Extracting low energy hadron-hadron physics from Lattice QCD

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Resources: JLaB & FermiLaB (DOE, USA), Livermore (USA), Tungsten (Illinois, USA), Mare Nostrum (BSC, Spain).

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Up to now we have done work in the meson-meson sector:

$\pi^+\pi^+$ scattering (I=2 $\pi\pi$)

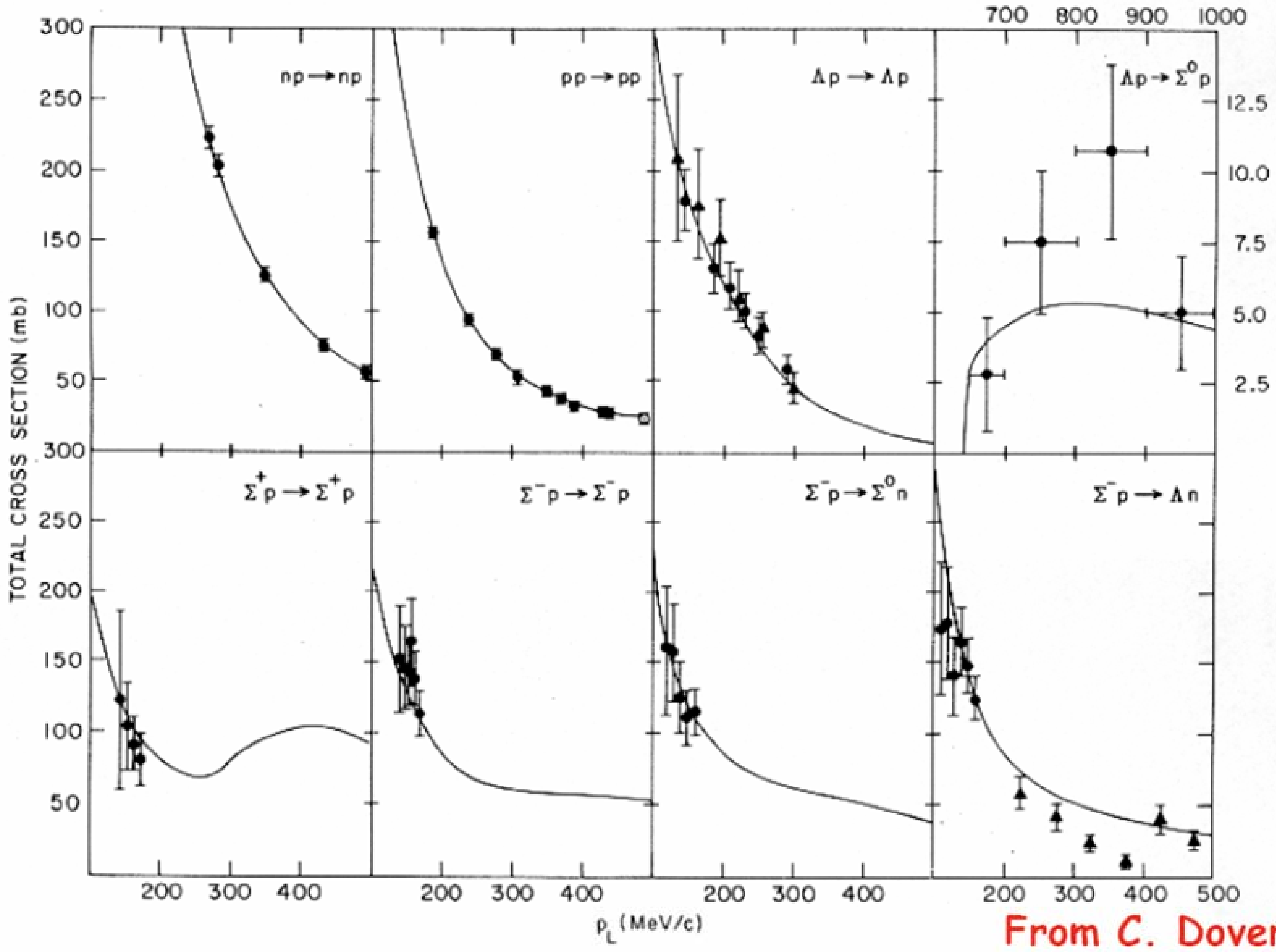
π^+K^+ scattering (I=3/2 and 1/2 πK)

K^+K^+ scattering (I=1 KK)

and in the baryon-baryon sector:

singlet and triplet NN, $\boxed{\Lambda\text{+N, }\Sigma\text{+N, YY}}$ scattering ⇒ { hypernuclear physics, neutron star interior }

Alternative/complementary source of information on those sectors where experiments are difficult/absent



TOTAL CROSS SECTION (mb)
$np \rightarrow np$
$pp \rightarrow pp$
$\Lambda p \rightarrow \Lambda p$
$\Lambda p \rightarrow \Sigma^0 p$
$\Sigma^+ p \rightarrow \Sigma^+ p$
$\Sigma^- p \rightarrow \Sigma^- p$
$\Sigma^- p \rightarrow \Sigma^0 n$
$\Sigma^- p \rightarrow \Lambda n$
p_L (MeV/c)
700 800 900 1000
12.5 10.0 7.5 5.0 2.5
300 250 200 150 100 50
200 300 400 500
From C. Dover

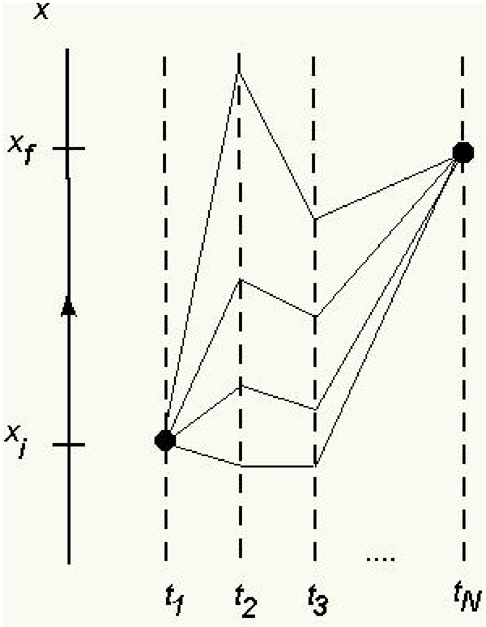
Why a LQCD calculation?

Absence of analytic solutions of QCD in the non-perturbative regime $\Rightarrow$ Approximations:

1. Models:
   - o formulate effective degrees of freedom (pions)
   - o formulate interactions retaining as much of the basic underlying theory as possible (phenomenological parameters)

2. First principles:
   - o keep the basic degrees of freedom (quarks, gluons)
   - o approximate the calculations (perturbative expansions, LQCD)

transition matrix element within the path integral formalism in QFT



action

$$\langle \theta_f(x,t_f) | \theta_i(x,t_i) \rangle \approx \sum_{\theta_P} \exp(iS(\{\theta_P\},\{\partial_\mu \theta_P\}))$$

$$= N_0 \lim_{N_x \to \infty} \lim_{N_t \to \infty} \int \prod_{m=1}^{N_x} \prod_{n=1}^{N-1} d\theta(x_m,t_n) \exp(iS[\{\theta(x_m,t_n), \partial_\mu \theta(x_m,t_n)\}])$$

Imaginary time $\rightarrow$ t = - i τ

Euclidean action $\rightarrow$ S^E = - iS

For real and positive actions $\exp(iS) \rightarrow \exp(-S^E)$

(PROBABILITY!) weighting factor

LQCD Non-perturbative implementation of ET

Uses the Feynman Path Integral approach

Starting point: Partition function in Euclidean space-time

$$\mathcal{Z} = \int \mathcal{D}A_\mu \, \mathcal{D}\bar{\Psi} \, \mathcal{D}\Psi \, e^{-S}$$

QCD action

$$\boxed{S = d^4x \, (1/4 \, F_{\mu\nu} F^{\mu\nu} - \bar{\Psi} M \Psi)}$$

By exact integration (Gaussian) on the fermion fields

$$\mathcal{Z} = \int \mathcal{D}A_\mu \, \det M \, \exp[\underbrace{-d^4x \, (1/4 \, F_{\mu\nu} F^{\mu\nu}}_{-S_{gluon}}]$$

nonlocal term which contains the fermionic contribution M(A)

HARD!!!

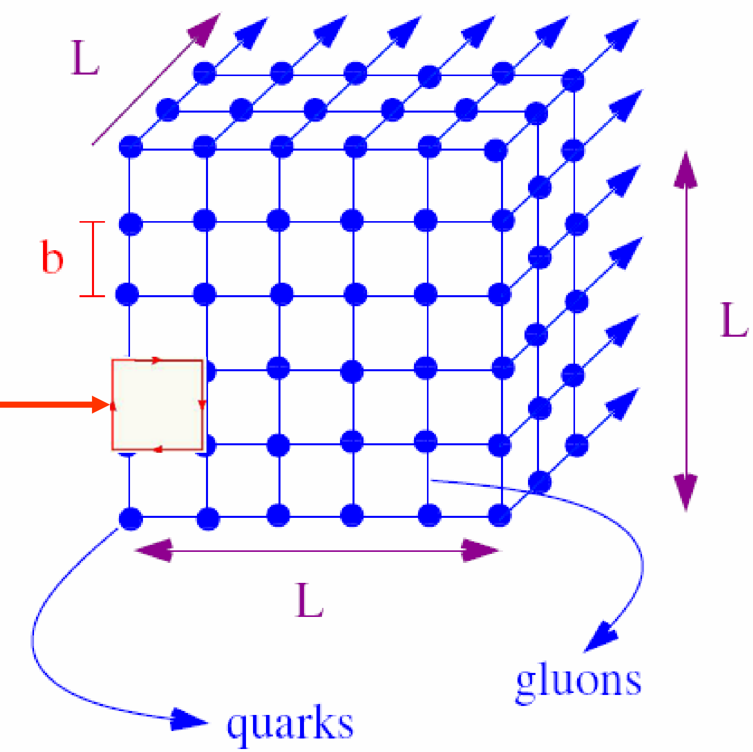
LQCD ↔ First principle calculations with *à priori less uncertainty*

Formulate QCD in an Euclidean lattice

Path-Integral Formalism:

$$\langle \hat{O} \rangle = \frac{1}{Z} \int \mathrm{D}U \, \hat{O} \det M[U] e^{-S[U]}$$

Plaquette



$$\langle G[\phi] \rangle_T = \frac{\sum_{\phi} e^{-\frac{E[\phi]}{kT}} G[\phi]}{\sum_{\phi} e^{-\frac{E[\phi]}{kT}}}$$

~Thermal average over configurations

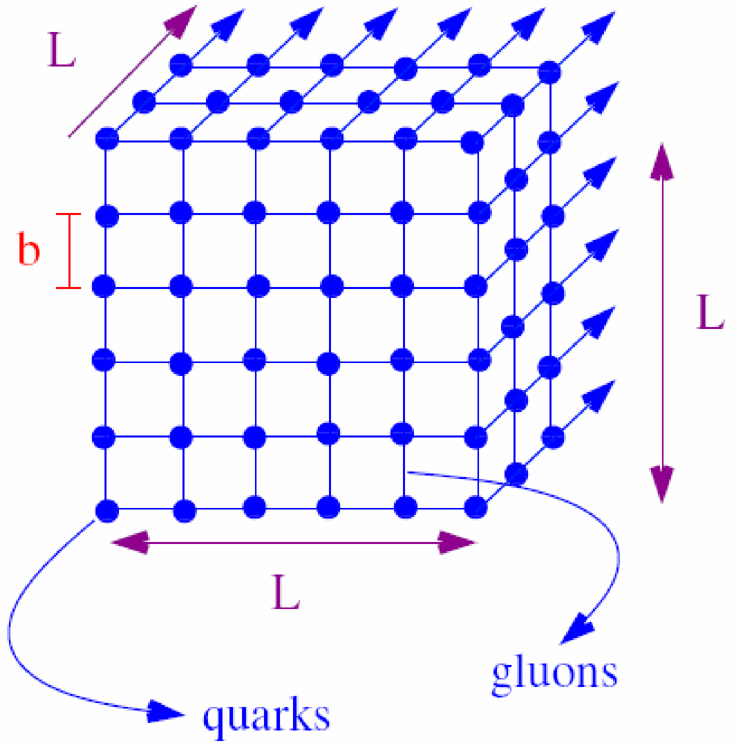
Monte-Carlo Evaluation

LQCD → Formulate QCD in an Euclidean lattice

$$L >> \text{relevant scales} >> b \qquad \left(\frac{1}{L} << m_\pi << \Lambda_\chi << \frac{1}{b} \right)$$

$$\text{Cost} \approx \left[\frac{\#\,\text{configs}}{1000}\right] \cdot \left[\frac{m_q}{20\ \text{MeV}}\right]^{-1} \cdot \left[\frac{V}{32\ \text{fm}^4}\right]^{\frac{5}{4}} \cdot \left[\frac{b}{0.08\ \text{fm}}\right]^{-6}$$

(L. Giusti, Lattice'06)



Present	Approaching nature	
$L \sim 2.5$ fm	⟷	large L
$b \sim 0.1$ fm	EFT	$b \to 0$
$m_q \sim m_s/2$		$m_q \to m_{u,d}^{phys}$

Computational resources and simulation

- Resources: JLaB (USA), FermiLaB (USA), Livermore (USA), Tungsten (Illinois, USA), Mare Nostrum (Barcelona, Spain).

DW fermion propagators

(sea) light quark mass

(sea) strange quark mass

Ensemble	bm_l	bm_s	bm_l^{dwf}	bm_s^{dwf}	# configurations	# sources
2064f21b676m007m050	0.007	0.050	0.0081	0.081	(468)	x 16
2064f21b676m010m050	0.010	0.050	0.0138	0.081	(658)	x 20
2064f21b679m020m050	0.020	0.050	0.0313	0.081	(486)	x 24
2064f21b681m030m050	0.030	0.050	0.0478	0.081	(564)	x 8

("chopped": 64 → 32)

20^3 x 64

(valence) light quark mass

(valence) strange quark mass

m_π = **295**, **357**, **495**, **595** MeV

MILC gauge configurations

Chroma software R.G. Edwards and B.Joo [SciDAC Collaboration]

¿What is simulated?

Lattice simulations → Evaluation of vacuum correlation functions:

$$\langle \Gamma_1(t)\Gamma_2(0)\rangle \equiv \langle 0|\Gamma_1(t)\Gamma_2(0)|0\rangle \qquad \boxed{\text{at large t}}$$

$$\langle \Gamma_1(t)\Gamma_2(0)\rangle = \langle 0|\Gamma_1(0)\, e^{-\hat{H}t}\,\Gamma_2(0)|0\rangle = \sum_n \langle 0|\Gamma_1(0)|E_n\rangle e^{-E_n t}\langle E_n|\Gamma_2(0)|0\rangle$$

$$\to \quad \langle 0|\Gamma_1(0)|\boxed{E_0}\rangle\langle \boxed{E_0}|\Gamma_2(0)|0\rangle e^{\boxed{-E_0 t}}, \qquad \text{as} \quad \boxed{t \to \infty}$$

lowest energy eigenstate

from the exponential decay → energies

Ensure that the (assimptotic) exponential dominates the correlation function

Ex:

$$C_{\pi^+}(t) = \sum_{\vec{x}} \langle \pi^-(t,\vec{x})\pi^+(0,\vec{0})\rangle, \qquad \pi^+(t,\vec{x}) = \overline{u}(t,\vec{x})\gamma_5 d(t,\vec{x})$$

Extracting masses and energy shifts

mass

One-baryon correlator:

$$C_A(t) = \sum_{\vec{x}} \left\langle A(t,\vec{x})\, A^{\dagger}(0,\vec{0}) \right\rangle = \sum_n C_A^n\, e^{-E_A^n t} \rightarrow C_A\, e^{-M_A t}$$

2-baryon correlator:

$$C_{AB}(t) = \sum_{\vec{x},\vec{y}} \left\langle A(t,\vec{x})\, B(t,\vec{x})\, B^{\dagger}(0,\vec{0})\, A^{\dagger}(0,\vec{0}) \right\rangle = \sum_n C_{AB}^n\, e^{-E_{AB}^n t} \rightarrow C_{AB}\, e^{-E_{AB} t}$$

$$\boxed{\text{Energy shift:} \quad \Delta E = E_{AB} - M_A - M_B}$$

$$G_{AB}(t) = \frac{C_{AB}(t)}{C_A(t)\, C_B(t)} = \sum_n C^n e^{-\Delta E^n t} \rightarrow \boxed{C e^{-\Delta E\, t}}$$

$\Delta E = E_{AB} - M_A - M_B$

effective ΔE: looking for plateaus

$\boxed{\text{Energy shift: } \Delta E = E_{AB} - M_A - M_B}$

$$G_{AB}(t) = \frac{C_{AB}(t)}{C_A(t)\,C_B(t)} = \sum_n C^n e^{-\Delta E^n t} \to C e^{\boxed{-\Delta E\, t}}$$

$\Delta E = E_{AB} - M_A - M_B$

Build: $$\log \frac{G_{AB}(t)}{G_{AB}(t+1)} \to \log \frac{Ce^{-\Delta E\, t}}{Ce^{-\Delta E\,(t+1)}} \to \Delta E^{eff}$$

Scattering from LQCD ?

$$\langle 0 | N_1(t,-\vec{k})\, N_2(t,\vec{k})\, N_1^+(0,-\vec{k})\, N_2^+(0,\vec{k}) | 0 \rangle =$$

$$\sum_n e^{-Ht} \langle 0 | N_1(0,-\vec{k})\, N_2(0,\vec{k}) | n \rangle \langle n | N_1^+(0,-\vec{k})\, N_2^+(0,\vec{k}) | 0 \rangle$$

$$\underset{(t\to\infty)}{\rightarrow} \quad e^{-2m_N t} \langle 0 | N_1(0,-\vec{k})\, N_2(0,\vec{k}) | (N_1 N_2)_{\text{rest}} \rangle \langle (N_1 N_2)_{\text{rest}} | N_1^+(0,-\vec{k})\, N_2^+(0,\vec{k}) | 0 \rangle$$

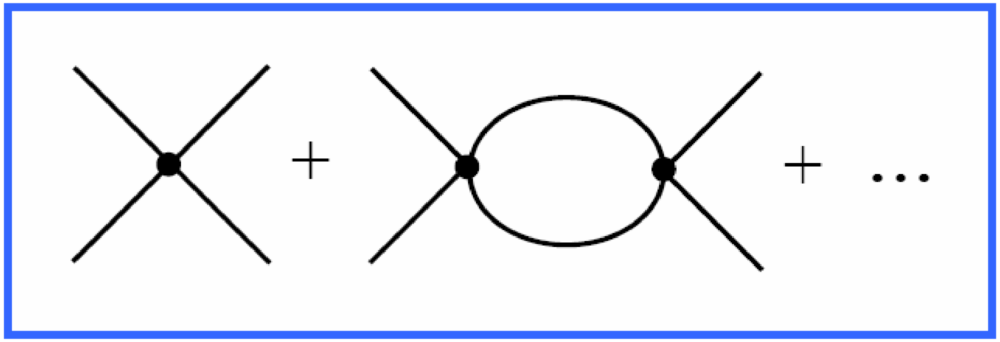
Forbidden except for kinematical thresholds!!

The Maiani-Testa theorem states that infinite-volume Euclidean space Green functions cannot be used to extract S-matrix elements except at kinematic thresholds.

Lüscher method

$$L >> |a|, \qquad E_0 = \frac{4\pi a}{ML^3}\left[1 - c_1 \frac{a}{L} + c_2 \left(\frac{a}{L}\right)^2 + \ldots\right] + O\left(L^{-6}\right)$$

Pionless theory of NN interactions



2B elastic scattering amplitude in the continuum of a $\mathrm{EFT}(\not\pi)$ of nrel baryons:

$$\mathcal{A} \;=\; \frac{\sum C_{2n}p^{2n}}{1-I_0\boxed{\sum C_{2n}p^{2n}}}\,, \qquad I_0 \;=\; \left(\frac{\mu}{2}\right)^{4-D}\int\frac{d^{D-1}\mathbf{q}}{(2\pi)^{D-1}}\,\frac{1}{E-\frac{|\mathbf{q}|^2}{M}+i\epsilon}$$

linearly divergent

PDS scheme to $I_0 \quad \Rightarrow \quad I_0^{(PDS)} = -\frac{M}{4\pi}\;(\mu+ip)\;+\;\mathcal{O}(D-4)$

$$p=\sqrt{ME} \quad \Rightarrow \quad \boxed{\mathcal{A}=\frac{4\pi}{M}\,\frac{1}{p\cot\delta-ip}}$$

$$p<\sqrt{m_\pi M}$$

Pionless theory of NN interactions in a box

We are interested in the energy eigenvalues of 2N placed in a box of size L with PBC

$$\mathcal{A} = \frac{4\pi}{M}\frac{1}{p\cot\delta - ip}$$

Energy-eigenvalues $\longleftrightarrow$ $\mathrm{Re}(\mathcal{A}^{-1}) = 0 \;\Rightarrow\; \frac{1}{\sum C_{2n}(\mu)\, p^{2n}} - \mathrm{Re}(I_0^{(PDS)}(L)) = 0$

$$I_0(L) = \tfrac{1}{L^3}\sum_{\mathbf{k}} \frac{1}{E - \frac{|\mathbf{k}|^2}{M}}$$

$$p\cot\delta(p) = -\frac{1}{\pi L} S\left(\frac{p^2L^2}{4\pi^2}\right) = -\frac{1}{\pi L}\left[\sum_{\vec{j}}^{|\vec{j}|<\Lambda} \frac{1}{|\vec{j}|^2 - \left(\frac{p^2L^2}{4\pi^2}\right)^2} - 4\pi\Lambda\right]$$

(valid for scattering and bound states)

Beane, Bedaque, Parreño, Savage, Phys. Lett. B 585,1-2, 106-114 **(**2004**)**

Procedure

taken from our fits
to lattice CF's

$$\boxed{\Delta E \equiv \sqrt{p^2 + M_A^2} + \sqrt{p^2 + M_B^2} - M_A - M_B}$$

Eigenvalue Equation
(below inelastic
thresholds)

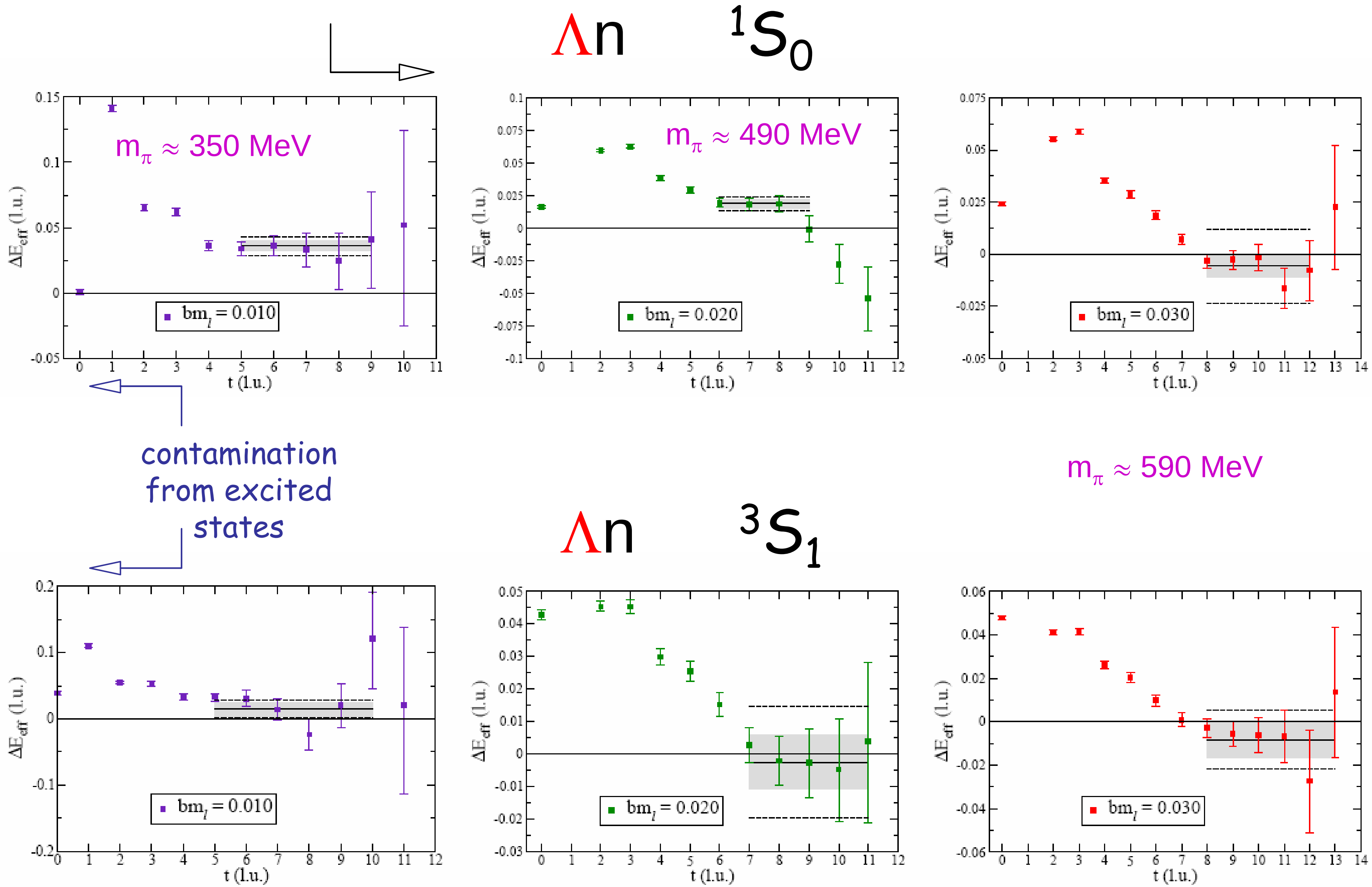
$$p \cot \delta(p) = -\frac{1}{\pi L} S\left(\frac{\boxed{p^2} L^2}{4\pi^2}\right) = \boxed{-\frac{1}{a} + \frac{1}{2} r_0 p^2}$$

extract

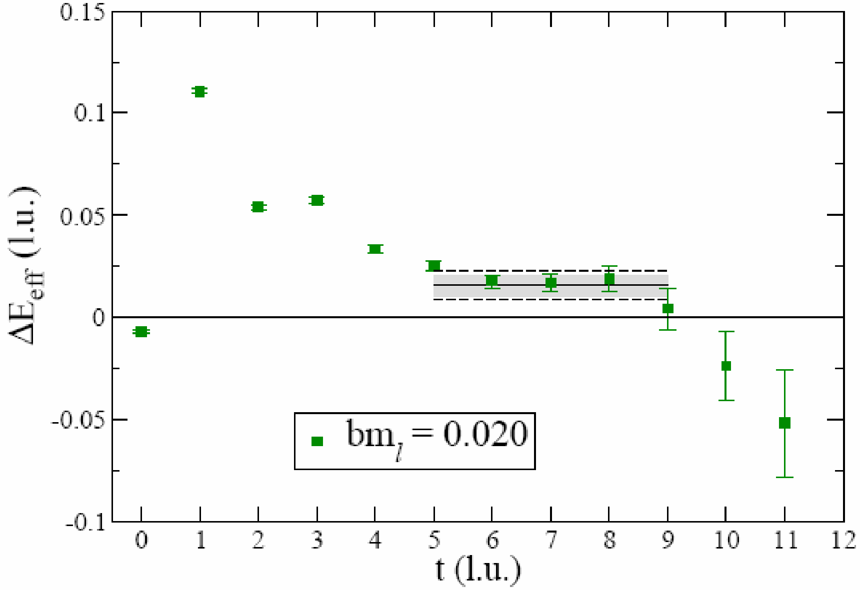
$$S(\eta) \equiv \sum_{\vec{j}}^{|\vec{j}|<\Lambda} \frac{1}{|\vec{j}|^2 - \eta^2} - 4\pi \Lambda$$

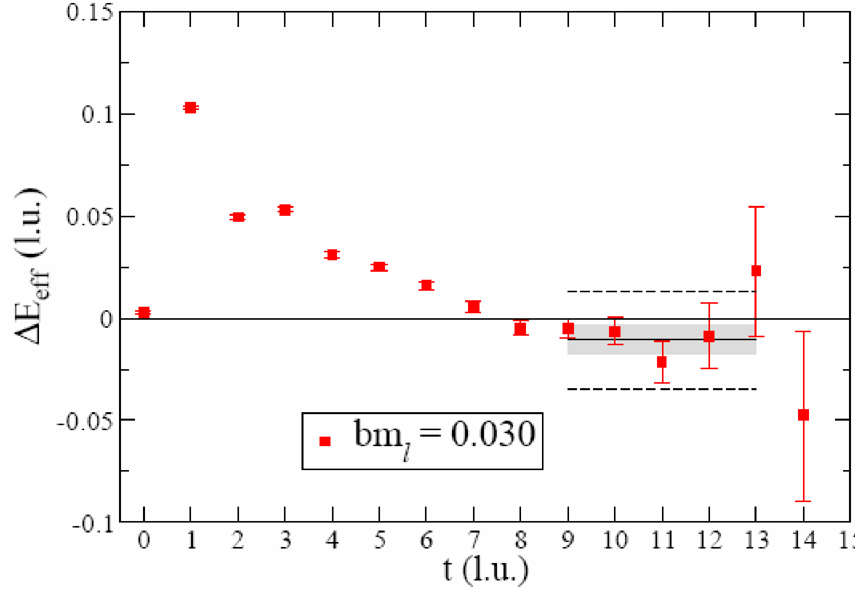
u.v. regulator

signal-to-noise ratio ~ $\sqrt{N_{conf}}\ e^{-(M_N + M_\Lambda - 3m_\pi)t}$

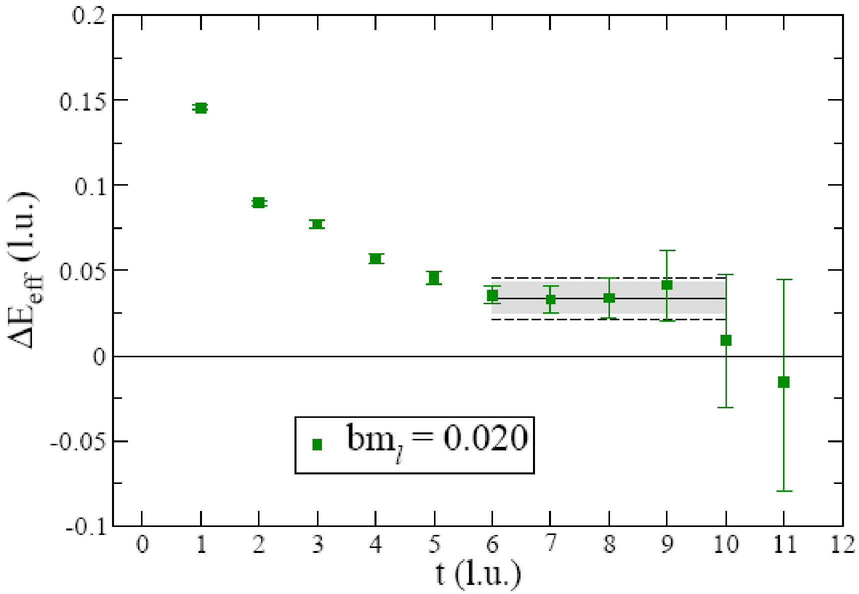


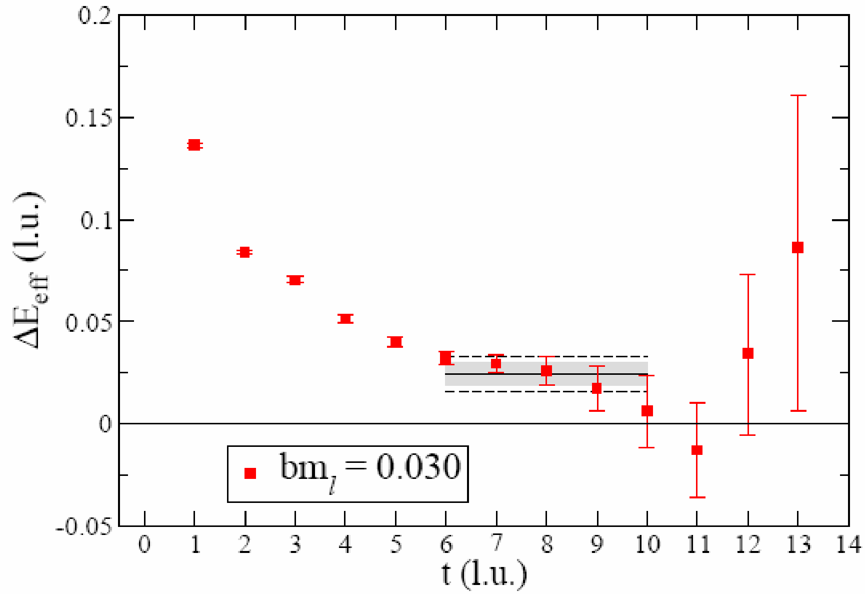
$\Sigma^- n$ $\quad {}^1S_0$





$m_\pi \approx 490$ MeV $\quad {}^3S_1 \quad$ $m_\pi \approx 590$ MeV





YN results

Channel	m_π (MeV)	Range	ΔE (MeV)	$\lvert\mathbf{k}\rvert$ (MeV)	δ (degrees)	$-(k\cot\delta)^{-1}$ (fm)
$n\Lambda$ $^1S_0$	$592 \pm 1 \pm 10$	8-12	$-9 \pm 8 \pm 20$	–	–	
	$493 \pm 1 \pm 8$	6-9	$29.8 \pm 5.4 \pm 2.5$	$197 \pm 24 \pm 4$	$-32.3 \pm 8.1 \pm 2.8$	$0.63 \pm 0.12 \pm 0.014$
	$354 \pm 1 \pm 6$	5-9	$56.8 \pm 6.0 \pm 5.5$	$255 \pm 22 \pm 13$	$-53.4 \pm 8.5 \pm 10.1$	$1.04 \pm 0.24 \pm 0.15$
$n\Lambda$ $^3S_1$	$592 \pm 1 \pm 10$	8-13	$-13 \pm 13 \pm 8$	–	–	
	$493 \pm 1 \pm 8$	7-11	$-4 \pm 13 \pm 14$	–	–	
	$354 \pm 1 \pm 6$	5-10	$23 \pm 17 \pm 4$	$168 \pm 62 \pm 14$	$-23 \pm 18 \pm 4$	$0.50 \pm 0.26 \pm 0.06$
$n\Sigma^-$ $^1S_0$	$592 \pm 1 \pm 10$	9-13	$-17 \pm 11 \pm 27$	–	–	
	$493 \pm 1 \pm 8$	5-9	$24.9 \pm 7.8 \pm 3.0$	$179 \pm 28 \pm 11$	$-27.2 \pm 9.0 \pm 3.8$	$0.57 \pm 0.13 \pm 0.05$
$n\Sigma^-$ $^3S_1$	$592 \pm 1 \pm 10$	6-10	$38.5 \pm 8.8 \pm 5.0$	$226 \pm 26 \pm 15$	$-44.3 \pm 9.8 \pm 5.4$	$0.85 \pm 0.20 \pm 0.10$
	$493 \pm 1 \pm 8$	6-10	$53 \pm 14 \pm 5$	$261 \pm 35 \pm 13$	$-58 \pm 15 \pm 5$	$1.19 \pm 0.51 \pm 0.15$

statistical

systematic

Infinite volume vs finite volume

Infinite volume

Binding energy: $\Delta E = M\text{-}2m < 0 \neq 0$

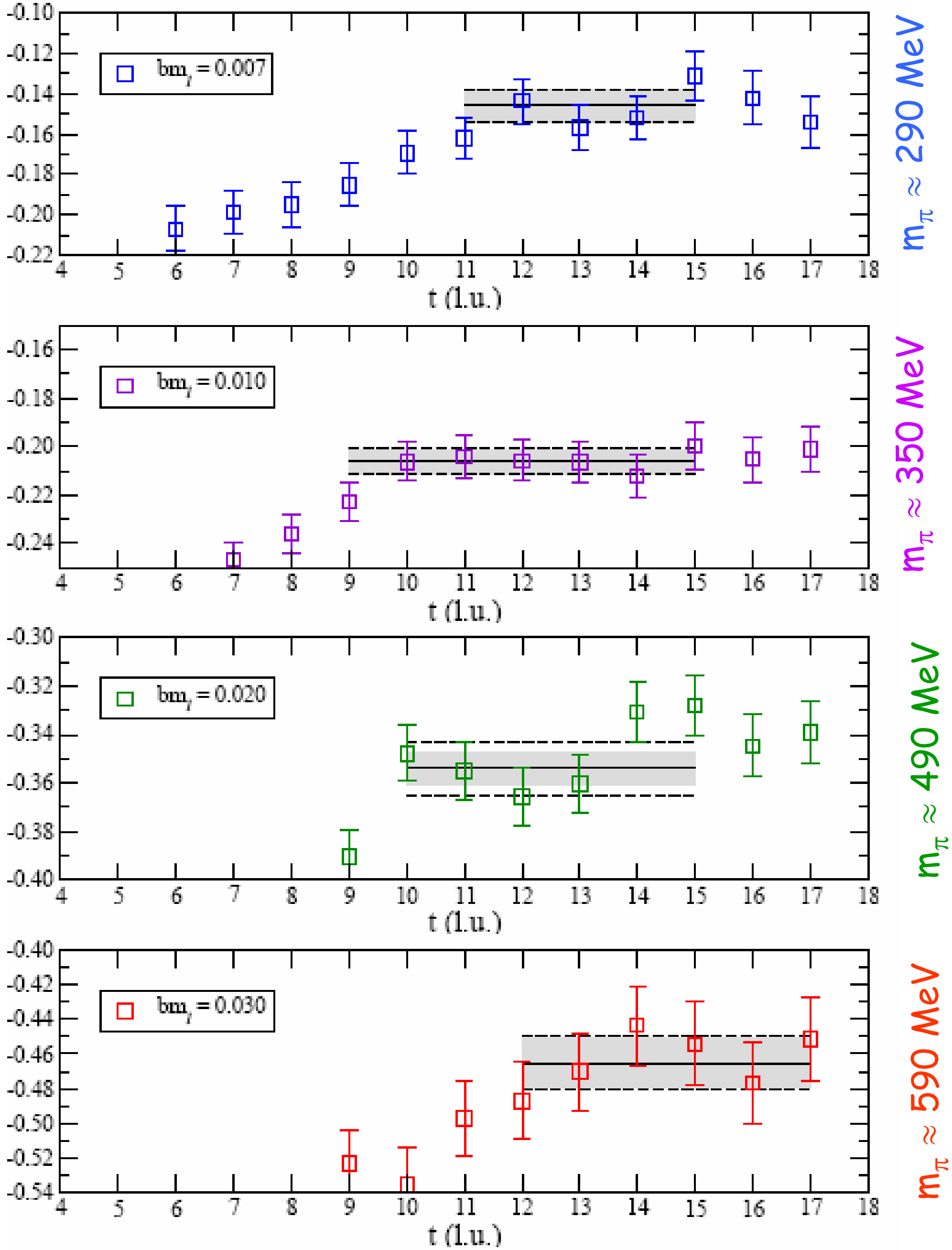
Scattering states: $\Delta E > 0$ (lowest=threshold)

Finite volume

Binding energy: $\Delta E = M\text{-}2m < 0 \neq 0$

Scattering states: $\Delta E = E_0 - 2m = O(1/L^3)$

$\Delta E < 0$ for attractive interaction
$\Delta E > 0$ for repulsive interaction



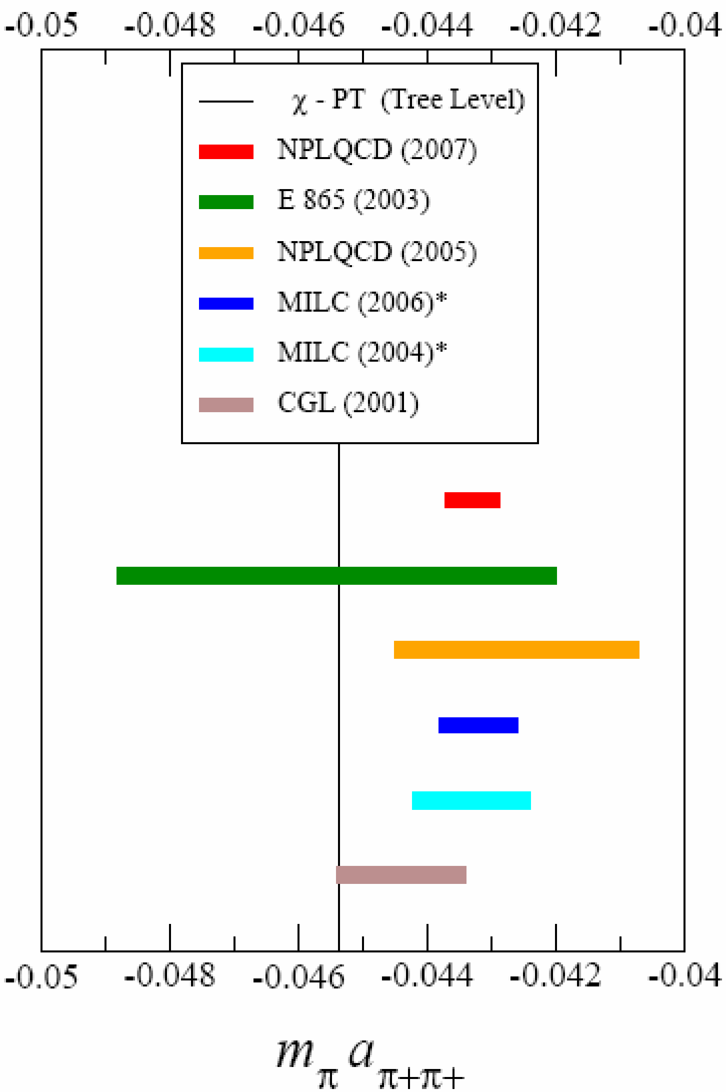
$\pi^+ \pi^+$ scattering

$(m_\pi a_{\pi^+\pi^+})^{EFF}$

combined with χPT

$$m_\pi a_{\pi\pi}^{I=2} = -0.04330 \pm 0.00042$$

NPLQCD, e-Print: **arXiv:0706.3026** [hep-lat]
(S. Beane, MENU07)



	$m_\pi\ a_{\pi\pi}^{I=2}$
χPT (Tree Level)	-0.04438
NPLQCD (2007)	-0.04330 ± 0.00042
E 865 (2003)	$-0.0454 \pm 0.0031 \pm 0.0010 \pm 0.0008$
NPLQCD (2005)	$-0.0426 \pm 0.0006 \pm 0.0003 \pm 0.0018$
MILC (2006)*	-0.0432 ± 0.0006
MILC (2004)*	-0.0433 ± 0.0009
CGL (2001)	-0.0444 ± 0.0010

Summary

- Increasing (super)computing capabilities $\rightarrow$ extensive use of dynamic simulations of the interactions between quarks and gluons $\rightarrow$ interactions between two hadrons.
- In particular, we have performed the first dynamical LQCD simulation of the strong low energy YN interaction.
- Our results in the baryon-baryon sector are limited by the present statistics.
- This work is in progress. We are completing the analysis for the YN and YY channels,
  - accumulating more statistics with same L and b
  - increase the signal/noise ratio
  - future: reduce the systematic sources of error:
    - smaller lattice spacings (continuum extrapolations)
    - larger volumes