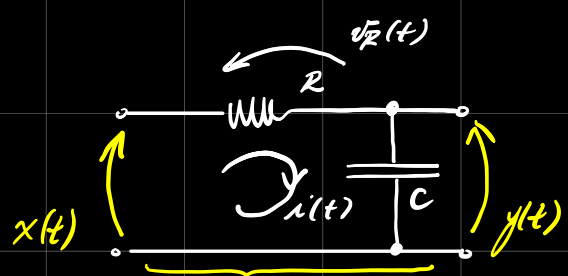
Sistemas Lineares - LIT

Linear Invariante no Tempo

(·) Como determinar a saída $y(t)$ sendo conhecido o sistema $h(t)$ e a entrada $x(t)$?

circuito elétrico



$$y(t) = f(R, C, x(t))$$

malha: $x(t) = R\,i(t) + y(t)$

$$i_c(t) = C\,\frac{dv_c(t)}{dt} = C\,\frac{dy(t)}{dt}$$

$$x(t) = R.C.\,\dot{y}(t) + y(t)$$

$$\dot{y}(t) = x(t)\cdot\frac{1}{RC} - \dot{y}(t)\,\frac{1}{RC}$$ ← equação diferencial que descreve o comportamento

tempo (

$\mathcal{L}$ do circuito

$$sY(s) = X(s)\frac{1}{RC} - Y(s)\frac{1}{RC}$$

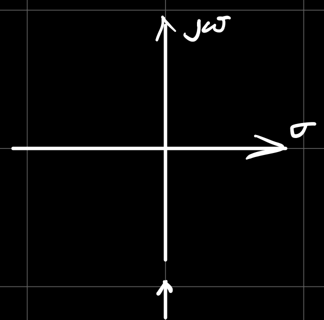
$y(0) = 0$

$$sY(s) + Y(s)\frac{1}{RC} = X(s)\frac{1}{RC} \qquad \frac{Y(s)}{X(s)} = \frac{1/RC}{s + 1/RC}$$

$$Y(s)\left(s + 1/RC\right) = X(s)\,1/RC$$

$$\frac{Y(s)}{X(s)} = \frac{1}{sRC + 1} = H(s) \;\ldots\; \text{Função de transferência}$$

$\mathcal{F}$



$s = j\omega$
$\sigma = 0$

$$H(\omega) = \frac{1}{j\omega RC + 1}$$

$$\lim_{\omega \to \infty} H(\omega) = 0$$

$$H(\omega = 0) = \frac{1}{j0RC + 1} = 1$$

F.P.B.

Exemplo: $R = 1\Omega$ $C = 1\Omega$

a. impulso unitário

b. degrau unitário $\longrightarrow y(t) = ?$

c. $\cos(t)$

a. Impulso $\longrightarrow \delta(t) \xrightarrow{\mathcal{F}} 1$

$$\frac{Y(\omega)}{\underbrace{X(\omega)}_{1}} = \frac{1}{j\omega + 1} \quad \therefore \quad Y(\omega) = \frac{1}{j\omega + 1}$$

$$H(\omega) = \frac{1}{j\omega + 1}$$

Se eu desejo descobrir o $H(\omega)$ é apropriado estimular o sistema com um impulso.

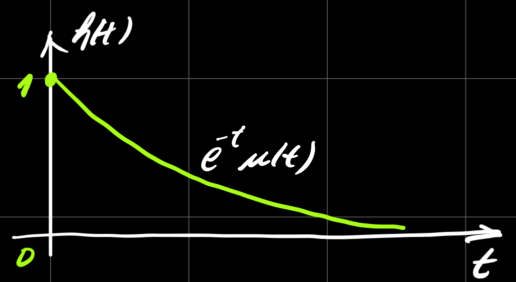
$$Y(\omega) = H(\omega)$$

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{j\omega + 1} e^{+j\omega t} d\omega \qquad \text{Resposta impulsiva}$$

$$\frac{1}{j\omega + a} \overset{\mathcal{F}^{-1}}{\longleftrightarrow} e^{-at}\mu(t)$$

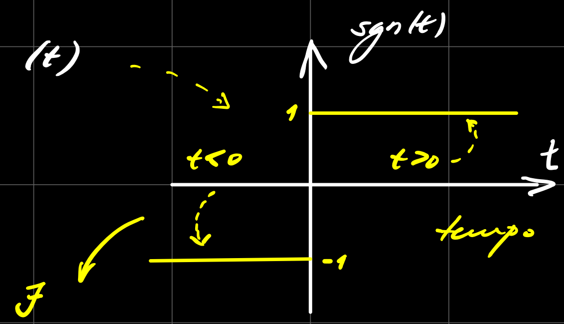
$$\int_0^{\infty} e^{-at} e^{-j\omega t} dt = \int_0^{\infty} e^{-(a+j\omega)t} dt = \left. \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right|_0^{\infty}$$

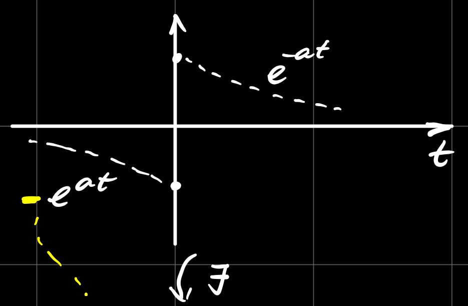
$$0 + \frac{e^{-(a+j\omega)\cdot 0}}{a + j\omega} = \frac{1}{j\omega + a} \quad \therefore \quad h(t) = e^{-t}\mu(t)$$



(b) Degrau unitário

$$u(t) = \frac{1}{2} + \frac{1}{2}\,\mathrm{sgn}(t)$$





$\lim_{a \to 0}$

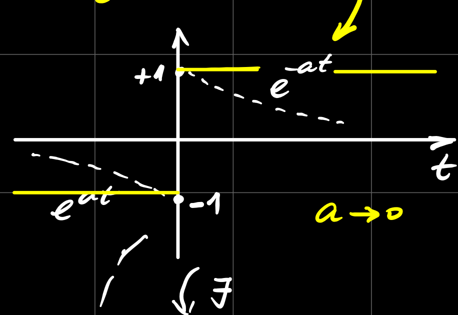
$$\int_{-\infty}^{0} -\underbrace{e^{at}\,e^{-j\omega t}}_{e^{(a-j\omega)t}}\,dt + \int_{0}^{+\infty} \underbrace{e^{-at}\,e^{-j\omega t}}_{e^{-(a+j\omega)t}}\,dt =$$

$$\left.\frac{e^{(a-j\omega)t}}{(a-j\omega)}\right|_{-\infty}^{0} + \left.\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right|_{0}^{\infty} =$$

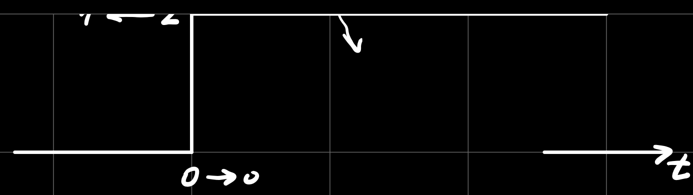
$0 +$

$$\frac{1}{a-j\omega} - 0 + 0 + \frac{1}{a+j\omega} = \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$-\frac{1}{-j\omega} + \frac{1}{j\omega} = \frac{2}{j\omega}$$

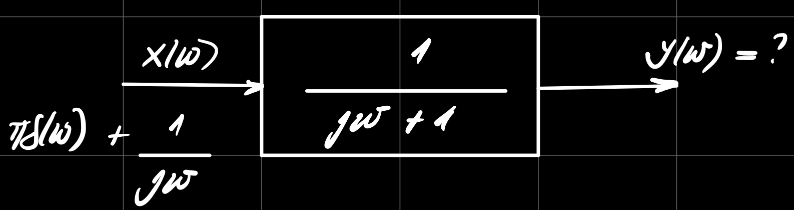


$$\mathcal{F}\{\mathrm{sgn}(t)\} = \frac{2}{j\omega}$$



$$\frac{1}{2} + \frac{\mathrm{sgn}(t)}{2}$$

$$\pi\delta(\omega) + \frac{1}{j\omega} \qquad \therefore \quad \mathcal{F}\{u(t)\} = \underbrace{\pi\delta(\omega)} + \underbrace{\frac{1}{j\omega}}$$



$$Y(\omega) = X(\omega) \cdot H(\omega) = \left[\pi\delta(\omega) + \frac{1}{j\omega}\right] \cdot \frac{1}{j\omega + 1}$$

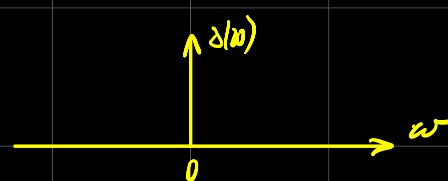
$$Y(\omega) = \frac{\pi\delta(\omega)}{j\omega + 1} + \frac{1}{j\omega(j\omega + 1)} \xrightarrow[\mathcal{F}^{-1}]{} y(t)$$

$\omega = 0$

$$\underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty} \pi\delta(\omega)\, e^{j\omega t}\, d\omega} + \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{j\omega(j\omega + 1)}\, e^{j\omega t}\, d\omega$$

$$\frac{1}{2\pi} \cdot \pi \cdot e^{0t} = 1/2 \checkmark$$

$\delta(\omega)$



$$\delta(\omega) \begin{cases} 1, & \omega = 0 \\ 0; & \text{c.c.} \end{cases}$$

$$\frac{1}{j\omega \cdot (j\omega + 1)} \Rightarrow \frac{A}{j\omega} + \frac{B}{j\omega + 1}$$

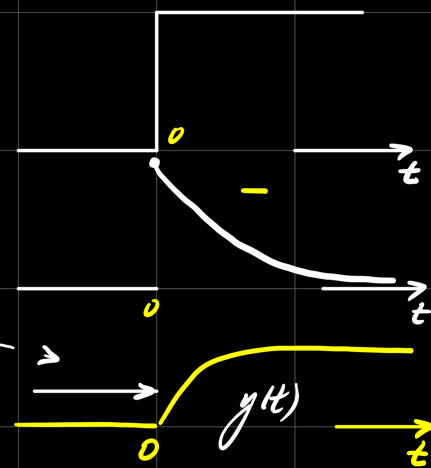
$$\longrightarrow Aj\omega + A + Bj\omega = 1 \quad \therefore \quad A = 1$$

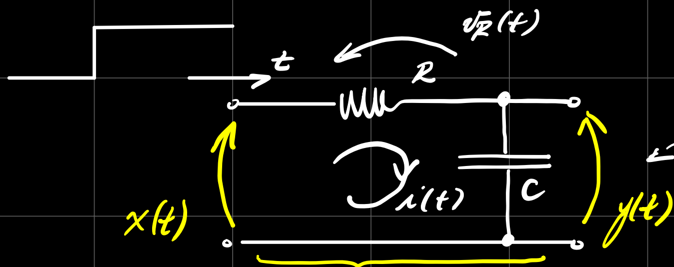
$$B = -1$$

$$\underbrace{\mathcal{F}^{-1}\{1/j\omega\}} - \mathcal{F}^{-1}\{1/j\omega+1\} =$$

$$\left[\frac{\text{sgn}(t)}{2} - e^{-t}\mu(t)\right] + 1/2$$

$$\left(\mu(t) - e^{-t}\mu(t)\right) =$$





c. $\underbrace{\cos t}$ $\therefore$ $\omega_0 = 1\,rd/s$

$$X(\omega) = \pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0) = \pi\delta(\omega-1) + \pi\delta(\omega+1)$$

$$\cos(\omega t) \to \cos(t) \quad \therefore \omega = 1$$

$$Y(\omega) = \frac{\pi\delta(\omega-1)}{j\omega+1} + \frac{\pi\delta(\omega+1)}{j\omega+1}$$

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\pi\delta(\omega-1)}{j\omega+1}e^{+j\omega t}\,d\omega +$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\pi\delta(\omega+1)}{j\omega+1}e^{+j\omega t}\,d\omega$$

$$=$$

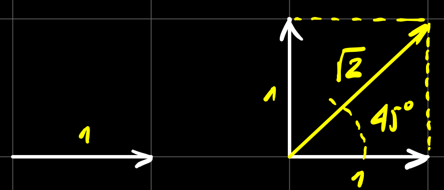
$$\frac{1}{2\pi}\cdot\frac{\pi\cdot e^{jt}}{j+1}+\frac{1}{2\pi}\frac{\pi e^{-jt}}{-j+1}$$

$$\frac{1}{2}\cdot\frac{e^{jt}}{j+1}+\frac{1}{2}\frac{e^{-jt}}{-j+1}$$

$$\frac{(-j+1)\cdot e^{jt}+(j+1)e^{-jt}}{1-j+j+1}=$$

$$\frac{1}{2}\cdot\frac{-j\cdot e^{jt}+e^{jt}+je^{-jt}+e^{-jt}}{2}=\frac{1}{2}\cdot\frac{2\cos(t)+2\,\mathrm{sen}(t)}{2}$$

$$1\cdot\cos(t)\longrightarrow\frac{\cos(t)+\mathrm{sen}(t)}{2}=0.707\cdot\cos(t-45^\circ)$$



$$\sqrt{2}/2=0.707$$

$$-je^{jt}+je^{-jt}+\underbrace{e^{jt}+e^{-jt}}_{2\cos t}=$$

$$e^{j\theta}=\cos\theta+j\,\mathrm{sen}\,\theta$$
$$e^{-j\theta}=\cos\theta-j\,\mathrm{sen}\,\theta$$

$$-j(\underbrace{e^{jt}-e^{-jt}}_{2j\,\mathrm{sen}(t)})$$

$$+2\,\mathrm{sen}(t)$$

$$H(\omega)=\frac{1}{j\omega+1}\quad\therefore\quad H(\omega=1)=\frac{1}{j+1}$$

$$\Longrightarrow H(\omega=1)=\frac{1}{\sqrt{2}\angle 45^\circ}=\sqrt{2}/2\angle-45^\circ$$

$$y(t)=\sqrt{2}/2\cdot\cos(t-45^\circ)\ ✓$$