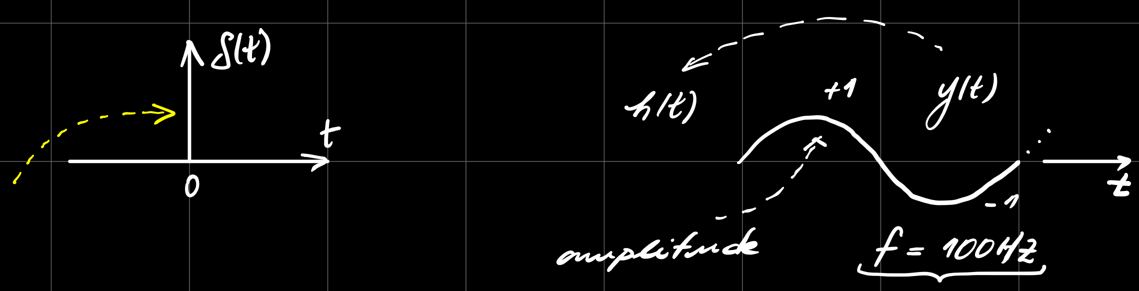
Domínio Digital

1. Teorema da amostragem
2. Transformada Z
3. mapeamento s em z
4. Aproximação s em z
5. Simulação: impulsiva, ao degrau, senoide



1. Quando $x(t)=\delta(t)$ temos $y(t)=h(t)$.
2. $h(t) = sen(\omega t)$, $\omega = 2.\pi.100 = 200\pi$ rd/s

   $h(t) = sen(200\pi t)$
3. $H(s) = \mathcal{L}\{h(t)\} = \mathcal{L}\{sen(200\pi t)\} =$

   $\mathcal{L}\{sen(\omega t)\} = \dfrac{\omega}{s^2+\omega^2}$

$$H(s) = \frac{200\pi}{\underbrace{s^2 + (200\pi)^2}} \longleftarrow sw?$$

4. a. $z = e^{sT} \longrightarrow s^2 + (200\pi)^2 = 0$

$$p_{1,2} = \pm\sqrt{-(200\pi)^2} = \pm j\sqrt{(200\pi)^2} =$$

$$p_{1,2} = \pm j\, 200\pi$$

$$z_{1,2} = e^{\pm j 200\pi \cdot T}$$ T: taxa de amostragem

$$f_s = 2 f_{máx} \therefore f_s > \underbrace{200Hz} \Rightarrow f_s = 4000Hz$$

20x

20 amostras em 1/2 período

$$z_{1,2} = e^{\pm j 200\pi \cdot 1/4000} = e^{\pm j\, \pi/20}$$

$$z_{1,2} = 1 \angle \pm \pi/20 \longrightarrow z_1 = \_\_\_\_\_\_\_\_ ,\ z_2 = \_\_\_\_\_\_\_\_$$

$$H(z) = \left[\frac{1}{z - z_1} + \frac{1}{z - z_2}\right] \times \underset{\uparrow \text{ganho}}{K_z}$$

b. Aproximação (equivalência)

$$\longrightarrow s \equiv \frac{2}{T}\frac{z-1}{z+1}, \quad s \equiv \frac{z-1}{T}, \quad s \equiv \frac{z-1}{Tz}$$

$$H(s) = \frac{200\pi}{\underbrace{s^2 + (200\pi)^2}} \longrightarrow \text{Qual é a melhor?}$$

?

$$H(s) = \frac{1}{s+1}$$

$$H(s) = \frac{a}{s^2 + a^2} \quad \ldots \quad a = 200\pi$$

$$s \equiv \frac{2}{T} \frac{z-1}{z+1} \longrightarrow \frac{a}{\left(\frac{2}{T}\frac{z-1}{z+1}\right)^2 + a^2} =$$

$$= \frac{a(z+1)^2}{(2/T)^2(z-1)^2 + a^2(z+1)^2} = \frac{a(z^2+2z+1)}{\mathrm{Den}(z)}$$

$$\frac{a(z^2+2z+1)}{(2/T)^2 \cdot \left[z^2 - 2z + 1\right] + a^2\left[z^2 + 2z + 1\right]} =$$

$$\frac{a(z^2+2z+1)}{\left((2/T)^2 + a^2\right)z^2 + \left(2a^2 - (2/T)^2 \cdot 2\right) \cdot z + \left((2/T)^2 + a^2\right)} \quad z^{-2}$$

$$\frac{y(z)}{x(z)} = \frac{a(1 + 2z^{-1} + z^{-2})}{\underbrace{\left((2/T)^2 + a^2\right)}_{K_1} + \underbrace{\left(2a^2 - (2/T)^2 \cdot 2\right)}_{K_2} z^{-1} + \underbrace{\left((2/T)^2 + a^2\right)}_{K_3} z^{-2}}$$

$$y(n) \cdot K_1 + y(n-1)K_2 + y(n-2) \cdot K_3 = x(n) \cdot a + 2a\,x(n-1) + x(n-2)a$$

$$y(n) = \frac{1}{K_1} \cdot \left(-y(n-1)K_2 - y(n-2)K_3 + x(n) \cdot a + 2a\,x(n-1) + a\,x(n-2)\right)$$

$$H(s) = \frac{a}{s^2 + a^2} \qquad s \equiv \frac{z-1}{T}$$

$$H(z) = \frac{a}{\left(\frac{z-1}{T}\right)^2 + a^2} = \frac{aT^2}{(z-1)^2 + a^2T^2}$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{aT^2 \quad z^{-2}}{z^{-2}(z^2 - 2z + 1 + a^2T^2)} = \frac{aT^2 z^{-2}}{1 - 2z^{-1} + (1+a^2T^2)z^{-2}}$$

$$aT^2 z^{-2} X(z) = Y(z) - 2z^{-1}Y(z) + (1+a^2T^2) z^{-2} Y(z)$$

$$aT^2 x(n-2) = y(n) - 2y(n-1) + (1+a^2T^2) y(n-2)$$

$$\therefore \; y(n) = aT^2 x(n-2) + 2y(n-1) - (1+a^2T^2) y(n-2)$$

$$H(s) = \frac{a}{s^2 + a^2} \qquad s \equiv \frac{z-1}{Tz}$$

$$H(z) = \frac{a}{\left(\frac{z-1}{Tz}\right)^2 + a^2} = \frac{aT^2z^2}{z^2 - 2z + 1 + a^2T^2z^2}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{aT^2z^2}{z^2(1+a^2T^2) - 2z + 1} \cdot \frac{z^{-2}}{z^{-2}}$$

$$\frac{Y(z)}{X(z)} = \frac{aT^2}{(1+a^2T^2) - 2z^{-1} + z^{-2}}$$

$$(1+a^2T^2)y(n) - 2y(n-1) + y(n-2) = x(n)aT^2$$

$$y(n) = \frac{1}{1+a^2T^2}\left(+2y(n-1) - y(n-2)\right) + x(n)aT^2$$

$$y(n) = aT^2x(n-2) + 2y(n-1) - (1+a^2T^2)\,y(n-2)$$