INMA 2471 – Optimization models and methods II

Homework I – AMPL modelling

[v1.1]

This homework consists in formulating and solving the following two optimization case studies, using AMPL models as the central tool. You will work in pairs, and produce for each case study AMPL models and a separate PDF report describing your modelling process, implementation, results and comments.

Many variants can be considered for each problem, and you have a lot of freedom in what to include in your report (you may even include content not requested in the problem description). The AMPL models themselves should be readable, well-structured and easy to maintain, and can be commented (in which case it is not necessary to repeat what is in the comments in the main report).

The total length of your report should not exceed five pages. The deadline for submitting your report and all accompanying files (source code, data files, etc.) in a single zip file is **Sunday October 23rd.**.

A. An optimal fast food diet. An AMPL dat file describing the cost and nutritional properties of items available at a well-known fast-food chain is provided in `FastFood.dat`. The data file also provides guidelines (i.e. lower and upper bounds) on the daily intake of calories and various nutriments.

1. Find the cheapest menu that satisfies the guidelines. Comment on the obtained solution and, if you estimate it necessary, add additional constraints to make the solution more realistic and to render the diet easier to follow.
2. Propose a menu for each day of the week that will aim at maximizing the variety among the seven daily menus, i.e., that will pick daily menus as different from each other as possible, while still trying to achieve a low total cost. Explain how you estimate and optimize variety.

B. Factory planning.

In production planning problems, choices must be made about how many of what products to produce using what resources in order to maximize profits or minimize costs, while meeting a range of constraints. These problems are common across a broad range of manufacturing situations.

In this particular situation, we'll model and solve a production mix problem: during each period we can manufacture a range of products. Each of the products needs a different amount of time to manufacture on different machines, and yields a different profit.The aim is to create an optimal multi-period production plan to maximize the profit. Some machines are not available in every period due to maintenance. There is a upper limit on the sales of each product in each month due to market limitations and the storage capacity is also restricted.

A factory makes seven products using a range of machines including four grinders, two vertical drills, three horizontal drills, one borer and one planer. Each product has a defined profit contribution per unit sold (defined as the sales price per unit minus the cost of raw materials). In addition, the manufacturing of each product requires a certain amount of time on each machine (in hours). The contribution and manufacturing time value are shown below. A dash indicates the manufacturing product for the given product does not require that machine.

	Prod. 1	Prod. 2	Prod. 3	Prod. 4	Prod. 5	Prod. 6	Prod. 7
Contribution to profit	10	6	8	4	11	9	3
Grinding	0.5	0.7	-	-	0.3	0.2	0.5
Vertical drilling	0.1	0.2	-	0.3	-	0.6	-
Horizontal drilling	0.2	-	0.8	-	-	-	0.6
Boring	0.05	0.03	-	0.07	0.1	-	0.08
Planing	-	-	0.01	-	0.05	-	0.05

In each of the six months covered by this model, one or more of the machines is scheduled to be down for maintenance and as a result will not be available to use for production that month. The maintenance schedule is as follows:

Month	Machine
January	One Grinder
February	Two Horizontal Drills
March	One borer
April	One vertical drill
May	One grinder and one vertical drill
June	One horizontal drill

There limitations to how many of each product can be sold in a given month. These limits are shown below:

Month	Prod. 1	Prod. 2	Prod. 3	Prod. 4	Prod. 5	Prod. 6	Prod. 7
January	500	1000	300	300	800	200	100
February	600	500	200	0	400	300	150
March	300	600	0	0	500	400	100
April	200	300	400	500	200	0	100
May	0	100	500	100	1000	300	0
June	500	500	100	300	1100	500	60

Up to 100 units of each product may be stored in inventory at a cost of 50 cents per unit per month. At the start of January there is no product inventory. However, by the end of June there should be 50 units of each product in inventory.

The factory produces product six days a week using two eight-hour shifts per day. It may be assumed that each month consists of 24 working days. Also, for the purposes of this model, there are no production sequencing issues that need to be taken into account.

1. What should the production plan look like ? Comment.
2. Suppose you are now also responsible for the planning of maintenance. Each machine must be down for maintenance during one month in the planning horizon, with the exception of the four grinding machines where only two of them need to be down during one month. How does this new flexibility affect the production plan ? Comment.

(This problem is based on a larger model built for the Cornish engineering company of Holman Brothers. Sources: GUROBI optimization and Model Building in Mathematical Programming by H. Paul Williams.)

Changelog. [v1.0, 2016-09-29] initial release. [v1.1 2016-10-12 typo in manufacturing requirements table corrected (some figures were shifted on the "boring" line)