Over-Determined Offset Short Calibration of a VNA

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***Abstract*— Electromagnetic modeling of coaxial 1.85 mm and 1.0 mm standards for Vector Network Analyzer (VNA) calibration shows only limited accuracy. The approach presented in this paper can overcome this accuracy limitation. Instead of handing over the standards' S-parameters to the calibration algorithm, parameterized models of the standards are used as input into the calibration algorithm. The technique is demonstrated with an offset short calibration where the unknown phase constant of the short's parameterized model is estimated with the calibration algorithm. No assumptions on the frequency dependency of the phase constant are needed. Further on we describe the best constellation of the offset shorts' reflection coefficients in the Smith chart when calibrating a one-port.**

***Index Terms*— Calibration, Vector Network Analyzer, measurement standards, parameter estimation, over-determined, coaxial connectors**

I. Introduction

Offset short Vector Network Analyzer (VNA) calibration has a number of advantages, particularly in the case of the coaxial 1.85 mm and 1.00 mm systems forming the focus of this paper. For this type of systems the manufacturing of offset shorts is easy and accurate as opposed to the production of air lines, sliding loads and fixed loads, [1]. From this point of view a calibration technique using only offset shorts is highly preferable.

On the other hand the electromagnetic modeling, i.e., the computation of electrical parameters from mechanical dimensions and material properties is more sophisticated for offset shorts than for air lines. To overcome this shortcoming and to maintain the maximum level of calibration accuracy we propose an over-determined approach using more than three partly unknown offset shorts of different length. The measurements performed during the calibration procedure deliver raw S-parameters for each offset short. This data is used to determine both the error box of the VNA and the remaining still unknown parameters of the offset shorts.

In previous work the parameters for physical characterization of a particular offset short have included the mechanical length l_k and the (complex) propagation constant γ of the offset line. The VNA calibration techniques using offset shorts as standards can be categorized according to the following scheme: A calibration technique is considered to be of type A if it requires both the propagation constants and the lengths of all offset shorts involved. Type B techniques assume given mechanical lengths and lossless line sections and estimate the (imaginary) propagation constant ($\gamma = j\beta$) by means of the raw calibration data. Finally a calibration technique is called of type C if both the complex propagation constants and the length of the line sections are estimated rather than computed from mechanical data.

A type A approach is described in [2] where a least squares technique is used to find error box parameters which minimize the effect of errors in the standards. The method requires only offset shorts as standards and is verified using Monte Carlo techniques.

For low loss lines it might be suitable to neglect the losses of the offset shorts' line section. In [3] a calibration method is proposed which requires a load, a short with defined phase and two or more unknown offset shorts. This type B technique essentially fits a circle to the raw results of the offset shorts in the Smith chart and determines the error box parameters using radius and center of this circle.

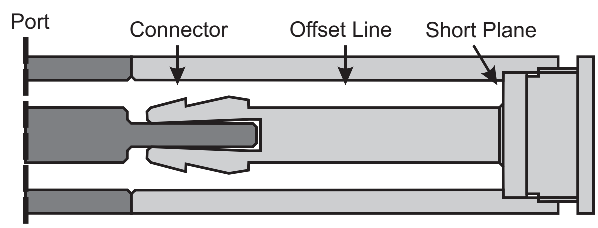

Fig. 1. Female short consisting of connector, line section and short plane. The geometry of the connector is exaggerated for illustration purposes.

Consequently, type C approaches fit a spiral to the raw measurement results of the offset short standards in the Smith chart. Engen [4] describes a technique using a sliding load and a sliding short. The $l_k\gamma$-products due to the different positions of sliding load and sliding short are estimated in an iterative least squares approach. A further development of this is described in [5] where a load, a flush short and several offset shorts are involved in the VNA calibration. This technique estimates the $l_k\gamma$-products for each offset short. More advanced is the approach proposed in [6] which requires only offset reflection standards for the calibration. The phase constant β of the standard's line sections is assumed to be proportional to the frequency and the respective attenuation constants are estimated for each frequency.

In this paper we propose an even more elaborated calibration technique which uses only offset shorts as standards. In section II we give a detailed description of the offset short and discuss which of its parameters should be estimated and which should be rather computed from physical data. Note that we use the phrase ”estimate parameters” if the parameters are a direct result of the calibration algorithm using the raw S-parameters. In contrast we use the wording ”compute parameters” when they are determined directly from physical properties of the standards. An important result of this distinction between estimating and computing consists in the design of the calibration algorithm. Section III describes the details of our calibration technique which estimates the phase constant for each frequency but requires both the computed attenuation constant α and the physical length l_k of the offset shorts’ line sections. This calibration method raises a nonlinear least squares problem which is solved by an optimization algorithm. A numerical test bench, described in section IV, is used to assess the accuracy of the algorithm. Finally the effects on calibration accuracy of the combination of different offset short lengths are treated.

II. Description of the Offset Short

Figure 1 shows a schematic view of an offset short. A wave entering the port first passes the connector then travels along the offset line and is finally reflected at the short plane. We identify three parts: the connector, the offset line and the short plane. Each of these blocks can be characterized separately. Since we are focusing on coaxial 1.85 mm and 1.0 mm offset shorts the connector is — from a geometrical point of view — the most complex part. Its S-parameters can be obtained by advanced 3D electromagnetic field simulations [7]. The available accuracy depends on the numerical technique and is reasonably high but nevertheless finite. At a first glance the line part is described by its physical dimensions (length, radii) and the conductivity of the conductors. However, for frequencies above 50 GHz the roughness of the conductor surfaces starts to play an important role. As shown in [8] the phase constant is no longer truly proportional to frequency and the losses are higher than that of an ideal line with smooth surfaces and equal conductivity. Similar statements hold for the surface impedance of the short plane which is also affected by the roughness.

The interaction of the short’s building blocks can be described by cascading the S-parameter matrices of all parts. Let S_{Ck} be the S-matrix of the k-th connector and

$$S_{Sk} = \begin{pmatrix} \Gamma e^{-2\gamma l_k} & 1 \\ 1 & 0 \end{pmatrix} \tag{1}$$

the combined S-matrix of the k-th offset line and short plane. Alternatively we can also refer to the respective T-matrices T_{Ck} and T_{Sk}. Returning to the S-matrix S_{Sk} we state that the short plane’s reflection factor Γ and

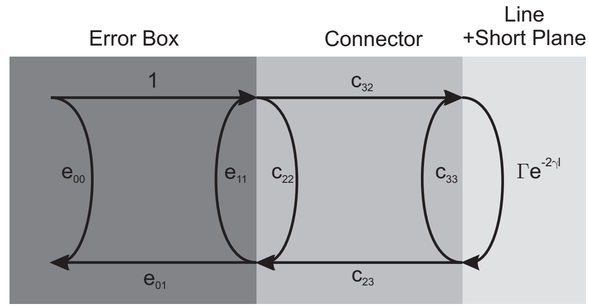

Fig. 2. Flow diagram of the offset short consisting of the blocks Error Box, Connector and Line + Short Plane.

the (complex) propagation constant γ of the line section are assumed to be the same for all shorts, whereas the offset short’s individual length is denoted as l_k. The $l_k\gamma$-product is difficult to compute not only due to roughness effects concerning γ but also due to uncertainties in the measurement of the offset short lengths l_k. E.g., an offset short length uncertainty of $10\,\mu$m translates at the maximum frequency (70 GHz) of a 1.85 mm system to a phase uncertainty of $1.6°$. The influences of uncertain length and incorrectly computed propagation constant have to be considered in the calibration algorithm.

III. Calibration Algorithm

The first key point of our calibration algorithm is that we pass the more detailed model parameters like γ, Γ or l_k to the calibration algorithm, instead of handing over only the total offset shorts’ S-parameters. Besides that the accuracy of these computed model parameters is handed over to the calibration algorithm via the parameters’ variance. The second key point consists in the treatment of those cases where the available accuracy of computed model parameters becomes that low compared to the needed accuracy that virtually infinite variance has to be assumed. These parameters then play a similar role in the calibration algorithm as the parameters of the error box. Thus it is not necessary to compute these model parameters from the offset shorts’ physical properties but rather estimate them during calibration.

However, the number of model parameters to be estimated is limited by the number of available excess S-parameter measurements. Each measurement delivers one complex number and determining the error box requires three complex values. The design goal is a calibration method working with four standards, which allows to determine the error box and to estimate two additional real-valued parameters. Hence the estimation of the individual lengths l_k is impossible due to a lack of excess measurements. In addition, also the structure of the standard itself restricts the number of assessable parameters.

Obviously during calibration one can never estimate the S-parameters of the connector block, see Fig. 2. Hence only attenuation constant α, phase constant β, and the short plane's reflection coefficient Γ are candidates for estimation. Monte Carlo simulations employing realistic ranges of the parameter variations, similar as in [9], showed that β is a much better candidate for estimation than α and Γ. This means that for the purpose of VNA calibration the computed Γ and α are accurate enough whereas the computation of β is too inaccurate.

To estimate error box parameters and phase constant β we have to find an equation which relates the latter with the raw measured T-parameters. To this end we arrange the error box parameters e_{00}, e_{01}, e_{10} and e_{11} to an S-parameter matrix S_E and transform it into the related T-matrix T_E, see Fig. 2. Then the raw T-parameters which we obtain theoretically by measuring the k-th offset short can be easily described by

$$T_{Mk} = T_E T_{Ck} T_{Sk}. \tag{2}$$

For practical measurements we have to describe the errors stemming from the physical modeling of the standards as well as errors from the connection process. These mating errors are mainly due to outer conductor offsets. We assume that this group of errors can be described by an additional matrix $T_{\epsilon k}$ to be introduced right after T_E in the above equation and obtain

$$T_{Mk} = T_E T_{\epsilon k} T_{Ck} T_{Sk} \tag{3}$$

instead of (2). For simplicity's sake we assume that the respective S-matrix has the form

$$S_{\epsilon k} = \begin{pmatrix} \epsilon_k & 1 \\ 1 & 0 \end{pmatrix} \tag{4}$$

where ϵ_k is a Gaussian random variable with zero mean value and known variance σ_k^2. In the following solution process only β and the matrix T_E are considered as unknowns. Writing matrix equation (3) for more than three offset shorts yields a set of matrix equations with more equations than unknowns.

According to the Gauss-Markov theorem the most likely values for the ϵ_ks satisfy the request

$$\sum_k \frac{|\epsilon_k|^2}{\sigma_k^2} \rightarrow minimum$$

Unfortunately the system of equations yielding the ϵ_k for given β and T_E is nonlinear. Starting from an approximate solution the system can be solved by means of nonlinear least squares optimizers such as the `lsqnonlin` algorithm provided by the Matlab toolbox.

IV. Numerical Testbench

For verification a numerical test using Monte Carlo techniques is performed. In a first step we generate the T-matrices needed to evaluate (3) and store the resulting raw data. After this initial step we reset the error box parameters and all parameters to be estimated to an unknown state. Subsequently we recalculate them using the stored raw data. As a result new error box values $\{\bar{e}_{nm}\}$ are obtained which can be compared with the original ones by the following error expression

$$\text{Squared Error} = \frac{1}{6} \sum_{nm=00,12,11} |\bar{e}_{nm} - e_{nm}|^2. \tag{5}$$

Note that the resulting Squared Error does not depend strongly on the error box T_E chosen for the evaluation of (3). In the same equation $T_{\epsilon k}$ represents the combined connection process and physical modeling errors by a Gaussian random variable with $\sigma_k^2 = 10^{-6}$. This corresponds to an error level of approx. -53 dB. Furthermore we assume the connector matrix T_{Ck} to be that of a simple through and for compiling T_{SK} we generate γ using [10], and set $\Gamma = -1$ for all shorts. Our particular test refers to a one-port calibration of a 1.85 mm system using a calibration kit with four offset shorts with lengths $l_k = 5.4,\ 6.3,\ 7.12$ and 7.6 mm. We assume the same partially unknown propagation constant $\gamma = \alpha + j\beta$ for all offset shorts involved.

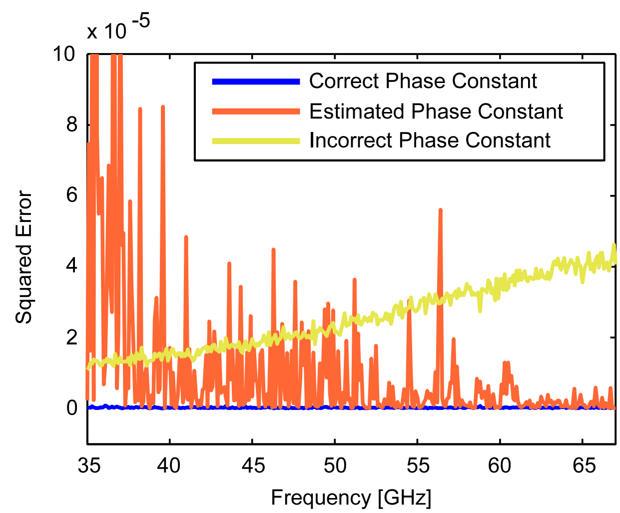

Fig. 3. Mean squared error of three calibration algorithms plotted over the frequency. The calibration using the correct phase constant is on an error level of approx. 10^{-6}. The second algorithm estimates the phase constant and the third assumes an incorrect phase constant.

Figure 3 shows the results of this calibration. The dark blue line reflects the very low resulting error when the phase constant is assumed to be perfectly known and only repeatability and electromagnetic modeling errors come into the picture. The setup producing the yellow

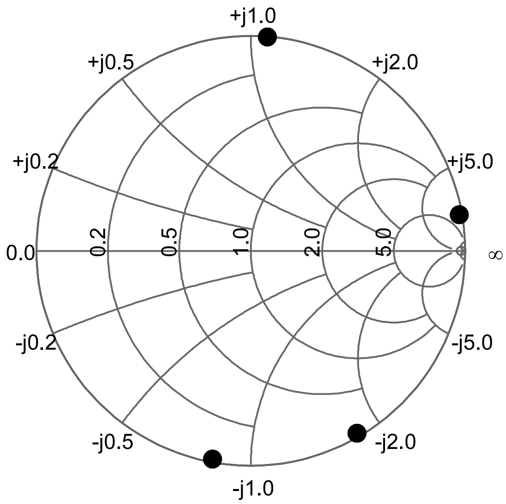

Fig. 4. Reflection coefficients of the four offset shorts plotted in the Smith chart for $f = 35\,\text{GHz}$. The reference impedance of the Smith chart is $Z_0 = 50\Omega$.

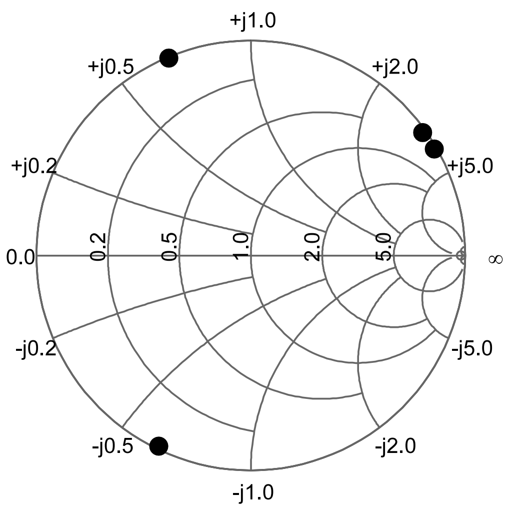

Fig. 5. Reflection coefficients of the four offset shorts plotted in the Smith chart for $f = 65\,\text{GHz}$. The reference impedance of the Smith chart is $Z_0 = 50\Omega$.

curve assumes a wrongly computed phase constant which deviates one tenth of a percent from the results of [10]. This assumed error is due to surface roughness of the offset line sections and is deemed to be realistic. Finally the orange curve shows the results when the phase constant is estimated. This curve is very noisy because phase constant estimation reduces the over-determinedness of the calibration significantly. It is shown that for lower frequencies it is better to calculate the propagation constant while for higher frequencies the estimation algorithm outperforms the precalculation.

This behavior can be described by a rule of thumb derived from error sensitivity analysis for determined error box and phase constant calculation. For high accuracy the following constellations of reflection coefficients in the Smith chart are required: To determine the error box parameters three points with maximum mutual distance are optimal. Ideal estimation of the phase constant is obtained with two reflection coefficients being close to each other, compare Figs. 3, 4 and 5.

V. Conclusion

For the purpose of VNA calibration a model of the offset short is introduced and its different building blocks are discussed. The line section's phase constant is difficult to compute from its physical properties for frequencies higher than $50\,\text{GHz}$. For this reason we use a set of four or more offset shorts to estimate the phase constant during calibration. In a simple numerical experiment the gain in calibration accuracy due to the phase constant estimation is demonstrated. A rule of thumb concerning the used offset lengths is derived to guarantee high calibration accuracy for this approach.

Acknowledgements

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