Aula Passada : $g(t) \rightarrow$ sinal a ser decomposto

$g(t) \Rightarrow c_n \, x_n(t)$ $\quad$ $x(t) \rightarrow$ sinal de base

$$\longrightarrow c_n = \frac{\int_{T_0} g(t) \cdot x_n(t)\, dt}{\int_{T_0} x_n^2(t)\, dt} \quad \ldots \text{ projeção}$$

$$g(t) = c_1 x_1(t) + c_2 x_2(t) + c_3 x_3(t) + \ldots$$

$$\longrightarrow \int_{T_0} x_n(t)\, x_m(t)\, dt = 0 , \quad m \neq n$$

ortogonais $\quad m, n = 1, 2, 3, \ldots$

Resumo:

ANÁLISE :
analisador

$$c_n = \frac{\int_{T_0} g(t)\, x_n(t)\, dt}{\int_{T_0} x_n^2(t)\, dt}$$

SÍNTESE :
sintetizador

$$g(t) = \sum_{n=1}^{\infty} c_n x_n(t)$$

Análise

$$a_n = \frac{\int_{T_0} g(t) \cdot \underbrace{\cos(n\omega_0 t)}\, dt}{\int_{T_0} \cos^2(n\omega_0 t)\, dt}$$

$$b_n = \frac{\int_{T_0} g(t)\, \underbrace{\operatorname{sen}(n\omega_0 t)}\, dt}{\int_{T_0} \operatorname{sen}^2(n\omega_0 t)\, dt}$$

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cdot \cos(n\omega_0 t) + b_n \operatorname{sen}(n\omega_0 t)$$

valor médio $\longleftarrow$ $a_0 = \frac{1}{T_0} \int_{T_0} g(t)\, dt$

Série trigonométrica de Fourier.

$$C_n = \frac{\int_{T_0} g(t)\, x_n(t)\, dt}{\int_{T_0} x_n^2(t)\, dt}$$

$$\underbrace{e^{j\theta}} = \underbrace{\cos\theta + j\operatorname{sen}\theta}$$ p. exemplo $\theta = \pi$

$$e^{j\pi} = \underbrace{\cos\pi}_{-1} + j\underbrace{\operatorname{sen}\pi}_{0} \quad \therefore \quad e^{j\pi} = -1$$

$$\frac{de^{j\theta}}{d\theta} \Rightarrow$$

$$D_n = \frac{\int_{T_0} g(t)\, e^{-jn\omega_0 t}\, dt}{\int_{T_0} \left(e^{-jn\omega_0 t}\right)^2 dt}$$

$$\underbrace{e^{jn\omega_0 t}}_{} = \underbrace{\cos(n\omega_0 t)}_{} + j\,\underbrace{\mathrm{sen}(n\omega_0 t)}_{}$$

$$\cos(\omega_0 t) \longrightarrow e^{j\omega_0 t} = \cos(\omega_0 t) + j\,\mathrm{sen}(\omega_0 t)$$

$$\overset{+}{-}\; e^{-j\omega_0 t} = \cos(\omega_0 t) - j\,\mathrm{sen}(\omega_0 t)$$

$$e^{j\omega_0 t} + e^{-j\omega_0 t} = 2 \cdot \cos(\omega_0 t)$$

$$\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} = \frac{e^{+j\omega_0 t}}{2} + \frac{e^{-j\omega_0 t}}{2}$$

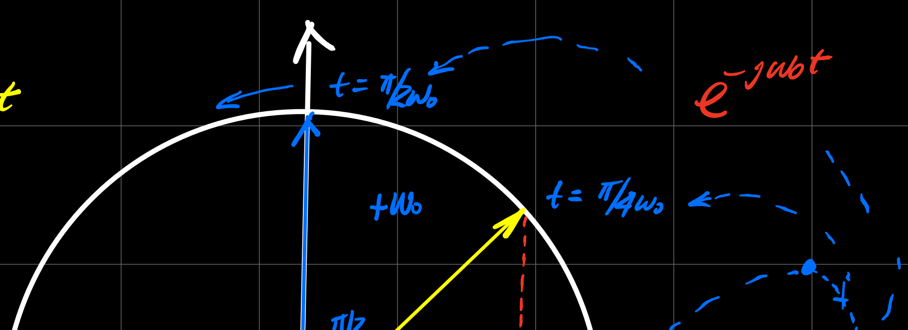
$$2j\,\underbrace{\mathrm{sen}\,\omega_0 t}_{}$$

$$\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} = 2j\,\mathrm{sen}(\omega_0 t) \qquad \frac{e^{+j\omega_0 t}}{2j} - \frac{e^{-j\omega_0 t}}{2j}$$

Interpretação.

$$e^{j\omega_0 t} \rightarrow \omega_0 t$$

t	$\omega_0 t$
0	0



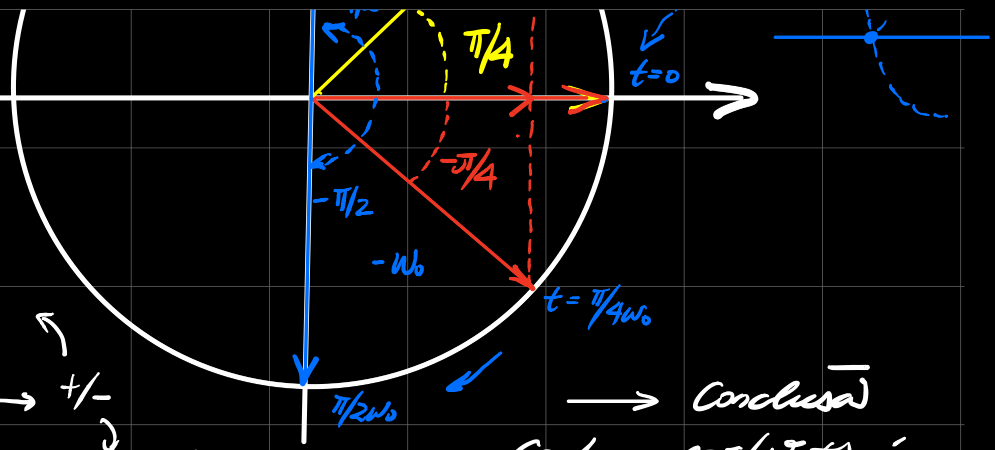
$\frac{\pi}{4w_0}$ $\pi/4$

$\frac{\pi}{2w_0}$ $\pi/2$

anti

rd/s → +/-

horário



→ Conclusão

Cada cos(w0t) é composto de duas exponenciais

$$g(t) = \sum_{n=-\infty}^{+\infty} D_n \underbrace{e^{jnw_0 t}}$$

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cdot \cos(n w_0 t) + b_n \, \text{sen}(n w_0 t)$$

$$e^{jnw_0 t} = \cos(n w_0 t) + j\,\text{sen}(n w_0 t)$$

$$\left| e^{jnw_0 t} \right| = \left[\cos^2(n w_0 t) + \text{sen}^2(n w_0 t)\right]^{1/2} = 1$$

$$e^{j325 \times \pi} \Rightarrow \text{módulo} = 1$$

$$\measuredangle e^{jnw_0 t} = \text{TAN}^{-1}\left[\frac{\text{sen}(n w_0 t)}{\cos(n w_0 t)}\right] = n w_0 t$$

$\rightarrow TAN(n\omega_0 t)$

$$\measuredangle e^{j325\pi} \Rightarrow 325\pi$$

$$e^{j325\pi} = 1 \measuredangle 325\pi \quad (!)$$

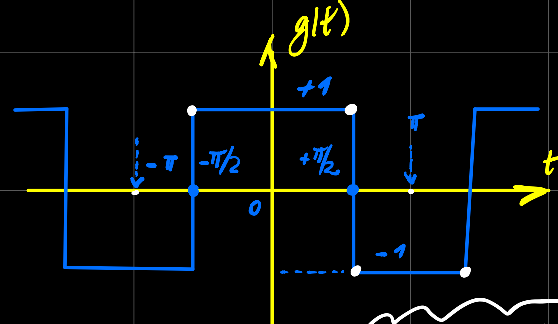
Série Exponencial de Fourier

Análise: $$D_n = \frac{1}{T_0}\int_{T_0} g(t)\, e^{-jn\omega_0 t}\, dt$$

Síntese: $$g(t) = \sum_{n=-\infty}^{+\infty} D_n\, e^{+jn\omega_0 t}\, dt$$

Exemplo

$$T_0 = 2\pi \therefore \omega_0 = \frac{2\pi}{T_0} \therefore \omega_0 = 1$$



$$D_n = \frac{1}{2\pi}\cdot\int_{-\pi/2}^{+\pi/2} 1\cdot e^{-jnt}\,dt + \frac{1}{2\pi}\cdot\int_{+\pi/2}^{3\pi/2} (-1)\, e^{-jnt}\,dt$$

$$D_n = \frac{1}{2\pi}\,\frac{e^{-jnt}}{(-jn)}\Bigg|_{-\pi/2}^{+\pi/2} - \frac{1}{2\pi}\,\frac{e^{-jnt}}{(-jn)}\Bigg|_{+\pi/2}^{+3\pi/2}$$

$$D_n = \frac{1}{2\pi}\cdot\left[\frac{e^{-jn\pi/2} - e^{+jn\pi/2}}{(-jn)} - \frac{e^{-jn3\pi/2} - e^{-jn\pi/2}}{(-jn)}\right]$$

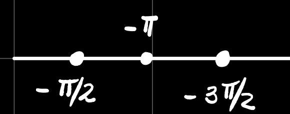
$$D_n = \frac{1}{2\pi}\cdot\left[\frac{-e^{-jn\pi/2}+e^{+jn\pi/2}}{(+jn)} + \frac{+e^{-jn3\pi/2}-e^{-jn\pi/2}}{(+jn)}\right]$$

euler: $e^{+jn\pi/2} - e^{-jn\pi/2} = 2j\,\text{sen}\,(n\pi/2)$

$$D_n = \frac{1}{2\pi}\left[\frac{2j\,\text{sen}\,(n\pi/2)}{+jn} - \frac{2j\,\text{sen}\,(n\pi/2)}{+jn}\cdot e^{-jn\pi}\right]$$

$\frac{0}{0}$!

$$\frac{+e^{-jn3\pi/2} - e^{-jn\pi/2}}{(+jn)}$$



$$e^{-j\pi n}\left[e^{-jn\pi/2} - e^{+jn\pi/2}\right] = e^{-jn\pi}\cdot\left(-2j\,\text{sen}\,n\pi/2\right)$$

e j

$$D_n = \frac{1}{2\pi}\left[\frac{2\cancel{j}\,\text{sen}\,(n\pi/2)}{+\cancel{j}n} - \frac{2j\,\text{sen}\,(n\pi/2)}{+jn}\cdot e^{-jn\pi}\right]$$

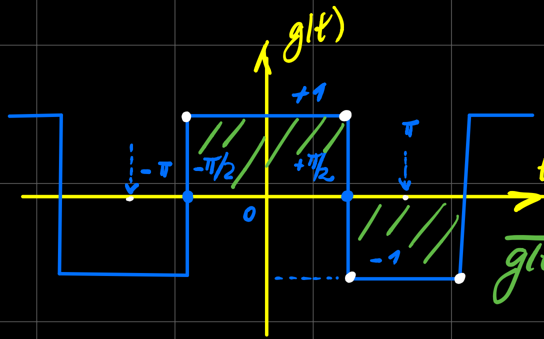
$\frac{0}{0}$!

$$\rightarrow \lim_{\underbrace{n\to0}} \frac{\text{sen}\,(n)}{n} = \lim_{n\to0}\frac{\cos(n)}{1} = \underbrace{1}$$

$$\underbrace{D_n} = \frac{1}{2}\left[\underbrace{\frac{\text{sen}\,(n\pi/2)}{\pi n/2}}_{1 \text{ qdo } n\to0} - \underbrace{\frac{\text{sen}\,(n\pi/2)}{\pi n/2}}_{1 \text{ qdo } n\to0}\cdot e^{-jn\pi}\right]$$

$n=0 \quad \therefore \quad \underbrace{\frac{\text{sen}(0)}{0}}_{0/0\,!} - \underbrace{\frac{\text{sen}(0)}{0}}_{0/0\,!} e^{j0}$

$D_0 = \frac{1}{2} \cdot \left[\underbrace{\frac{\text{sen}(0)}{0}}_{1} - \underbrace{\frac{\text{sen}(0)}{0}}_{1} e^{0} \right] = \underline{\underline{0}}$



$\overline{g(t)} = 0$

$D_0 = \frac{\int_{T_0} g(t) e^{-j0\omega_0 t} dt}{T_0}$

$D_0 = \frac{1}{T_0} \int_{T_0} g(t)\, dt \;!$

valor médio

$\underbrace{D_n} = \frac{1}{2} \left[\underbrace{\frac{\text{sen}(n\pi/2)}{\pi n/2}}_{1 \text{ qdo } n \to 0} - \underbrace{\frac{\text{sen}(n\pi/2)}{\pi n/2}}_{1 \text{ qdo } n \to 0} \cdot e^{-jn\pi} \right]$

$D_1 = 1/2 \cdot \left[\frac{\text{sen}(\pi/2)^{\swarrow 1}}{\pi/2} - \frac{\text{sen}(\pi/2)}{\pi/2} \underbrace{e^{-j\pi}}_{\cos\pi - j\,\text{sen}\pi \,=\, -1} \right]$

$D_1 = 1/2 \left[\frac{2}{\pi} + \frac{2}{\pi}(+1) \right] = 1/2 \cdot 4/\pi = \underbrace{2/\pi}$

$\underbrace{g(t)}_{?} = \frac{2}{\pi} \cdot e^{+jt} = \underbrace{\frac{2}{\pi}} \cdot (\underbrace{\cos t} + j \underbrace{\text{sen}(t)}_{?\ \text{valor}}) + 0 \qquad \omega_0 = 1$

$D_{-1} = \frac{1}{2} \left[\frac{\text{sen}(-\pi/2)^{-1}}{-\pi/2} - \frac{\text{sen}(-\pi/2)^{-1}}{-\pi/2} \cdot e^{+j\pi} \right]$

1 q$\underline{\text{do}}$ $n \to 0$ $\qquad$ 1 q$\underline{\text{do}}$ $n \to 0$ $\qquad$ $\cos \pi + \operatorname{sen} \pi$

$$\frac{1}{2} \cdot \left[\frac{2}{\pi} + \frac{2}{\pi} \right] = 2/\pi$$

$$g(t) = \underbrace{\underbrace{\frac{2}{\pi}}_{D_1}} \cdot \underbrace{\left(\cos t + j \operatorname{sen}(t) \right)}_{e^{+jt}} + \underbrace{\underbrace{\frac{2}{\pi}}_{D_{-1}}} \cdot \underbrace{\left(\cos t - j \operatorname{sen}(t) \right)}_{e^{-jt}}$$

$\omega_0 - 1$ $\qquad$ $\omega_0 - 1$

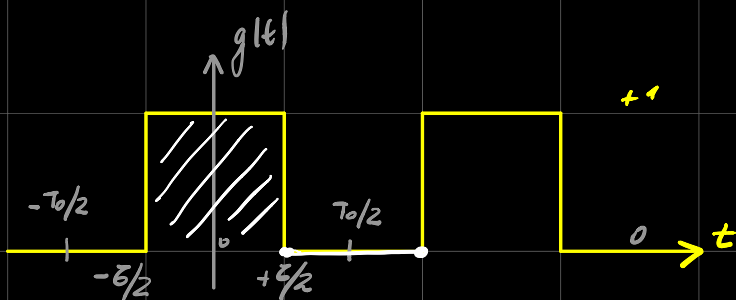
$$g(t) = \frac{2}{\pi} \cos t + \frac{2}{\pi} \cos t = \frac{4}{\pi} \cdot \cos t \; !$$

1.27

$$g(t) = \dots + D_{-2} e^{-j2t} + D_{-1} e^{-jt} + D_0 + D_1 e^{+jt} + D_2 e^{+j2t} + \dots$$

$$\cos(n\omega_0 t) = \frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2}$$

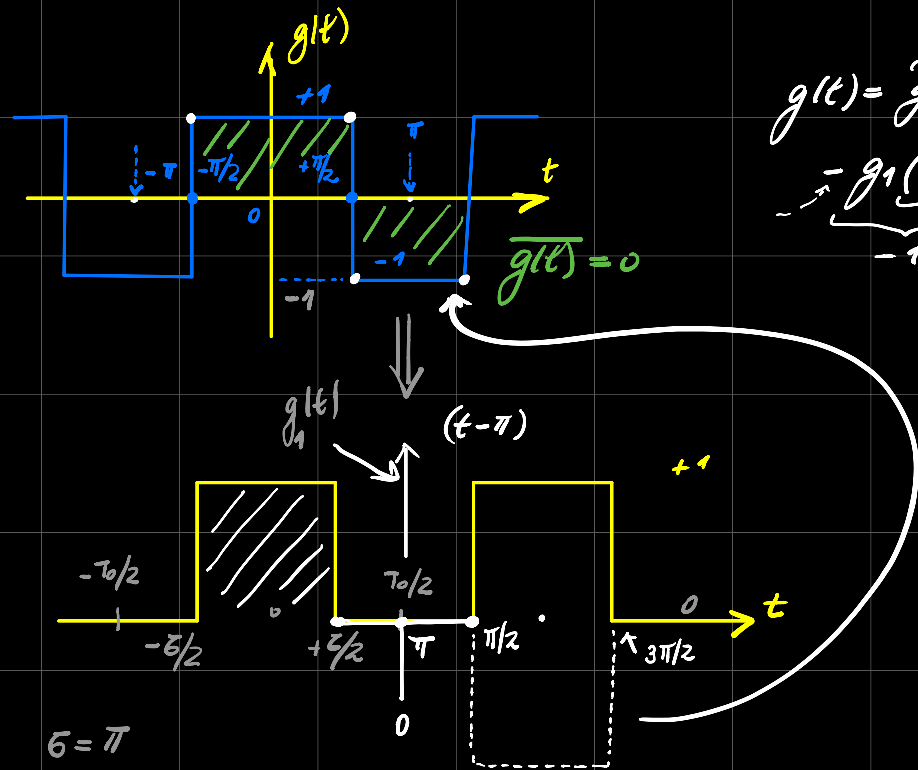
Geral



$$D_n = \frac{1}{T_0} \int_{-\tau/2}^{+\tau/2} 1 \cdot e^{-jn\omega_0 t} \, dt = \frac{1}{T_0} \cdot \frac{e^{-jn\omega_0 t}}{-jn\omega_0} \Bigg|_{-\tau/2}^{+\tau/2}$$

$$D_n = \frac{1}{T_0} \cdot \frac{-e^{-jn\omega_0 \tau/2} + e^{+jn\omega_0 \tau/2}}{+jn\omega_0} = \frac{\tau/2 \cdot \operatorname{sen}(n\omega_0 \tau/2)}{jn\omega_0 \cdot T_0 \; \tau/2}$$

$$D_n = \frac{\tau}{2T_0} \cdot \frac{\text{sen}(n\omega_0\tau/2)}{n\omega_0\tau/2}$$



$$g(t) = \overbrace{g_1(t)}^{+1} + \underbrace{-\,g_1(t-\pi)}_{-1}$$

$\tau = \pi$

$T_0 = 2\pi$

$$g(t) = g_1(t) - g_1(t-\pi)$$

$$\downarrow \mathcal{F}$$

$$D_n = \frac{\tau}{2T_0} \cdot \frac{\text{sen}(n\omega_0\tau/2)}{n\omega_0\tau/2}$$

$$D_n = \frac{1}{2\pi}\int_{\pi/2}^{3\pi/2} -1\, e^{-jnt}\,dt \Rightarrow \underbrace{\tau/2T_0 \cdot \frac{\text{sen}(n\omega_0\tau/2)}{n\omega_0\tau/2}}\cdot \underbrace{e^{-jn\pi}}_{}$$

t_0

$$D_n = \frac{1}{T_0}\int_{T_0} g(\underbrace{t-\pi}_{x}) \cdot e^{-jn\omega_0 t}\, dt$$

$t - \pi = x \quad \therefore \quad t = x + \pi \quad \therefore \quad dt = dx$

$$D_n = \frac{1}{T_0}\int_{T_0} g(x) \cdot e^{-jn\omega_0 x} \cdot e^{-jn\omega_0 \pi}\, dx =$$

$$D_n = \underbrace{e^{-jn\omega_0 \pi}}_{} \cdot \underbrace{\frac{1}{T_0}\int_{T_0} g(t)\, e^{-jn\omega_0 t}\, dt}_{D_n}$$

deslocamento temporal!