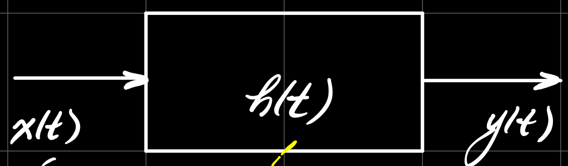


$h(t)$ ... resposta impulsiva

$x(t)$ ⇢ $\delta(t) \xrightarrow{\mathcal{F}} X(\omega)$

$h(t) \xrightarrow{\mathcal{F}} H(\omega)$

$y(t) = h(t) \xrightarrow{\mathcal{F}} Y(\omega)$

assinatura do sistema

$$Y(\omega) = H(\omega) \cdot X(\omega) \xrightarrow{\mathcal{F}^{-1}} y(t)$$

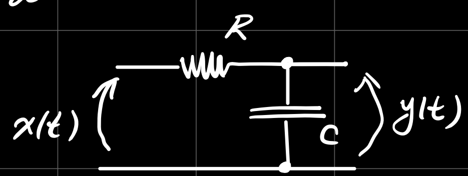
Domínio da frequência

CONVOLUÇÃO

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau$$

OK!

$$\left(u(t) - e^{-t} u(t)\right) \Longrightarrow$$



$x(t) = u(t)$ ... degrau unitário

(1a) risposta impulsiva: $Y(s) = \dfrac{1/sC}{R + 1/sC} \cdot X(s)$

$$Y(s) = \frac{1}{sRC+1} \cdot X(s) = \frac{1}{s+1} \cdot X(s) = \frac{1}{s+1}$$

$\mathcal{L}\{\delta(t)\} = 1$

$y(t) = h(t) = e^{-t}\mu(t)$ $\qquad x(t) = \mu(t)$

$$y(t) = \int_{-\infty}^{\infty} \underbrace{\mu(\tau)}\cdot e^{-(t-\tau)} \cdot \mu(t-\tau) \cdot \underbrace{d\tau}$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$\mu(\tau) \quad \begin{cases} \tau \geq 0 & \cdots 1 \\ \tau < 0 & \cdots 0 \end{cases}$$

$$y(t) = \int_{0}^{\infty} e^{-(t-\tau)} \underbrace{\mu(t-\tau)}\, \underbrace{d\tau}$$

$$\mu(t-\tau) \quad \begin{cases} \underbrace{t-\tau \geq 0} & \cdots 1 \dashrightarrow t \geq \tau \quad \leftarrow \quad \underbrace{\tau \leq t}_{\tau_{max} = t} \\ t-\tau < 0 & \cdots 0 \dashrightarrow t < \tau \end{cases}$$

$$y(t) = \int_{0}^{t} e^{-(t-\tau)} d\tau =$$

$$= \int_{0}^{t} e^{-t} e^{+\tau} \underline{d\tau} = e^{-t} \int_{0}^{t} e^{+\tau} d\tau =$$

$$e^{-t} \cdot \left(e^{\tau}\right)\Big|_{0}^{t} = e^{-t}\left(e^{t} - e^{0}\right) = 1 - e^{-t}.$$

$t \geq 0$

$(1-e^{-t})\cdot\mu(t)$

Laplace

Analogia entre vetores e sinais

$$C_n = \frac{\int_{T_0} g(t)x(t)\,dt}{\int_{T_0} x^2(t)\,dt}$$

$x(t) \longrightarrow$ ∿

harmônicas ω_0 $D_n = \frac{1}{T_0}\int_{T_0} g(t)\cdot e^{-jn\omega_0 t}\,dt$

discreto

$n \Rightarrow -\infty \cdots +\infty$

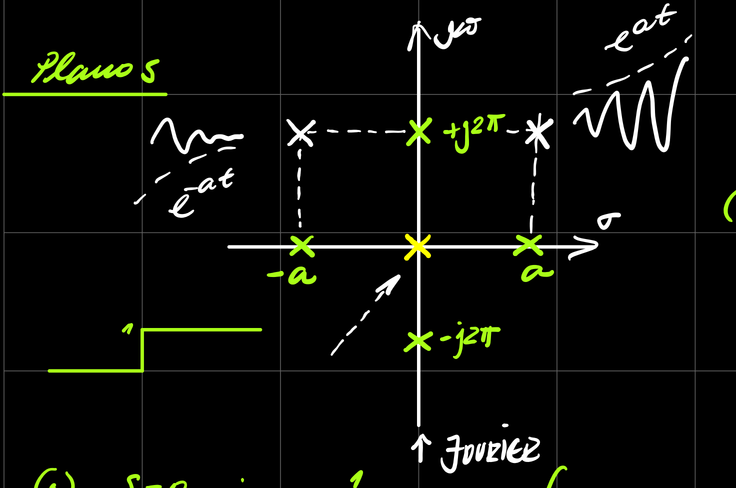
$\ldots, -2, -1, 0, 1, 2 \ldots$

infinitas harmônicas $G(\omega) = \int_{-\infty}^{\infty} g(t)\,e^{-j\omega t}\,dt$

$e^{-j\omega t}\cdot e^{+\sigma t}$

contínuo

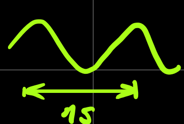
Plano S



$x(t) = \delta(t)$

(·) $s_{1,2} = \pm j2\pi$

$\omega = 2\pi$



$e^{-\sigma t}\cdot e^{-j\omega t}$

(·) $s = 0 \therefore \frac{1}{s}, \longrightarrow \int \longrightarrow \int \delta(t)\,dt = 1!$

(·) $s = -a \longrightarrow e^{-at}\mu(t)$

(1) $s = a \longrightarrow e^{at}\mu(t)$

$$G(s) = \int_{-\infty}^{\infty} g(t)\, e^{-\sigma t} \cdot e^{-j\omega t}\, dt =$$

$$\int_{-\infty}^{\infty} g(t)\, e^{-(\sigma + j\omega)t}\, dt \quad \ldots \quad \sigma + j\omega = s$$

$$G(s) = \int_{-\infty}^{\infty} g(t)\, e^{-st}\, dt \quad \ldots \quad G(\omega) = \int_{-\infty} g(t)\, e^{-j\omega t}\, dt$$

$\sigma = 0$

SÉRIE DE FOURIER : periódico
Transformada : não periódico
Laplace : transitório

Impulso : $\delta(t) \longrightarrow \mathcal{L}\{\delta(t)\}$

$$\mathcal{L}\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t)\, e^{-st}\, dt = \int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$

$e^{-s0} = 1$

$$\mathcal{L}\{\delta(t-T)\} = \int_{-\infty}^{\infty} \delta(\underbrace{t-T}_{t'})\, e^{-st}\, dt = \int_{-\infty}^{\infty} \delta(t')\, e^{-s(t'+T)}\, dt'$$

$t' = t - T \quad \therefore \quad t = t' + T \quad \therefore \quad dt = dt'$

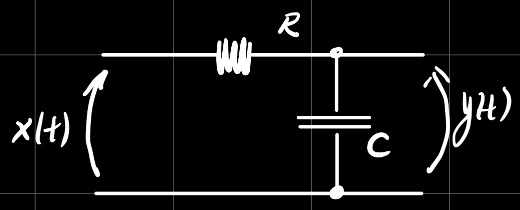
$$= \underbrace{\int_{-\infty}^{\infty} \delta(t') \cdot e^{-st'} \cdot dt'}\ \cdot\ e^{-sT}$$

$$\int_0^a x^2 dx = \int_0^a y^2 dy$$

$$\int^{\infty} \ldots e^{-st}$$

$$\int_{-\infty} \delta(t) e^{-st} \, dt =$$

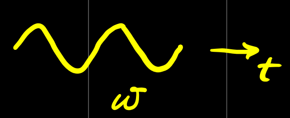
$$\mathcal{L}\{\delta(t-T)\} = e^{-sT} \quad \ldots \quad \mathcal{L}\{\delta(t+T)\} = e^{+sT}$$

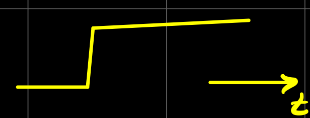


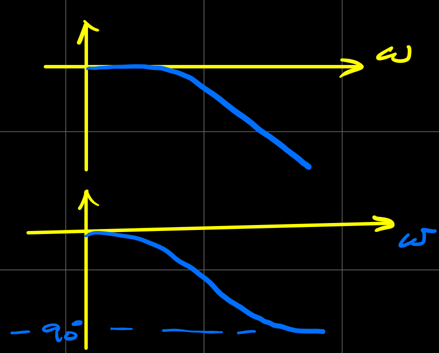
$\mathcal{F}$ $\mathcal{L}$

$$H(\omega) = \frac{1}{j\omega RC + 1} \qquad\qquad H(s) = \frac{1}{sRC + 1}$$

Resposta em frequência   Transitória



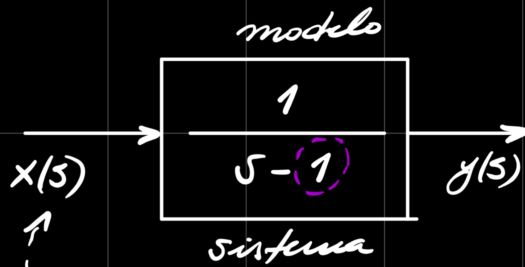




estabilidade

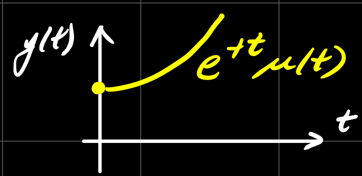
↓

resposta transitória



$x(t) = \delta(t)$

$$y(s) = \frac{1}{s-1}$$



$$y(t) = e^{+t}\,\mu(t)$$

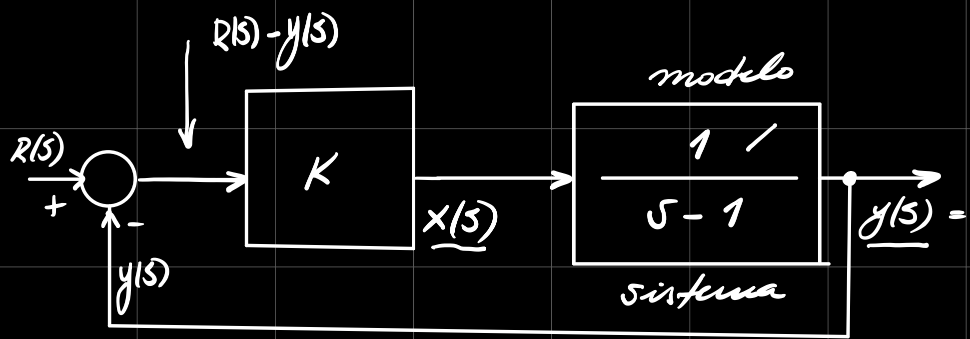
instável

$D(s) = 0 \longrightarrow$ raízes $\longrightarrow \mathrm{Re}\{\ \} > 0$

instável

$s - 1 = 0 \longrightarrow p_1 = +1 \longrightarrow +1 > 0$

$\therefore$ instável



$$y(s) = X(s) \cdot \frac{1}{s-1} \quad , \quad X(s) = K \cdot (R(s) - y(s))$$

$$y(s) = \frac{1}{s-1} \cdot K \cdot (R(s) - y(s)) =$$

$$y(s) + \frac{K}{s-1}\, y(s) = \frac{K}{s-1}\, R(s)$$

$$\frac{y(s)}{R(s)} = \frac{K/(s-1)}{1 + K/(s-1)} = \frac{K}{s-1+K}$$

$$\frac{K}{s-1+K}$$

$s-1+K = 0 \quad \therefore \quad p_1 = \underbrace{1-K}$

estável $\quad 1-K < 0 \quad \therefore \quad \underbrace{K > 1}$