Analysis of the SEPIC Converter

Ben Schaeffer, Dennis Gilbert
bms@pdx.edu, dennis@pdx.edu
Group II
ECE-445

February 16, 2010

1 Analysis Summary

$$
\begin{aligned}
I_1 &= \left(\frac{D^2}{D'^2}\right)\frac{V_g}{R} \\
I_2 &= \left(\frac{D}{D'}\right)\frac{V_g}{R} \\
V_1 &= V_g \\
V_2 &= \left(\frac{D}{D'}\right)Vg
\end{aligned}
$$

$$
\begin{aligned}
\Delta i_1 &= \frac{DV_gT_s}{L_1} \\
\Delta i_2 &= \frac{DV_gT_s}{L_2} \\
\Delta v_1 &= \left(\frac{D^2}{D'}\right)\frac{V_gT_s}{RC_1} \\
\Delta v_2 &= \left(\frac{D^2}{D'}\right)\frac{V_gT_s}{RC_2}
\end{aligned}
$$

$$
\begin{aligned}
\text{Transistor Peak Inverse Voltage} &= \frac{V_g}{D'}\left(1+\frac{D^2T_s(C_1+C_2)}{2RC_1C_2}\right) \\
\text{Transistor Peak Current} &= V_gD\left(\frac{1}{RD'^2}+\frac{T_s(L_1+L_2)}{2L_1L_2}\right) \\
\text{Diode Peak Inverse Voltage} &= \frac{V_g}{D'}\left(1+\frac{D^2T_s(C_1+C_2)}{2RC_1C_2}\right) \\
\text{Diode Peak Current} &= V_gD\left(\frac{1}{RD'^2}+\frac{T_s(L_1+L_2)}{2L_1L_2}\right)
\end{aligned}
$$

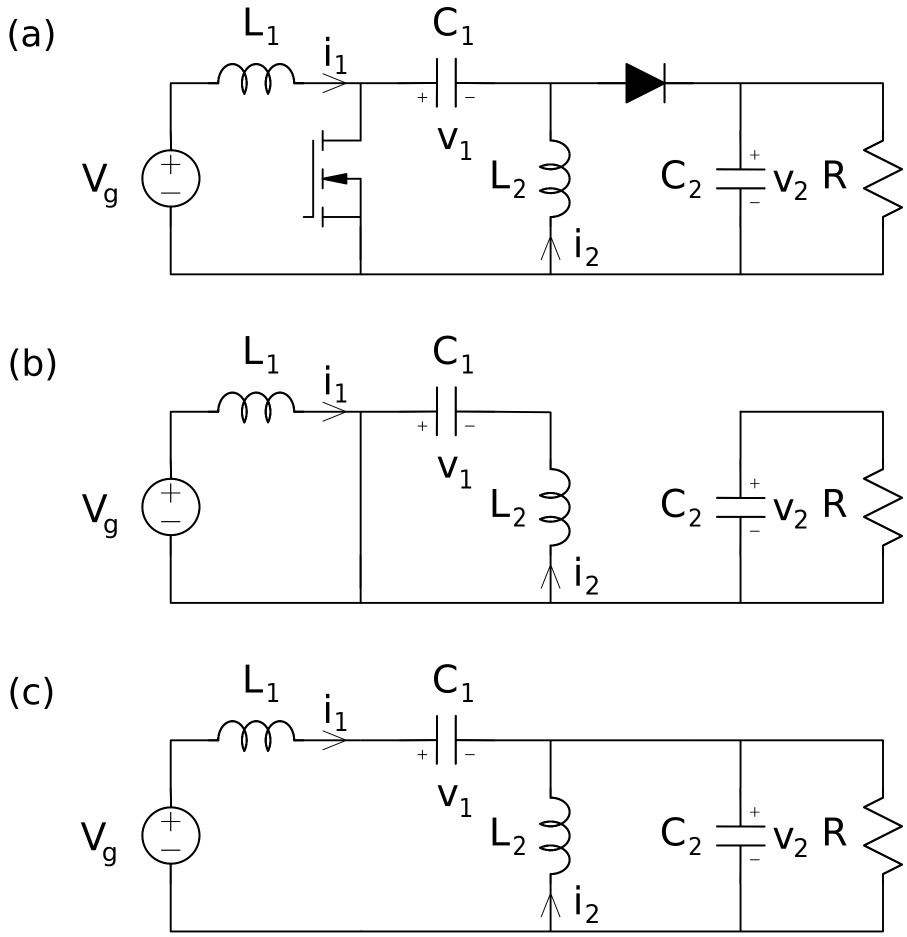

Figure 1 An ideal SEPIC converter: (a) practical realization using MOSFET and diode, (b) when the MOSFET conducts, (c) when the diode conducts.

2 Analysis

During DT_s:

$$\begin{aligned} i_{C_1} &= -i_{L_2} \\ i_{C_2} &= \frac{-v_{C_2}}{R} \\ v_{L_1} &= V_g \\ v_{L_2} &= v_{C_1} \end{aligned}$$

During $D'T_s$:

$$\begin{aligned}
i_{C_1} &= i_{L_1} \\
i_{C_2} &= i_{L_1} + i_{L_2} - \frac{v_{C_2}}{R} \\
v_{L_1} &= V_g - v_{C_1} - v_{C_2} \\
v_{L_2} &= -v_{C_2}
\end{aligned}$$

Find linear ripple approximations, let $V = V_{C_2}$, switching period $= T_s$, $D' = 1 - D$

Using volt-second balance on $\langle V_{L_1} \rangle, \langle V_{L_2} \rangle$:

$$\begin{aligned}
\langle V_{L_1} \rangle &= DV_g + D'(V_g - V - V_{C_1}) = 0 \\
V_g &= D'(V + V_{C_1}) \\
V_{C_1} &= \frac{V_g}{D'} - V
\end{aligned}$$

$$\begin{aligned}
\langle V_{L_2} \rangle &= DV_{C_1} - D'V = 0 \\
V_{C_1} &= \frac{D'V}{D} \\
\text{using substitution} \frac{D'V}{D} &= \frac{V_g}{D'} - V \\
\frac{D'V}{D} + V &= \frac{V_g}{D'} \\
D'V + DV &= \frac{V_g D}{D'} \\
V &= \frac{V_g D}{D'} \\
\frac{V}{V_g} &= \frac{D}{D'} = M(D) \\
V_{C_1} &= \frac{D'}{D}\left(\frac{DV_g}{D'}\right) = V_g
\end{aligned}$$

Using charge balance on $\langle I_{C_1} \rangle, \langle I_{C_2} \rangle$:

$$\begin{aligned}
\langle I_{C_1} \rangle &= -DI_{L_2} + D'I_{L_1} = 0 \\
I_{L_1} &= I_{L_2}\frac{D}{D'} \\
\langle I_{C_2} \rangle &= D\frac{-V}{R} + D'\left(I_{L_1} + I_{L_2} - \frac{V}{R}\right) = 0 \\
\frac{V}{R} &= D'(I_{L_1} + I_{L_2}) \\
I_{L_1} &= \frac{V}{RD'} - I_{L_2}
\end{aligned}$$

Using substitution $\quad I_{L_2}\frac{D}{D'} = \dfrac{V}{RD'} - I_{L_2}$

$$\begin{aligned}
I_{L_2}\frac{D}{D'} + I_{L_2} &= \frac{V}{RD'} \\
I_{L_2}\left(\frac{D}{D'} + 1\right) &= \frac{V}{RD'} \\
I_{L_2}\left(\frac{D}{D'}\frac{D'}{D'}\right) &= \frac{V}{RD'} \\
I_{L_2}\left(\frac{1}{D'}\right) &= \frac{V}{RD'} \\
I_{L_2} &= \tfrac{V}{R} = \frac{V_g D}{RD'} \\
I_{L_1} &= I_{L_2}\frac{D}{D'} = \left(\frac{V_g D}{RD'}\right)\frac{D}{D'} = \frac{V_g D^2}{RD'^2}
\end{aligned}$$

Peak to peak ripple calculations:

$$\begin{aligned}
\Delta i_1 &= \frac{DV_g T_s}{L_1} \\
\Delta i_2 &= \frac{DV_g T_s}{L_2} \\
\Delta v_1 &= \left(\frac{D^2}{D'}\right)\frac{V_g T_s}{RC_1} \\
\Delta v &= \left(\frac{D^2}{D'}\right)\frac{V_g T_s}{RC_2}
\end{aligned}$$

Transistor Peak Inverse Voltage $= V_{C_1} + V + \frac{\Delta v_1}{2} + \frac{\Delta v}{2}$

$$\begin{aligned}
&= V_g + V_g \frac{D}{D'} + \frac{V_g D^2 T_s \left(\frac{1}{C_1} + \frac{1}{C_2}\right)}{2RD'} \\
&= \frac{V_g}{D'} + \frac{V_g D^2 T_s \left(C_1 + C_2\right)}{2RD'C_1C_2} \\
&= \frac{V_g}{D'} \left(1 + \frac{D^2 T_s (C_1 + C_2)}{2RC_1C_2}\right)
\end{aligned}$$

Transistor Peak Current $= I_{L_1} + I_{L_2} + \frac{\Delta i_1}{2} + \frac{\Delta i_2}{2}$

$$\begin{aligned}
&= I_{L_2} \frac{D}{D'} + I_{L_2} + \frac{V_g D T_s}{2L_1} + \frac{V_g D T_s}{2L_2} \\
&= \frac{I_{L_2}}{D'} + \frac{V_g D T_s (L_1 + L_2)}{2L_1L_2} \\
&= \frac{V_g D}{RD'^2} + \frac{V_g D T_s (L_1 + L_2)}{2L_1L_2} \\
&= V_g D \left(\frac{1}{RD'^2} + \frac{T_s (L_1 + L_2)}{2L_1L_2}\right)
\end{aligned}$$

Diode Peak Inverse Voltage $= V_{C_1} + v + \frac{\Delta v_1}{2} + \frac{\Delta v}{2}$

$$\begin{aligned}
&= \text{Transistor Peak Inverse Voltage} \\
&= \frac{V_g}{D'} \left(1 + \frac{D^2 T_s (C_1 + C_2)}{2RC_1C_2}\right)
\end{aligned}$$

Diode Peak Current $= I_{L_1} + I_{L_2} + \frac{\Delta i_1}{2} + \frac{\Delta i_2}{2}$

$$\begin{aligned}
&= \text{Transistor Peak Current} \\
&= V_g D \left(\frac{1}{RD'^2} + \frac{T_s (L_1 + L_2)}{2L_1L_2}\right)
\end{aligned}$$

The ripple for both capacitor voltages are in phase with each other as well as the both inductor ripple currents are in phase with each other.

3 State Space Analysis

During $0 < t < DT_s$ interval as shown in figure 1b:

$$\begin{aligned}
V_g + L_1\frac{di_1}{dt} = 0 \quad &\rightarrow \quad \frac{di_1}{dt} = \frac{V_g}{L_1} \\
L_2\frac{di_2}{dt} - v_1 = 0 \quad &\rightarrow \quad \frac{di_2}{dt} = \frac{v_1}{L_2} \\
C_1\frac{dv_1}{dt} + i_2 = 0 \quad &\rightarrow \quad \frac{dv_1}{dt} = -\frac{i_2}{C_1} \\
C_2\frac{dv_2}{dt} + \frac{v_2}{R} = 0 \quad &\rightarrow \quad \frac{dv_2}{dt} = -\frac{v_2}{RC_2}
\end{aligned}$$

During $D'T_s$

$$\begin{aligned}
V_g + L_1\frac{di_1}{dt} + v_1 + v_2 \quad &\rightarrow \quad \frac{di_1}{dt} = \frac{V_g}{L_1} - \frac{v_1}{L_1} - \frac{v_2}{L_1} \\
L_2\frac{di_2}{dt} + v_2 = 0 \quad &\rightarrow \quad \frac{di_2}{dt} = -\frac{v_2}{L_2} \\
C_1\frac{dv_1}{dt} - i_1 = 0 \quad &\rightarrow \quad \frac{dv_1}{dt} = \frac{i_1}{C_1} \\
C_2\frac{dv_2}{dt} - i_1 - i_2 + \frac{v_{C_2}}{R} = 0 \quad &\rightarrow \quad \frac{dv_2}{dt} = \frac{i_1}{C_2} + \frac{i_2}{C_2} - \frac{v_2}{RC_2}
\end{aligned}$$

Expression for A_1, B_1 :

$$\begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \\ \frac{dv_1}{dt} \\ \frac{dv_2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{L_2} & 0 \\ 0 & -\frac{1}{C_1} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} [V_g] \tag{1}$$

Expression for A_2, B_2 :

$$\begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \\ \frac{dv_1}{dt} \\ \frac{dv_2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1}{L_1} & -\frac{1}{L_1} \\ 0 & 0 & 0 & -\frac{1}{L_2} \\ \frac{1}{C_1} & 0 & 0 & 0 \\ \frac{1}{C_2} & \frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} [V_g] \tag{2}$$

Combining expressions for A, B, and x in equations (1) and (2):

$$\begin{aligned}
A = DA_1 + D'A_2, B = DB_1 + D'B_2 \\
0 = Ax + Bu \rightarrow X = -A^{-1}BV_g
\end{aligned}$$

The vector X now represents the steady state DC value of x.

$$[X] = \begin{bmatrix} I_1 \\ I_2 \\ V_1 \\ V_2 \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\frac{D'}{L_1} & -\frac{D'}{L_1} \\ 0 & 0 & \frac{D}{L_2} & -\frac{D'}{L_2} \\ \frac{D'}{C_1} & -\frac{D}{C_1} & 0 & 0 \\ \frac{D'}{C_2} & \frac{D'}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{V_g}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

The A^{-1} factor is found by dividing the adjoint of A by the determinant of A:

$$\begin{aligned}
|A| &= -\frac{D'}{L_1}\frac{-D'}{L_2}\frac{D'}{C_1}\frac{D'}{C_2} - \frac{-D'}{L_1}\frac{-D'}{L_2}\frac{-D}{C_1}\frac{D'}{C_2} + \frac{-D'}{L_1}\frac{D}{L_2}\frac{-D}{C_1}\frac{D'}{C_2} - \frac{-D'}{L_1}\frac{D}{L_2}\frac{-D'}{C_1}\frac{D'}{C_2} \\
&= \frac{D'^4 + 2D'^3 D + D'^2 D^2}{L_1 L_2 C_1 C_2} \\
&= D'^2 \frac{D'^2 + 2D' D + D^2}{L_1 L_2 C_1 C_2} \\
&= D'^2 \frac{(D + D')^2}{L_1 L_2 C_1 C_2} \\
&= \frac{D'^2}{L_1 L_2 C_1 C_2}
\end{aligned}$$

The above determinant of A divides into the adjoint of A producing A^{-1} within the expression:

$$X = - \begin{bmatrix} \frac{-D^2 L_1}{D'^2 R} & -\frac{D L_2}{D' R} & C_1 & -\frac{D C_2}{D'} \\ \frac{-D L_1}{D' R} & \frac{-L_2}{R} & -C_1 & C_2 \\ -L_1 & L_2 & 0 & 0 \\ \frac{-D L_1}{D'} & -L_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{V_g}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{D^2 V_g}{D'^2 R} \\ \frac{D V_g}{D' R} \\ V_g \\ \frac{D V_g}{D'} \end{bmatrix} \tag{4}$$

Calculate dx for interval DT_s:

$$\Delta x = |(A_1 X + B_1 V_g) DT_s)| = \left| \left(\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{L_2} & 0 \\ 0 & -\frac{1}{C_1} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} \frac{D^2 V_g}{D'^2 R} \\ \frac{D V_g}{D' R} \\ V_g \\ \frac{D V_g}{D'} \end{bmatrix} + \begin{bmatrix} \frac{V_g}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} \right) [DT_s] \right| = \begin{bmatrix} \frac{DT_s V_g}{L_1} \\ \frac{DT_s V_g}{L_2} \\ \frac{D^2 T_s V_g}{D' R C_1} \\ \frac{D^2 T_s V_g}{D' R C_2} \end{bmatrix}$$

Calculate $\Delta^{(2)}x$ (ignoring sign):

$$\Delta^{(2)}x = \frac{A\Delta x T_s}{8} = \begin{bmatrix} 0 & 0 & \frac{-D'}{L_1} & \frac{-D'}{L_1} \\ 0 & 0 & \frac{D}{L_2} & \frac{-D'}{L_2} \\ \frac{D'}{C_1} & -\frac{D}{C_1} & 0 & 0 \\ \frac{D'}{C_2} & \frac{D'}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} \frac{DT_sV_g}{L_1} \\ \frac{DT_sV_g}{L_2} \\ \frac{D^2T_sV_g}{D'RC_1} \\ \frac{D^2T_sV_g}{D'RC_2} \end{bmatrix} = \begin{bmatrix} (\frac{1}{C_1} + \frac{1}{C_2})\frac{D^2T_s^2V_g}{8RL_1} \\ (\frac{D}{C_1} - \frac{D'}{C_2})\frac{D^2T_s^2V_g}{8D'RL_2} \\ (\frac{D'}{L_1} - \frac{D}{L_2})\frac{DT_s^2V_g}{8C_1} \\ (\frac{D'}{L_1} + \frac{D'}{L_2} - \frac{D}{D'R^2C_2})\frac{DT_s^2V_g}{8C_2} \end{bmatrix}$$