Nuclear symmetry energy deduced from dipole excitations: a comparison with other constraints

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• This work is part of a longer-term research plan.

• The goal is:
understanding which are the observables that can constrain a universal Energy Density Functional (EDF).

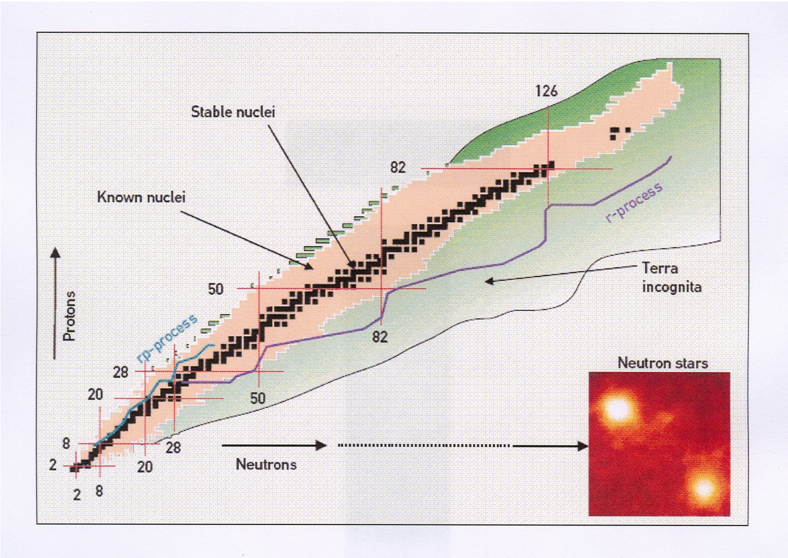
• In particular:
we emphasize the role of EDFs in providing links between finite nuclei and the nuclear EOS, or the nuclear astrophysics.

• One specific point consists in:
assessing what are the limits of EDFs. What are the correlations to be inserted explicitly ?

Nuclear theory and EDFs

Strong uncertainties affect *at the same time* the nuclear effective Hamiltonian and the many-body correlations. This naturally generates complementary nuclear models.

Models based on the **energy density functionals** (self-consistent mean field and extensions) allow systematic exploration of the nuclear chart – they also provide links with the **equation of state (EOS) of uniform matter**.



EDF parametrization

Finite nuclei

Nuclear EOS and astrophysics

Energy density functionals (EDFs)

$$E = \left\langle \Psi \right| \hat{H} \left| \Psi \right\rangle = \left\langle \Phi \right| \hat{H}_{eff} \left| \Phi \right\rangle = E\left[\hat{\rho} \right]$$

$\left| \Phi \right\rangle$ **Slater determinant** $\Leftrightarrow$ $\hat{\rho}$ **1-body density matrix**

• Minimization of E can be performed either within the nonrelativistic or relativistic framework → Hartree-Fock or Hartree equations

• In the former case one often uses a two-body effective force and defines a starting Hamiltonian; in the latter case a Lagrangian is written, including nucleons as Dirac spinors and effective mesons as exchanged particles.

• 8-10 free parameters (typically). Skyrme/Gogny vs. RMF/RHF.

• The linear response theory describes the small oscillations, the Giant Resonances or other multipole strength → (Quasiparticle) Random Phase Approximation or (Q)RPA

• Self-consistency !

Difference between self-consistent mean field (SCMF) and energy density functionals

In the self-consistent mean field (SCMF) one starts really from an effective Hamiltonian $H_{eff} = T + V_{eff}$, and <u>THEN</u> builds $< \Phi \mid H_{eff} \mid \Phi >$ and defines this as E.

In DFT, one builds directly E[ρ]. → More general !

In the nonrelativistic HF case:

$$\frac{1}{2}\sum_{ij}\langle ij|V_{\text{eff}}|ij\rangle = \frac{1}{2}\sum_{ij}\int \phi_i^\dagger(1)\phi_j^\dagger(2)v(1,2)\phi_i(1)\phi_j(2).$$

With Skyrme, for instance, the term in t_0 (t_3) provides a term ρ^2 ($\rho^{\alpha+2}$).
DFT provides a more general functional, in which there may be any dependence on the density.

A few details about the implementation

• HF (or HFB) equations are solved in coordinate space. Standard Skyrme sets are employed.

$$V_{pp}^{eff} = V_0 \left[1 - x \cdot \frac{\rho(r)}{\rho_0}\right] \delta(r - r')$$

x=0	volume pairing
x=1	surface pairing
x=0.5	mixed pairing

• RPA and QRPA are solved in configuration space (matrix formulation). Full self-consistency is achieved (2-body Coulomb, including Slater exchange, and spin-orbit are taken care of). For QRPA, we use the canonical basis.

• Powerful numerical tests are provided not only by the stability of the results, but also by the Thouless theorem and dielectric theorem.

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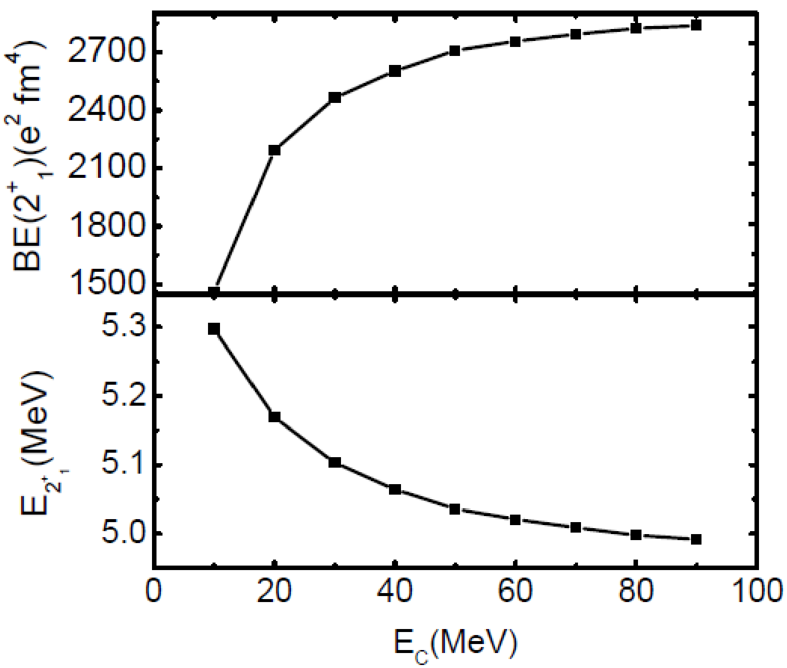
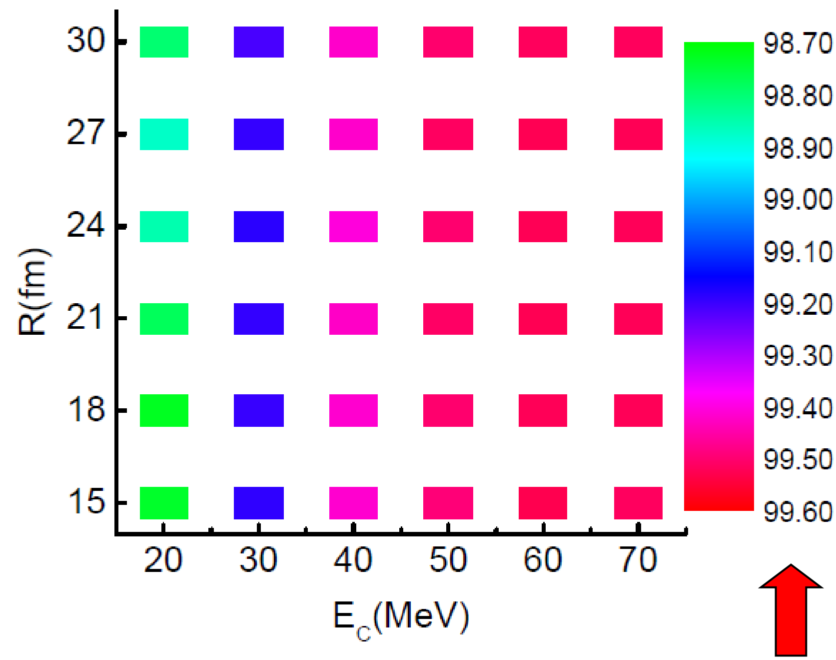
The continuum is discretized. The basis must be large due to the zero-range character of the force.

The energy-weighted sum rule should be equal to the double-commutator value: well fulfilled !

$$m_1(\hat{O}) = \sum_\nu E_\nu |\langle \nu|\hat{O}|\tilde{0}\rangle|^2 = \frac{1}{2}\langle 0|[\hat{O},[H,\hat{O}]]|0\rangle$$

^{208}Pb - SGII





Percentages m_1(RPA)/m_1(DC) [%]

- The dielectric theorem states that the INVERSE energy weighted sum rule m_{-1} can be obtained from CONSTRAINED calculations, that is, by finding the minimum of the energy with a constraint added to the Hamiltonian (H + λO).

$$
\begin{aligned}
m_{-1}(\hat{O}) &= -\frac{1}{2}\left[\frac{\partial}{\partial\lambda}\langle\phi(\lambda)|\hat{O}|\phi(\lambda)\rangle\right]_{\lambda=0} \\
&= \frac{1}{2}\left[\frac{\partial^2}{\partial\lambda^2}\langle\phi(\lambda)|H|\phi(\lambda)\rangle\right]_{\lambda=0}
\end{aligned}
$$

^{42}Ca

m_{-1}(CHFB-1)	m_{-1}(CHFB-2)	m_{-1}(QRPA)
95.98 fm^4/MeV	95.68 fm^4/MeV	93.99 fm^4/MeV

Cf. L. Capelli, G.C., J. Li, Phys. Rev. C79, 054329 (2009)

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Constrained energy: $\boxed{E_M^C = \sqrt{\frac{m_1}{m_{-1}}}}$

From the functional E[ρ_n,ρ_p] to the EOS

In uniform matter, spatial densities are simple numbers. If we translate E[ρ] into P[ρ] we have the Equation Of State.

Nuclear matter EOS

Symmetric matter EOS

Symmetry energy S

$$\frac{E}{A}(\rho,\delta) = \frac{E}{A}(\rho,\delta=0) + S(\rho)\,\delta^2$$

where $\delta = (\rho_n - \rho_p)/\rho$.

Around saturation: the symmetric matter EOS is reasonably known.

The asymmetric matter is not !

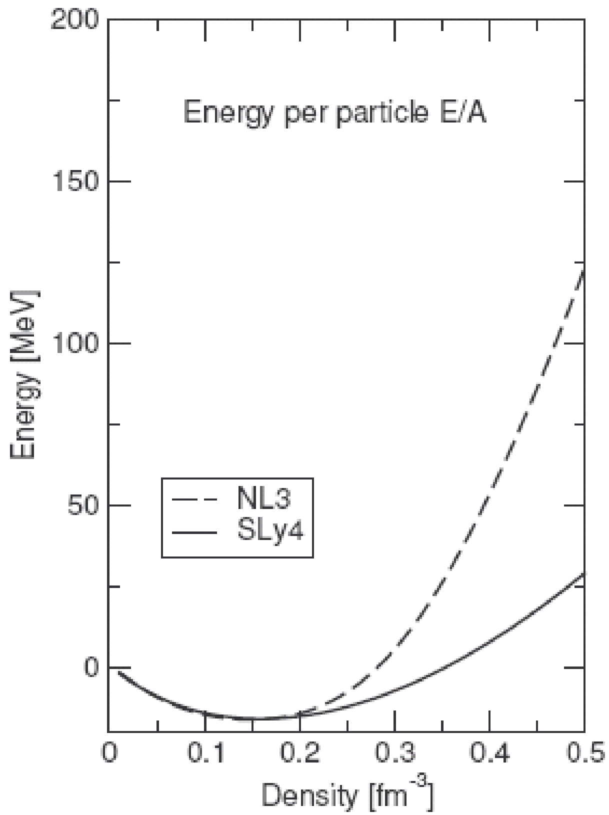
Symmetric matter incompressibility:

$$K_\infty = 9\rho_0^2 \frac{d^2(E/A)}{d\rho^2}\bigg|_{\rho_0}$$

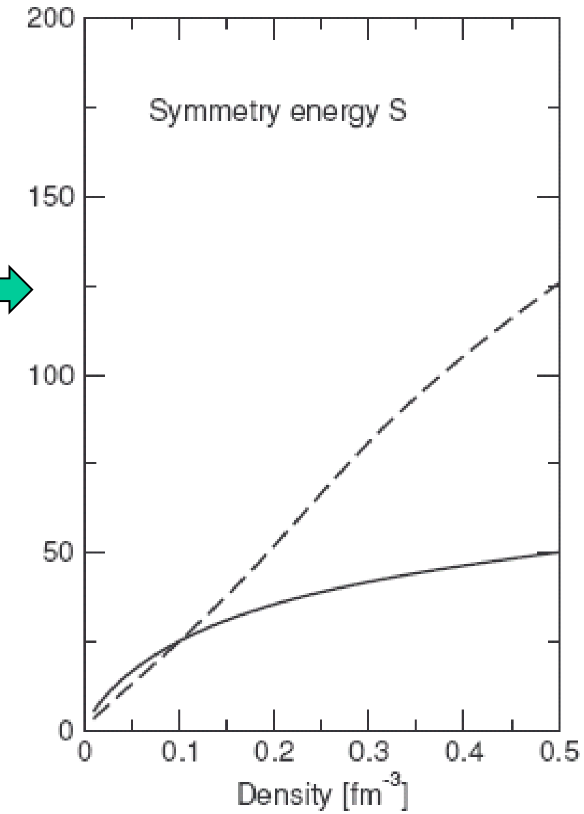
Uncertainty of the order of ± 10%

Larger uncertainty on:

$$\begin{aligned} S(\rho_0) &= J \\ S'(\rho_0) &= L/3\rho_0 \\ S''(\rho_0) &= K_{\rm sym}/9\rho_0^2 \end{aligned}$$

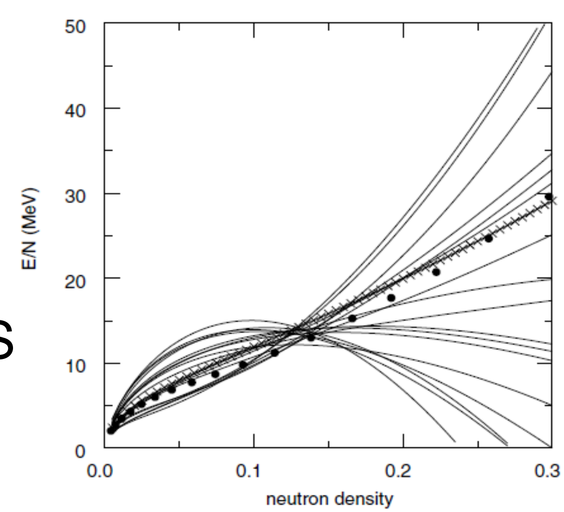


Symmetric and asymmetric matter EOS



$$\chi = -\frac{1}{V}\left(\frac{\partial P}{\partial V}\right)^{-1} \quad K_\infty = \frac{9}{\rho_0}\chi^{-1}$$

E/A (n. matter)
= E/A(sym) + S



How the ISGMR constraints the symmetric matter EOS

The link comes (only) from the use of an Energy Functional E[ρ].

IT PROVIDES AT THE SAME TIME

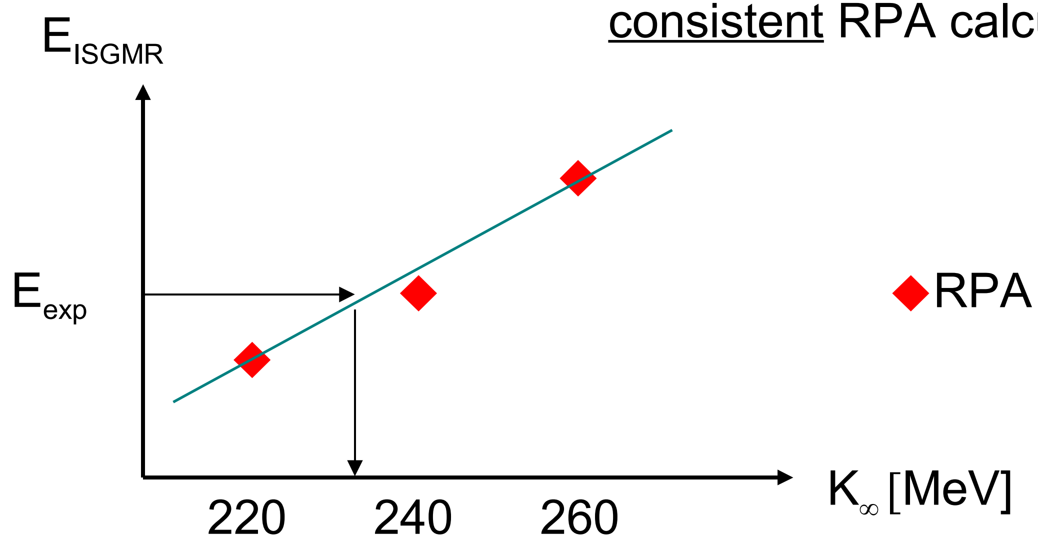
K_∞ in nuclear matter (analytic)

E_{ISGMR} (by means of <u>self-consistent</u> RPA calculations)

Skyrme

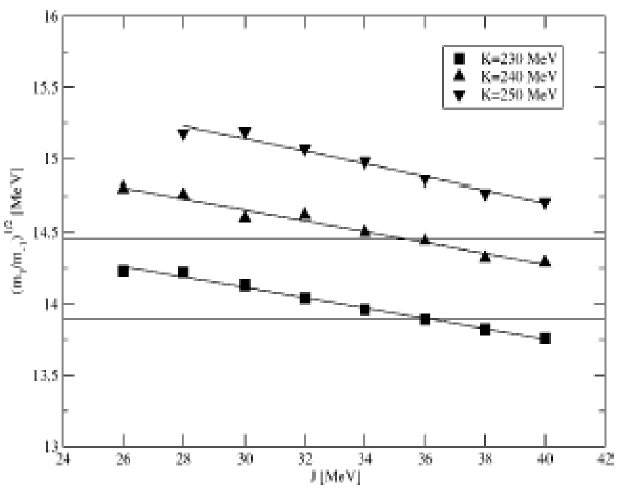
Gogny

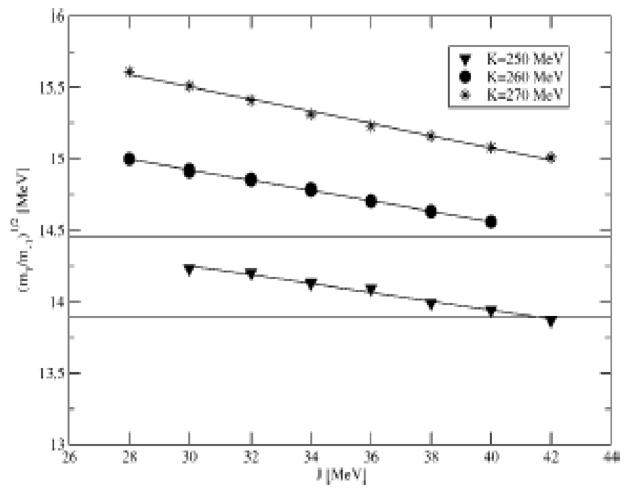
RMF



Extracted value of K_∞

$J \equiv S(\rho_0)$





α = 1/6 implies K around 230-240 MeV α = 1/3 implies K around 250 MeV

G.C., N. Van Giai, J. Meyer, K. Bennaceur, P. Bonche, *Phys. Rev.* **C70**, 024307 (2004)

Constraint from the ISGMR in ^{208}Pb :

E_{GMR} constrains K_∞ = 240 ± 20 MeV. The error comes from the choice of the density dependence, not from the relativistic or nonrelativistic framework.

S. Shlomo, V.M. Kolomietz, G.C., *Eur. Phys. J.* **A30**, 23 (2006)

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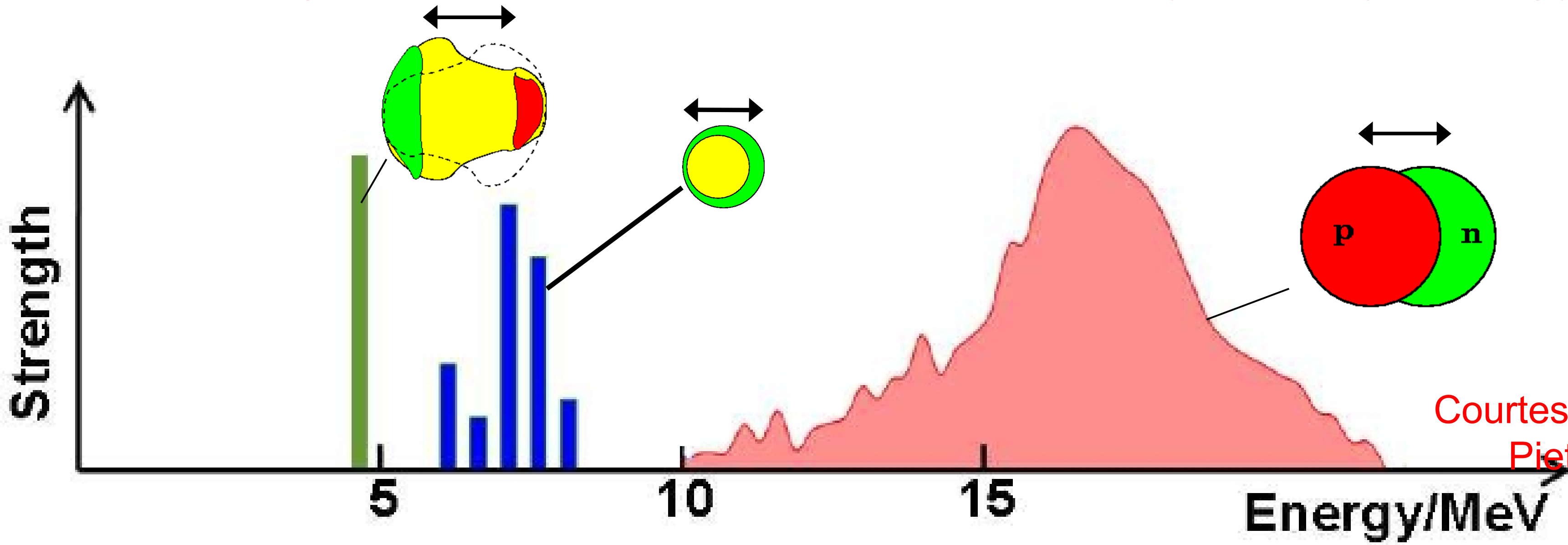
Electromagnetic field → Dipole modes → Symmetry energy



Courtesy: N. Pietralla

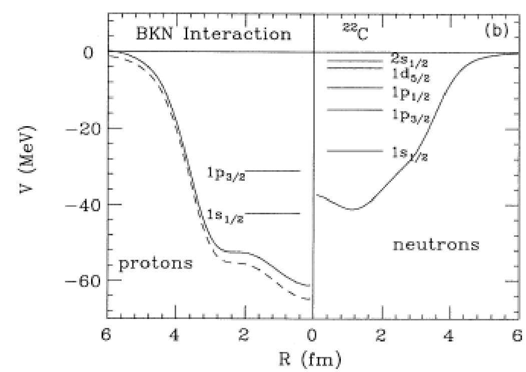
Giant Dipole Resonance (GDR) ↔ S[ρ]

Pygmy Dipole Resonance (PDR) ?

Two-Phonon excitation ($2^+ \otimes 3^-$) Unrelated with S[ρ]

In light nuclei low-lying dipole strength may be due to continuum transitions of weakly bound orbitals. No collective oscillation !

In medium-heavy nuclei the collectivity of the PDR should be assessed.



What **precisely** is the GDR correlated with ?

In the case in which the GDR exhausts the whole sum rule, its energy can be deduced following the formulas given by E. Lipparini and S. Stringari [Phys. Rep. 175, 103 (1989)]. Employing a simplified, yet realistic functional they arrive at

$$E_{-1} \equiv \sqrt{\frac{m_1}{m_{-1}}} = \sqrt{\frac{3\hbar^2}{m\langle r^2\rangle}\frac{b_{\rm vol}}{\left[1+\frac{5}{3}\frac{b_{\rm surf}}{b_{\rm vol}}A^{-\frac{1}{3}}\right]}(1+\kappa)}.$$

Cf. also G.C., N. Van Giai, H. Sagawa, PLB 363 (1995)
5.

$$E = \int d_3r\ \mathcal{E}\left[\rho(\vec{r})\right] \qquad \boxed{\textbf{LDA}}$$

$$\int d_3r\ S(\rho)\frac{(\rho_n(\vec{r})-\rho_p(\vec{r}))^2}{\rho} \quad \leftrightarrow \quad b\frac{(N-Z)^2}{A}$$

If only volume, b is only b_{vol} and equals $S(\rho_0)$=J.

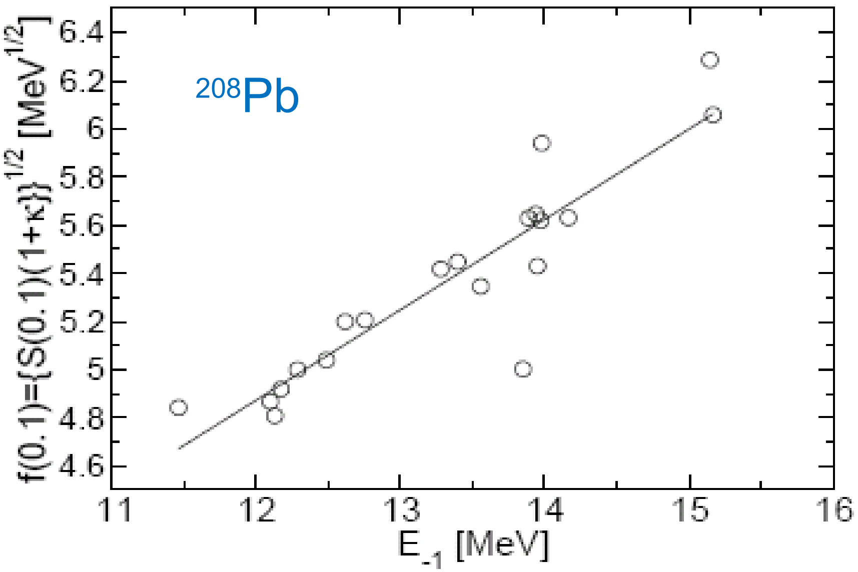
The surface correction is not strictly analytic but several results agree in stating that it produces b_{eff} = S(0.1 fm^{-3}) !

The Giant Dipole Resonance as a quantitative constraint on the symmetry energy

Luca Trippa, Gianluca Colò and Enrico Vigezzi

Phys. Rev. C77, 061304(R) (2008)

It is assumed that the previous formula holds for S at some sub-saturation density. The best value comes from χ^2_{min}.



23.3 < S(0.1) < 24.9 MeV

- x-axis: E_{GDR} from RPA;
- y-axis: $\left[S(\rho = 0.1\ \mathrm{fm}^{-3})(1+\kappa)\right]^{1/2}$; κ is the enhancement factor.

This result, namely 24.1 ± 0.8 MeV is based on an estimate of κ. Most of the error is coming from the uncertainty on this quantity.

PRL 102, 122502 (2009) PHYSICAL REVIEW LETTERS week ending 27 MARCH 2009

Nuclear Symmetry Energy Probed by Neutron Skin Thickness of Nuclei

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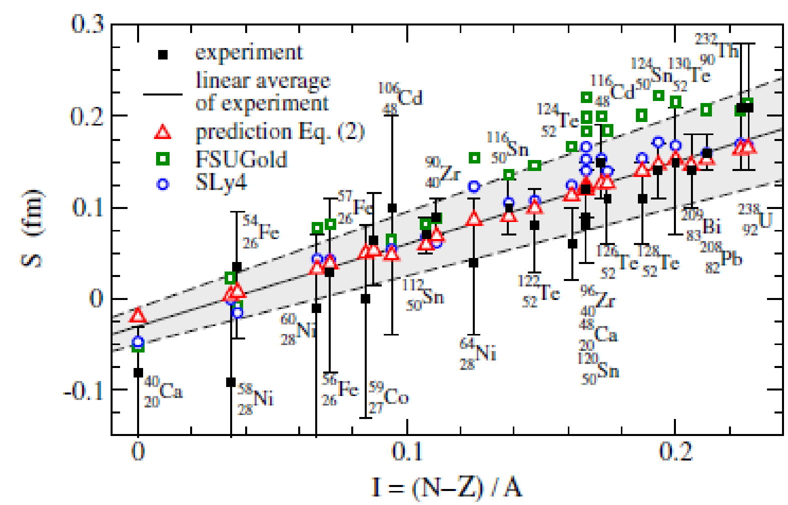
(Received 3 June 2008; revised manuscript received 1 February 2009; published 26 March 2009)

The authors show, independently (!), that

"b_{sym} in finite nuclei ≈ $S(\rho=0.1\ \mathrm{fm}^{-3})$"

and that in fact the value of the density is slightly decreasing in lighter systems.

They also use the liquid drop model to relate the neutron skin and b_{sym}, and on top of it the above relation in order to infer conclusions of the symmetry energy.



A third way to understand GDR $\leftrightarrow$ S[ρ]

If one builds dipole excitations with the Goldhaber-Teller model, by starting from

$$\rho(\vec{r}, R_i) = \frac{\rho_0 i}{1 + exp[(r - R_i)/a]}$$

and shift these densities by separating p and n,

$$\rho(\vec{r} + z\vec{e}_z, R_i) = \left(1 + z\frac{d}{dz} + \frac{1}{2}z^2\frac{d^2}{dz^2}\right)\rho(\vec{r}, R_i)$$

then by calculating the energy change we arrive at

$$\delta E = 2\pi \int dr\ r^2 \frac{2S(\rho)}{\rho}\left(\rho_n - \rho_p\right)\left(\delta\rho_n - \delta\rho_p\right)$$

This is an effective average of S which is peaked around 0.1 fm^{-3}. (Once more !)

We would need (yet miss) a similar understanding for the PDR !

PHYSICAL REVIEW C 76, 051603(R) (2007)

Nuclear symmetry energy and neutron skins derived from pygmy dipole resonances

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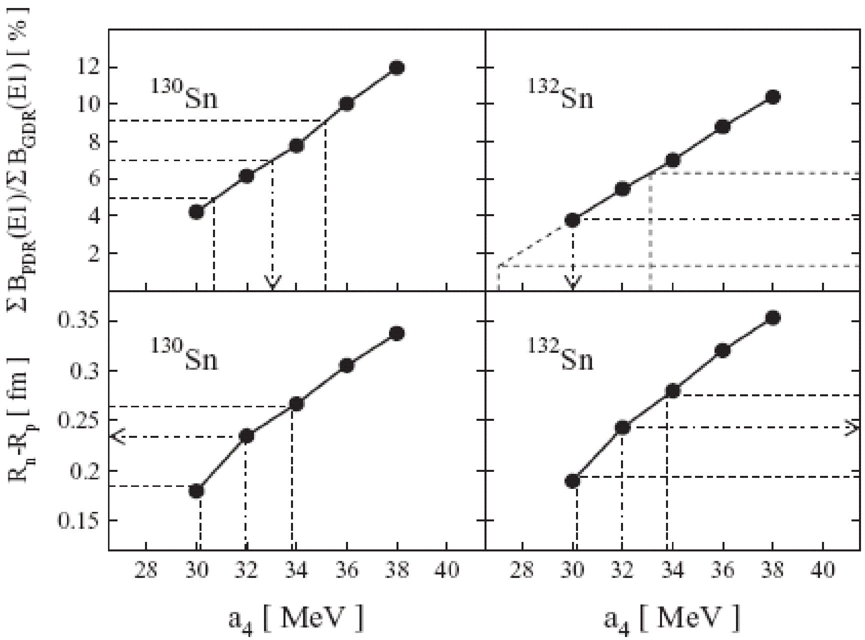
(LAND Collaboration)

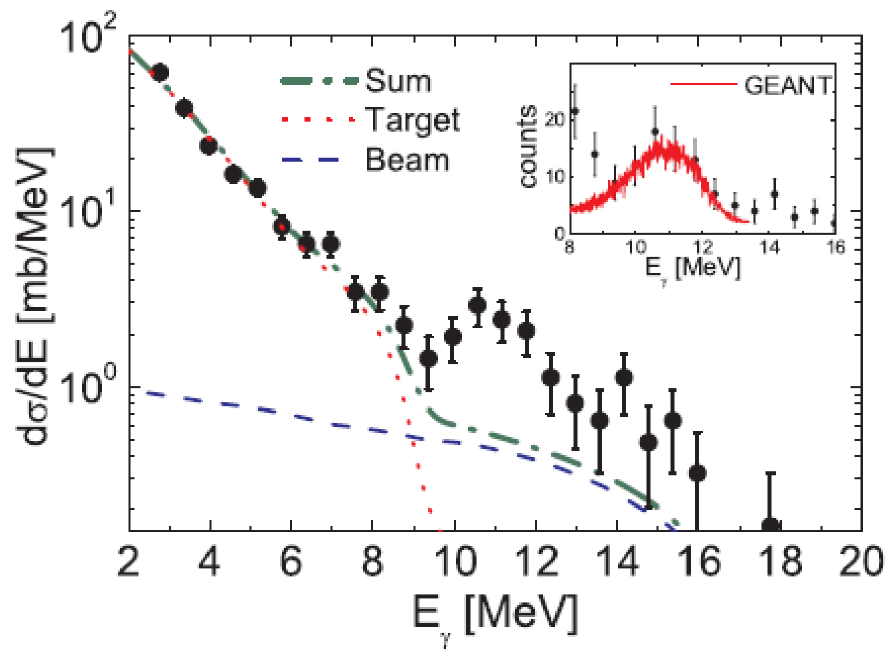


- Few interactions (all belonging to the same “class”) have been used to check correlations.
- Pairing in ^{130}Sn ?
- Other pygmy dipole states in different mass regions should be looked at.
- Is this really a “resonance” ? Need of exclusive experiments (γ-decay).

Recently, a Coulomb excitation measurement has been carried out by the experimental group of Milano U.: ^{68}Ni at 600 MeV/A on a Au target. Low-lying (or “pygmy”) dipole strength has been found around 11 MeV. O.Wieland *et al.*, PRL 102, 092502 (2009)

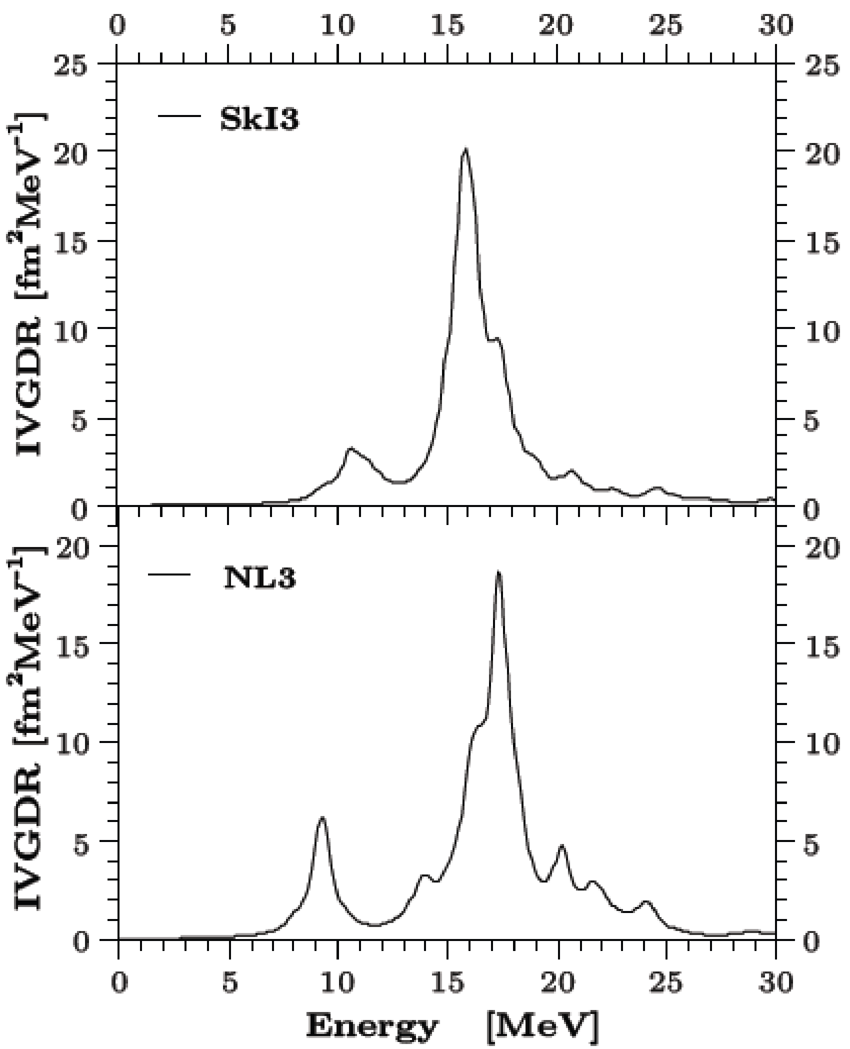
RISING

The Gamma ray spectrum shows an excess with respect to statistical emission



Dipole strength in ^{68}Ni

- Theoretical calculations show also a well-defined PDR !
- Several configurations that originate from neutron hole states $f_{5/2}$, $p_{1/2}$ and $p_{3/2}$ contribute.
- Interactions characterized by larger values of the symmetry energy seem to produce a state that is (a) more collective and (b) more decoupled from the GDR.



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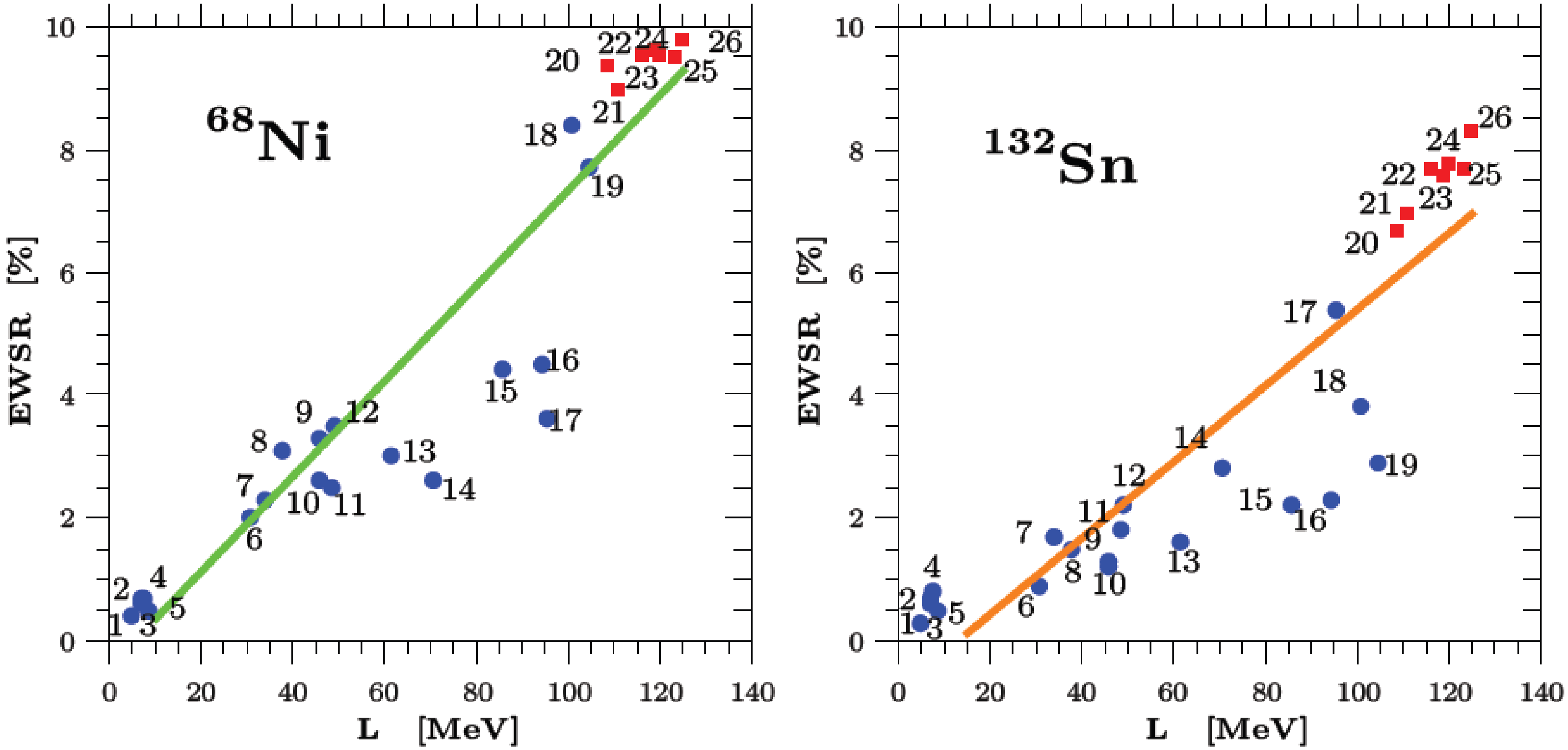
Correlation between L and the PDR



For the first time the approach has been pursued with different nuclei and different classes of EDFs. Blue=Skyrme; red=RMF.

Using experimental data → L = 65.1 ± 15.5 MeV

Skyrme forces:

1=v0902, 2=MSk3, 3=BSk1, 4=v110, 5=v100, 6=Tond6, 7=Tond9, 8=SGII, 9=SkM*, 10=SLy4, 11=SLy5, 12=SLy230a, 13=LNS, 14=SkMP, 15=SkRs, 16=SkGs, 17=SK255, 18=SkI3, 19=SkI2

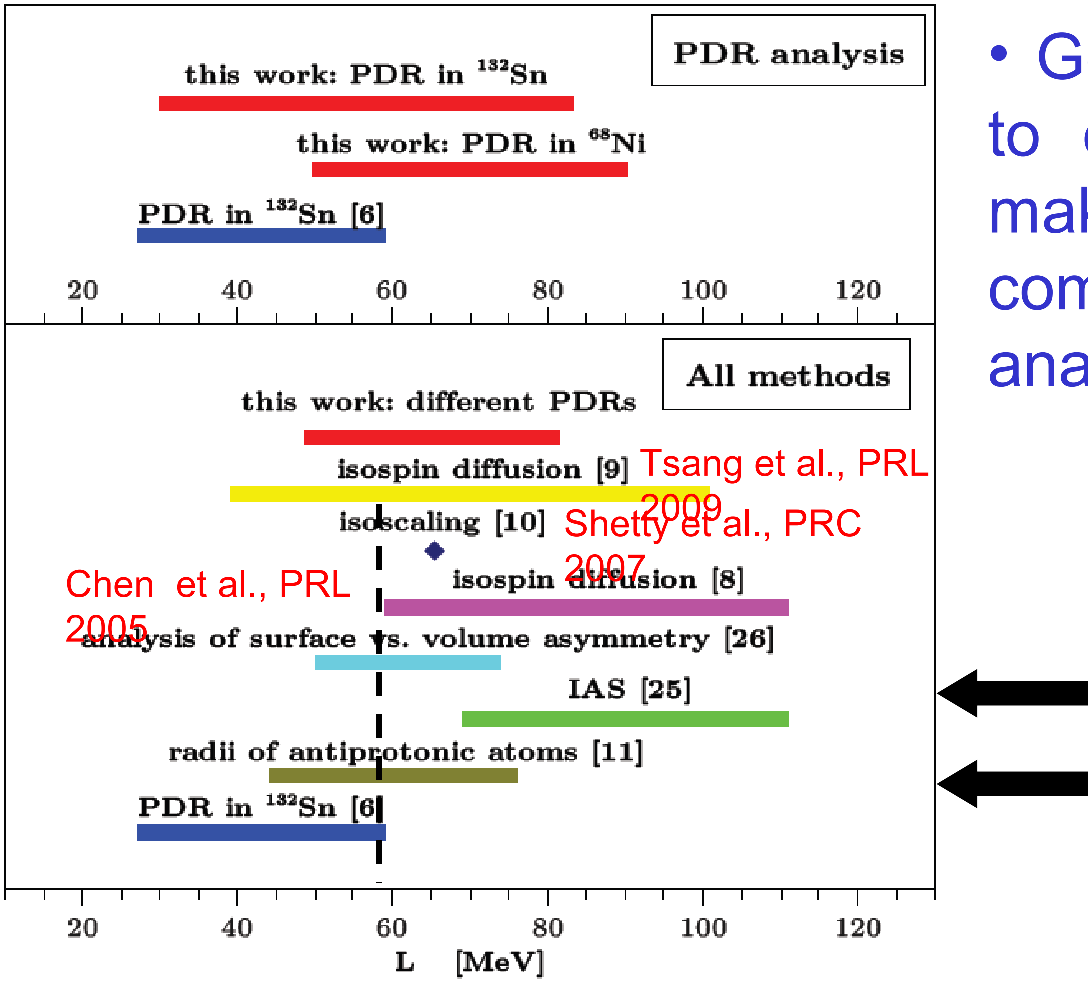
RMF (meson exchange) Lagrangians:

20=NLC, 21=TM1, 22=PK1, 23=NL3, 24=NLBA 25=NL3+ 26=NLE.

PHYSICAL REVIEW C 81, 041301(R) (2010)

Constraints on the symmetry energy and neutron skins from pygmy resonances in ^{68}Ni and ^{132}Sn

Andrea Carbone,[1] Gianluca Colò,[1,2] Angela Bracco,[1,2] Li-Gang Cao,[1,2,3,4] Pier Francesco Bortignon,[1,2]
Franco Camera,[1,2] and Oliver Wieland[2]

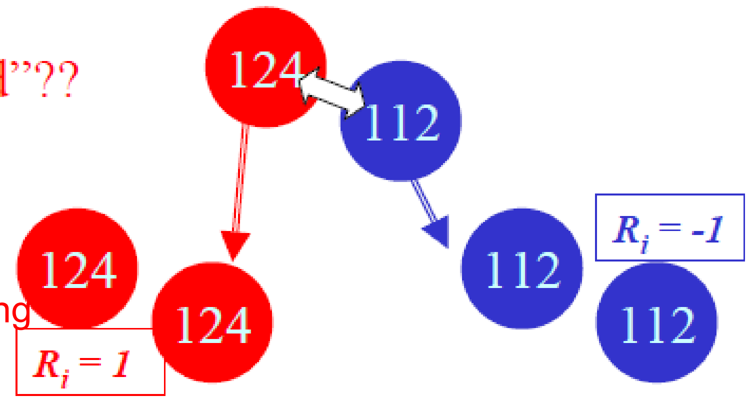


- Generalizing the approach to extract L from the PDR makes its value (more) compatible with those from analysis of HI collisions.

$$R_i = 2\,\frac{x_{AB} - (x_{AA} + x_{BB})/2}{x_{AA} - x_{BB}}$$

x_{AB}, y_{AB} experimental or theoretical observable for AB

Non-isospin transport effects are "cancelled"??



Courtesy: B. Tsang

If x is the asymmetry δ, we speak of "isospin diffusion" and a stiff symmetry energy causes small diffusion (too much energy cost !), whereas a soft symmetry energy causes a rather good degree of equilibration.

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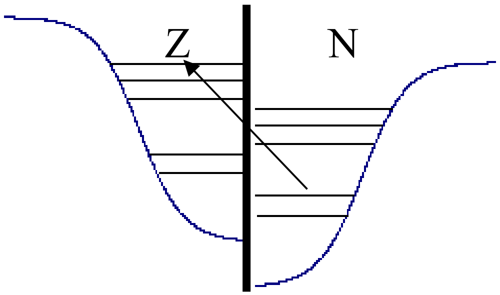
Time evolution of the one-body distribution function $f(r,p,t)$

$$\frac{\partial}{\partial t} f(r,p) - \underbrace{\{h(f), f(r,p)\}}_{\text{Vlasov}} = \underbrace{K(f)}_{\text{Boltzmann}} + \underbrace{\delta K(r,p,t)}_{\text{Langevin}}$$

Symmetry energy from charge-exchange states



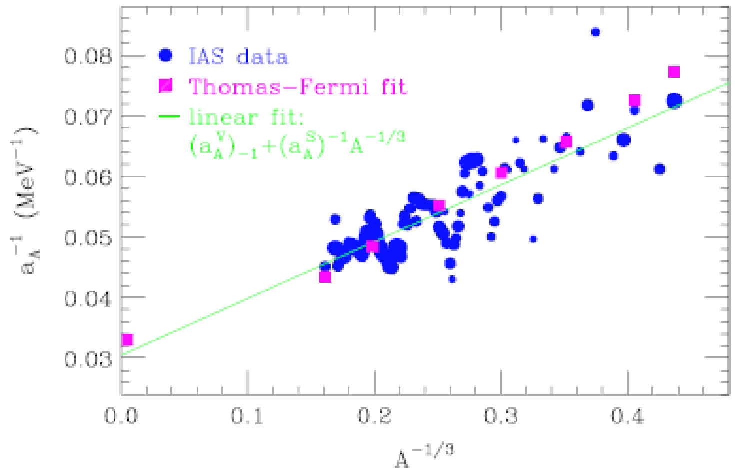
- Quite natural application of isospin symmetry
- Which states are collective and well-known ?

1 → IAS

P. Danielewicz and J. Lee, Nucl. Phys. A **818**, 36 (2009).
P. Danielewicz, Nucl. Phys. A **727**, 233 (2003).

With respect to mother nucleus: only Coulomb energy. In daughter nucleus: generalization of mass formula

$$E_A = a_A(A)\,\frac{(N-Z)^2}{A} = 4\,a_A(A)\,\frac{T_z^2}{A}$$
$$\rightarrow 4\,a_A(A)\,\frac{T^2}{A} = 4\,a_A(A)\,\frac{T(T+1)}{A}$$



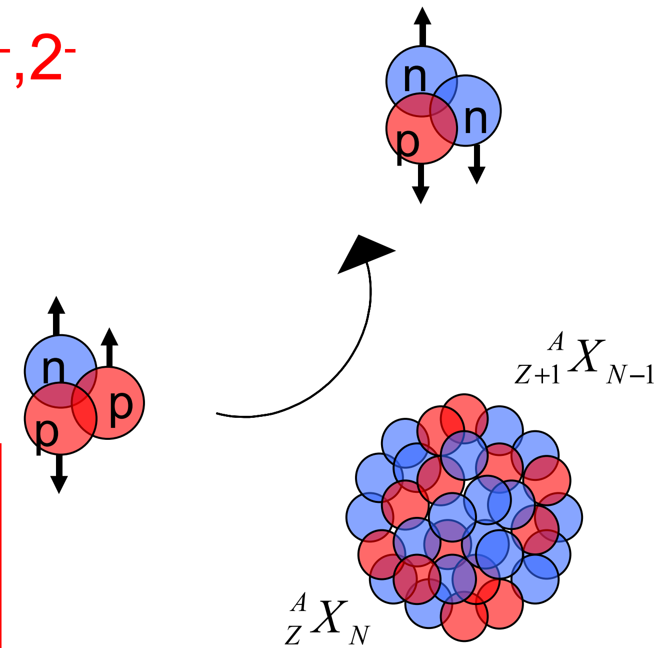
Macroscopic formulae ↔ shell effects

2 → Spin dipole states : L=1 S=1 $J^{\pi}=0^{-},1^{-},2^{-}$

$$\hat{S}_{\pm} = \sum_{im\mu} t^i_{\pm}\sigma^i_m r_i Y^{\mu}_1(\hat{r}_i),$$

$$S_- - S_+ = \sum_{\lambda}(S^{\lambda}_- - S^{\lambda}_+) = \frac{9}{4\pi}(N\langle r^2\rangle_n - Z\langle r^2\rangle_p).$$

- The measurement of the (charge-exchange) spin-dipole sum rules allows also the determination of ΔR.



PHYSICAL REVIEW C 76, 024301 (2007)

Charge exchange spin-dipole excitations in ^{90}Zr and ^{208}Pb and the neutron matter equation of state

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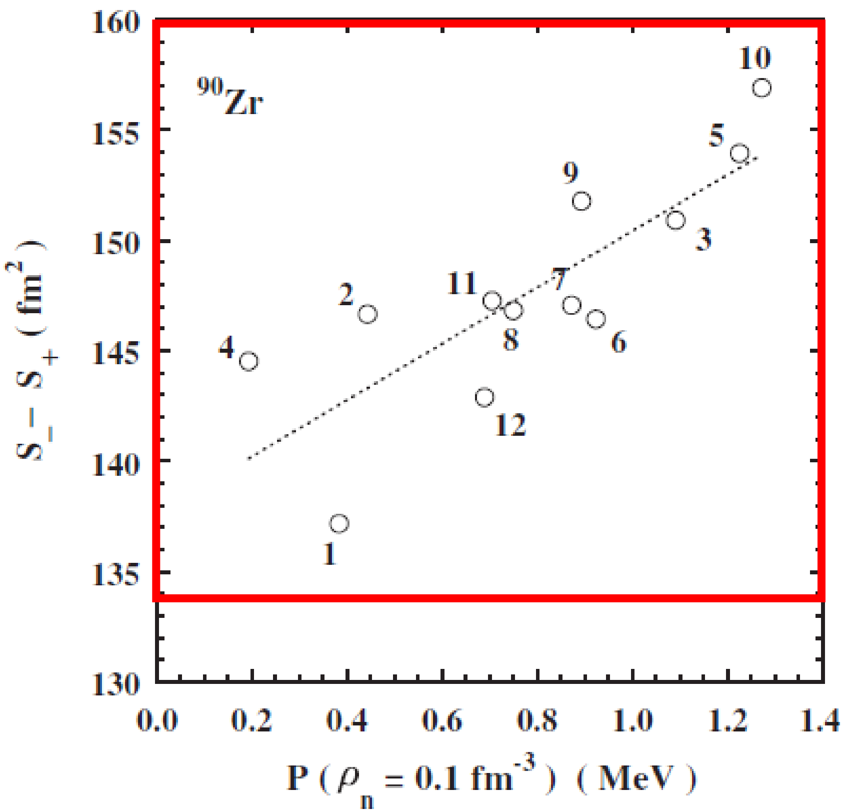
(Received 5 February 2007; published 1 August 2007)

Exp.: $S_{-} - S_{+}$ = 147±13 fm^2.

Allowed band...



Reduction of uncertainties related to this specific type of reaction welcome (optical potentials, relationship between cross section and dipole strength).

PHYSICAL REVIEW C **80**, 045806 (2009)

Density dependence of the nuclear symmetry energy: A microscopic perspective

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We perform a systematic analysis of the density dependence of nuclear symmetry energy within the microscopic Brueckner-Hartree-Fock (BHF) approach using the realistic Argonne V18 nucleon-nucleon potential plus a phenomenological three-body force of Urbana type. Our results are compared thoroughly with those arising from several Skyrme and relativistic effective models. The values of the parameters characterizing the BHF equation of state of isospin asymmetric nuclear matter fall within the trends predicted by those models and are compatible with recent constraints coming from heavy ion collisions, giant monopole resonances, or isobaric analog states. In particular we find a value of the slope parameter $L = 66.5$ MeV, compatible with recent experimental constraints from isospin diffusion, $L = 88 \pm 25$ MeV. The correlation between the neutron skin thickness of neutron-rich isotopes and the slope L and curvature K_{sym} parameters of the symmetry energy is studied. Our BHF results are in very good agreement with the correlations already predicted by other authors using nonrelativistic and relativistic effective models. The correlations of these two parameters and the neutron skin thickness with the transition density from nonuniform to β-stable matter in neutron stars are also analyzed. Our results confirm that there is an inverse correlation between the neutron skin thickness and the transition density.

Extraction of the neutron radii from L

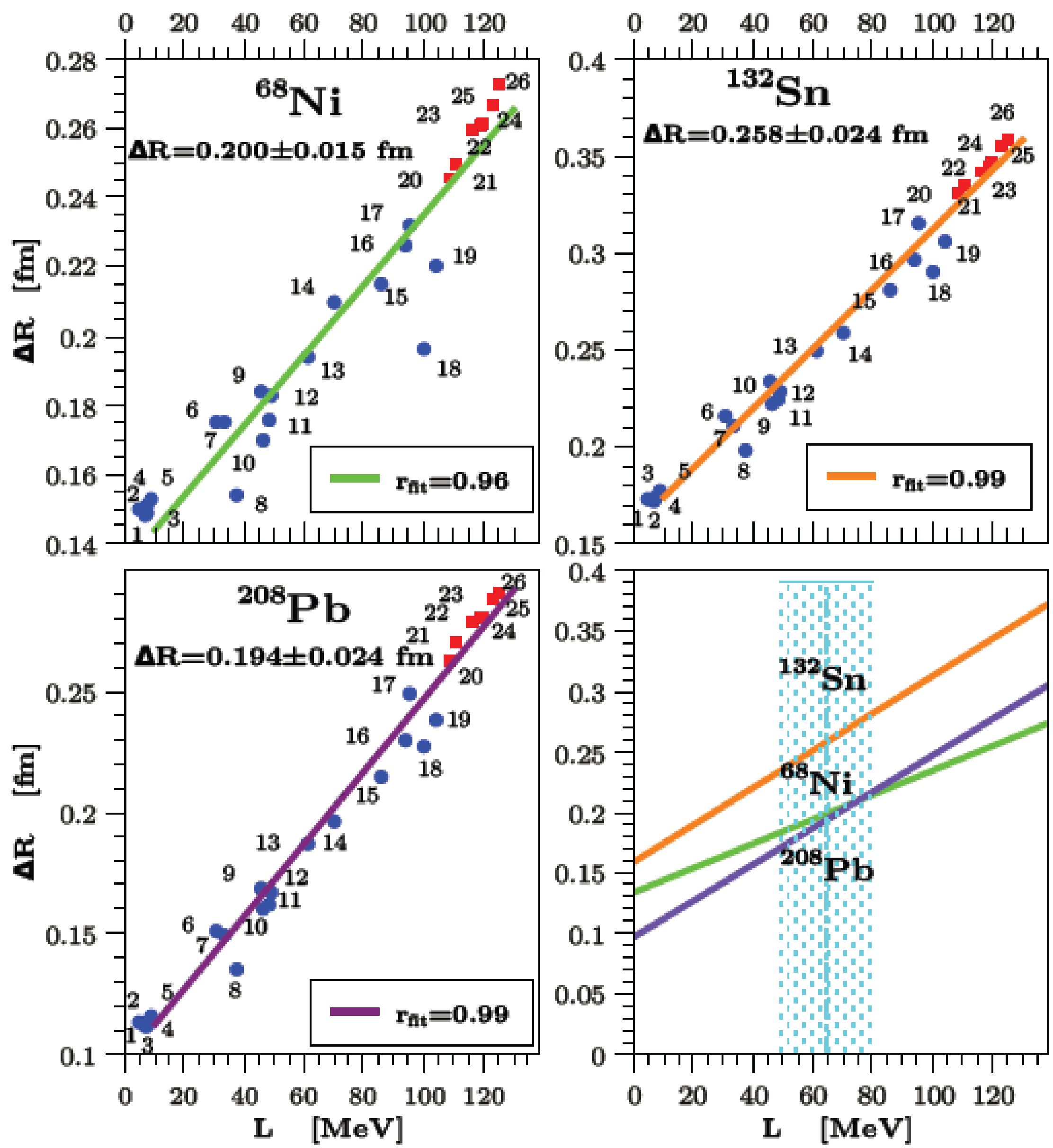
Strong correlations between L and ΔR (the neutron skin thickness) have been noticed previously.

B.A. Brown, PRL 85, 5296 (2000); S. Typel and B.A. Brown, PRC 64, 027302(R) (2001).
R.J. Furnstahl, NPA 706, 85 (2002);
S. Yoshida and H. Sagawa, PRC 69, 024318 (2004).

By using our range for L, we find ΔR with its error.

^{68}Ni: 0.200 ± 0.015 fm

^{132}Sn: 0.258 ± 0.024 fm



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Conclusions

- Mean field models allow correlations between nuclear structure (ground-state properties, giant resonances etc.) and the nuclear EOS – and consequently, with nuclear astrophysics.

- Within these models one can find a correlation between the PDR strength and one of the important parameters governing the density dependence of the symmetry energy, namely the slope at ρ_0.

- In this way one can extract a constraint on this slope. Many groups now converge to values around 60 MeV for L.

- Open question: beyond mean field ?