LETTER

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The complexity of classical music networks

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Abstract – Previous works suggest that musical networks often present the scale-free and the small-world properties. From a musician's perspective, the most important aspect missing in those studies was harmony. In addition to that, the previous works made use of outdated statistical methods. Traditionally, least-squares linear regression is utilised to fit a power law to a given data set. However, according to Clauset *et al.* such a traditional method can produce inaccurate estimates for the power law exponent. In this paper, we present an analysis of musical networks which considers the existence of chords (an essential element of harmony). Here we show that only 52.5% of music in our database presents the scale-free property, while 62.5% of those pieces present the small-world property. Previous works argue that music is highly scale-free; consequently, it sounds appealing and coherent. In contrast, our results show that not all pieces of music present the scale-free and the small-world properties. In summary, this research is focused on the relationship between musical notes (Do, Re, Mi, Fa, Sol, La, Si, and their sharps) and accompaniment in classical music compositions. More information about this research project is available at https://eden.dei.uc.pt/~vitorgr/MS.html.

Introduction. – This research aims to investigate whether or not classical music networks present the scale-free and the small-world properties. In order to find a consistent answer, forty pieces of classical music were selected and represented as musical networks.

When a network presents the scale-free property, its node degree distribution follows a power law [1]. It is important to highlight that the node degree consists of the number of edges connected to that node. Basically, when a network presents the small-world property, the average number of edges between any two vertices will be small. At the same time the clustering coefficient will be large compared to similar random networks [2].

Perkins *et al.* [3] argue that most networks created by humans present the scale-free property, for example, in linguistics, physics, biology, and music. In general, those claims are based on the least-squares linear regression method [4] or basic random walk analysis [5]. Clauset *et al.*'s [6] method refutes many of those claims. We have applied this latter statistical method to our musical networks to evaluate the scale-free property. Typically, the estimated power law exponent (a) of a network which presents the scale-free property is in the range $2 < a < 3$, because usually it has an ultra-small diameter [7]. A power law distribution only has a well-defined mean over $x \in [1, \infty]$, if $a > 2$. When $a > 3$, it has a finite variance that diverges with the upper integration limit $x_{\max}$ as $\langle x^2 \rangle = \int_{x_{\max}}^{x_{\min}} x^2 P(x) \sim x_{\max}^{3-a}$. Liu *et al.* [8] estimated power law exponents with the traditional least-squares linear regression method and reported musical networks with exponents in the range $1 < a < 2$. Perkins *et al.* [3] also utilised the same method to report an exponent in the range $1.05 < a < 1.28$ for their restricted musical network. Perkins *et al.* reported a simple log-log plot. This result is the first indication of a data set that follows a power law distribution. However, according to Clauset *et al.* [6] this idea is not enough to confirm if a particular data set follows a power law, because alternative distributions must be tested. Ferretti [9] did not report the a exponent for his musical networks.

In a small-world network, the mean shortest path length (MSPL) is the average number of edges between any two vertices. The MSPL is given by $\sum_{(s,t)\in V} \frac{d(s,t)}{n(n-1)}$, where V is the set of nodes in the network, $d(s,t)$ is the shortest

Table 1: First bar of Mozart's Sonata No. 16 (KV 545). Time = 480 PPQN.

Time	Event	Note
0	Note-ON	72
0	Note-ON	60
240	Note-OFF	60
240	Note-ON	67
480	Note-OFF	67
480	Note-ON	64
720	Note-OFF	64
720	Note-ON	67
960	Note-OFF	67
960	Note-OFF	72
960	Note-ON	76
960	Note-ON	60
1200	Note-OFF	60
1200	Note-ON	67
1440	Note-OFF	67
1440	Note-OFF	76
1440	Note-ON	79
1440	Note-ON	64
1680	Note-OFF	64
1680	Note-ON	67
1920	Note-OFF	67

path from s to t, and n is the number of nodes in the network. For a network to be considered of small-world type, its MSPL must be very small, at the same time its average clustering coefficient (ACC) must be considerably larger when compared with random networks of the same size in terms of the number of nodes and edges [2]. The local clustering of each node in a network is the fraction of triangles that actually exist over all possible triangles in its neighbourhood. The average clustering coefficient is calculated according to [10]. Perkins *et al.* [3] did not evaluate the small-world property in their musical network. Liu *et al.* [8] argue that their musical networks present the small-world property without comparing MSPL and ACC with alternative random networks. Finally, Ferretti [9] shows that his musical networks present the small-world property comparing MSPL and ACC with alternative random networks.

Musical networks. – The pieces used to build our musical networks are available in Musical Instrument Digital Interface (MIDI) format [11]. They were downloaded from Bernd Krueger's website [12]. Before executing the procedure to create the musical network, it is necessary to convert the MIDI file into a text file and select the appropriate information: i) time; ii) Note-On and Note-Off events; and iii) note number. In a MIDI file, the time is registered in pulses per quarter note (PPQN). The Note-On and Note-Off events determine whether the piano key is pressed or not. Finally, the MIDI note numbers specify 128 distinct pitches, *i.e.*, there is a unique number to each piano key. Table 1 shows a representation of the first bar of Mozart's Sonata No. 16 (KV 545) after selecting the appropriate information from its MIDI file. Figure 1 shows other three representations of the same piece.

It is necessary to define nodes and edges to build a network from a piece of music. Nodes are defined as individual or combined notes matched by their duration (semibreve, minim, crotchet, quaver, semi-quaver, etc). So, each note number inside a network node is paired with its respective duration —refer to fig. 1(c). In our musical networks, we consider broken chords[1].

Edges are defined chronologically through the connections between notes and/or broken chords as the music is played. When a new edge is created between two network nodes, its weight is equal to 1. Whenever an edge is re-used by the composer, the weight related to that particular edge is incremented by 1. The weight of an edge refers to the most used transitions among network nodes within a musical network.

Liu *et al.* [8] and Perkins *et al.* [3] combined several pieces of music into one musical network. Such an approach was taken to provide a consistent numerical analysis in terms of network size, *i.e.*, number of nodes and edges. In order to achieve this, they built musical networks with pieces within the same musical key. Both authors disregarded the existence of chords, as did Ferretti [9]. Uniquely, Perkins *et al.* [3] transposed all pieces to the C major key and built just one musical network.

We have built musical networks taking broken chords into account. Furthermore, we have built musical networks that represent only one piece of music per network. Since we are incorporating harmonic elements, our musical networks are considerably larger in terms of number of nodes and number of edges, when compared to the musical networks built by Liu *et al.* [8], Ferretti [9], and Perkins *et al.* [3]. The source code used to create the forty musical networks as well as our music database are available at `https://eden.dei.uc.pt/~vitorgr/MS.html`. The source code was written in Python/NetworkX language [14].

Results regarding the scale-free property. – In summary, Clauset *et al.*'s [6] statistical method comprises three steps. The first step consists of estimating the power law exponent with maximum likelihood estimators [15]. The second step consists of applying the Kolmogorov-Smirnov (KS) test [16]. This test returns a p-value. If the p-value is greater than 0.1, it can be said that the power law distribution is a plausible hypothesis for the input data set in question (the node degree distribution). If the p-value is less than 0.1, the data set is rejected as a power law distribution. Figure 2 shows the results after

[1]A broken chord is a chord that is broken up in a specific way by the composer. This musical element is utilised by the composer to create rhythmic interest and accompaniment. It can also be utilised by the musician for improvisation. A broken chord may repeat some of the notes from the chord and span one or more octaves [13].

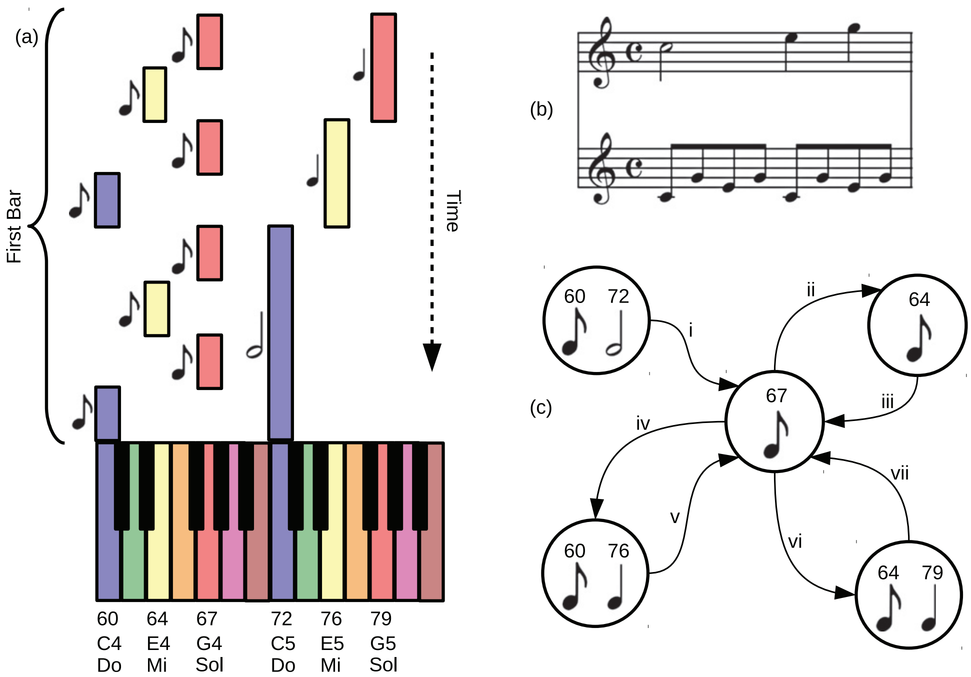

Fig. 1: (Colour online) Three different representations of Mozart's Sonata No. 16 (KV 545) first bar: (a) a piece of music can be modelled as a dynamic system with a set of musical notes evolving over time; (b) traditional musical score; (c) a musical network that takes notes and broken chords into account. The roman numerals represent the order in which each edge was created. In that particular case, all edges have weight equal to 1, because none of them were used more than once.

Table 2: Likelihood ratio (LR) test for the same six musical networks: power law *vs.* exponential, power law *vs.* log-normal, power law *vs.* stretched exponential.

Likelihood ratio test								
Musical Network	Power law (KS) p-value	Exponential		Log-normal		Stretched exponential		Scafe-free?
		LR	p-value	LR	p-value	LR	p-value	
(a)	0.590	9.88	0.00	6.36	0.00	4.09	0.00	YES
(b)	0.424	4.82	0.00	2.67	0.00	1.61	0.10	YES
(c)	0.285	5.54	0.00	3.78	0.00	2.87	0.00	YES
(d)	0.004	3.16	0.00	−0.03	0.97	−1.15	0.24	No
(e)	0.090	−0.28	0.77	−0.42	0.67	−0.44	0.65	No
(f)	0.000	4.91	0.00	3.84	0.00	2.55	0.01	No

applying the first and second steps of Clauset's statistical method on six musical networks. It is evident that the prediction is consistently aligned with the data sets in figs. 2(a), (b), and (c). This is no longer true for figs. 2(d), (e), and (f). This observation is reflected by the p-values in figs. 2(d), (e), and (f), all below 0.1. The exponents for all musical networks are close to the range $2 < a < 3$, which indicates a trend for ultra-small diameters.

Finally, the third step of Clauset's statistical method compares the power law with alternative hypotheses by means of a likelihood ratio (LR) tests [17]. For each alternative tested, if the calculated likelihood ratio is significantly different from zero, its sign indicates whether the alternative distribution is favoured over the power law model. A positive LR indicates power law preference, while a negative LR indicates the alternative hypothesis. Table 2 shows the results after applying the third step of Clauset's statistical method on the same six musical networks. The LR tests confirm the presence of the scale-free property in musical networks (a), (b), and (c). This is not the case for musical networks (d), (e), and (f). The musical network (d) behaves more like a log-normal distribution than a power law, while the musical network (e) behaves more like an exponential distribution. The musical network (f) does not behave like any distribution tested.

Although we provide a precise evaluation of the power law, our musical networks do not present a long tail as many scale-free networks, *i.e.*, we could not identify a small number of nodes with very high degree. On the other hand, according to Janssen [18] due to the finite

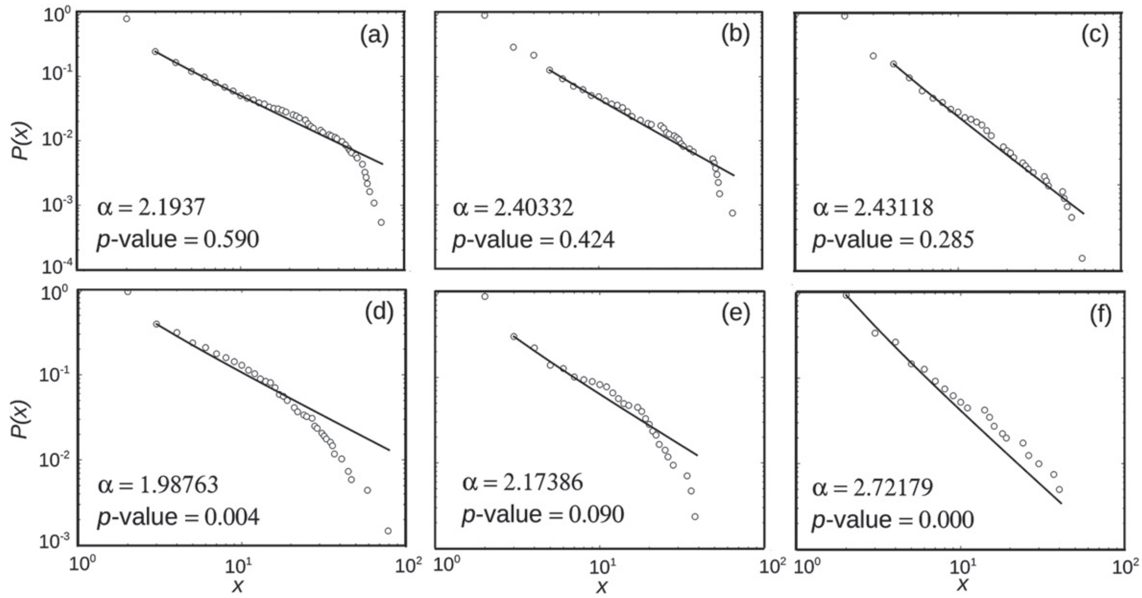

Fig. 2: Log-log plot of node degrees distribution $P(x)$ and their power law fitting process for six musical networks: (a) Sonata No. 23 in F minor (Appassionata) Opus 57 (1804) composed by Beethoven; (b) Sonata No. 12 in F major KV 332 (1783) composed by Mozart; (c) Piano Sonata in D major Hoboken XVI:33 (1778) composed by Haydn; (d) Violin partita No. 2 in D minor BWV 1004 (1720) composed by Bach; (e) Sonatina in F major Opus 36 No. 4 Opus 36 (1797) composed by Clementi; and (f) Sonatina in C major Opus 36 No. 3 Opus 36 (1797) also composed by Clementi.

size of real-world networks the power law inevitably has a cutoff at some maximum degree. Such a cutoff can be clearly verified in figs. 2(a), (b), and (c). The Supplementary Material `Supplementarymaterial.pdf` gives the details of our analyses of each piece of music.

Results regarding the small-world property. – Table 3 presents the mean shortest path length and the average clustering coefficient calculated from i) our musical networks, ii) random networks, iii) small-world networks. A set of twenty random networks and a set of twenty small-world networks were generated for each of our musical networks. The average MSPL and the average ACC were calculated for the set of random networks and for the set of small-world networks. The random networks present equivalent sizes in terms of number of nodes and edges. The small-world networks present near-equivalent sizes in terms of number of nodes and edges, because it is not trivial to build a small-world network with a specific number of nodes and edges. The small-world networks were created according to [19].

From table 3, it is easy to see that the ACC result from our musical networks is always greater than the ACC result from the equivalent random networks. Often, the ACC result from our musical networks is greater than the ACC result from the small-world networks. However, not all musical networks present smaller MSPLs compared to their random and small-world networks. If i) the MSPL result from the musical network is within a confidence interval; and, if ii) its average clustering coefficient is close to or greater than the ACC result from the small-world networks, we consider this musical network compatible with the small-world property. More statistics about our musical networks can be found in table 4.

Discussion on the fractal nature of music. – The scale-free property in occidental music is particular interesting because this style of music typically resolves to a predefined tonal centre. Thus, the hypothesis that classical music presents the scale-free property due to successive returns to the tonal centre needs investigation. Scale invariance is an inherent characteristic observed in fractals. For almost thirty years researchers have stated that music has a fractal nature [20]. Henderson-Sellers and Cooper [21] disagree on this point. Given that 52.5% of music in our database is scale-free, our work suggests that music may or may not present the scale-free property found in fractals.

Most statements [22–26] affirming that music has a fractal nature were based on fractal dimension methods, *i.e.*, self-similarity. Usually, fractal dimension [27] is applied in the spectral analysis of music. Complex network analysis and fractal dimension are two topics that are related to scale invariance and power laws. Song *et al.* [28] outline a very interesting connection between self-similarity in complex networks and fractal dimension. A significant limitation of fractal dimension is that it does not necessarily prove that a pattern is fractal. According to Mandelbrot [29], at least a few other essential characteristics must be identified, for instance: different types of self-similarities [30] (exact, quasi, or qualitative) and/or multifractal scaling [31,32]. In addition to that, the majority

Table 3: Mean shortest path length, average clustering coefficient for musical networks, random networks, and small-world networks.

Musical networks			Random networks		Small-world networks		
	MSPL	ACC	MSPL	ACC	MSPL	ACC	Small-world?
Beethoven Opus 81	6.02	0.15	6.24	0.00	5.91	0.07	YES
Brahms Opus 1	9.33	0.07	7.55	0.00	6.53	0.07	No
Chopin Opus 35	12.64	0.09	6.50	0.00	5.95	0.08	No
Clementi No.1	6.37	0.14	5.14	0.01	4.51	0.07	No
Mozart KV330	4.89	0.09	5.42	0.00	5.47	0.06	YES
Mozart KV331	5.24	0.11	5.78	0.00	5.74	0.08	YES
Mozart KV332	5.51	0.11	5.96	0.00	5.84	0.07	YES
Mozart KV333	5.02	0.18	5.77	0.00	5.96	0.06	YES
Schubert D784	13.67	0.06	7.10	0.00	5.91	0.08	No
Shostakovich Opus 57	9.68	0.05	6.93	0.00	5.86	0.08	No

Table 4: Number of nodes, number of edges, diameter, average degree, and maximum degree.

Musical network	Nodes	Edges	Diameter	Avg. Degree	Max. Degree
Beethoven Opus 81	1432	2317	33	3.23	65
Brahms Opus 1	2903	4166	49	2.87	131
Chopin Opus 35	1487	2319	45	3.11	36
Clementi No.1	297	466	15	3.13	21
Mozart KV330	947	1719	19	3.63	68
Mozart KV331	1144	1977	17	3.45	92
Mozart KV332	1340	2299	19	3.43	66
Mozart KV333	1532	2815	29	3.67	99
Schubert D784	1349	1905	66	2.82	33
Shostakovich Opus 57	1293	1842	48	2.84	39

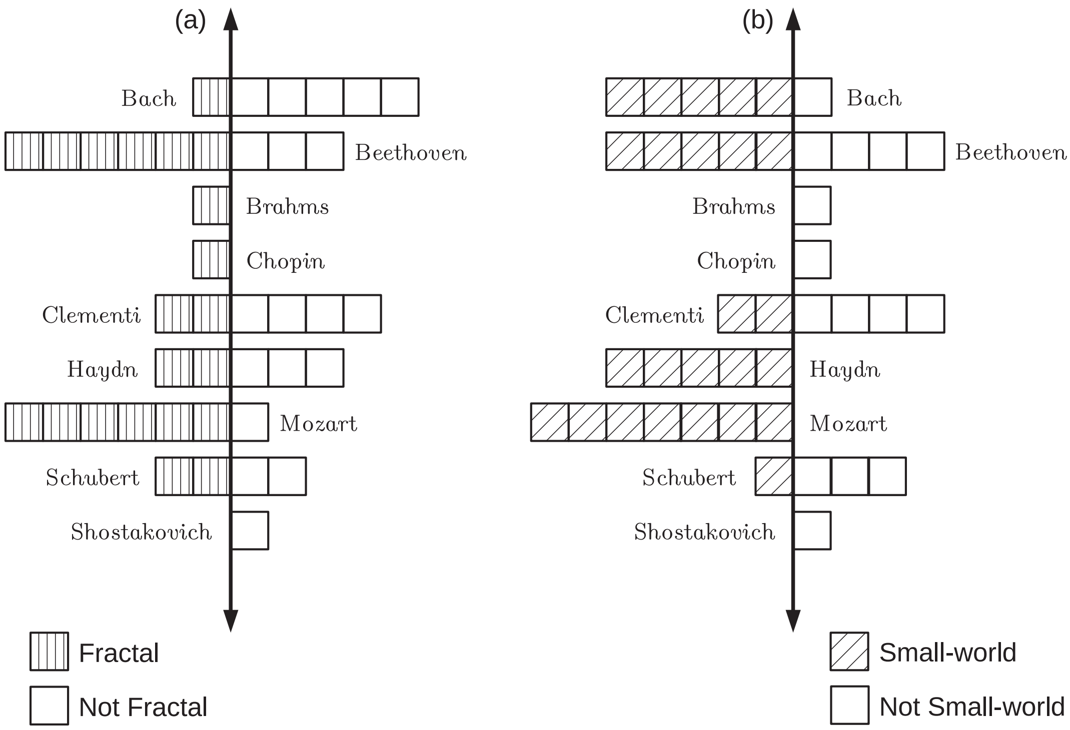

Fig. 3: (a) Results regarding the scale-free property. (b) Results regarding the small-world property.

of papers based on fractal dimension admit that only segments of pieces are scale-free, *i.e.*, self-similar.

Other statements [33–35] asserting that music has a fractal nature were based on pitch fluctuations. For example, Hsü [33,34] analysed the variations in pitch interval between successive notes in twelve pieces composed by Bach, Chopin, and Mozart. They show that the frequency of appearance of each pitch interval approximately follows a power law relation. Liu *et al.* [35] also show evidence for scale invariance over pitch fluctuations in pieces composed by Bach, Mozart, Beethoven, Mendelssohn, and Chopin.

Liu *et al.* [8], Ferretti [9], and Perkins *et al.* [3] are the main references of our work, because they have made use of complex network analysis to show that several musical networks have fractal nature. From a musician's perspective, it is richer and more interesting to represent a piece within a network than simply count the variations in pitch or rhythm intervals. In fact, because nodes and edges can represent different musical attributes within a network, we believe that complex network analysis is the ideal way to evaluate the scale-free property in music, *i.e.*, if music has a fractal nature.

Conclusions and future work. – Previous works [3,8,9] suggest that different pieces of music are fractal compatible because of the presence of the scale-free property in their musical networks. However, the previous works disregarded three substantial aspects in its analysis: i) the presence of broken chords; ii) the use of a single piece of music per musical network; and iii) the use of updated statistical methods. Perkins *et al.* [3] did not evaluate the small-world property of their musical network. Liu *et al.* [8] argue that their musical networks present the small-world property without comparing MSPL and ACC with alternative random networks. Ferretti [9] did not report the scale-free exponent of his six musical networks.

In contrast, our studies show that not every musical piece presents the scale-free property or the small-world property. The hypothesis that classical music presents the scale-free property due to successive returns to the tonal centre is proven to be false, given that only 52.5% of music submitted to Clauset's test presents the scale-free property. Therefore, a tonal structure is not enough to create musical pieces which present the scale-free property. Mozart and Beethoven seemed to know better than others how to compose musical pieces with the scale-free property —see fig. 3(a). Only 62.5% of music in our database has the small-world property. All Mozart's and Haydn's compositions strongly present the said property —see fig. 3(b).

Future research will include: i) evaluation of other music genres; ii) investigation of edge weight distribution; iii) evaluation of musical networks fractal dimension according to Song *et al.*'s [28,36] algorithms; and iv) understanding the community structure of our musical networks.

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