02.01-statistic_models

April 11, 2021

1 Modelos Econométricos

Neste notebook tem os seguintes modelos estatísticos: - AR - ARIMA - SARIMA

1.1 Importações

```python
[1]: # Data analysis and data wrangling
     import numpy as np
     import pandas as pd

     # Metrics
     from sklearn.metrics import mean_squared_error
     from sklearn.metrics import mean_absolute_percentage_error

     # Plotting
     import seaborn as sns
     import matplotlib.pyplot as plt

     # statsmodels
     from statsmodels.tsa.ar_model import AutoReg
     from statsmodels.tsa.arima_model import ARIMA
     from statsmodels.tsa.statespace.sarimax import SARIMAX
     import statsmodels.api as sm

     # autoarima
     import pmdarima as pm

     # Other
     from IPython.display import Image
     import warnings
     import pprint
     import datetime
     import itertools
     import os
```

1.2 Preparação do Diretório Principal

```
[2]: def prepare_directory_work(end_directory: str='notebooks'):
         # Current path
         curr_dir = os.path.dirname(os.path.realpath("__file__"))

         if curr_dir.endswith(end_directory):
             os.chdir('..')
             return curr_dir

         return f'Current working directory: {curr_dir}'
```

```
[3]: prepare_directory_work(end_directory='notebooks')
```

```
[3]: '/home/campos/projects/tcc-ufsc-grad/notebooks'
```

1.3 Formatação das células

```
[4]: # OPTIONAL: Load the "autoreload" extension so that code can change
     %load_ext autoreload

     # Guarantees visualization inside the jupyter
     %matplotlib inline

     # Print xxxx rows and columns
     pd.set_option('display.max_rows', None)
     pd.set_option('display.max_columns', None)
     pd.set_option('float_format', '{:f}'.format)

     # Supress unnecessary warnings so that presentation looks clean
     warnings.filterwarnings('ignore')

     # pretty print
     pp = pprint.PrettyPrinter(indent=4)
```

```
[5]: plt.style.use('seaborn')  # fivethirtyeight
     plt.rc('figure',figsize=(16,8))
     plt.rc('font',size=15)
     plt.rc('legend',fontsize=15)

     # Seaborn rcParams
     # ===================
     sns.set(context='poster',  # notebook
             style='darkgrid',
             palette='deep',
             color_codes=True)
```

```
# graph style
sns.set(style='dark', palette='deep')

plt.style.use('fivethirtyeight')
```

1.4   Carregamento dos dados

```
[6]: %%time

df_vale3 = pd.read_csv('data/cleansing/df_vale3_cleansing.csv',
                       encoding='utf8',
                       delimiter=',',
                       parse_dates=True,
                       index_col=0,
                       verbose=True)
```

```
Tokenization took: 2.94 ms
Type conversion took: 3.39 ms
Parser memory cleanup took: 0.01 ms
CPU times: user 15.2 ms, sys: 1.52 ms, total: 16.7 ms
Wall time: 15.2 ms
```

```
[7]: df_vale3.head()
```

```
[7]:              preco   residuos  tendencia  sazonalidade    diff_1     diff_2  \
     data
     2010-07-12 40.000000  1.002310  41.827333      1.000149 -0.600000 -0.460000
     2010-07-13 40.070000  1.036654  41.910833      0.998563  0.070000 -0.530000
     2010-07-14 40.080000  1.028377  41.977833      1.000439  0.010000  0.080000
     2010-07-15 39.760000  1.044658  42.045833      1.000935 -0.320000 -0.310000
     2010-07-16 38.880000  1.028132  42.123500      1.001784 -0.880000 -1.200000


                  diff_3    diff_4    diff_5
     data
     2010-07-12  0.490000  0.980000  0.420000
     2010-07-13 -0.390000  0.560000  1.050000
     2010-07-14 -0.520000 -0.380000  0.570000
     2010-07-15 -0.240000 -0.840000 -0.700000
     2010-07-16 -1.190000 -1.120000 -1.720000
```

1.5 Divisão dos Dados

```
[8]: size_train = 2132
     size_test = 313
     print(size_train)
     print(size_test)

     df_train = df_vale3['preco'].iloc[:size_train]
     df_test = df_vale3['preco'].iloc[size_train:]
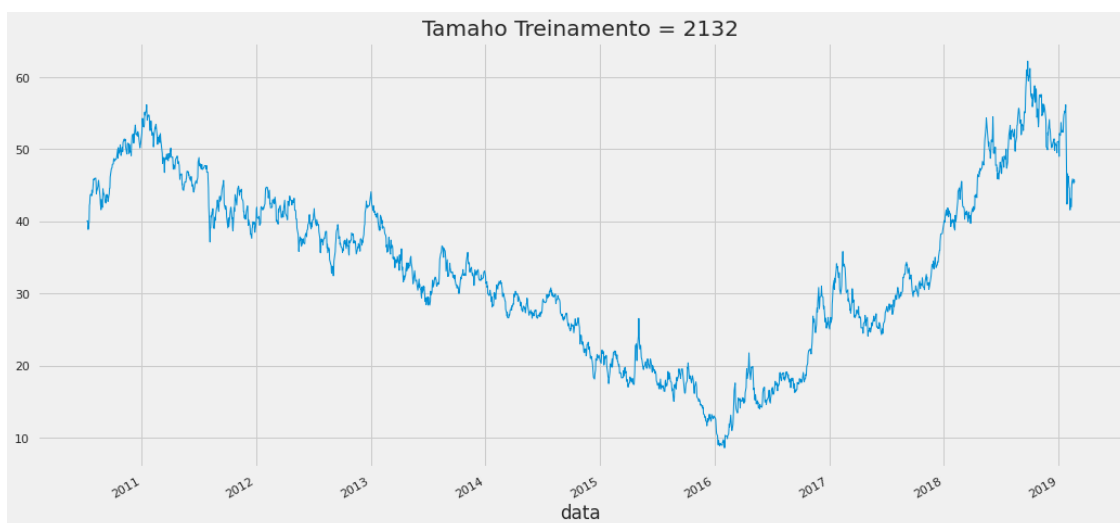```

```
2132
313
```

```
[9]: df_train.tail()
```

```
[9]: data
     2019-02-15    45.880000
     2019-02-18    45.250000
     2019-02-19    45.490000
     2019-02-20    45.800000
     2019-02-21    45.380000
     Name: preco, dtype: float64
```

```
[10]: df_train.plot(linewidth=1)
      plt.grid(True)
      plt.title(f'Tamaho Treinamento = {len(df_train)}')
```

```
[10]: Text(0.5, 1.0, 'Tamaho Treinamento = 2132')
```
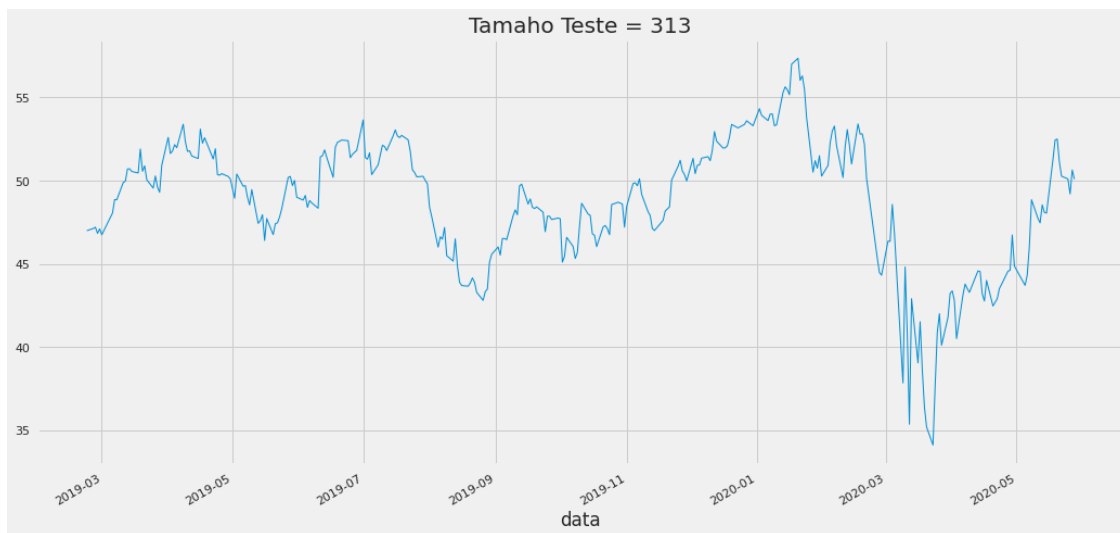


```
[11]: df_test.head()
```

```
[11]: data
      2019-02-22    46.990000
      2019-02-25    47.120000
      2019-02-26    47.200000
      2019-02-27    46.830000
      2019-02-28    47.100000
      Name: preco, dtype: float64
```

```
[12]: df_test.plot(linewidth=1)
      plt.grid(True)
      plt.title(f'Tamaho Teste = {len(df_test)}')
```

```
[12]: Text(0.5, 1.0, 'Tamaho Teste = 313')
```


1.5.1 Manipulação do índice

```
[13]: df_train.index
```

```
[13]: DatetimeIndex(['2010-07-12', '2010-07-13', '2010-07-14', '2010-07-15',
                     '2010-07-16', '2010-07-19', '2010-07-20', '2010-07-21',
                     '2010-07-22', '2010-07-23',
                     ...
                     '2019-02-08', '2019-02-11', '2019-02-12', '2019-02-13',
                     '2019-02-14', '2019-02-15', '2019-02-18', '2019-02-19',
                     '2019-02-20', '2019-02-21'],
                    dtype='datetime64[ns]', name='data', length=2132, freq=None)
```

```
[14]: df_test.index
```

```
[14]: DatetimeIndex(['2019-02-22', '2019-02-25', '2019-02-26', '2019-02-27',
                      '2019-02-28', '2019-03-01', '2019-03-06', '2019-03-07',
                      '2019-03-08', '2019-03-11',
                      ...
                      '2020-05-15', '2020-05-18', '2020-05-19', '2020-05-20',
                      '2020-05-21', '2020-05-22', '2020-05-25', '2020-05-26',
                      '2020-05-27', '2020-05-28'],
                     dtype='datetime64[ns]', name='data', length=313, freq=None)
```

```python
[15]: df_train.reset_index(drop=True, inplace=True)
      df_train.index
```

```
[15]: RangeIndex(start=0, stop=2132, step=1)
```

```python
[16]: df_test.reset_index(drop=True, inplace=True)
      df_test.index
```

```
[16]: RangeIndex(start=0, stop=313, step=1)
```

```python
[17]: df_train.index = pd.RangeIndex(start=0, stop=len(df_train), step=1)
      df_train.index
```

```
[17]: RangeIndex(start=0, stop=2132, step=1)
```

```python
[18]: df_test.index = pd.RangeIndex(start=2132, stop=len(df_vale3), step=1)
      df_test.index
```

```
[18]: RangeIndex(start=2132, stop=2445, step=1)
```

1.6 Dicionário de Resultados

```python
[19]: dict_results = {}
```

1.7 Impressão dos Resutados

```python
[20]: def show_result_model(df_train, df_test, y_forecast, model_name):
          future_forecast = pd.DataFrame(y_forecast,
                                         index=df_test.index,
                                         columns=['previsao'])
          #mape = mean_absolute_percentage_error(df_test, y_forecast)
          mape = mean_absolute_percentage_error(df_test, y_forecast)*100

          mse = mean_squared_error(df_test, y_forecast, squared=True)
          dict_results[model_name] = [mape, mse]
```

```
pd.concat([df_test, future_forecast], axis=1).plot()

plt.legend()
plt.grid(True)
plt.xlabel("Tempo (dias)", fontsize=20)
plt.ylabel("Preço (R$)", fontsize=20)
plt.title(f'MAPE = {mape:.2f} % | MSE = {mse:.2f}', fontsize=25)
```

1.8 Busca dos Melhores Parâmetros

Grid Search

```
[21]:  # Define the p, d and q parameters to take any value between 0 and 2
       p = q = range(0, 3)
       d = range(0, 3)

       # Generate all different combinations of p, q and q triplets
       list_pdq = list(itertools.product(p, d, q))
       print(f'All different combinations of p, q and q:\n {list_pdq}')

       # Generate all different combinations of seasonal p, q and q triplets
       seasonal_pdq = [(x[0], x[1], x[2], 12) for x in list(itertools.product(p, d,
         →q))]
       print(f'\n\nAll different combinations of seasonal p, q and q:\n
         →{seasonal_pdq}')
```

All different combinations of p, q and q:
 [(0, 0, 0), (0, 0, 1), (0, 0, 2), (0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0),
(0, 2, 1), (0, 2, 2), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 1, 0), (1, 1, 1), (1,
1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2), (2, 0, 0), (2, 0, 1), (2, 0, 2), (2, 1,
0), (2, 1, 1), (2, 1, 2), (2, 2, 0), (2, 2, 1), (2, 2, 2)]

All different combinations of seasonal p, q and q:
 [(0, 0, 0, 12), (0, 0, 1, 12), (0, 0, 2, 12), (0, 1, 0, 12), (0, 1, 1, 12), (0,
1, 2, 12), (0, 2, 0, 12), (0, 2, 1, 12), (0, 2, 2, 12), (1, 0, 0, 12), (1, 0, 1,
12), (1, 0, 2, 12), (1, 1, 0, 12), (1, 1, 1, 12), (1, 1, 2, 12), (1, 2, 0, 12),
(1, 2, 1, 12), (1, 2, 2, 12), (2, 0, 0, 12), (2, 0, 1, 12), (2, 0, 2, 12), (2,
1, 0, 12), (2, 1, 1, 12), (2, 1, 2, 12), (2, 2, 0, 12), (2, 2, 1, 12), (2, 2, 2,
12)]

```
[22]:  def search_best_params_arima_model(df_train: 'Dataframe', pdq: list) -> list:
           best_model = 99999
           best_params = (0, 0, 0)
           param = ()
```

```python
    for param in pdq:
        try:
            arima_model = ARIMA(df_train, order=param)
            results = arima_model.fit()
            print(f'pdq = {param} | AIC = {results.aic}')

            if results.aic < best_model:
                best_model = results.aic
                best_params = param
        except:
            continue

    print(f'best ARIMA: {best_params} | AIC:{best_model}')
    return [best_params, best_model]
```

```python
[23]: def search_best_params_sarima_model(df_train, trend, pdq):
    best_model = 99999
    best_param_seasonal = ()
    param = ()
    param_seasonal = ()

    for param_seasonal in seasonal_pdq:
        try:
            sarima_model = SARIMAX(df_train,
                                   order=pdq,
                                   seasonal_order=param_seasonal,
                                   trend=trend,
                                   enforce_stationarity=True,
                                   enforce_invertibility=False)
            results = sarima_model.fit()
            print(f'pdq = {pdq} | param_seasonal = {param_seasonal} | AIC =␣
 ↪{results.aic}')

            if results.aic < best_model:
                best_model = results.aic
                best_param_seasonal = param_seasonal
        except:
            continue

    print(f'\n\nBest SARIMA: {pdq}x{param_seasonal}12 | AIC:{best_model}')
    return [best_param_seasonal, best_model]
```

1.9 Modelos Estatísticos

1.10 AR

- A ST não é estacionária
- Não há tendência, (trend='n')
- Não aá sazonalidade, (seasonal=False) no período de 30 dias

```
[24]: ar_model = AutoReg(df_train,
                         lags=313,
                         trend='n',
                         seasonal=False,
                         period=len(df_test))
      ar_fit = ar_model.fit()
```
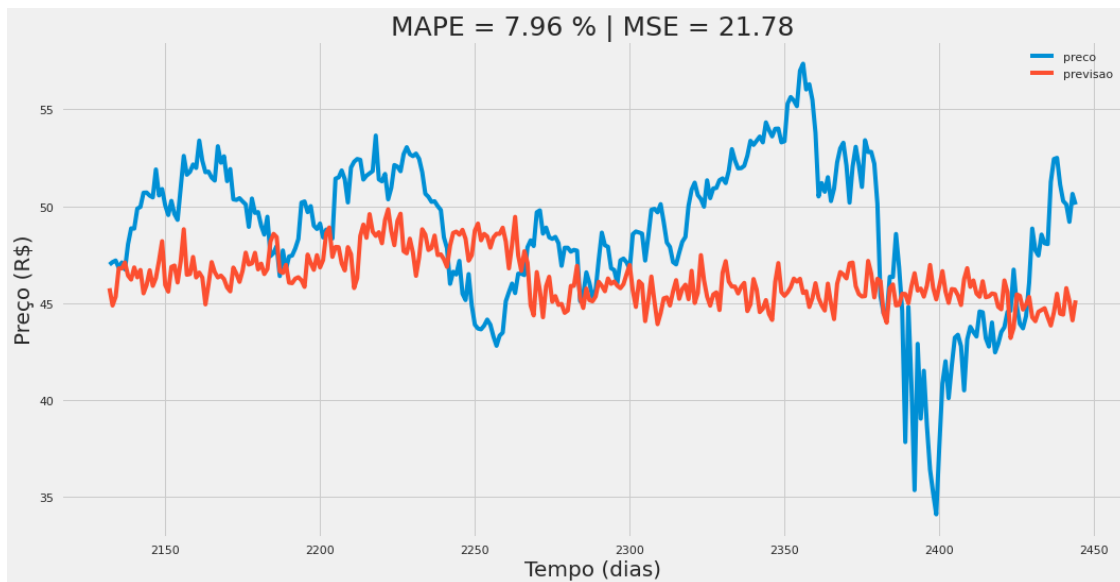
```
[25]: # forecast
      y_ar_forecast = ar_fit.predict(start=(df_test.index[0]), end=df_test.index[-1])
```

```
[26]: show_result_model(df_train=df_train,
                        df_test=df_test,
                        y_forecast=y_ar_forecast,
                        model_name='ar_model')
```
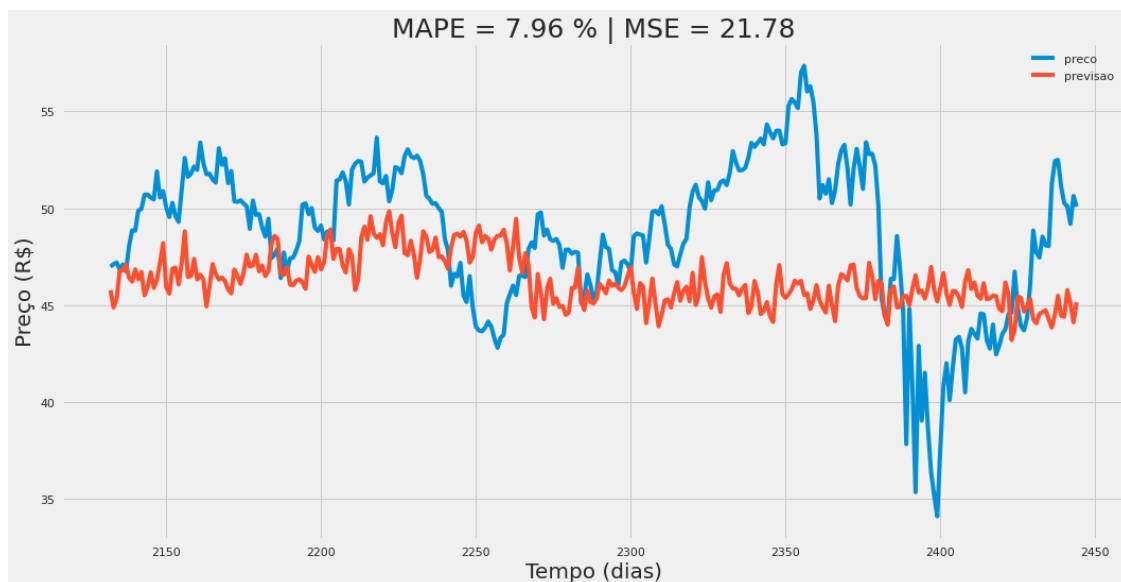

zoom

```
[27]: show_result_model(df_train=df_test,
                        df_test=df_test,
                        y_forecast=y_ar_forecast,
                        model_name='ar_model')
```

MAPE = 7.96 % | MSE = 21.78

1.10.1 ARIMA

1.11 Librarie: `pmdarima`

- Tips: https://alkaline-ml.com/pmdarima/tips_and_tricks.html

```
[28]: %%time

list_order_arima = search_best_params_arima_model(df_train=df_train,
                                                  pdq=list_pdq)
print(list_order_arima)
```

```
pdq = (0, 0, 0) | AIC = 16618.529921052803
pdq = (0, 0, 1) | AIC = 13847.15668983633
pdq = (0, 1, 0) | AIC = 5202.4005815196615
pdq = (0, 1, 1) | AIC = 5202.427062935278
pdq = (0, 1, 2) | AIC = 5190.195445269178
pdq = (0, 2, 0) | AIC = 6616.986469503832
pdq = (0, 2, 1) | AIC = 5209.942045808245
pdq = (0, 2, 2) | AIC = 5209.948411382415
pdq = (1, 0, 0) | AIC = 5209.139906527847
pdq = (1, 0, 1) | AIC = 5209.001459488028
pdq = (1, 0, 2) | AIC = 5197.200090236441
pdq = (1, 1, 0) | AIC = 5202.743560213121
pdq = (1, 1, 1) | AIC = 5195.7933811977655
pdq = (1, 1, 2) | AIC = 5191.786433609438
pdq = (1, 2, 0) | AIC = 6150.321210862042
```

```
pdq = (1, 2, 2) | AIC = 5203.335114958441
pdq = (2, 0, 0) | AIC = 5209.340449357725
pdq = (2, 0, 1) | AIC = 5202.394724547465
pdq = (2, 0, 2) | AIC = 5198.922147313669
pdq = (2, 1, 0) | AIC = 5190.807259240531
pdq = (2, 1, 1) | AIC = 5192.782074290211
pdq = (2, 1, 2) | AIC = 5189.626133250238
pdq = (2, 2, 0) | AIC = 5852.928438526969
pdq = (2, 2, 1) | AIC = 5198.377858813686
pdq = (2, 2, 2) | AIC = 5200.355168190182
best ARIMA: (2, 1, 2) | AIC:5189.626133250238
[(2, 1, 2), 5189.626133250238]
CPU times: user 17.5 s, sys: 8.89 s, total: 26.4 s
Wall time: 9.95 s
```

[33]:
```python
%%time

autoarima_model = pm.auto_arima(df_train,
                                stepwise=True,
                                suppress_warnings=True,
                                error_action="ignore",
                                information_criterion='aic',
                                start_p=2,
                                start_d=1,
                                start_q=2,
                                lags=313,
                                seasonal=False,
                                trace=True)
```

```
Performing stepwise search to minimize aic
 ARIMA(2,1,2)(0,0,0)[0] intercept   : AIC=5189.626, Time=1.78 sec
 ARIMA(0,1,0)(0,0,0)[0] intercept   : AIC=5202.401, Time=0.14 sec
 ARIMA(1,1,0)(0,0,0)[0] intercept   : AIC=5202.744, Time=0.12 sec
 ARIMA(0,1,1)(0,0,0)[0] intercept   : AIC=5202.427, Time=0.14 sec
 ARIMA(0,1,0)(0,0,0)[0]             : AIC=5200.421, Time=0.03 sec
 ARIMA(1,1,2)(0,0,0)[0] intercept   : AIC=5191.786, Time=0.49 sec
 ARIMA(2,1,1)(0,0,0)[0] intercept   : AIC=5192.782, Time=0.56 sec
 ARIMA(3,1,2)(0,0,0)[0] intercept   : AIC=5191.420, Time=1.94 sec
 ARIMA(2,1,3)(0,0,0)[0] intercept   : AIC=5191.381, Time=2.26 sec
 ARIMA(1,1,1)(0,0,0)[0] intercept   : AIC=5195.793, Time=0.42 sec
 ARIMA(1,1,3)(0,0,0)[0] intercept   : AIC=5189.941, Time=2.74 sec
 ARIMA(3,1,1)(0,0,0)[0] intercept   : AIC=5194.800, Time=0.66 sec
 ARIMA(3,1,3)(0,0,0)[0] intercept   : AIC=5193.582, Time=1.96 sec
 ARIMA(2,1,2)(0,0,0)[0]             : AIC=5187.657, Time=0.38 sec
 ARIMA(1,1,2)(0,0,0)[0]             : AIC=5189.811, Time=0.24 sec
 ARIMA(2,1,1)(0,0,0)[0]             : AIC=5190.805, Time=0.49 sec
 ARIMA(3,1,2)(0,0,0)[0]             : AIC=5189.452, Time=0.72 sec
 ARIMA(2,1,3)(0,0,0)[0]             : AIC=5189.413, Time=0.85 sec
```

```
ARIMA(1,1,1)(0,0,0)[0]                    : AIC=5193.812, Time=0.17 sec
ARIMA(1,1,3)(0,0,0)[0]                    : AIC=5192.206, Time=0.23 sec
ARIMA(3,1,1)(0,0,0)[0]                    : AIC=5192.823, Time=0.13 sec
ARIMA(3,1,3)(0,0,0)[0]                    : AIC=5191.614, Time=0.72 sec

Best model:  ARIMA(2,1,2)(0,0,0)[0]
Total fit time: 17.189 seconds
CPU times: user 42 s, sys: 21.7 s, total: 1min 3s
Wall time: 17.2 s
```

[34]:
```python
print(autoarima_model.order)
print(autoarima_model.aic())
```

```
(2, 1, 2)
5187.656935415547
```

[36]:
```python
# fit
autoarima_model_fit = autoarima_model.fit(y=df_train)
autoarima_model_fit
```

[36]:
```
ARIMA(order=(2, 1, 2), scoring_args={}, suppress_warnings=True,
      with_intercept=False)
```

[37]:
```python
# forecast
y_forecast = autoarima_model_fit.predict(n_periods=len(df_test[:313]))
len(y_forecast)
```
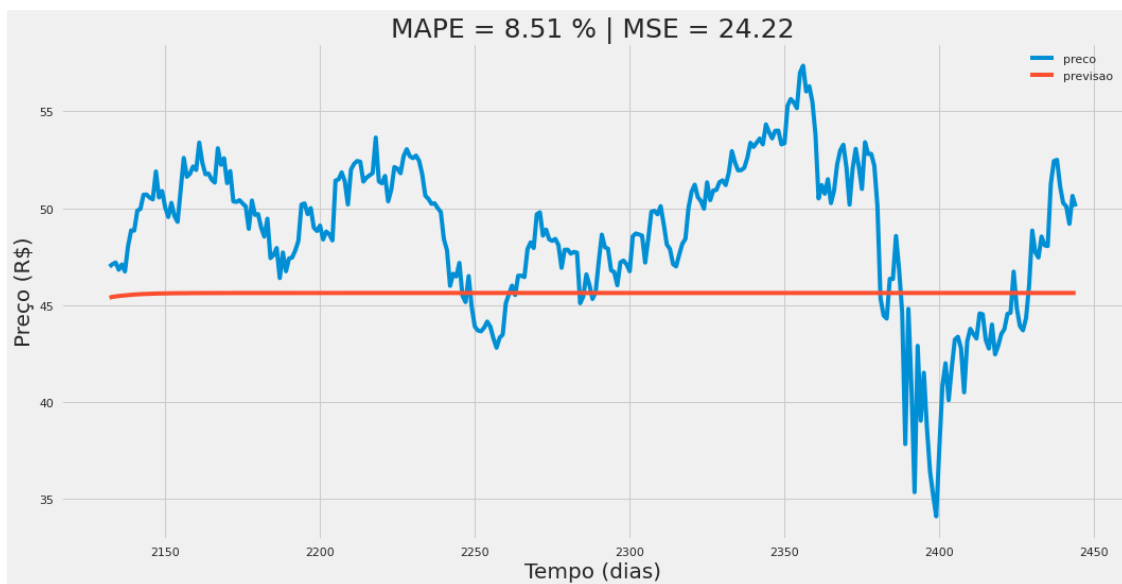
[37]: 313

[39]:
```python
show_result_model(df_train=df_train,
                  df_test=df_test,
                  y_forecast=y_forecast,
                  model_name='arima_model')
```

MAPE = 8.51 % | MSE = 24.22
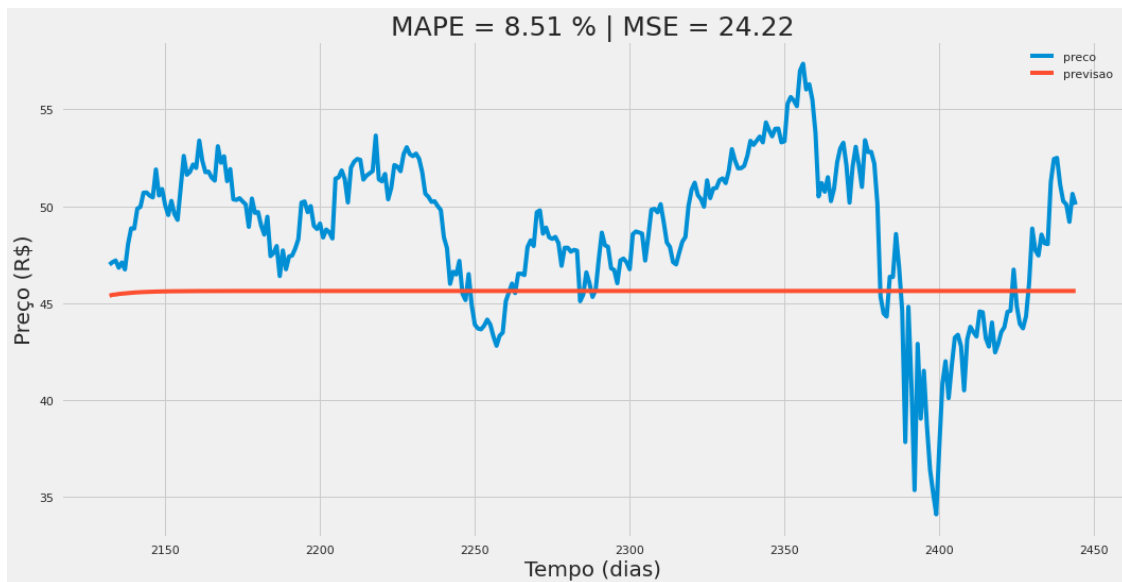
Zoom

```
[40]: show_result_model(df_train=df_test,
                        df_test=df_test,
                        y_forecast=y_forecast,
                        model_name='arima_model')
```


MAPE = 8.51 % | MSE = 24.22

13

1.12 SARIMA

```
[49]: %%time

list_order_seasonal_aic = search_best_params_sarima_model(df_train=df_train,
                                                          trend='t',
                                                          pdq=(2, 1, 2))
print(list_order_seasonal_aic)
```

```
pdq = (2, 1, 2) | param_seasonal = (0, 0, 0, 12) | AIC = 5191.5422018697
pdq = (2, 1, 2) | param_seasonal = (0, 0, 1, 12) | AIC = 5191.568584251327
pdq = (2, 1, 2) | param_seasonal = (0, 0, 2, 12) | AIC = 5191.023297688418
pdq = (2, 1, 2) | param_seasonal = (0, 1, 0, 12) | AIC = 6669.048884663625
pdq = (2, 1, 2) | param_seasonal = (0, 1, 1, 12) | AIC = 6018.448897661481
pdq = (2, 1, 2) | param_seasonal = (0, 1, 2, 12) | AIC = 7730.417661667607
pdq = (2, 1, 2) | param_seasonal = (0, 2, 0, 12) | AIC = 8901.911156543076
pdq = (2, 1, 2) | param_seasonal = (0, 2, 1, 12) | AIC = 8225.333911230264
pdq = (2, 1, 2) | param_seasonal = (0, 2, 2, 12) | AIC = 7669.739406840514
pdq = (2, 1, 2) | param_seasonal = (1, 0, 0, 12) | AIC = 5191.430740851928
pdq = (2, 1, 2) | param_seasonal = (1, 0, 1, 12) | AIC = 5194.0387448148795
pdq = (2, 1, 2) | param_seasonal = (1, 0, 2, 12) | AIC = 5193.801789566946
pdq = (2, 1, 2) | param_seasonal = (1, 1, 0, 12) | AIC = 6126.856861312977
pdq = (2, 1, 2) | param_seasonal = (1, 1, 1, 12) | AIC = 5969.51317179567
pdq = (2, 1, 2) | param_seasonal = (1, 1, 2, 12) | AIC = 6195.0245268799035
pdq = (2, 1, 2) | param_seasonal = (1, 2, 0, 12) | AIC = 7989.770158383615
pdq = (2, 1, 2) | param_seasonal = (1, 2, 1, 12) | AIC = 7472.563750855076
pdq = (2, 1, 2) | param_seasonal = (1, 2, 2, 12) | AIC = 8155.590108751798
pdq = (2, 1, 2) | param_seasonal = (2, 0, 0, 12) | AIC = 5190.676621773375
pdq = (2, 1, 2) | param_seasonal = (2, 0, 1, 12) | AIC = 5194.585803420131
pdq = (2, 1, 2) | param_seasonal = (2, 0, 2, 12) | AIC = 5195.639738795778
pdq = (2, 1, 2) | param_seasonal = (2, 1, 0, 12) | AIC = 5963.883309501448
pdq = (2, 1, 2) | param_seasonal = (2, 1, 1, 12) | AIC = 5933.311399586823
pdq = (2, 1, 2) | param_seasonal = (2, 1, 2, 12) | AIC = 6237.863137339467
pdq = (2, 1, 2) | param_seasonal = (2, 2, 0, 12) | AIC = 7712.475147540053
pdq = (2, 1, 2) | param_seasonal = (2, 2, 1, 12) | AIC = 7468.894534182369
pdq = (2, 1, 2) | param_seasonal = (2, 2, 2, 12) | AIC = 7794.323067797779


Best SARIMA: (2, 1, 2)x(2, 2, 2, 12)12 | AIC:5190.676621773375
[(2, 0, 0, 12), 5190.676621773375]
CPU times: user 47min 37s, sys: 38min 31s, total: 1h 26min 8s
Wall time: 12min 52s
```

```
[50]: list_order_seasonal_aic
```

```
[50]: [(2, 0, 0, 12), 5190.676621773375]
```

```
[52]: sarima_model = SARIMAX(df_train,
                             order=(2, 1, 2),
                             seasonal_order=list_order_seasonal_aic[0],
                             trend='c',
                             enforce_stationarity=False,
                             enforce_invertibility=False)
```

```
[53]: # fit
      sarima_fit = sarima_model.fit()
      print(sarima_fit)
```

```
<statsmodels.tsa.statespace.sarimax.SARIMAXResultsWrapper object at
0x7fcac5d07940>
```

```
[54]: # forecast
      y_forecast_sarima = sarima_fit.predict(start=df_test.index[0], end=df_test.
       ↪index[-1]).to_numpy()
      len(y_forecast_sarima)
```

[54]: 313

```
[55]: show_result_model(df_train=df_test,
                        df_test=df_test,
                        y_forecast=y_forecast_sarima,
                        model_name='sarima_model')
```
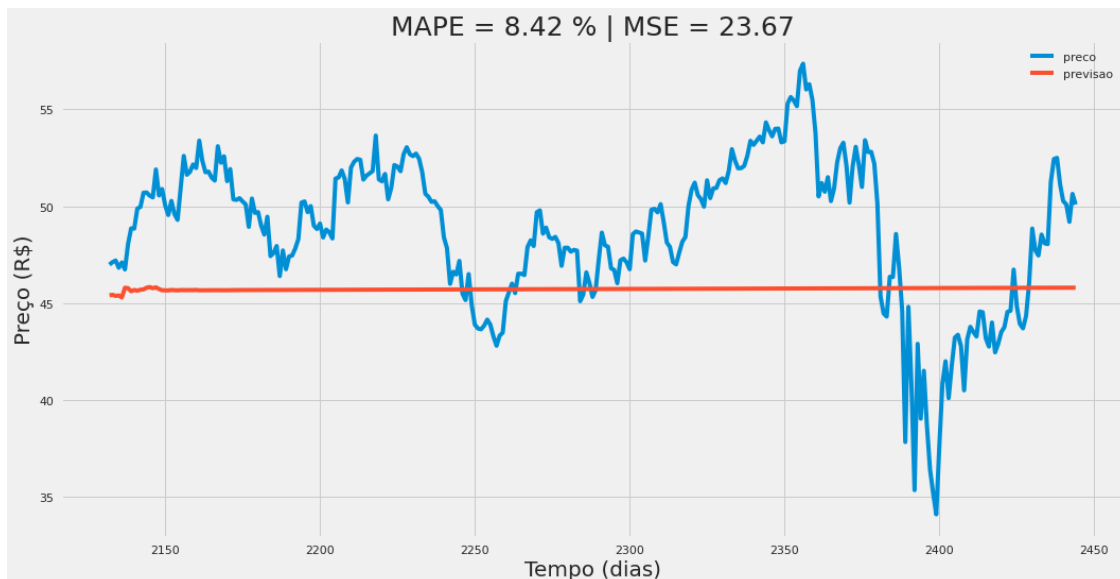
1.13 Resultados

```
[48]: dict_results
```

```
[48]: {'ar_model': [7.955813872394689, 21.780365013206687],
       'arima_model': [8.514572215444968, 24.220464782643145],
       'sarima_model': [9.535109047752895, 33.967392880412575]}
```