Mathematics II	
029	
23/07/2024	8.30 AM-11.30 AM

NESA NATIONAL EXAMINATION AND SCHOOL INSPECTION AUTHORITY

ADVANCED LEVEL NATIONAL EXAMINATIONS, 2023-2024

SUBJECT: MATHEMATICS II

COMBINATIONS:

- MATHEMATICS-CHEMISTRY-BIOLOGY **(MCB)**
- MATHEMATICS -COMPUTER SCIENCE-ECONOMICS **(MCE)**
- MATHEMATICS-ECONOMICS-GEOGRAPHY **(MEG)**
- MATHEMATICS -PHYSICS-COMPUTER SCIENCE **(MPC)**
- MATHEMATICS-PHYSICS-GEOGRAPHY **(MPG)**
- PHYSICS-CHEMISTRY-MATHEMATICS **(PCM)**

DURATION: 3 HOURS

Instructions to candidates:

1) Write your names and index number on the answer booklet as written on your registration form, and **DO NOT** write your names and index number on additional answer sheets if provided.
2) Do not open this question paper until you are told to do so.
3) This paper consists of **two** sections: **A** and **B**.
   Section A: Attempt **all** questions. **(55 marks)**
   Section B: Attempt **only three** questions. **(45 marks)**
4) **Geometrical instruments and silent non-programmable calculators may be used.**
5) Use only a **blue** or **black** pen.

Section A: Attempt all questions (55 marks)

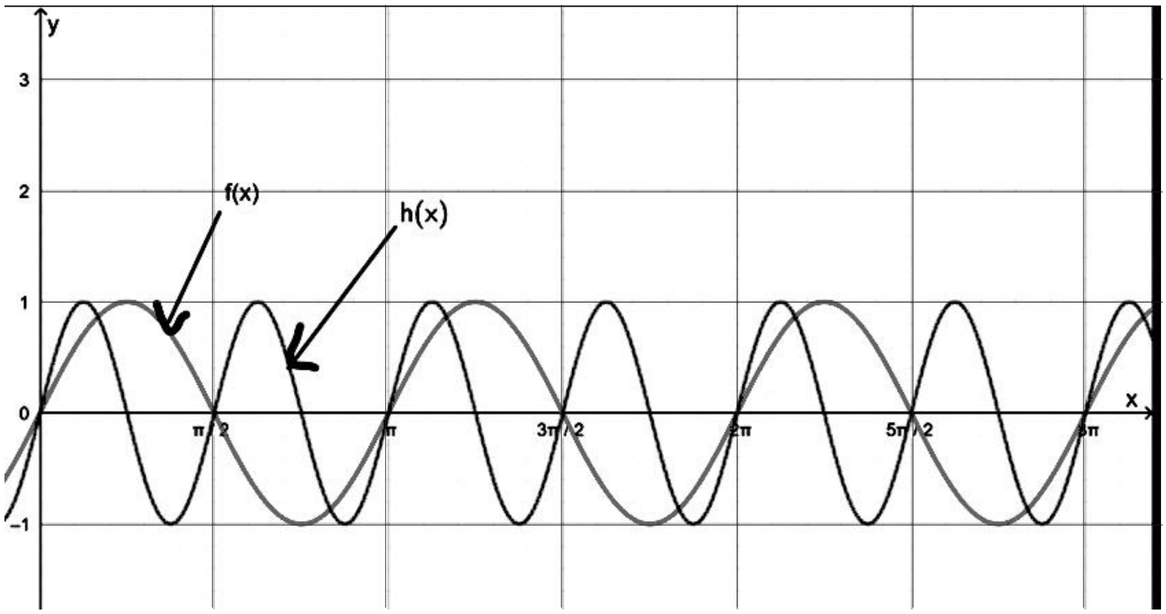
1) State whether each of the following statements is **true** or **false**.
   a) A binary operator or a binary operation combines two elements to give a unique third element. **(1 mark)**
   b) A compound proposition identically true for all possible truth values of its components is called a contradiction. **(1 mark)**

2) Solve in the set of real numbers, the equation:
   $\boldsymbol{2\log x = \log(x+12)}$
   (3 marks)

3) Find the value of the real number n in the Polynomial $\boldsymbol{P(x) = x^3 + nx^2 + 3x - 1}$ **, if** $P(x)$ leaves a remainder of 1 when divided by $\boldsymbol{x-2}$. **(4 marks)**

4) If $\boldsymbol{\tan A = \frac{a}{a+1}}$ *and* $\boldsymbol{\tan B = \frac{1}{2a+1}}$ **,** prove that $\boldsymbol{A + B = \frac{\pi}{4}}$.
   (3 marks)

5) By observing the following graphs of functions $\boldsymbol{f(x)}$**and** $\boldsymbol{h(x)}$:



   a) Write the period of each function. **(2 marks)**
   b) State any difference refer to those functions. **(1 mark)**

6) Solve the equation $\boldsymbol{3\cos x + 3 = 2\sin^2 x, 0 \le x \le 2\pi}$ **(4 marks)**

7) Determine the domain of definition for the function

$$\boldsymbol{f(x) = \ln\left(\frac{e^x}{1-e^x}\right).}$$ **(3 marks)**

8) Show that $\left(i + \sqrt{3}\right)^{96} = 2^{96}$. **(4 marks)**

9) Consider $A = \{(-2x, 0{,}5x), x \in \mathbb{R}\}$,show that $(\mathbb{R}, A, +)$ is a sub vector space of $\mathbb{R}^3$. **(3 marks)**

10) In a geometric progression, the sixth term is 8 times the third term and the sum of the seventh and eighth terms is 192. Determine:
   a) the common ratio. **(2 marks)**
   b) the first term. **(2 marks)**
   c) the sum of the fifth to eleventh terms. **(2 marks)**

11) Each student in a class gives each other a handshake. If there are 66 handshakes done in general, how many students are in the class? **(4 marks)**

12) Evaluate $\int \sqrt{1 - \sin x}\, dx$. **(4 marks)**

13) Determine whether the integral $\int_{-\infty}^{0} xe^x\, dx$ is convergent or divergent. **(5 marks)**

14) Find the mean value with respect to x of the function $(5x^2 - 4x)$ for $1 \leq x \leq 3$. **(4 marks)**

15) If the focus of a standard ellipse is at $(1,0)$ and corresponding directrix has the equation $x = 4$, find its equation. **(3 marks)**

Section B: Attempt three questions only (45 marks)

16) Consider the function f defined by $f(x) = ln(1 + x)$.
   a) Determine the domain and the range of f. **(2 marks)**
   b) Write down the first four non-zero terms of the Maclaurin expansion of $ln(1 + x)$ in ascending powers of x. **(3 marks)**

c) Without using hospital's rule, evaluate $\boldsymbol{lim_{x\to 0}\frac{ln(1+x)}{x}}$. **(2 marks)**

d) From the results in b) above, find an approximation to the integral $\int_0^1 \frac{ln(1+x)}{x}\,dx$, giving your answer correct to 4 decimal places.

(3 marks)

e) Write down the power series for $ln(1+x)$. **(2 marks)**

f) Determine the radius and interval of convergence of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$ **(3 marks)**

17) Find the general solution of the following differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=\frac{e^x}{x^2+1}$. **(15 marks)**

18) Using the method of matrix, solve the following system of equations:

$$\begin{cases}2x+y+z=5\\ x+y+z=4\\ x-y+2z=1\end{cases}$$

(15 marks)

19) A conic is given by the polar equation $r=\frac{10}{3-2\cos\theta}$.

a) Find the eccentricity, **(4 marks)**
b) Identify the conic, **(2 marks)**
c) Locate its directrix, **(4 marks)**
d) Sketch the conic. **(5 marks)**

20) A random variable x has probability density function

$$F(x)=\begin{cases}Ax(6-x)^2; 0\le x\le 6\\ 0 \qquad\qquad elsewhere\end{cases}$$

a) Find the value of the constant A . **(5 marks)**

b) Calculate:

i) The mean; **(4 marks)**
ii) The variance; **(4 marks)**
iii) The standard deviation. **(2 marks)**

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