Climate in Economic Models

humberto.llavador@upf.edu

3 APPROACHES

1. A LINEAR CARBON-CIRCULATION MODEL (NORDHAUS DICE AND RICE)
2. DEPRECIATION MODEL (GOLOSOV ET AL 2014)
3. LINEAR RELATIONSHIP BETWEEN EMISSIONS AND TEMPERATURE

humberto.llavador@upf.edu

A LINEAR CARBON–CIRCULATION MODEL

A. A simple two-reservoir linear model: Atmosphere and Ocean (discrete time)

- Atmosphere: $S_t - S_{t-1} = -\phi_1 S_{t-1} + \phi_2 S_{t-1}^L + E_{t-1}$
- Ocean: $S_t^L - S_{t-1}^L = \phi_1 S_{t-1} - \phi_2 S_{t-1}^L$

Steady-state ($S_t = S_{t-1} = S, S_t^L = S_{t-1}^L = S^L$ and $E_t = 0$ for all t)

$$\underbrace{\phi_1 S}_{\text{inflow}} = \underbrace{\phi_2 S^L}_{\text{outflow}} \longleftrightarrow \frac{S}{S^L} = \frac{\phi_2}{\phi_1}$$

Relative size at steady-state

- No unique solution. Compatible with different concentration levels, and hence temperature increases.

A linear carbon–Circulation model

A simple two-reservoir linear model: Atmosphere and Ocean (discrete time)

- Atmosphere: $S_t - S_{t-1} = -\phi_1 S_{t-1} + \phi_2 S_{t-1}^L + E_{t-1}$
- Ocean: $S_t^L - S_{t-1}^L = \phi_1 S_{t-1} - \phi_2 S_{t-1}^L$

These linear systems can be solved analytically, and we can find the path of convergence (check how to solve 'homogeneous first-order linear equations')

At a given t, if $E_{t+s} = 0$ for all $s > 0$:

$$S_{t+s} = \frac{\phi_2}{\phi_1}(S_t + S_t^L) - \frac{\phi_2 S_t^L - \phi_1 S_t}{\phi_1 + \phi_2}(1 - \phi_1 - \phi_2)^s$$

$$S_{t+s}^L = \frac{\phi_1}{\phi_2}(S_t + S_t^L) - \frac{\phi_2 S_t^L - \phi_1 S_t}{\phi_1 + \phi_2}(1 - \phi_1 - \phi_2)^s$$

humberto.llavador@upf.edu

A linear carbon–Circulation model

$$S_{t+s} = \frac{\phi_2}{\phi_1}(S_t + S_t^L) - \frac{\phi_2 S_t^L - \phi_1 S_t}{\phi_1 + \phi_2}(1 - \phi_1 - \phi_2)^s$$

$$S_{t+s}^L = \frac{\phi_1}{\phi_2}(S_t + S_t^L) - \frac{\phi_2 S_t^L - \phi_1 S_t}{\phi_1 + \phi_2}(1 - \phi_1 - \phi_2)^s$$

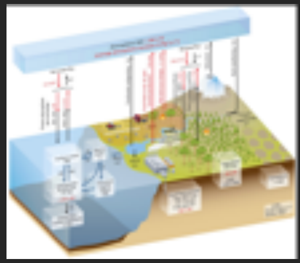
Observations:

- The stock of carbon is constant: $S_{t+s} = S_{t+s}^L$ for all $s > 0$
- The system converges to the steady-state: $\frac{S_{t+s}}{S_{t+s}^L} \to \frac{\phi_2}{\phi_1}$ as $s \to \infty$
- The rate of convergence is $(1 - \phi_1 - \phi_2)$

The product of the eigenvalues: 1 and $(1 - \phi_1 - \phi_2)$

Calibration:

- Pre-industrial stocks: $S = 589\ GtC$ and $S^L = 37{,}100\ GtC \Rightarrow \frac{\phi_2}{\phi_1} \sim 1.59\%$
- Choose a reasonable rate of convergence



humberto.llavador@upf.edu

A linear carbon–Circulation model

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B. A three-reservoir linear model (Nordhaus DICE model)

- Atmosphere: $S_t - S_{t-1} = -\phi_{12} S_{t-1} + \phi_{21} S^U_{t-1} + E_{t-1}$
- Upper Ocean: $S^U_t - S^U_{t-1} = \phi_{12} S_{t-1} - (\phi_{21} + \phi_{23})\, S^U_{t-1} + \phi_{32} S^L_{t-1}$
- Lower Ocean: $S^L_t - S^L_{t-1} = \phi_{23} S^U_{t-1} - \phi_{32} S^L_{t-1}$

Steady-state ($S^k_t = S^k_{t-1} = S^k$ for $k \in \{\emptyset, U, L\}$, and $E_t = 0$ for all t)

- $\dfrac{S}{S^U} = \dfrac{\phi_{21}}{\phi_{12}}$
- $\dfrac{S^U}{S^L} = \dfrac{\phi_{32}}{\phi_{23}}$
- therefore: $\dfrac{S}{S^L} = \dfrac{\phi_{21}}{\phi_{12}} \dfrac{\phi_{32}}{\phi_{23}}$

Relative sizes at steady-state

(inflows = outflows)

humberto.llavador@upf.edu

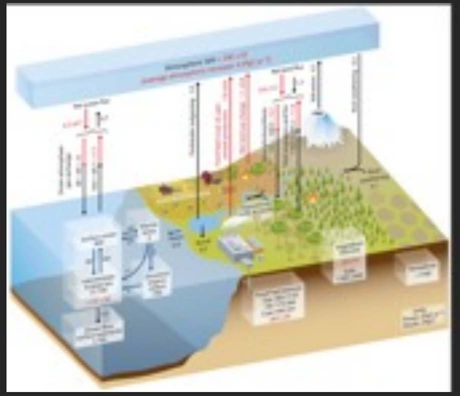
A linear carbon–Circulation model

B. A three-reservoir linear model (Nordhaus DICE model)

- Atmosphere: $S_t - S_{t-1} = -\phi_{12} S_{t-1} + \phi_{21} S^U_{t-1} + E_{t-1}$
- Upper Ocean: $S^U_t - S^U_{t-1} = \phi_{12} S_{t-1} - (\phi_{21} + \phi_{23})\, S^U_{t-1} + \phi_{32} S^L_{t-1}$
- Lower Ocean: $S^L_t - S^L_{t-1} = \phi_{23} S^U_{t-1} - \phi_{32} S^L_{t-1}$

Calibration (two approaches):

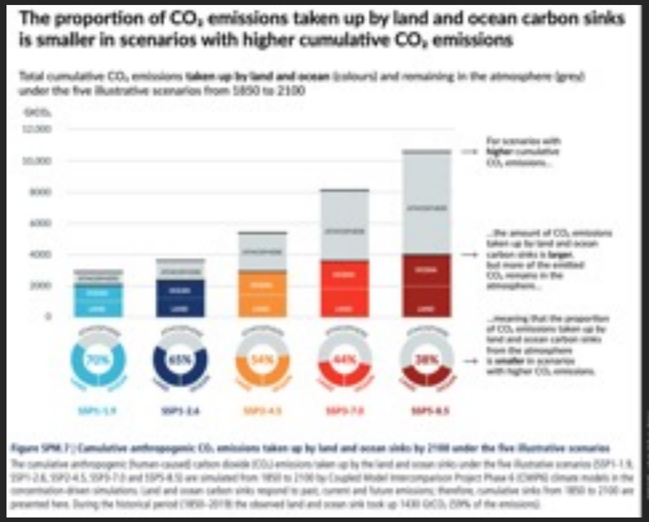
1. Match the dynamics of more complicated models (Nordhaus..)
2. Use measured flows and stocks.
   We need 4 pieces of information $\{\phi_{12}, \phi_{21}, \phi_{23}, \phi_{32}\}$
   Example (not unique)
   - Outflow from atmosphere to upper-ocean $\phi_{12} = \frac{60}{589} \sim 0.102$
   - Outflow from upper-ocean to atmosphere $\phi_{21} = \frac{60}{900} \sim 0.0667$
   - Outflow from upper-ocean to lower-ocean $\phi_{23} = \frac{90}{900} \sim 0.1$
   - Outflow from lower-ocean to upper-ocean $\phi_{32} = \frac{90}{37{,}100} \sim 0.00243$



humberto.llavador@upf.edu

Problems with linear models

- They may be too simplified, missing non-linearities, and other relevant variables.
  - For example, the capacity to store carbon may depend on temperature (and so on the stock of carbon), hence ϕ_{ij} may not be constant (non-linearities).



- They yield too much and too fast removal of CO_2
  - At steady-state (pre-industrial levels)

$$\frac{S}{S+S^L+S^U}=\frac{597}{597+3200+37100}\approx 1.46\%$$

  - After 100 years ~98% of emissions have been removed
  - However (Archer 2005; IPCC AR6)
    - 50% of CO_2 pulse emissions is remove from the atmosphere in decades
    - 30% in centuries
    - 20% remain "forever" (millennia)

B. Non–Structural Approach: Depreciation model

Golosov, Hassler, Krusell and Tsyvinski (2014)

- An alternative is to try to match key characteristics directly

$$S_t = \bar{S} + \sum_{s=0}^{t} (1 - d_s) E_{t-s}$$

  - $1 - d_s$: fraction of emissions emitted s periods ago that stay in the atmosphere
    - A φ_L of an emission pulse stays in the atmosphere for thousands of years.
    - A fraction $(1 - \varphi_0)$ of $(1 - \varphi_L)$ is immediately removed
    - The remaining $\varphi_0(1 - \varphi_L)$ decays at a geometric rate φ.

$$1 - d_s = \varphi_L + (1 - \varphi_L)\varphi_0(1 - \varphi)^s$$

humberto.llavador@upf.edu

B. Non–Structural Approach: Depreciation model

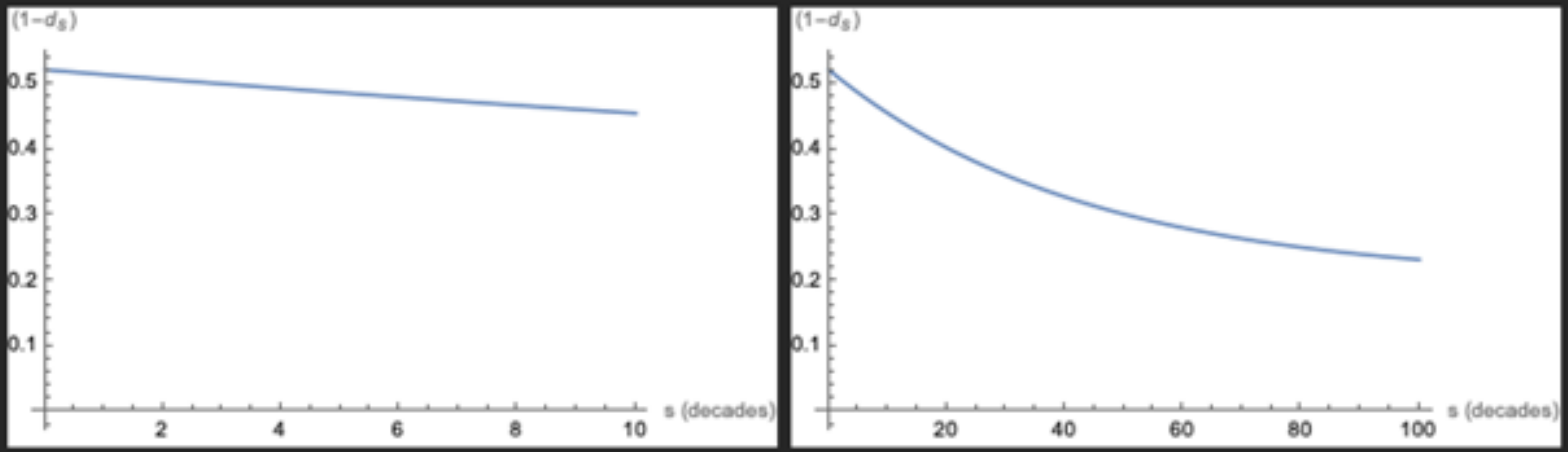
Golosov, Hassler, Krusell and Tsyvinski (2014)

$$1 - d_s = \varphi_L + (1 - \varphi_L)\varphi_0(1 - \varphi)^s$$

Calibration ($t = 10$ years)

- $\varphi_L = 0.2$: 20% of an emission pulse stays "forever"
- $d_2 = 0.5$: 50% is removed in 30 years
- $(1 - \varphi)^{30} = 0.5$: 300 years is the mean life of the excess carbon that does not stay in the atmosphere forever

- $\varphi_L = 0.2$
- $\varphi_L = 0.393$
- $\varphi = 0.0228$



humberto.llavador@upf.edu

From CO2 concentration to temperature change

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A. Higher concentration of CO2 in the atmosphere increases forcing: surplus of the energy budget (inflow-outflow).
Good approximation by Arrhenius equation (Arrhenius 1889)

$$F(S) = \frac{\eta}{\log 2} \log\left(\frac{S}{S_0}\right)$$

Stock of CO_2 in the atmosphere

Pre-industrial level

B. Human forcing F must be balanced by increases in outflow (a function of temperature: $k \cdot T$):

$$F = k \cdot \Delta T$$

Increase in the outflow

(Human) FORCING:
Variation in the inflow

Temperature perturbation

C. Combining (A) and (B)

$$\Delta T(S) = \frac{\eta}{k} \frac{\log\left(\frac{S}{S_0}\right)}{\log 2} \equiv \lambda \frac{\log\left(\frac{S}{S_0}\right)}{\log 2}$$

Climate-sensitivity parameter

Temperature changed associated to doubling the concentration of CO2 in the atmosphere (above pre-industrial levels)

humberto.llavador@upf.edu

From CO2 concentration to temperature change

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Long-term temperature increase

$$\Delta T(S) = \lambda \frac{\log\left(\frac{S}{S_0}\right)}{\log_2}$$

Climate-sensitivity parameter

Temperature changed associated to doubling the concentration of CO_2 in the atmosphere (above pre-industrial levels

$\lambda \in [2.5^oC, 4^oC]$ with best estimate 3^oC (IPCC AR6, 2022)

Simple calculation:

- $S_{1850} = 280ppm$
- $S_{2020} = 420ppm$ (keelingcurve.ucsd.edu/)
- $\Delta T = 1.75^o C[1.46, 2.34]$

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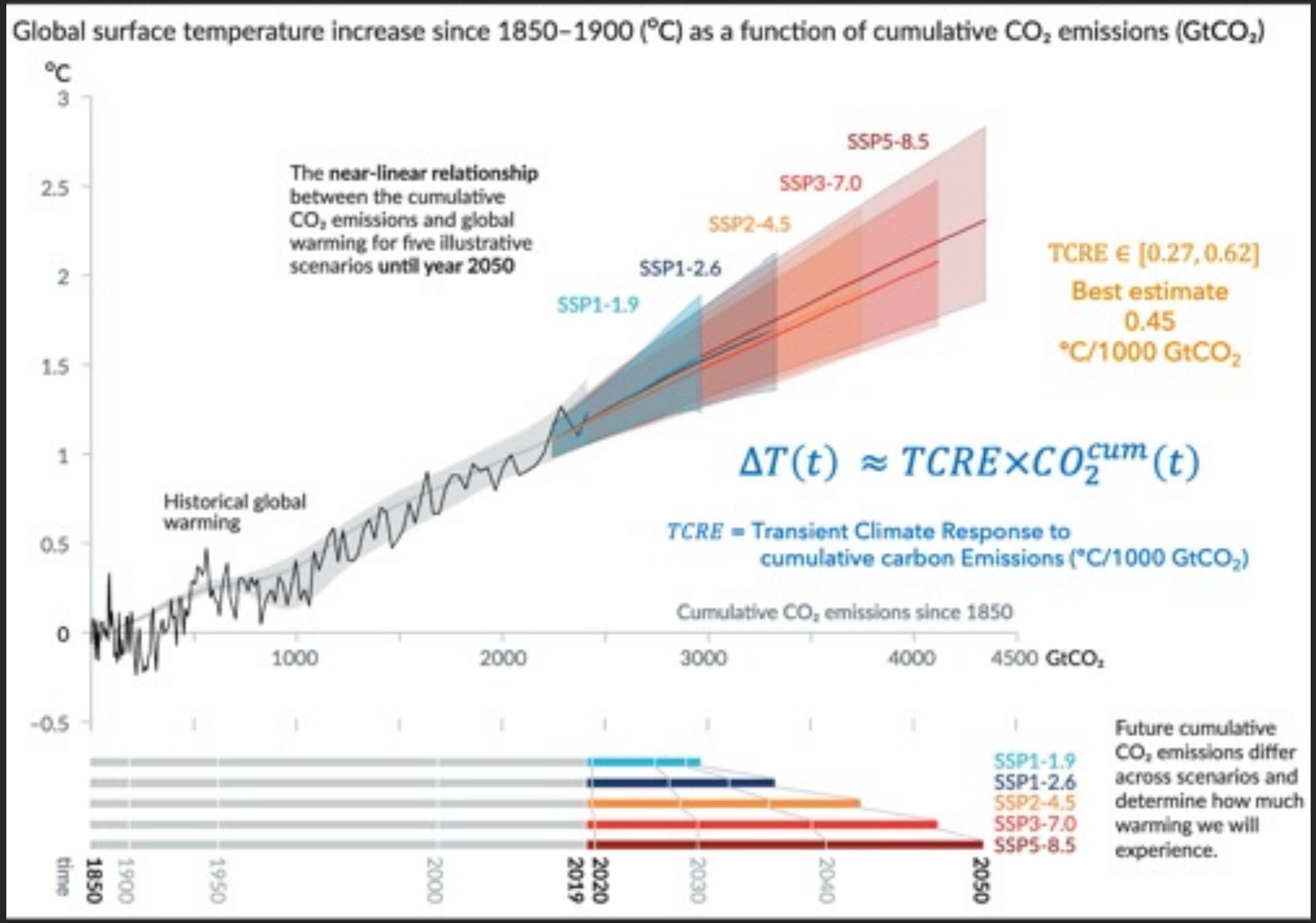
C. Almost linear relationship between cumulative emissions and temperature

Dietz and Veemans (2019); Matthews et al. (2009)

$$\Delta T_{\mathrm{t}} = \mathrm{TCRE} \times \sum_{s=1850}^{t} E_t$$

$TCRE \in [0.27, 0.52]$°C/1000 GtCO2 with best estimate $0.45^{\mathrm{o}}\mathrm{C}$

(IPCC AR6, 2022)



Simple calculation:

- Emissions 1850-2019 = 2,390GtCO_2
- $\Delta T_{\mathrm{t}} = 1.07\ [0.64, 1.24]$

humberto.llavador@upf.edu

Summary

- Structural linear model (Nordhaus DICE)
  - Emissions → Concentrations: Explicit carbon cycle model with 3 reservoirs: atmosphere, upper ocean and lower ocean
  - Concentrations → long-term temperature: Arrhenius equation and proportional forcing
- Non-structural depreciation model (Hassler et al. 2014)
  - Emission → Concentrations: $S_t = \overline{S} + \sum_{s=0}^{t}(1 - d_s)E_{t-s}$
  - Concentrations → long-term temperature: Arrhenius equation and proportional forcing
- Linear relationship cumulative emission-temperature increase
  - Cumulative emissions → temperature increase: $\Delta T_{\mathrm{t}} = \mathrm{TCRE} \times \sum_{s=1850}^{t} E_t$

humberto.llavador@upf.edu

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humberto.llavador@upf.edu