

02.01-statistic_models

April 11, 2021

1 Modelos Econométricos

Neste notebook tem os seguintes modelos estatísticos: - AR - ARIMA - SARIMA

1.1 Importações

```
[1]: # Data analysis and data wrangling
import numpy as np
import pandas as pd

# Metrics
from sklearn.metrics import mean_squared_error
from sklearn.metrics import mean_absolute_percentage_error

# Plotting
import seaborn as sns
import matplotlib.pyplot as plt

# statsmodels
from statsmodels.tsa.ar_model import AutoReg
from statsmodels.tsa.arima_model import ARIMA
from statsmodels.tsa.statespace.sarimax import SARIMAX
import statsmodels.api as sm

# autoarima
import pmdarima as pm

# Other
from IPython.display import Image
import warnings
import pprint
import datetime
import itertools
import os
```

1.2 Preparação do Diretório Principal

```
[2]: def prepare_directory_work(end_directory: str='notebooks'):
      # Current path
      curr_dir = os.path.dirname(os.path.realpath("__file__"))

      if curr_dir.endswith(end_directory):
          os.chdir('..')
          return curr_dir

      return f'Current working directory: {curr_dir}'
```

```
[3]: prepare_directory_work(end_directory='notebooks')
```

```
[3]: '/home/campos/projects/tcc-ufsc-grad/notebooks'
```

1.3 Formatação das células

```
[4]: # OPTIONAL: Load the "autoreload" extension so that code can change
      %load_ext autoreload

      # Guarantees visualization inside the jupyter
      %matplotlib inline

      # Print xxxx rows and columns
      pd.set_option('display.max_rows', None)
      pd.set_option('display.max_columns', None)
      pd.set_option('float_format', '{:f}'.format)

      # Suppress unnecessary warnings so that presentation looks clean
      warnings.filterwarnings('ignore')

      # pretty print
      pp = pprint.PrettyPrinter(indent=4)
```

```
[5]: plt.style.use('seaborn') # fivethirtyeight
      plt.rc('figure',figsize=(16,8))
      plt.rc('font',size=15)
      plt.rc('legend',fontsize=15)

      # Seaborn rcParams
      # =====
      sns.set(context='poster', # notebook
              style='darkgrid',
              palette='deep',
              color_codes=True)
```

```
# graph style
sns.set(style='dark', palette='deep')

plt.style.use('fivethirtyeight')
```

1.4 Carregamento dos dados

```
[6]: %%time

df_vale3 = pd.read_csv('data/cleansing/df_vale3_cleansing.csv',
                        encoding='utf8',
                        delimiter=',',
                        parse_dates=True,
                        index_col=0,
                        verbose=True)
```

Tokenization took: 2.94 ms

Type conversion took: 3.39 ms

Parser memory cleanup took: 0.01 ms

CPU times: user 15.2 ms, sys: 1.52 ms, total: 16.7 ms

Wall time: 15.2 ms

```
[7]: df_vale3.head()
```

```
[7]:
```

	preco	residuos	tendencia	sazonalidade	diff_1	diff_2	\
data							
2010-07-12	40.000000	1.002310	41.827333	1.000149	-0.600000	-0.460000	
2010-07-13	40.070000	1.036654	41.910833	0.998563	0.070000	-0.530000	
2010-07-14	40.080000	1.028377	41.977833	1.000439	0.010000	0.080000	
2010-07-15	39.760000	1.044658	42.045833	1.000935	-0.320000	-0.310000	
2010-07-16	38.880000	1.028132	42.123500	1.001784	-0.880000	-1.200000	

	diff_3	diff_4	diff_5
data			
2010-07-12	0.490000	0.980000	0.420000
2010-07-13	-0.390000	0.560000	1.050000
2010-07-14	-0.520000	-0.380000	0.570000
2010-07-15	-0.240000	-0.840000	-0.700000
2010-07-16	-1.190000	-1.120000	-1.720000

1.5 Divisão dos Dados

```
[8]: size_train = 2132
size_test = 313
print(size_train)
print(size_test)

df_train = df_vale3['preco'].iloc[:size_train]
df_test = df_vale3['preco'].iloc[size_train:]
```

2132

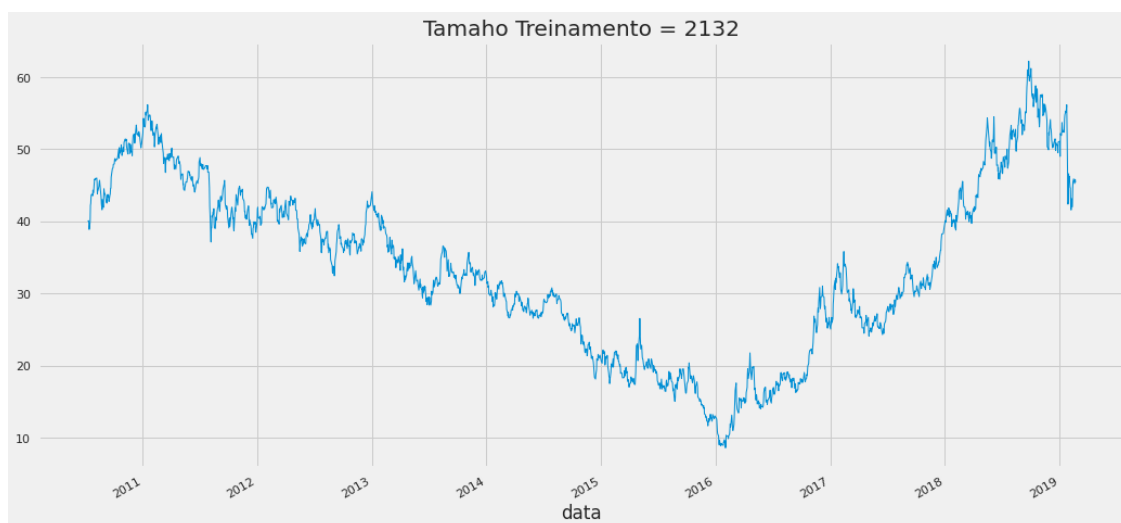
313

```
[9]: df_train.tail()
```

```
[9]: data
2019-02-15    45.880000
2019-02-18    45.250000
2019-02-19    45.490000
2019-02-20    45.800000
2019-02-21    45.380000
Name: preco, dtype: float64
```

```
[10]: df_train.plot(linewidth=1)
plt.grid(True)
plt.title(f'Tamaho Treinamento = {len(df_train)}')
```

```
[10]: Text(0.5, 1.0, 'Tamaho Treinamento = 2132')
```

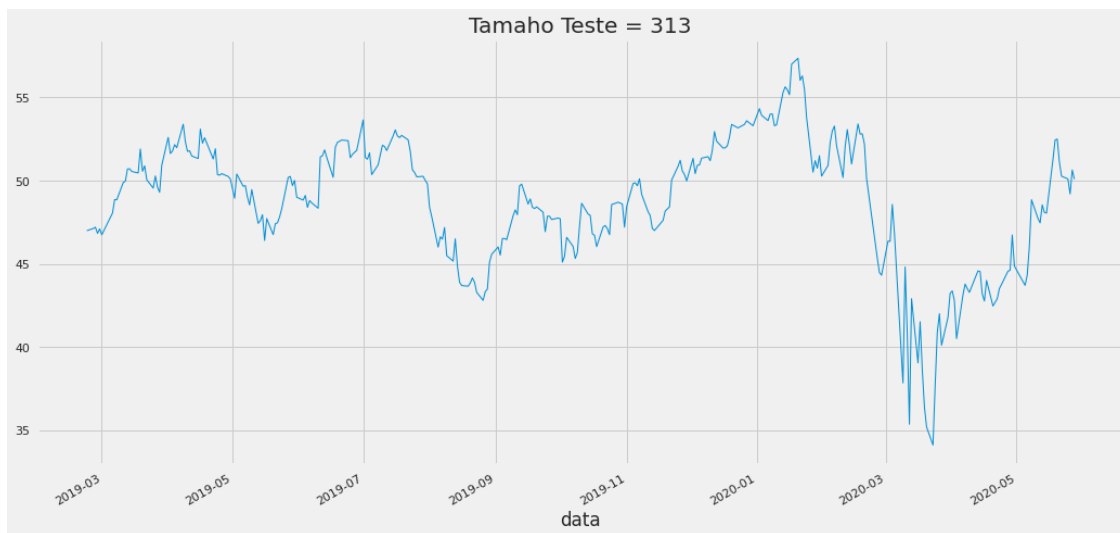


```
[11]: df_test.head()
```

```
[11]: data
      2019-02-22    46.990000
      2019-02-25    47.120000
      2019-02-26    47.200000
      2019-02-27    46.830000
      2019-02-28    47.100000
      Name: preco, dtype: float64
```

```
[12]: df_test.plot(linewidth=1)
      plt.grid(True)
      plt.title(f'Tamaho Teste = {len(df_test)}')
```

```
[12]: Text(0.5, 1.0, 'Tamaho Teste = 313')
```



1.5.1 Manipulação do índice

```
[13]: df_train.index
```

```
[13]: DatetimeIndex(['2010-07-12', '2010-07-13', '2010-07-14', '2010-07-15',
                    '2010-07-16', '2010-07-19', '2010-07-20', '2010-07-21',
                    '2010-07-22', '2010-07-23',
                    ...,
                    '2019-02-08', '2019-02-11', '2019-02-12', '2019-02-13',
                    '2019-02-14', '2019-02-15', '2019-02-18', '2019-02-19',
                    '2019-02-20', '2019-02-21'],
                    dtype='datetime64[ns]', name='data', length=2132, freq=None)
```

```
[14]: df_test.index
```

```
[14]: DatetimeIndex(['2019-02-22', '2019-02-25', '2019-02-26', '2019-02-27',
                    '2019-02-28', '2019-03-01', '2019-03-06', '2019-03-07',
                    '2019-03-08', '2019-03-11',
                    ...
                    '2020-05-15', '2020-05-18', '2020-05-19', '2020-05-20',
                    '2020-05-21', '2020-05-22', '2020-05-25', '2020-05-26',
                    '2020-05-27', '2020-05-28'],
                    dtype='datetime64[ns]', name='data', length=313, freq=None)
```

```
[15]: df_train.reset_index(drop=True, inplace=True)
      df_train.index
```

```
[15]: RangeIndex(start=0, stop=2132, step=1)
```

```
[16]: df_test.reset_index(drop=True, inplace=True)
      df_test.index
```

```
[16]: RangeIndex(start=0, stop=313, step=1)
```

```
[17]: df_train.index = pd.RangeIndex(start=0, stop=len(df_train), step=1)
      df_train.index
```

```
[17]: RangeIndex(start=0, stop=2132, step=1)
```

```
[18]: df_test.index = pd.RangeIndex(start=2132, stop=len(df_vale3), step=1)
      df_test.index
```

```
[18]: RangeIndex(start=2132, stop=2445, step=1)
```

1.6 Dicionário de Resultados

```
[19]: dict_results = {}
```

1.7 Impressão dos Resultados

```
[20]: def show_result_model(df_train, df_test, y_forecast, model_name):
      future_forecast = pd.DataFrame(y_forecast,
                                     index=df_test.index,
                                     columns=['previsao'])

      #mape = mean_absolute_percentage_error(df_test, y_forecast)
      mape = mean_absolute_percentage_error(df_test, y_forecast)*100

      mse = mean_squared_error(df_test, y_forecast, squared=True)
      dict_results[model_name] = [mape, mse]
```

```

pd.concat([df_test, future_forecast], axis=1).plot()

plt.legend()
plt.grid(True)
plt.xlabel("Tempo (dias)", fontsize=20)
plt.ylabel("Preço (R$)", fontsize=20)
plt.title(f'MAPE = {mape:.2f} % | MSE = {mse:.2f}', fontsize=25)

```

1.8 Busca dos Melhores Parâmetros

Grid Search

```

[21]: # Define the p, d and q parameters to take any value between 0 and 2
p = q = range(0, 3)
d = range(0, 3)

# Generate all different combinations of p, q and q triplets
list_pdq = list(itertools.product(p, d, q))
print(f'All different combinations of p, q and q:\n {list_pdq}')

# Generate all different combinations of seasonal p, q and q triplets
seasonal_pdq = [(x[0], x[1], x[2], 12) for x in list(itertools.product(p, d,
    ↪q))]
print(f'\n\nAll different combinations of seasonal p, q and q:\n
    ↪{seasonal_pdq}')

```

All different combinations of p, q and q:

```

[(0, 0, 0), (0, 0, 1), (0, 0, 2), (0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0),
(0, 2, 1), (0, 2, 2), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 1, 0), (1, 1, 1), (1,
1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2), (2, 0, 0), (2, 0, 1), (2, 0, 2), (2, 1,
0), (2, 1, 1), (2, 1, 2), (2, 2, 0), (2, 2, 1), (2, 2, 2)]

```

All different combinations of seasonal p, q and q:

```

[(0, 0, 0, 12), (0, 0, 1, 12), (0, 0, 2, 12), (0, 1, 0, 12), (0, 1, 1, 12), (0,
1, 2, 12), (0, 2, 0, 12), (0, 2, 1, 12), (0, 2, 2, 12), (1, 0, 0, 12), (1, 0, 1,
12), (1, 0, 2, 12), (1, 1, 0, 12), (1, 1, 1, 12), (1, 1, 2, 12), (1, 2, 0, 12),
(1, 2, 1, 12), (1, 2, 2, 12), (2, 0, 0, 12), (2, 0, 1, 12), (2, 0, 2, 12), (2,
1, 0, 12), (2, 1, 1, 12), (2, 1, 2, 12), (2, 2, 0, 12), (2, 2, 1, 12), (2, 2, 2,
12)]

```

```

[22]: def search_best_params_arima_model(df_train: 'Dataframe', pdq: list) -> list:
    best_model = 99999
    best_params = (0, 0, 0)
    param = ()

```

```

for param in pdq:
    try:
        arima_model = ARIMA(df_train, order=param)
        results = arima_model.fit()
        print(f'pdq = {param} | AIC = {results.aic}')

        if results.aic < best_model:
            best_model = results.aic
            best_params = param
    except:
        continue

print(f'best ARIMA: {best_params} | AIC:{best_model}')
return [best_params, best_model]

```

```

[23]: def search_best_params_sarima_model(df_train, trend, pdq):
    best_model = 99999
    best_param_seasonal = ()
    param = ()
    param_seasonal = ()

    for param_seasonal in seasonal_pdq:
        try:
            sarima_model = SARIMAX(df_train,
                                    order=pdq,
                                    seasonal_order=param_seasonal,
                                    trend=trend,
                                    enforce_stationarity=True,
                                    enforce_invertibility=False)

            results = sarima_model.fit()
            print(f'pdq = {pdq} | param_seasonal = {param_seasonal} | AIC = ↪
            {results.aic}')

            if results.aic < best_model:
                best_model = results.aic
                best_param_seasonal = param_seasonal
        except:
            continue

    print(f'\n\nBest SARIMA: {pdq}x{param_seasonal}12 | AIC:{best_model}')
    return [best_param_seasonal, best_model]

```


1.9 Modelos Estatísticos

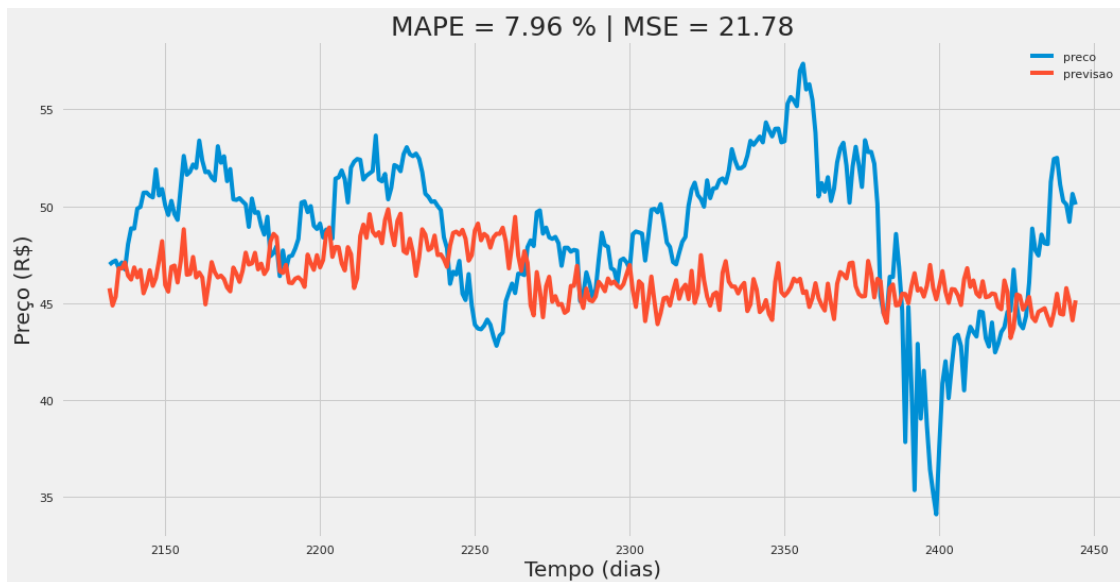
1.10 AR

- A ST não é estacionária
- Não há tendência, (trend='n')
- Não há sazonalidade, (seasonal=False) no período de 30 dias

```
[24]: ar_model = AutoReg(df_train,
                        lags=313,
                        trend='n',
                        seasonal=False,
                        period=len(df_test))
ar_fit = ar_model.fit()
```

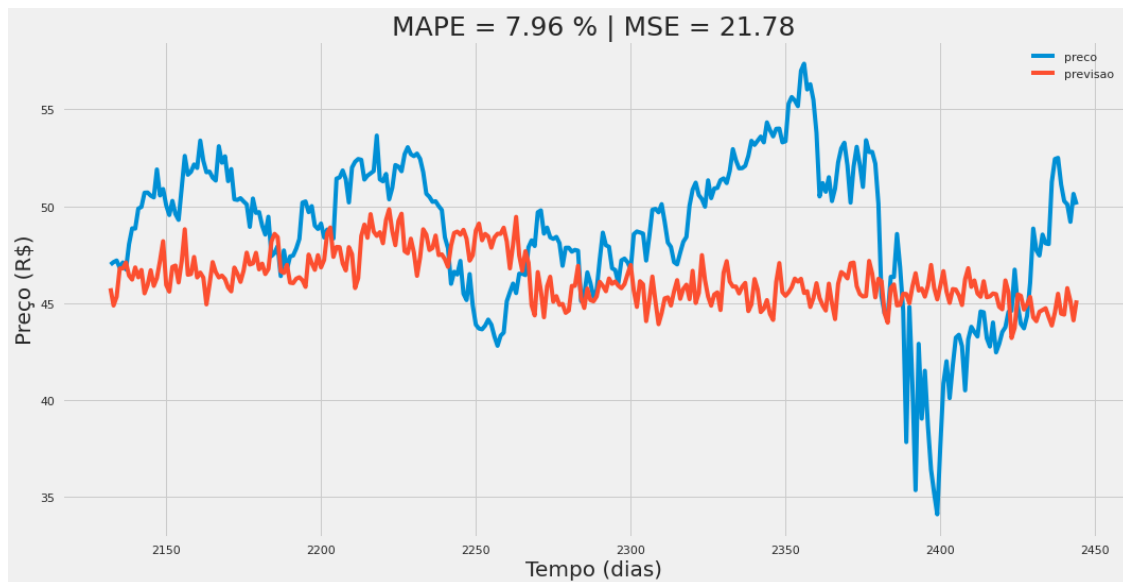
```
[25]: # forecast
y_ar_forecast = ar_fit.predict(start=(df_test.index[0]), end=df_test.index[-1])
```

```
[26]: show_result_model(df_train=df_train,
                        df_test=df_test,
                        y_forecast=y_ar_forecast,
                        model_name='ar_model')
```



zoom

```
[27]: show_result_model(df_train=df_test,
                        df_test=df_test,
                        y_forecast=y_ar_forecast,
                        model_name='ar_model')
```



1.10.1 ARIMA

1.11 Librarie: `pmdarima`

- Tips: https://alkaline-ml.com/pmdarima/tips_and_tricks.html

```

pdq = (1, 2, 2) | AIC = 5203.335114958441
pdq = (2, 0, 0) | AIC = 5209.340449357725
pdq = (2, 0, 1) | AIC = 5202.394724547465
pdq = (2, 0, 2) | AIC = 5198.922147313669
pdq = (2, 1, 0) | AIC = 5190.807259240531
pdq = (2, 1, 1) | AIC = 5192.782074290211
pdq = (2, 1, 2) | AIC = 5189.626133250238
pdq = (2, 2, 0) | AIC = 5852.928438526969
pdq = (2, 2, 1) | AIC = 5198.377858813686
pdq = (2, 2, 2) | AIC = 5200.355168190182
best ARIMA: (2, 1, 2) | AIC:5189.626133250238
[(2, 1, 2), 5189.626133250238]
CPU times: user 17.5 s, sys: 8.89 s, total: 26.4 s
Wall time: 9.95 s

```

```

[33]: %%time

autoarima_model = pm.auto_arima(df_train,
                                stepwise=True,
                                suppress_warnings=True,
                                error_action="ignore",
                                information_criterion='aic',
                                start_p=2,
                                start_d=1,
                                start_q=2,
                                lags=313,
                                seasonal=False,
                                trace=True)

```

Performing stepwise search to minimize aic

```

ARIMA(2,1,2)(0,0,0)[0] intercept : AIC=5189.626, Time=1.78 sec
ARIMA(0,1,0)(0,0,0)[0] intercept : AIC=5202.401, Time=0.14 sec
ARIMA(1,1,0)(0,0,0)[0] intercept : AIC=5202.744, Time=0.12 sec
ARIMA(0,1,1)(0,0,0)[0] intercept : AIC=5202.427, Time=0.14 sec
ARIMA(0,1,0)(0,0,0)[0]          : AIC=5200.421, Time=0.03 sec
ARIMA(1,1,2)(0,0,0)[0] intercept : AIC=5191.786, Time=0.49 sec
ARIMA(2,1,1)(0,0,0)[0] intercept : AIC=5192.782, Time=0.56 sec
ARIMA(3,1,2)(0,0,0)[0] intercept : AIC=5191.420, Time=1.94 sec
ARIMA(2,1,3)(0,0,0)[0] intercept : AIC=5191.381, Time=2.26 sec
ARIMA(1,1,1)(0,0,0)[0] intercept : AIC=5195.793, Time=0.42 sec
ARIMA(1,1,3)(0,0,0)[0] intercept : AIC=5189.941, Time=2.74 sec
ARIMA(3,1,1)(0,0,0)[0] intercept : AIC=5194.800, Time=0.66 sec
ARIMA(3,1,3)(0,0,0)[0] intercept : AIC=5193.582, Time=1.96 sec
ARIMA(2,1,2)(0,0,0)[0]          : AIC=5187.657, Time=0.38 sec
ARIMA(1,1,2)(0,0,0)[0]          : AIC=5189.811, Time=0.24 sec
ARIMA(2,1,1)(0,0,0)[0]          : AIC=5190.805, Time=0.49 sec
ARIMA(3,1,2)(0,0,0)[0]          : AIC=5189.452, Time=0.72 sec
ARIMA(2,1,3)(0,0,0)[0]          : AIC=5189.413, Time=0.85 sec

```

```
ARIMA(1,1,1)(0,0,0)[0]      : AIC=5193.812, Time=0.17 sec
ARIMA(1,1,3)(0,0,0)[0]      : AIC=5192.206, Time=0.23 sec
ARIMA(3,1,1)(0,0,0)[0]      : AIC=5192.823, Time=0.13 sec
ARIMA(3,1,3)(0,0,0)[0]      : AIC=5191.614, Time=0.72 sec
```

```
Best model:  ARIMA(2,1,2)(0,0,0)[0]
Total fit time: 17.189 seconds
CPU times: user 42 s, sys: 21.7 s, total: 1min 3s
Wall time: 17.2 s
```

```
[34]: print(autoarima_model.order)
      print(autoarima_model.aic())
```

```
(2, 1, 2)
5187.656935415547
```

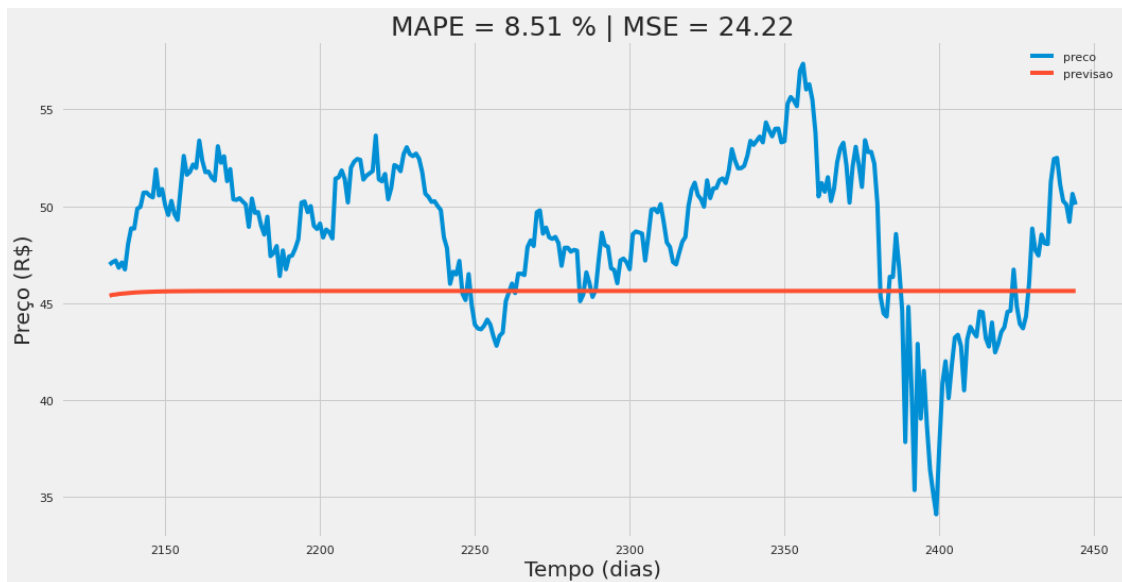
```
[36]: # fit
      autoarima_model_fit = autoarima_model.fit(y=df_train)
      autoarima_model_fit
```

```
[36]: ARIMA(order=(2, 1, 2), scoring_args={}, suppress_warnings=True,
          with_intercept=False)
```

```
[37]: # forecast
      y_forecast = autoarima_model_fit.predict(n_periods=len(df_test[:313]))
      len(y_forecast)
```

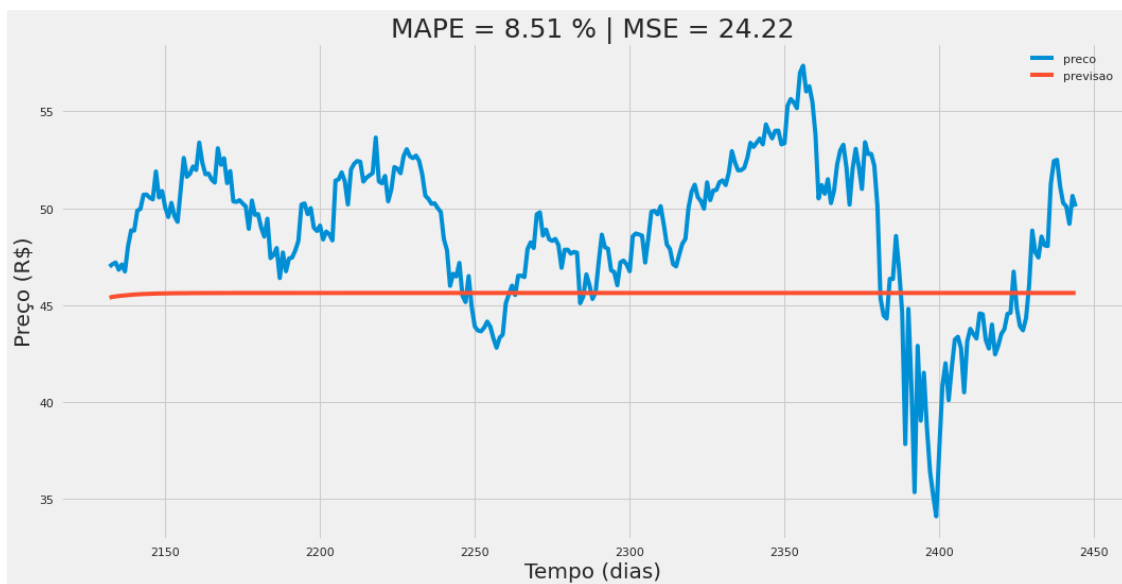
```
[37]: 313
```

```
[39]: show_result_model(df_train=df_train,
                        df_test=df_test,
                        y_forecast=y_forecast,
                        model_name='arima_model')
```



Zoom

```
[40]: show_result_model(df_train=df_test,  
                        df_test=df_test,  
                        y_forecast=y_forecast,  
                        model_name='arima_model')
```



1.12 SARIMA

```
[49]: %%time

list_order_seasonal_aic = search_best_params_sarima_model(df_train=df_train,
                                                         trend='t',
                                                         pdq=(2, 1, 2))

print(list_order_seasonal_aic)
```

```
pdq = (2, 1, 2) | param_seasonal = (0, 0, 0, 12) | AIC = 5191.5422018697
pdq = (2, 1, 2) | param_seasonal = (0, 0, 1, 12) | AIC = 5191.568584251327
pdq = (2, 1, 2) | param_seasonal = (0, 0, 2, 12) | AIC = 5191.023297688418
pdq = (2, 1, 2) | param_seasonal = (0, 1, 0, 12) | AIC = 6669.048884663625
pdq = (2, 1, 2) | param_seasonal = (0, 1, 1, 12) | AIC = 6018.448897661481
pdq = (2, 1, 2) | param_seasonal = (0, 1, 2, 12) | AIC = 7730.417661667607
pdq = (2, 1, 2) | param_seasonal = (0, 2, 0, 12) | AIC = 8901.911156543076
pdq = (2, 1, 2) | param_seasonal = (0, 2, 1, 12) | AIC = 8225.333911230264
pdq = (2, 1, 2) | param_seasonal = (0, 2, 2, 12) | AIC = 7669.739406840514
pdq = (2, 1, 2) | param_seasonal = (1, 0, 0, 12) | AIC = 5191.430740851928
pdq = (2, 1, 2) | param_seasonal = (1, 0, 1, 12) | AIC = 5194.0387448148795
pdq = (2, 1, 2) | param_seasonal = (1, 0, 2, 12) | AIC = 5193.801789566946
pdq = (2, 1, 2) | param_seasonal = (1, 1, 0, 12) | AIC = 6126.856861312977
pdq = (2, 1, 2) | param_seasonal = (1, 1, 1, 12) | AIC = 5969.51317179567
pdq = (2, 1, 2) | param_seasonal = (1, 1, 2, 12) | AIC = 6195.0245268799035
pdq = (2, 1, 2) | param_seasonal = (1, 2, 0, 12) | AIC = 7989.770158383615
pdq = (2, 1, 2) | param_seasonal = (1, 2, 1, 12) | AIC = 7472.563750855076
pdq = (2, 1, 2) | param_seasonal = (1, 2, 2, 12) | AIC = 8155.590108751798
pdq = (2, 1, 2) | param_seasonal = (2, 0, 0, 12) | AIC = 5190.676621773375
pdq = (2, 1, 2) | param_seasonal = (2, 0, 1, 12) | AIC = 5194.585803420131
pdq = (2, 1, 2) | param_seasonal = (2, 0, 2, 12) | AIC = 5195.639738795778
pdq = (2, 1, 2) | param_seasonal = (2, 1, 0, 12) | AIC = 5963.883309501448
pdq = (2, 1, 2) | param_seasonal = (2, 1, 1, 12) | AIC = 5933.311399586823
pdq = (2, 1, 2) | param_seasonal = (2, 1, 2, 12) | AIC = 6237.863137339467
pdq = (2, 1, 2) | param_seasonal = (2, 2, 0, 12) | AIC = 7712.475147540053
pdq = (2, 1, 2) | param_seasonal = (2, 2, 1, 12) | AIC = 7468.894534182369
pdq = (2, 1, 2) | param_seasonal = (2, 2, 2, 12) | AIC = 7794.323067797779
```

```
Best SARIMA: (2, 1, 2)x(2, 2, 2, 12)12 | AIC:5190.676621773375
[(2, 0, 0, 12), 5190.676621773375]
CPU times: user 47min 37s, sys: 38min 31s, total: 1h 26min 8s
Wall time: 12min 52s
```

```
[50]: list_order_seasonal_aic
```

```
[50]: [(2, 0, 0, 12), 5190.676621773375]
```

```
[52]: sarima_model = SARIMAX(df_train,
                             order=(2, 1, 2),
                             seasonal_order=list_order_seasonal_aic[0],
                             trend='c',
                             enforce_stationarity=False,
                             enforce_invertibility=False)
```

```
[53]: # fit
sarima_fit = sarima_model.fit()
print(sarima_fit)
```

<statsmodels.tsa.statespace.sarimax.SARIMAXResultsWrapper object at 0x7fcac5d07940>

O 7 B i

]:

fit

=

1.13 Resultados

```
[48]: dict_results
```

```
[48]: {'ar_model': [7.955813872394689, 21.780365013206687],  
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