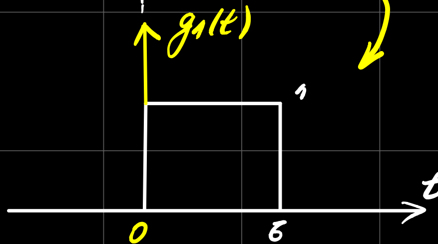
$$\frac{\tau}{T_0}\cdot\frac{\text{sen}\, n\omega_0\tau/2}{n\omega_0\tau/2} \quad \ldots \quad \omega_0 = \frac{2\pi}{T_0}$$

Deslocamento no tempo



$$g(t) \longleftrightarrow G(\omega)$$

$$\tau\cdot\frac{\text{sen}\,(\omega\tau/2)}{\omega\tau/2}$$



$$\longrightarrow g_1(t) = g(t-\tau/2)$$

$$\begin{cases} g_1(0) = g(-\tau/2) \\ g_1(\tau/2) = g(0) \\ g_1(\tau) = g(\tau/2) \end{cases}$$

$\downarrow$ $G_1(\omega)$

$$G_1(\omega) = \int_0^{\tau} 1\cdot e^{-j\omega t}\,dt = \left.\frac{e^{-j\omega t}}{-j\omega}\right|_0^{\tau} = \frac{e^{-j\omega\tau}-1}{-j\omega}$$

$$e^{-j\omega\tau/2}\frac{\left(e^{-j\omega\tau}-e^{-j\omega 0}\right)}{-j\omega} = e^{-j\omega\tau/2}\cdot\frac{\left(-e^{-j\omega\tau/2}+e^{+j\omega\tau/2}\right)}{+j\omega}$$

$$e^{-j\omega\tau/2}\cdot 2j\frac{\operatorname{sen}\omega\tau/2}{j\omega\tau/2}\cdot\tau/2 = \tau\cdot\frac{\operatorname{sen}\omega\tau/2}{\omega\tau/2}\cdot e^{-j\omega\tau/2}$$

$$g(t) \longleftrightarrow G(\omega) \qquad\qquad g_1(t) \longleftrightarrow G_1(\omega)$$

$$\tau\cdot\frac{\operatorname{sen}(\omega\tau/2)}{\omega\tau/2} \qquad\qquad \tau\cdot\frac{\operatorname{sen}\omega\tau/2}{\omega\tau/2}\cdot e^{-j\omega\tau/2}$$

$$g(t-\tau/2)$$

Generalizar:

$$G_1(\omega) = \int_{-\infty}^{\infty} g\underbrace{(t-t_0)}_{x}\,e^{-j\omega t}\,dt \qquad t-t_0 = x$$

$$t = x+t_0$$

$$dt = dx$$

$$\int_{-\infty}^{\infty} g(x)\,e^{-j\omega(x+t_0)}\,\underbrace{dx} = \underbrace{e^{-j\omega t_0}\cdot\int_{-\infty}^{\infty}\underbrace{g(x)\,e^{-j\omega x}\,dx}_{G(\omega)}}$$

$$\Longrightarrow \int_{-\infty}^{+\infty} g(t)\,e^{-j\omega t}\,dt = G(\omega) = \int_{-\infty}^{+\infty} g(x)\,e^{-j\omega x}\,dx$$

$$g(t) \Longleftrightarrow G(\omega)$$

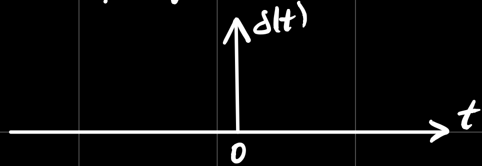
$$g(t-t_0) \Longleftrightarrow G(\omega)\cdot e^{-j\omega t_0}$$

$$\underbrace{\tau\cdot\frac{\operatorname{sen}(\omega\tau/2)}{\omega\tau/2}}_{g(t)} \quad G(\omega) \longleftrightarrow \underbrace{\tau\cdot\frac{\operatorname{sen}(\omega\tau/2)}{\omega\tau/2}\times e^{-j\omega t_0}}_{g(t-t_0)} \quad G_1(\omega)$$

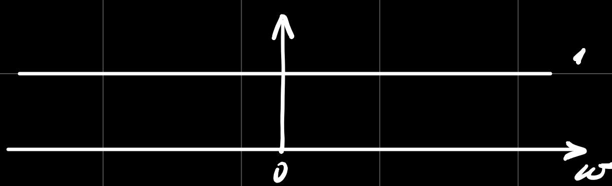
$$\mathcal{F}\{\delta(t)\} = G_\delta(w) = 1$$

Impulso

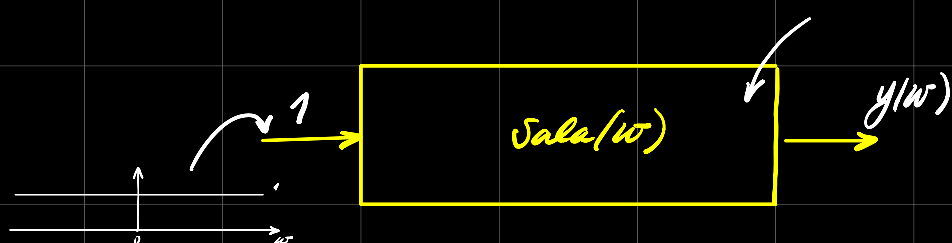
$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$



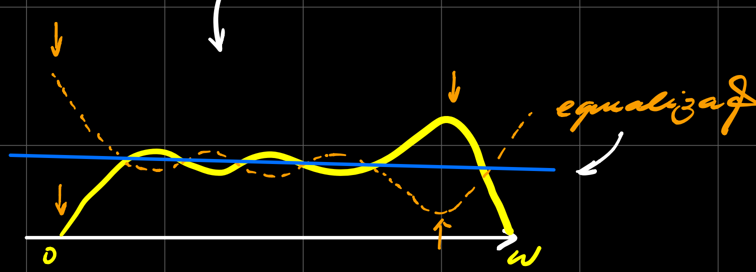
$$G_\delta(w) = \int_{-\infty}^{+\infty} \delta(t)\underbrace{e^{-jwt}}_{e^{-jw0}=1} dt = \int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$



(Sala de som) $\longleftrightarrow$ Equalizaç



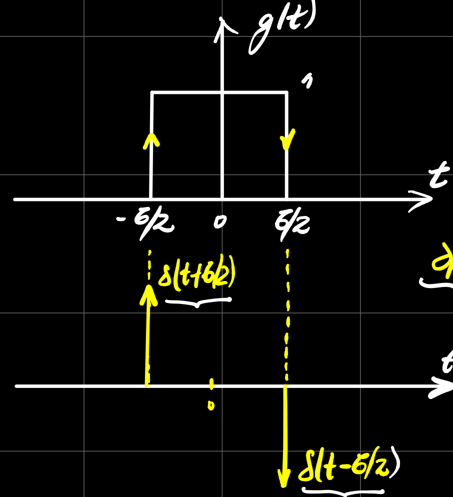
$$y(w) = 1 \cdot \text{Sala}(w) = \text{Sala}(w)$$



$$\delta(t) \longleftrightarrow 1$$

$$\delta(t-t_0) \longleftrightarrow 1 \cdot e^{-j\omega t_0}$$

anterior



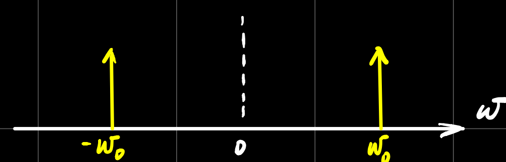
$\partial/\partial t$

$$\xrightarrow{\mathcal{F}} \underbrace{e^{+j\omega\tau/2} - e^{-j\omega\tau/2}}_{2j\sin\omega\tau/2}$$

$$\frac{\tau/2 \cdot 2j \sin \omega\tau/2}{j\omega\,\tau/2} = \tau\,\frac{\sin \omega\tau/2}{\omega\tau/2}$$

$\int \cdots 1/j\omega$

$$g(t) \xleftarrow{\mathcal{F}^{-1}}$$



$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} G(\omega)\, e^{j\omega t}\, d\omega$$

$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \delta(\omega+\omega_0)\, e^{j\omega t}\, d\omega + \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0)\, e^{j\omega t}\, d\omega$$

$x_1 = \omega + \omega_0$ $\qquad$ $x_2 = \omega - \omega_0$

$\omega = x_1 - \omega_0$ $\qquad$ $\omega = x_2 + \omega_0$

$d\omega = dx_1$ $\qquad$ $d\omega = dx_2$

$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(x_1)\, e^{jx_1 t} \cdot \underbrace{e^{-j\omega_0 t}}\, dx_1 + \frac{1}{2\pi}\int_{-\infty}^{+\infty} \delta(x_2)\, e^{jx_2 t}\, \underbrace{e^{j\omega_0 t}}\, dx_2$$

$d\omega \;\; dx_1$ $\qquad$ $d\omega = dx_2$

$$g(t) = e^{-j\omega_0 t} \cdot \underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(x_1)\, e^{jx_1 t}\, dx_1}_{1/2\pi} + e^{+j\omega_0 t}\underbrace{\frac{1}{2\pi}\int_{-\infty}^{+\infty} \delta(x_2)\, e^{jx_2 t}\, dx_2}_{1/2\pi}$$

$$g(t) = \frac{1}{2\pi}\cdot\underbrace{\left(e^{-j\omega_0 t} + e^{+j\omega_0 t}\right)}_{2\cos\omega_0 t} = \frac{1}{\pi}\cdot\cos(\omega_0 t)$$