```
Quiet[Remove["Global`*"], {Remove::rmnsm}];
Print["Mathematica $Version = “", $Version, "”"];
Print["Execution time = ", DateString[DateList[], {"Hour", ":", "Minute",
   " on ", "DayNameShort", " ", "Day", " ", "MonthNameShort", " ", "Year"}]];
```

```
Mathematica $Version = “9.0 for Mac OS X x86 (64-bit) (January 24, 2013)”

Execution time = 21:40 on Sun 03 Jan 2016
```

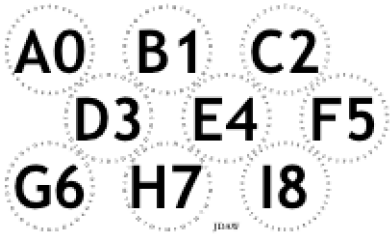
```
(*
    http://www.jdawiseman.com/papers/placemat/placemat.html#PermittedPackingStyles
    http://www.jdawiseman.com/papers/placemat/PackingStyles_formulae.nb
    http://www.jdawiseman.com/papers/placemat/PackingStyles_formulae.pdf
*)

(*
    Two stage process.

    Stage one, calculate maximum possible r. Generally this is done by having
 adjacent circles touch, by solving polynomial(s) of the form something == 4 r^2.
    But other constraints about non-
  adjacent circles not overlapping are also needed. E.g., in PostsAndLintel,
also r ≤ w/4, otherwise in all narrow pages the first and last circles
 might overlap even though ‘adjacent’ circles only touch.
    Typically this stage entails creating and solving a polynomial in r,
with coefficients involving the likes of h, w, nR, nC, etc.

    Stage 2, r is further lowered by the likes of MaxRadius (a hard maximum)
 or ShrinkRadii (pages with different layouts to have same radius).
    Then, using this smaller r, circles are positioned.
    The broad principle is that the gap all
 pairs of ‘adjacent’ circles are the same distance apart.
*)

(* r = radius; h = usable Height; w = usable Width; nC= numCols;
nSC = numShortColumns; nR= numRows; nG = numGlasses (or numGlassesAtEdge);
gC = gap between Cols; gR = gap between Rows; gRD = gap between rows at dislocation;
gCD = gap between columns in Diamond pattern *)
$Assumptions = Apply[And, Map[# ∈ Reals && # > 0 &, {w, h, r, nR, nC, nSC, nG, gC,
      gR, gRD, gCD, x, y, x0, y0, x1, y2, x2, y2, x3, y3, x4, y4, x5, y5, x6, y6}]];

With[{mgn = 24., header = 0.},
  (* For analytic solutions set mgn and header to exact value *)

    {PortraitA4, LandscapeA4, LandscapeA3, LandscapeLegal, LandscapeLedger,
     LandscapePoint6} =
    {{w → 210 × 360 / 127 - 2 mgn, h → 297 × 360 / 127 - 2 mgn - header},
     {w → 297 × 360 / 127 - 2 mgn, h → 210 × 360 / 127 - 2 mgn - header},
     {w → 420 × 360 / 127 - 2 mgn, h → 297 × 360 / 127 - 2 mgn - header},
     {w → 14 × 72 - 2 mgn, h → (8 + 1 / 2) 72 - 2 mgn - header},
     {w → 17 × 72 - 2 mgn, h → 11 × 72 - 2 mgn - header},
     {w → 1000., s → 600.}};

];

(* PostScriptForm[] *)
(*
    http://mathematica.stackexchange.com/questions/101954/postscriptform-or-forthform
    http://mathematica.stackexchange.com/questions/102894/multi-
  case-function-many-single-case-delayed-assignments-or-one-which
*)
Remove[PostScriptForm];
PostScriptForm[thing_Rational] := ToString[N[thing, 20], InputForm, NumberMarks → False];
PostScriptForm[thing_?AtomQ] := ToString[thing];
PostScriptForm[thing_List] := StringJoin @@ Riffle[Map[PostScriptForm, thing], "\r\n"];
PostScriptForm[MatrixForm[thing_]] := PostScriptForm[thing];
PostScriptForm[Times[-1, thing_]] := StringJoin[PostScriptForm[thing], " neg"];

PostScriptForm[thing_Power] := (
    psExponent := Which[
```

```
        # > 5 && Divisible[#, 3], psExponent[# / 3] <> " dup dup mul mul",
        # ≥ 5 && OddQ[#], "dup " <> psExponent[(# - 1) / 2] <> " dup mul mul",
        # ≥ 4 && EvenQ[#], psExponent[# / 2] <> " dup mul",
        # == 3, "dup dup mul mul",
        # == 2, "dup mul",
        # == 1 / 2, "sqrt",
        (Rational === Head[#]) && (Log[2, # // Denominator] // IntegerQ),
      psExponent[Simplify[2 #]] <> " sqrt",
        Not[IntegerQ[#]], PostScriptForm[#] <> " exp",
        # == 1, "",
        True, " !•Error•! "
    ] &;
    Which[
        thing[[2]] > 0 || Not[IntegerQ[thing[[2]]],
     PostScriptForm[thing[[1]] <> " " <> psExponent[thing[[2]],
        thing[[2]] == -1, "1 " <> PostScriptForm[thing[[1]] <> " div",
        thing[[2]] == 0, "1",
        True, "1 " <> PostScriptForm[thing[[1]] <> " " <> psExponent[-thing[[2]] <> " div"
    ]);

PostScriptForm[thing_Times] :=
  StringJoin@Riffle[Reap[If[MatchQ[thing[[1]], Power[_, n_Integer /; n < 0]],
       (Sow["1 " <> PostScriptForm[thing[[1, 1]] <> " div"];),
       (Sow[PostScriptForm[thing[[1]]];)]; Map[(If[MatchQ[#, Power[_, n_Integer /; n < 0]],
          (Sow[PostScriptForm[#[[1]]^(-#[[2]])] <> " div"];),
          (Sow[PostScriptForm[#] <> " mul"]; )]) &, Drop[List @@ thing, 1]]][[2, 1]], " "];

PostScriptForm[thing_Plus] :=
StringJoin @@ If[FreeQ[thing, _^n_],
    (* Simple expression, no powers, to be summed one item at a time *)
    Module[{i},
        i = Position[thing, Except[Times[-1, _] | (_?Negative)], 1, Heads → False];
        If[Length[i] > 0, i = i[[1, 1]],
      (i = Position[thing, Not[MatchQ[#, Times[-1, _]]] &, 1, Heads → False];
       i = If[Length[i] > 0, i[[1, 1]], 1])];
     Prepend[Map[(" " <> Replace[#, {(n_Integer /; n < 0 :> ToString[-n] <> " sub"),
            (Times[-1, _] :> PostScriptForm[Times @@ Drop[#, 1]] <> " sub"),
            (Times[n_ /; n < 0, _] :> PostScriptForm[Times @@ Drop[#, 1]] <>
               " " <> ToString[-#[[1]]] <> " mul sub"), (Times[n_ /; n > 0, _] :>
              PostScriptForm[Times @@ Drop[#, 1]] <> " " <> ToString[#[[1]]] <> " mul add"),
            (_ :> PostScriptForm[#] <> " add")}]) &, Drop[List @@ thing, {i}]],
      Replace[thing[[i]], {Times[-1, _] :> PostScriptForm[-thing[[i]]] <> " neg",
        _ :> PostScriptForm[thing[[i]]]}] ],
    (* Polynomial *)
    Module[{vars, exps, v, rcl, i, firstMul},
        vars = Variables[thing];
        exps = Exponent[thing, vars];
        v = Select[Transpose[{vars, exps}], (#[[2]] == Max @@ exps) &][[1, 1]];
        rcl = Reverse[Map[Factor, CoefficientList[thing, v]]];
        Reap[
            i = 1; firstMul = True; If[rcl[[1]] =!= 1, Sow[PostScriptForm[rcl[[1]]]];
Map[If[# === 0, i++,
           (Sow[If[firstMul && rcl[[1]] === 1, PostScriptForm[v^i] <> " ", " " <> PostScriptForm[
                 v^i] <> " mul "] <> If[MatchQ[#, (Times[_?Negative, _] | (_?Negative))],
              PostScriptForm[-#] <> " sub", PostScriptForm[#] <> " add"]];
            i = 1; firstMul = False)] &, Drop[rcl, 1]];
            If[i > 1, Sow[" " <> PostScriptForm[v^(i - 1)] <> " mul "]];
        ][[2, 1]]
]];
```

# Diamonds

Diamonds

```
Print[ A0 B1 C2
         D3 E4 F5
       G6 H7 I8 ];
```




```
(* http://
 www.jdawiseman.com/papers/placemat/placemat.html#PermittedPackingStyles_Diamonds *)

(* Also r bounded above by h/(nR+1) and by w/(nC+1) *)

Module[{poly},
    poly = (((w - 2 r)/(nC - 1))^2 + ((h - 2 r)/(nR - 1))^2 - 4 r^2) // Together // Numerator // Factor;
    Print[poly];
    Print[Map[Factor,
      CoefficientList[If[SameQ[Head[poly], Times], poly[[-1]], poly], r]] // MatrixForm];
    Print[Min[Select[r /. Solve[0 == poly /. LandscapeLegal /. {nR → 3, nC → 6}, {r}],
      # ∈ Reals && # > 0 &]]];
];
```

$$h^2 - 2\,h^2\,nC + h^2\,nC^2 - 4\,h\,r + 8\,h\,nC\,r - 4\,h\,nC^2\,r + 4\,r^2 - 16\,nC\,nR\,r^2 + 8\,nC^2\,nR\,r^2 + 8\,nC\,nR^2\,r^2 - 4\,nC^2\,nR^2\,r^2 - 4\,r\,w + 8\,nR\,r\,w - 4\,nR^2\,r\,w + w^2 - 2\,nR\,w^2 + nR^2\,w^2$$

$$\begin{pmatrix} h^2 - 2\,h^2\,nC + h^2\,nC^2 + w^2 - 2\,nR\,w^2 + nR^2\,w^2 \\ -4\,(h - 2\,h\,nC + h\,nC^2 + w - 2\,nR\,w + nR^2\,w) \\ -4\,(-1 + 4\,nC\,nR - 2\,nC^2\,nR - 2\,nC\,nR^2 + nC^2\,nR^2) \end{pmatrix}$$

112.289

```
With[{params = {gC, gR}}, Do[(
    Module[{poly},
    poly = Factor[GroebnerBasis[{
         h == 2 r + (nR - 1) gR,
         w == 2 r + (nC - 1) gC}, params[[i]], Drop[params, {i}]][[1]];
    Print[params[[i]], ":\t", Map[Factor, CoefficientList[
          If[SameQ[Head[poly], Times], poly[[-1]], poly], params[[i]]]] // MatrixForm];
    Print[Solve[0 == poly /. LandscapeLegal /. {nR → 3, nC → 6, r → 110}, params[[i]]]
];), {i, Length[params]}]]
```

gC: $\begin{pmatrix} 2\,r - w \\ -1 + nC \end{pmatrix}$

{{gC → 148.}}

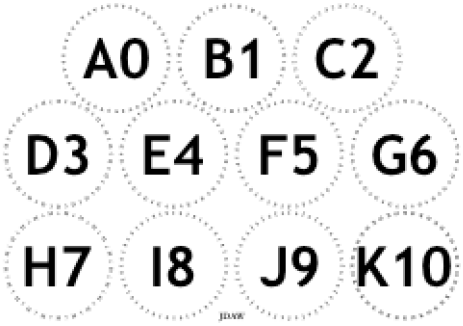
gR: $\begin{pmatrix} -h + 2\,r \\ -1 + nR \end{pmatrix}$

{{gR → 172.}}

# RectangularDislocation

RectangularDislocation

```
Print[ A0 B1 C2
       D3 E4 F5 G6
       H7 I8 J9 K10 ];
```




```
(* http://www.jdawiseman.com/papers/placemat/placemat.html
  #PermittedPackingStyles_RectangularDislocation *)

Module[{poly},
    poly = Factor[GroebnerBasis[{
        h == (2 nR - 2) r + gR,
        w == 2 r + 2 (nC - 1) gC,
        gR^2 + gC^2 == 4 r^2}, {r}, {gR, gC}][[1]]];
    Print[poly];
    Print[Map[Factor,
      CoefficientList[If[SameQ[Head[poly], Times], poly[[-1]], poly], r]] // MatrixForm];
    Print[Min[Select[r /. Solve[0 == poly /. LandscapeA3 /. {nR → 3, nC → 4}, {r}],
      # ∈ Reals && # > 0 &]]];
];
```

$$4\,h^2 - 8\,h^2\,nC + 4\,h^2\,nC^2 + 16\,h\,r - 32\,h\,nC\,r + 16\,h\,nC^2\,r - 16\,h\,nR\,r + 32\,h\,nC\,nR\,r - 16\,h\,nC^2\,nR\,r + 4\,r^2 - 32\,nR\,r^2 + 64\,nC\,nR\,r^2 - 32\,nC^2\,nR\,r^2 + 16\,nR^2\,r^2 - 32\,nC\,nR^2\,r^2 + 16\,nC^2\,nR^2\,r^2 - 4\,r\,w + w^2$$

$$\begin{pmatrix} 4\,h^2 - 8\,h^2\,nC + 4\,h^2\,nC^2 + w^2 \\ -4\,(-4\,h + 8\,h\,nC - 4\,h\,nC^2 + 4\,h\,nR - 8\,h\,nC\,nR + 4\,h\,nC^2\,nR + w) \\ 4\,(1 - 8\,nR + 16\,nC\,nR - 8\,nC^2\,nR + 4\,nR^2 - 8\,nC\,nR^2 + 4\,nC^2\,nR^2) \end{pmatrix}$$

139.023

```
(* But r might be shrunk by ShrinkRadii or by MaxRadius. So need to solve
 separate equations for the positions (gC,gR,gRD) which will depend on r. *)

With[{params = {gC, gR, gRD}}, Do[(
    Module[{poly},
    poly = Factor[GroebnerBasis[{
        h == 2 r + (nR - 2) gR + gRD,
        w == 2 r + 2 (nC - 1) gC,
        gRD^2 + gC^2 == gR^2}, params[[i]], Drop[params, {i}]][[1]]];
    Print[params[[i]], ":\t", Map[Factor, CoefficientList[
          If[SameQ[Head[poly], Times], poly[[-1]], poly], params[[i]]]] // MatrixForm];
    Print[Solve[0 == poly /. LandscapeA3 /. {nR → 3, nC → 4, r → 135}, params[[i]]]]
];), {i, Length[params]}]]
```

$$\text{gC:} \quad \begin{pmatrix} 2\,r - w \\ 2\,(-1 + nC) \end{pmatrix}$$

$$\{\{gC \to 145.425\}\}$$

$$\text{gR:} \quad \begin{pmatrix} 4\,h^2 - 8\,h^2\,nC + 4\,h^2\,nC^2 - 16\,h\,r + 32\,h\,nC\,r - 16\,h\,nC^2\,r + 20\,r^2 - 32\,nC\,r^2 + 16\,nC^2\,r^2 - 4\,r\,w + w^2 \\ -8\,(-1 + nC)^2\,(-2 + nR)\,(h - 2\,r) \\ 4\,(-1 + nC)^2\,(-3 + nR)\,(-1 + nR) \end{pmatrix}$$

$$\{\{gR \to 282.129\}\}$$

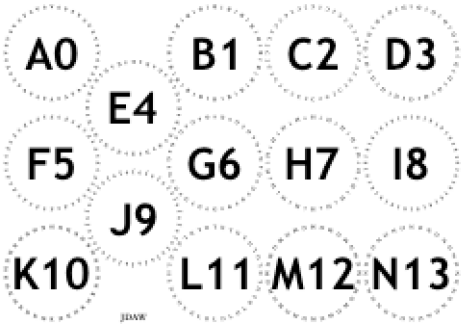
$$\text{gRD:} \quad \begin{pmatrix} -(-2\,h + 2\,h\,nC - 4\,nC\,r + 2\,nR\,r + 2\,w - nR\,w)\,(-2\,h + 2\,h\,nC + 8\,r - 4\,nC\,r - 2\,nR\,r - 2\,w + nR\,w) \\ 8\,(-1 + nC)^2\,(h - 2\,r) \\ 4\,(-1 + nC)^2\,(-3 + nR)\,(-1 + nR) \end{pmatrix}$$

$$\{\{gRD \to 241.761\}\}$$

# DiamondsAndRectangular

DiamondsAndRectangular

```
Print[ A0 B1 C2 D3
         E4
       F5 G6 H7 I8
         J9
       K10 L11 M12 N13 ];
```




```
(* http://www.jdawiseman.com/papers/placemat/placemat.html
  #PermittedPackingStyles_DiamondsAndRectangular *)

(* Also r bounded above by h/(nR+1) and by w/(nC-nSC)/2 *)

(* It is obvious that this cannot be quite the maximum
 r. Imagine moving C2 and H7 vertically towards each other,
and nudging M12 up by the same distance. Then there would be gaps
 either side of that column, which could be filled by increasing
 r. But only by a mite. And that equation is not solved here. *)

Module[{poly},
    poly = Factor[GroebnerBasis[{
        (* Equations need doing *)
        h == 2 r + gR (nR - 1),
        w == 2 gC nSC + 2 r (nC - 2 nSC),
        gR^2 + gC^2 == 4 r^2}, {r}, {gR, gC}][[1]]];
    Print[poly];
    Print[Map[Factor,
      CoefficientList[If[SameQ[Head[poly], Times], poly[[-1]], poly], r]] // MatrixForm];
    Print[Min[Select[r /. Solve[0 == poly /. LandscapeA3 /. {nR → 5, nC → 5, nSC → 1}, {r}],
      # ∈ Reals && # > 0 &]]];
];
```

$$-4\,h^2\,nSC^2+16\,h\,nSC^2\,r-4\,nC^2\,r^2+8\,nC^2\,nR\,r^2-4\,nC^2\,nR^2\,r^2+16\,nC\,nSC\,r^2-32\,nC\,nR\,nSC\,r^2+16\,nC\,nR^2\,nSC\,r^2-16\,nSC^2\,r^2+4\,nC\,r\,w-8\,nC\,nR\,r\,w+4\,nC\,nR^2\,r\,w-8\,nSC\,r\,w+16\,nR\,nSC\,r\,w-8\,nR^2\,nSC\,r\,w-w^2+2\,nR\,w^2-nR^2\,w^2$$

$$\begin{pmatrix} -4\,h^2\,nSC^2-w^2+2\,nR\,w^2-nR^2\,w^2 \\ 4\,(4\,h\,nSC^2+nC\,w-2\,nC\,nR\,w+nC\,nR^2\,w-2\,nSC\,w+4\,nR\,nSC\,w-2\,nR^2\,nSC\,w) \\ -4\,(nC^2-2\,nC^2\,nR+nC^2\,nR^2-4\,nC\,nSC+8\,nC\,nR\,nSC-4\,nC\,nR^2\,nSC+4\,nSC^2) \end{pmatrix}$$

122.641

```
With[{params = {gR, gC, gCD}}, Do[(
    Module[{poly},
    poly = Factor[GroebnerBasis[{
        h == 2 r + (nR - 1) gR,
        w == 2 r + 2 nSC gCD + (nC - 1 - 2 nSC) gC,
        gCD^2 + gR^2 == gC^2}, params[[i]], Drop[params, {i}]][[1]]];
    Print[params[[i]], ":\t", Map[Factor, CoefficientList[
          If[SameQ[Head[poly], Times], poly[[-1]], poly], params[[i]]]] // MatrixForm];
    Print[Solve[0 == poly /. LandscapeA3 /. {nR → 5, nC → 5, nSC → 1, r → 120}, params[[i]]]]
];), {i, Length[params]}]]
```

gR:

$$\begin{pmatrix} -h+2r \\ -1+nR \end{pmatrix}$$

$$\{\{gR \to 138.472\}\}$$

gC:

$$\begin{pmatrix} 4h^2 nSC^2 - 16h\,nSC^2 r + 4r^2 - 8\,nR\,r^2 + 4\,nR^2 r^2 + 16\,nSC^2 r^2 - 4rw + 8\,nR\,rw - 4\,nR^2 rw + w^2 - 2\,nR\,w^2 + nR^2 w^2 \\ 2(-1+nR)^2(-1+nC-2\,nSC)(2r-w) \\ (-1+nC)(-1+nR)^2(-1+nC-4\,nSC) \end{pmatrix}$$

$$\{\{gC \to 246.883\}\}$$

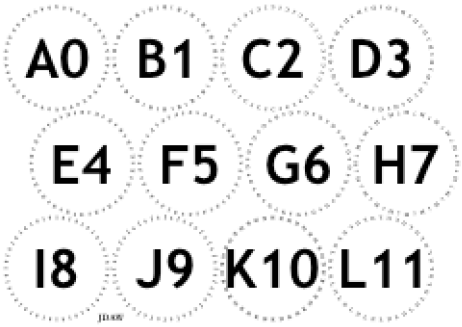
gCD:

$$\begin{pmatrix} (-h+h\,nC-2h\,nSC-2\,nC\,r+2\,nR\,r+4\,nSC\,r+w-nR\,w)(-h+h\,nC-2h\,nSC+4r-2\,nC\,r-2\,nR\,r+4\,nSC\,r- \\ -4(-1+nR)^2\,nSC(2r-w) \\ (-1+nC)(-1+nR)^2(-1+nC-4\,nSC) \end{pmatrix}$$

$$\{\{gCD \to 204.393\}\}$$

# RectangularAlternateNudge

```
Print[ A0 B1 C2 D3
       E4 F5 G6 H7
       I8 J9 K10 L11 ];
```




```
(* http://www.jdawiseman.com/papers/placemat/placemat.html
  #PermittedPackingStyles_RectangularAlternateNudge *)

(* Also r bounded above by h/(2+Sqrt[3] (nR-1)) and by w/(2 nC) *)

Module[{poly},
    poly = Factor[GroebnerBasis[{
        h == 2 r + gR (nR - 1),
        w == 2 nC r + gC,
        gR^2 + gC^2 == 4 r^2}, {r}, {gR, gC}][[1]]];
    Print[poly];
    Print[Map[Factor,
     CoefficientList[If[SameQ[Head[poly], Times], poly[[-1]], poly], r]] // MatrixForm];
    Print[Min[Select[r /. Solve[0 == poly /. LandscapeA3 /. {nR → 3, nC → 4}, {r}],
     # ∈ Reals && # > 0 &]]];
];
```

$$h^2 - 4\,h\,r + 4\,nC^2\,r^2 + 8\,nR\,r^2 - 8\,nC^2\,nR\,r^2 - 4\,nR^2\,r^2 + 4\,nC^2\,nR^2\,r^2 - 4\,nC\,r\,w + 8\,nC\,nR\,r\,w - 4\,nC\,nR^2\,r\,w + w^2 - 2\,nR\,w^2 + nR^2\,w^2$$

$$\begin{pmatrix} h^2 + w^2 - 2\,nR\,w^2 + nR^2\,w^2 \\ -4\,(h + nC\,w - 2\,nC\,nR\,w + nC\,nR^2\,w) \\ 4\,(nC^2 + 2\,nR - 2\,nC^2\,nR - nR^2 + nC^2\,nR^2) \end{pmatrix}$$

```
134.858
```

```
With[{params = {gR, gC}}, Do[(
    Module[{poly},
    poly = Factor[GroebnerBasis[{
        h == 2 r + gR (nR - 1),
        w == 2 nC r + gC}, params[[i]], Drop[params, {i}]][[1]]];
    Print[params[[i]], ":\t", Map[Factor, CoefficientList[
          If[SameQ[Head[poly], Times], poly[[-1]], poly], params[[i]]]] // MatrixForm];
    Print[Solve[0 == poly /. LandscapeA3 /. {nR → 3, nC → 4, r → 130}, params[[i]]]]
];), {i, Length[params]}]]
```

gR: $\begin{pmatrix} -h + 2\,r \\ -1 + nR \end{pmatrix}$

```
{{gR → 266.945}}
```

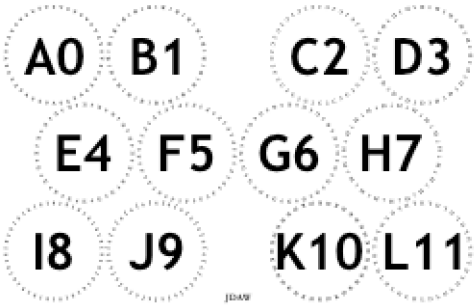
gC: $\begin{pmatrix} 2\,nC\,r - w \\ 1 \end{pmatrix}$

```
{{gC → 102.551}}
```

# RectangularAlternateSplitNudge

RectangularAlternateSplitNudge

```
Print[ A0 B1  C2 D3
        E4 F5 G6 H7
        I8 J9  K10 L11 ];
```




```
(* http://www.jdawiseman.com/papers/placemat/placemat.html
  #PermittedPackingStyles_RectangularAlternateSplitNudge *)

(* Also r bounded above by h/(2+Sqrt[3] (nR-1)) and by w/(2 nC) *)

Module[{poly},
    poly = Factor[GroebnerBasis[{
        h == 2 r + gR (nR - 1),
        w == 2 nC r + 2 gC,
        gR^2 + gC^2 == 4 r^2}, {r}, {gR, gC}][[1]]];
    Print[poly];
    Print[Map[Factor,
      CoefficientList[If[SameQ[Head[poly], Times], poly[[-1]], poly], r]] // MatrixForm];
    Print[Min[Select[r /. Solve[0 == poly /. LandscapeLedger /. {nR → 3, nC → 4}, {r}],
      # ∈ Reals && # > 0 &]]];
];
```

$$4\,h^2 - 16\,h\,r + 4\,nC^2\,r^2 + 32\,nR\,r^2 - 8\,nC^2\,nR\,r^2 - 16\,nR^2\,r^2 + 4\,nC^2\,nR^2\,r^2 - 4\,nC\,r\,w + 8\,nC\,nR\,r\,w - 4\,nC\,nR^2\,r\,w + w^2 - 2\,nR\,w^2 + nR^2\,w^2$$

$$\begin{pmatrix} 4\,h^2 + w^2 - 2\,nR\,w^2 + nR^2\,w^2 \\ -4\,(4\,h + nC\,w - 2\,nC\,nR\,w + nC\,nR^2\,w) \\ 4\,(nC^2 + 8\,nR - 2\,nC^2\,nR - 4\,nR^2 + nC^2\,nR^2) \end{pmatrix}$$

127.894

```
With[{params = {gR, gC}}, Do[(
    Module[{poly},
    poly = Factor[GroebnerBasis[{
        h == 2 r + gR (nR - 1),
        w == 2 nC r + 2 gC}, params[[i]], Drop[params, {i}]][[1]];
    Print[params[[i]], ":\t", Map[Factor, CoefficientList[
          If[SameQ[Head[poly], Times], poly[[-1]], poly], params[[i]]]] // MatrixForm];
    Print[Solve[0 == poly /. LandscapeA3 /. {nR → 3, nC → 4, r → 120}, params[[i]]]]
];), {i, Length[params]}]]
```

gR:
$$\begin{pmatrix} -h + 2\,r \\ -1 + nR \end{pmatrix}$$

{{gR → 276.945}}

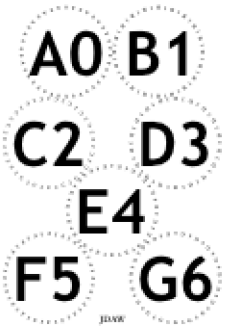
gC:
$$\begin{pmatrix} 2\,nC\,r - w \\ 2 \end{pmatrix}$$

{{gC → 91.2756}}

# DiamondsPlus

DiamondsPlus

```
Print[ A0 B1
       C2 D3
          E4
       F5 G6 ];
```




```
(* www.jdawiseman.com/papers/placemat/placemat.html
 #PermittedPackingStyles_DiamondsPlus *)

(* Also r bounded above by h/(nR+Sqrt[3]) and by w/4 *)

Module[{poly},
    poly = Factor[GroebnerBasis[{
        w == 2 r + 2 gC,
        h == 2 r + (nR - 2) gR + gRD,
        gR^2 + gC^2 == 4 r^2,
        gRD^2 + (w / 2 - 2 r)^2 == 4 r^2}, {r}, {gR, gC, gRD}][[1]]];
    Print[poly];
    Print[Map[Factor,
      CoefficientList[If[SameQ[Head[poly], Times], poly[[-1]], poly], r]] // MatrixForm];
    Print[Min[Select[r /. Solve[0 == poly /. PortraitA4 /. {nR → 4}, {r}],
      # ∈ Reals && # > 0 &]]];
];
```

$$16 h^4 - 128 h^3 r + 384 h^2 nR r^2 - 96 h^2 nR^2 r^2 + 1024 h r^3 - 1536 h nR r^3 + 384 h nR^2 r^3 + 1024 r^4 - 3072 nR r^4 + 3072 nR^2 r^4 - 1152 nR^3 r^4 + 144 nR^4 r^4 - 192 h^2 r w + 128 h^2 nR r w - 32 h^2 nR^2 r w + 768 h r^2 w - 512 h nR r^2 w + 128 h nR^2 r^2 w - 1792 nR r^3 w + 1984 nR^2 r^3 w - 768 nR^3 r^3 w + 96 nR^4 r^3 w + 40 h^2 w^2 - 32 h^2 nR w^2 + 8 h^2 nR^2 w^2 - 160 h r w^2 + 128 h nR r w^2 - 32 h nR^2 r w^2 - 64 r^2 w^2 + 288 nR r^2 w^2 - 200 nR^2 r^2 w^2 + 64 nR^3 r^2 w^2 - 8 nR^4 r^2 w^2 - 48 r w^3 + 160 nR r w^3 - 168 nR^2 r w^3 + 64 nR^3 r w^3 - 8 nR^4 r w^3 + 9 w^4 - 24 nR w^4 + 22 nR^2 w^4 - 8 nR^3 w^4 + nR^4 w^4$$

$$\begin{pmatrix} \left(4 h^2 + 9 w^2 - 6 nR w^2 + nR^2 w^2\right)\left(4 h^2 + w^2 - 2 nR w^2 + nR^2 w^2\right) \\ -8\left(16 h^3 + 24 h^2 w - 16 h^2 nR w + 4 h^2 nR^2 w + 20 h w^2 - 16 h nR w^2 + 4 h nR^2 w^2 + 6 w^3 - 20 nR w^3 + 21 nR^2 w^3 - 8 nR^3 w^3 + nR^4\right. \\ \left. -8\left(-48 h^2 nR + 12 h^2 nR^2 - 96 h w + 64 h nR w - 16 h nR^2 w + 8 w^2 - 36 nR w^2 + 25 nR^2 w^2 - 8 nR^3 w^2 + nR^4 w^2\right)\right. \\ 32\left(32 h - 48 h nR + 12 h nR^2 - 56 nR w + 62 nR^2 w - 24 nR^3 w + 3 nR^4 w\right) \\ 16\left(8 - 12 nR + 3 nR^2\right)^2 \end{pmatrix}$$

115.423

```
With[{params = {gR, gC, gRD, gCD}}, Do[(
    Module[{poly},
    poly = Factor[GroebnerBasis[{
        w == 2 r + 2 gC,
        h == 2 r + (nR - 2) gR + gRD,
        gR^2 + gC^2 == gRD^2 + gCD^2,
        gR^2 + gC^2 == 0^2 + (w - 2 r - 2 gCD)^2}, params[[i]], Drop[params, {i}]][[1]]];
    Print[params[[i]], ":\t", Map[Factor, CoefficientList[
          If[SameQ[Head[poly], Times], poly[[-1]], poly], params[[i]]]] // MatrixForm];
    Print[Solve[0 == poly /. PortraitA4 /. {nR → 4, r → 110}, params[[i]], Reals]];
];), {i, Length[params]}]]
```

$$gR: \begin{pmatrix} \left(16\,h^2 - 64\,h\,r + 52\,r^2 + 12\,r\,w - 3\,w^2\right)\left(16\,h^2 - 64\,h\,r + 84\,r^2 - 20\right. \\ -64\,(-2+nR)\,(h-2\,r)\left(16\,h^2 - 64\,h\,r + 68\,r^2 - 4\,r\,w + w^2\right. \\ 8\left(720\,h^2 - 768\,h^2\,nR + 192\,h^2\,nR^2 - 2880\,h\,r + 3072\,h\,nR\,r - 768\,h\,nR^2\,r + 2900\,r^2 - 3136\,nR\,r^2 + 784\,nR^2\,r^2 - 20\right. \\ -256\,(-2+nR)\left(13 - 16\,nR + 4\,nR^2\right)(h-2\,r) \\ 16\left(13 - 16\,nR + 4\,nR^2\right)^2 \end{pmatrix}$$

$$\{\{gR \to 170.851\}, \{gR \to 283.588\}, \{gR \to 377.685\}, \{gR \to 580.527\}\}$$

$$gC: \begin{pmatrix} 2\,r - w \\ 2 \end{pmatrix}$$

$$\{\{gC \to 163.638\}\}$$

$$gRD: \begin{pmatrix} \left(4\,h^2 - 16\,h\,r - 32\,r^2 + 48\,nR\,r^2 - 12\,nR^2\,r^2 + 48\,r\,w - 48\,nR\,r\,w + 12\,nR^2\,r\,w - 12\,w^2 + 12\,nR\,w^2 - 3\,n\right. \\ -16\,(h-2\,r)\left(36\,h^2 - 144\,h\,r - 32\,r^2 + 176\,nR\,r^2 - 44\right. \\ -8\left(84\,h^2 - 192\,h^2\,nR + 48\,h^2\,nR^2 - 336\,h\,r + 768\,h\,nR\,r - 192\,h\,nR^2\,r + 256\,r^2 - 432\,nR\,r^2 - 148\,nR^2\,r^2 + 128\,nR\right. \\ 192\left(13\right. \\ 16 \end{pmatrix}$$

$$\{\{gRD \to -587.165\}, \{gRD \to -181.48\}, \{gRD \to 6.71386\}, \{gRD \to 232.187\}\}$$

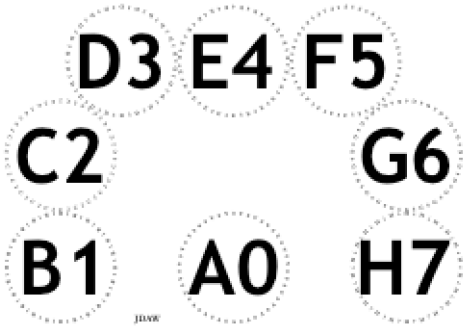
$$gCD: \begin{pmatrix} \left(4\,h^2 - 32\,h\,r + 16\,r^2 + 48\,nR\,r^2 - 12\,nR^2\,r^2 + 8\,h\,w + 16\,r\,w - 48\,nR\,r\,w + 12\,nR^2\,r\,w - 8\,w^2\right. \\ -32\,(2\,r - w)\left(20\,h^2 - 16\,h^2\,nR + 4\,h^2\,nR^2 - 80\,h\,r + 64\,h\,nR\,r - 16\,h\,nR^2\,r - 16\,r^2 + 208\,nR\,r^2 - 244\,nR^2\right. \\ -8\left(76\,h^2 - 64\,h^2\,nR + 16\,h^2\,nR^2 - 304\,h\,r + 256\,h\,nR\,r - 64\,h\,nR^2\,r - 1264\,r^2 + 3952\,nR\,r^2 - 3804\,nR^2\,r^2 + 1408\right. \\ 128\,(-3+ \end{pmatrix}$$

$$\{\{gCD \to -137.937\}, \{gCD \to 45.3505\}, \{gCD \to 327.344\}, \{gCD \to 369.443\}\}$$

# PostsAndLintel

PostsAndLintel

```
(* Also r bounded above by h/(nR-1)/2 and by w/4, and if one central glass by w/6,
and if two or three by w/8. Central glass(es) also cause
 bounds in h that vary according to whether odd or even on edge. *)

Print[ D3 E4 F5
       C2    G6
       B1 A0 H7 ];
```




```
(* http:///www.jdawiseman.com/papers/placemat/placemat.html
  #PermittedPackingStyles_Arch_PostsAndLintel *)

With[{
    x2 = r,
    y2 = (2 nR - 3) r ,
    x3 = w / 2 - (nG + 1 - 2 nR) r,
    y3 = h - r},
    Module[{poly},
        poly = Factor[GroebnerBasis[{
            (x2 - x3)^2 + (y2 - y3)^2 == 4 r^2}, {r}, {}][[1]]];
        Print[poly];
        Print[Map[Factor,
      CoefficientList[If[SameQ[Head[poly], Times], poly[[-1]], poly], r]] // MatrixForm];
        Print[Min[Select[r /. Solve[0 == poly /. LandscapeA3 /. {nR → 3, nG → 7}, {r}],
      # ∈ Reals && # > 0 &]]];
]];
```

$$4\,h^2 + 16\,h\,r - 16\,h\,nR\,r + 16\,r^2 + 16\,nG\,r^2 + 4\,nG^2\,r^2 - 64\,nR\,r^2 - 16\,nG\,nR\,r^2 + 32\,nR^2\,r^2 - 8\,r\,w - 4\,nG\,r\,w + 8\,nR\,r\,w + w^2$$

$$\begin{pmatrix} 4\,h^2 + w^2 \\ -4\,(-4\,h + 4\,h\,nR + 2\,w + nG\,w - 2\,nR\,w) \\ 4\,(4 + 4\,nG + nG^2 - 16\,nR - 4\,nG\,nR + 8\,nR^2) \end{pmatrix}$$

139.792

```
With[{params = {gR, gC, gRD, gCD}}, Do[(
    Module[{poly},
    poly = Factor[GroebnerBasis[{
        h == 2 r + (nR - 2) gR + gRD,
        w == 2 r + 2 gCD + (nG + 1 - 2 nR) gC,
        gC == gR,
        gR^2 == gRD^2 + gCD^2}, params[[i]], Drop[params, {i}]][[1]]];
    Print[params[[i]], ":\t", Map[Factor, CoefficientList[
          If[SameQ[Head[poly], Times], poly[[-1]], poly], params[[i]]]] // MatrixForm];
    Print[Min[Select[params[[i]] /. Solve[0 == poly /. LandscapeA4 /. {nR → 3, nG → 7, r → 130},
           params[[i]]], # ∈ Reals && # > 0 &]]];
);, {i, Length[params]}]]
```

gR:

$$\begin{pmatrix} 4\,h^2 - 16\,h\,r + 20\,r^2 - 4\,r\,w + w^2 \\ -2\,(-8\,h + 4\,h\,nR + 14\,r - 2\,nG\,r - 4\,nR\,r + w + nG\,w - 2\,nR\,w) \\ 13 + 2\,nG + nG^2 - 20\,nR - 4\,nG\,nR + 8\,nR^2 \end{pmatrix}$$

162.591

gC:

$$\begin{pmatrix} 4\,h^2 - 16\,h\,r + 20\,r^2 - 4\,r\,w + w^2 \\ -2\,(-8\,h + 4\,h\,nR + 14\,r - 2\,nG\,r - 4\,nR\,r + w + nG\,w - 2\,nR\,w) \\ 13 + 2\,nG + nG^2 - 20\,nR - 4\,nG\,nR + 8\,nR^2 \end{pmatrix}$$

162.591

gRD:

$$\begin{pmatrix} (3\,h + h\,nG - 2\,h\,nR - 10\,r - 2\,nG\,r + 6\,nR\,r + 2\,w - nR\,w)\,(-h + h\,nG - 2\,h\,nR - 2\,r - 2\,nG\,r + \\ -2\,\left(-3\,h + 2\,h\,nG + h\,nG^2 - 4\,h\,nR - 4\,h\,nG\,nR + 4\,h\,nR^2 + 2\,r - 8\,nG\,r - 2\,nG^2\,r + 18\,nR\,r + 10\,nG\,nR\,r - 12\,nR^2\,r + \right. \\ 13 + 2\,nG + nG^2 - 20\,nR - 4\,nG\,nR + 8\,nR^2 \end{pmatrix}$$

124.684

gCD:

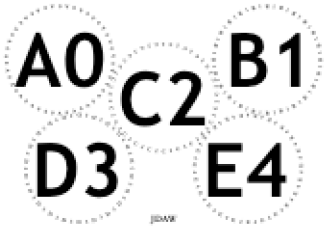
$$\begin{pmatrix} (h + h\,nG - 2\,h\,nR - 4\,r - 2\,nG\,r + 6\,nR\,r + w - nR\,w)\,(h + h\,nG - 2\,h\,nR - 8\,r - 2\,nG\,r + 6\,nR\,r + 3\,w - nR\,w) \\ 4\,\left(-2\,h - 2\,h\,nG + 5\,h\,nR + h\,nG\,nR - 2\,h\,nR^2 + 10\,r + 4\,nG\,r - 18\,nR\,r - 2\,nG\,nR\,r + 6\,nR^2\,r - 3\,w + 4\,nR\,w - nR^2\,w\right) \\ 13 + 2\,nG + nG^2 - 20\,nR - 4\,nG\,nR + 8\,nR^2 \end{pmatrix}$$

104.354

# Bespoke5

Adjusted5

```
Print[ A0 C2 B1
       D3   E4 ];
```




```
(* http://
 www.jdawiseman.com/papers/placemat/placemat.html#PermittedPackingStyles_Bespoke *)

(* Also r bounded above by h/(2+√3) and by w/(2+2√3) *)

With[{
    x0 = r,
    y0 = h - r,
    x2 = w / 2,
    y3 = r},
    Module[{poly},
        poly = Factor[GroebnerBasis[{
            (x0 - x2)^2 + (y0 - y2)^2 == 4 r^2,
            (x2 - x3)^2 + (y2 - y3)^2 == 4 r^2,
            (x3 - x0)^2 + (y3 - y0)^2 == 4 r^2}, {r}, {y2, x3}][[1]]];
        Print[poly];
        Print[Map[Factor,
      CoefficientList[If[SameQ[Head[poly], Times], poly[[-1]], poly], r]] // MatrixForm];
        Print[Min[Select[r /. Solve[0 == poly /. LandscapeA4, {r}], # ∈ Reals && # > 0 &]]];
]];
```

$$r^2 \left(16 h^4 - 128 h^3 r + 336 h^2 r^2 - 320 h r^3 + 64 r^4 + 16 h^2 r w - 64 h r^2 w + 64 r^3 w - 4 h^2 w^2 + 16 h r w^2 - 8 r w^3 + w^4\right)$$

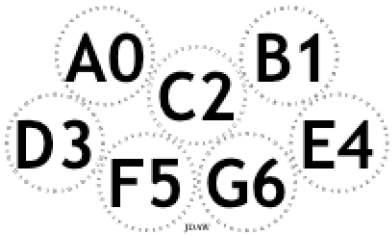
$$\begin{pmatrix} 16 h^4 - 4 h^2 w^2 + w^4 \\ -8 \left(16 h^3 - 2 h^2 w - 2 h w^2 + w^3\right) \\ 16 h (21 h - 4 w) \\ -64 (5 h - w) \\ 64 \end{pmatrix}$$

137.864

# Bespoke7

Adjusted7

```
Print[ A0 C2 B1
       D3 F5 G6 E4 ];
```




```
(* http://
 www.jdawiseman.com/papers/placemat/placemat.html#PermittedPackingStyles_Bespoke *)

(* Also r bounded above by h/(2+Sqrt[3]) and by w/6 *)

With[{
    y0 = h - r,
    x2 = w / 2,
    x3 = r,
    x5 = w / 2 - r,
    y5 = r},
    Module[{poly},
        poly = Factor[GroebnerBasis[{
            (x0 - x2)^2 + (y0 - y2)^2 == 4 r^2,
            (x2 - x5)^2 + (y2 - y5)^2 == 4 r^2,
            (x5 - x3)^2 + (y5 - y3)^2 == 4 r^2,
            (x3 - x0)^2 + (y3 - y0)^2 == 4 r^2}, {r}, {x0, y2, y3}][[1]]];
        Print[poly];
        Print[Map[Factor,
     CoefficientList[If[SameQ[Head[poly], Times], poly[[-1]], poly], r]] // MatrixForm];
        Print[Min[Select[r /. Solve[0 == poly /. LandscapeLegal, {r}], # ∈ Reals && # > 0 &]]];
]]
```

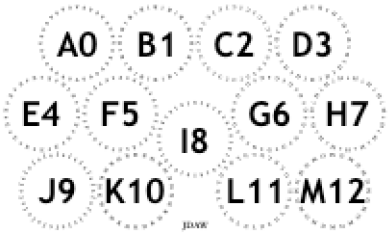
$$-(h-2r)\,r^2\,(2r-w)\,\left(16h^4-128h^3r+288h^2r^2-128hr^3+16r^4-64h^2rw+256hr^2w-448r^3w+8h^2w^2-32hrw^2+120r^2w^2-16rw^3+w^4\right)$$

$$\begin{pmatrix} \left(4h^2+w^2\right)^2 \\ -16\,(2h+w)\,\left(4h^2+w^2\right) \\ 8\left(36h^2+32hw+15w^2\right) \\ -64\,(2h+7w) \\ 16 \end{pmatrix}$$

```
124.964
```

# Temple

```
Print[ A0 B1 C2 D3
       E4 F5 I8 G6 H7
       J9 K10 _ L11 M12 ];
```




```
(* http://
 www.jdawiseman.com/papers/placemat/placemat.html#PermittedPackingStyles_Temple *)

(* Octic has a solution in a narrow range of aspect ratios. Other
  aspect ratios need spaced-out variant, coping with which is fiddly. *)

Module[{boundariesDist, polyRadiusEqns, polyR, specialR},
    With[{dist = 2 r}, With[{
        row0X = (l - dist (n - 1)) / 2,
        row0Y = s - r,
        row1X = r,
        (* row1Y *)
        row2X = l / 2,
        (* row2Y *)
        (* row3X *)
        row3Y = r,
        centreOffsetX = dist (n - 2) / 2
    },
        boundariesDist = {r > 0, row1Y > 0, row1Y < s, row2Y > 0, row2Y < s, row3X > 0};
        polyRadiusEqns = {
            (row1Y - row0Y)^2 + (row1X - row0X)^2 == dist^2,
            (row3Y - row1Y)^2 + (row3X - row1X)^2 == dist^2,
            (row1Y - row2Y)^2 + (row1X + centreOffsetX - row2X)^2 == dist^2,
            (row3Y - row2Y)^2 + (row3X + centreOffsetX - row2X)^2 == dist^2
        };
    ]];
    Print[polyRadiusEqns // MatrixForm];
    polyR = Factor[GroebnerBasis[polyRadiusEqns,
       {r}, {row1Y, row2Y, row3X}, MonomialOrder -> EliminationOrder][[1]]];
    Print[polyR];
    Print[
   Map[Factor, polyRcfl = CoefficientList[If[Head[polyR] === Times, polyR[[-1]], polyR], r]] //
    MatrixForm];
    Print[N[polyRcfl /. LandscapePoint6 /. {n → 4}, 12] // MatrixForm];
    Print["0.6×1: ", N[specialR =
      Min[Select[r /. Solve[0 == polyR /. (LandscapePoint6 /. {w → l, h → s}) /. {n → 4}, {r}],
        # ∈ Reals && # > 0 &]], 12]];
Print[specialLedgerR = Min[Select[r /. Solve[0 == polyR /. {n → 4} /. LandscapeLedger /.
          {w → l, h → s}, {r}], # ∈ Reals && # > 0 &]]];
StringJoin @@ Riffle[StringJoin /@ Transpose[{Map[StringTrim[
           # // Simplify // Factor // PostScriptForm] &, polyRcfl /. {s → S, l → L, n → N}],
        Table["   % Coeff " <> ToString[i], {i, 0, Length[polyRcfl] - 1}]}], "\r\n"]
];
```

$$\begin{pmatrix} \left(r + \frac{1}{2}\,(-l + 2\,(-1 + n)\,r)\right)^2 + (r + \text{row1Y} - s)^2 == 4\,r^2 \\ (r - \text{row1Y})^2 + (-r + \text{row3X})^2 == 4\,r^2 \\ \left(-\frac{1}{2} + r + (-2 + n)\,r\right)^2 + (\text{row1Y} - \text{row2Y})^2 == 4\,r^2 \\ (r - \text{row2Y})^2 + \left(-\frac{1}{2} + (-2 + n)\,r + \text{row3X}\right)^2 == 4\,r^2 \end{pmatrix}$$

$$r^2 \left(9\,l^8 + 72\,l^7\,r - 144\,l^7\,n\,r - 72\,l^6\,r^2 - 1008\,l^6\,n\,r^2 + 1008\,l^6\,n^2\,r^2 - 2400\,l^5\,r^3 + 864\,l^5\,n\,r^3 + 6048\,l^5\,n^2\,r^3 - 4032\,l^5\,n^3\,r^3 - 3824\,l^4\,r^4 + 24\,000\,l^4\,n\,r^4 - 4320\,l^4\,n^2\,r^4 - 20\,160\,l^4\,n^3\,r^4 + 10\,080\,l^4\,n^4\,r^4 + 22\,528\,l^3\,r^5 + 30\,592\,l^3\,n\,r^5 - 96\,000\,l^3\,n^2\,r^5 + 11\,520\,l^3\,n^3\,r^5 + 40\,320\,l^3\,n^4\,r^5 - 16\,128\,l^3\,n^5\,r^5 + 53\,248\,l^2\,r^6 - 135\,168\,l^2\,n\,r^6 - 91\,776\,l^2\,n^2\,r^6 + 192\,000\,l^2\,n^3\,r^6 - 17\,280\,l^2\,n^4\,r^6 - 48\,384\,l^2\,n^5\,r^6 + 16\,128\,l^2\,n^6\,r^6 - 212\,992\,l\,n\,r^7 + 270\,336\,l\,n^2\,r^7 + 122\,368\,l\,n^3\,r^7 - 192\,000\,l\,n^4\,r^7 + 13\,824\,l\,n^5\,r^7 + 32\,256\,l\,n^6\,r^7 - 9216\,l\,n^7\,r^7 + 212\,992\,n^2\,r^8 - 180\,224\,n^3\,r^8 - 61\,184\,n^4\,r^8 + 76\,800\,n^5\,r^8 - 4608\,n^6\,r^8 - 9216\,n^7\,r^8 + 2304\,n^8\,r^8 + 96\,l^6\,r\,s + 1536\,l^5\,r^2\,s - 1152\,l^5\,n\,r^2\,s + 1280\,l^4\,r^3\,s - 15\,360\,l^4\,n\,r^3\,s + 5760\,l^4\,n^2\,r^3\,s - 22\,528\,l^3\,r^4\,s - 10\,240\,l^3\,n\,r^4\,s + 61\,440\,l^3\,n^2\,r^4\,s - 15\,360\,l^3\,n^3\,r^4\,s - 39\,424\,l^2\,r^5\,s + 135\,168\,l^2\,n\,r^5\,s + 30\,720\,l^2\,n^2\,r^5\,s - 122\,880\,l^2\,n^3\,r^5\,s + 23\,040\,l^2\,n^4\,r^5\,s + 157\,696\,l\,n\,r^6\,s - 270\,336\,l\,n^2\,r^6\,s - 40\,960\,l\,n^3\,r^6\,s + 122\,880\,l\,n^4\,r^6\,s - 18\,432\,l\,n^5\,r^6\,s - 157\,696\,n^2\,r^7\,s + 180\,224\,n^3\,r^7\,s + 20\,480\,n^4\,r^7\,s - 49\,152\,n^5\,r^7\,s + 6144\,n^6\,r^7\,s - 24\,l^6\,s^2 - 384\,l^5\,r\,s^2 + 288\,l^5\,n\,r\,s^2 + 1472\,l^4\,r^2\,s^2 + 3840\,l^4\,n\,r^2\,s^2 - 1440\,l^4\,n^2\,r^2\,s^2 + 9728\,l^3\,r^3\,s^2 - 11\,776\,l^3\,n\,r^3\,s^2 - 15\,360\,l^3\,n^2\,r^3\,s^2 + 3840\,l^3\,n^3\,r^3\,s^2 + 5760\,l^2\,r^4\,s^2 - 58\,368\,l^2\,n\,r^4\,s^2 + 35\,328\,l^2\,n^2\,r^4\,s^2 + 30\,720\,l^2\,n^3\,r^4\,s^2 - 5760\,l^2\,n^4\,r^4\,s^2 - 40\,960\,l\,r^5\,s^2 - 23\,040\,l\,n\,r^5\,s^2 + 116\,736\,l\,n^2\,r^5\,s^2 - 47\,104\,l\,n^3\,r^5\,s^2 - 30\,720\,l\,n^4\,r^5\,s^2 + 4608\,l\,n^5\,r^5\,s^2 - 159\,744\,r^6\,s^2 + 81\,920\,n\,r^6\,s^2 + 23\,040\,n^2\,r^6\,s^2 - 77\,824\,n^3\,r^6\,s^2 + 23\,552\,n^4\,r^6\,s^2 + 12\,288\,n^5\,r^6\,s^2 - 1536\,n^6\,r^6\,s^2 - 896\,l^4\,r\,s^3 - 2048\,l^3\,r^2\,s^3 + 7168\,l^3\,n\,r^2\,s^3 - 6144\,l^2\,r^3\,s^3 + 12\,288\,l^2\,n\,r^3\,s^3 - 21\,504\,l^2\,n^2\,r^3\,s^3 + 53\,248\,l\,r^4\,s^3 + 24\,576\,l\,n\,r^4\,s^3 - 24\,576\,l\,n^2\,r^4\,s^3 + 28\,672\,l\,n^3\,r^4\,s^3 + 178\,176\,r^5\,s^3 - 106\,496\,n\,r^5\,s^3 - 24\,576\,n^2\,r^5\,s^3 + 16\,384\,n^3\,r^5\,s^3 - 14\,336\,n^4\,r^5\,s^3 + 112\,l^4\,s^4 + 256\,l^3\,r\,s^4 - 896\,l^3\,n\,r\,s^4 + 5888\,l^2\,r^2\,s^4 - 1536\,l^2\,n\,r^2\,s^4 + 2688\,l^2\,n^2\,r^2\,s^4 - 27\,136\,l\,r^3\,s^4 - 23\,552\,l\,n\,r^3\,s^4 + 3072\,l\,n^2\,r^3\,s^4 - 3584\,l\,n^3\,r^3\,s^4 - 18\,176\,r^4\,s^4 + 54\,272\,n\,r^4\,s^4 + 23\,552\,n^2\,r^4\,s^4 - 2048\,n^3\,r^4\,s^4 + 1792\,n^4\,r^4\,s^4 - 1536\,l^2\,r\,s^5 + 6144\,l\,r^2\,s^5 + 6144\,l\,n\,r^2\,s^5 - 47\,104\,r^3\,s^5 - 12\,288\,n\,r^3\,s^5 - 6144\,n^2\,r^3\,s^5 + 128\,l^2\,s^6 - 512\,l\,r\,s^6 - 512\,l\,n\,r\,s^6 + 23\,040\,r^2\,s^6 + 1024\,n\,r^2\,s^6 + 512\,n^2\,r^2\,s^6 - 4096\,r\,s^7 + 256\,s^8\right)$$

$$\begin{pmatrix} -8\,\left(-9\,l^7 + 18\,l^7\,n - 12\,l^6\,s + 48\,l^5\,\right. \\ 8\,\left(-9\,l^6 - 126\,l^6\,n + 126\,l^6\,n^2 + 192\,l^5\,s - 144\,l^5\,n\,s + 184\,l^4\,s^2 + 480\,l^4\,n\,s^2\right. \\ -32\,\left(75\,l^5 - 27\,l^5\,n - 189\,l^5\,n^2 + 126\,l^5\,n^3 - 40\,l^4\,s + 480\,l^4\,n\,s - 180\,l^4\,n^2\,s - 304\,l^3\,s^2 + 368\,l^3\right. \\ 16\,\left(-239\,l^4 + 1500\,l^4\,n - 270\,l^4\,n^2 - 1260\,l^4\,n^3 + 630\,l^4\,n^4 - 1408\,l^3\,s - 640\,l^3\,n\,s + 3840\,l^3\,n^2\,s - 960\,l^3\,n^3\,s + 360\,l^2\right. \\ -128\,\left(-176\,l^3 - 239\,l^3\,n + 750\,l^3\,n^2 - 90\,l^3\,n^3 - 315\,l^3\,n^4 + 126\,l^3\,n^5 + 308\,l^2\,s - 1056\,l^2\,n\,s - 240\,l^2\,n^2\right. \\ 128\,\left(416\,l^2 - 1056\,l^2\,n - 717\,l^2\,n^2 + 1500\,l^2\,n^3 - 135\,l^2\,n^4 - 378\,l^2\,n^5 + 126\,l^2\,n^6 + \right. \\ -512\,n\,\left(416\,l - 528\,l\,n - 239\,l\,n^2\right. \\ 25\end{pmatrix}$$

$$\begin{pmatrix} 4.29982 \times 10^{24} + 5.97197 \times 10^{18}\,l^2 + 1.45152 \times 10^{13}\,l^4 - 8.64 \times 10^6\,l^6 + 9.00000000000\,l^8 \\ -1.14662 \times 10^{23} - 1.19439 \times 10^{20}\,l - 1.19439 \times 10^{17}\,l^2 - 4.31309 \times 10^{14}\,l^3 - 1.93536 \times 10^{11}\,l^4 + 2.7648 \times 10^8\,l^5 + 57\,6 \\ 1.64826 \times 10^{21} + 2.38879 \times 10^{18}\,l + 5.54066 \times 10^{15}\,l^2 + 5.75078 \times 10^{12}\,l^3 - 2.23488 \times 10^9\,l^4 - 1.8432 \times 10^6\,l^5 - \\ -1.5129 \times 10^{19} - 3.90832 \times 10^{16}\,l - 6.50281 \times 10^{13}\,l^2 - 1.34554 \times 10^{10}\,l^3 + 1.92 \times 10^7\,l^4 - 160\,224.0 \\ 1.17084 \times 10^{17} + 3.44162 \times 10^{14}\,l + 2.9846 \times 10^{11}\,l^2 - 3.80928 \times 10^7\,l^3 + 1.31329600000 \times 10^6 \\ -7.04692 \times 10^{14} - 1.59326 \times 10^{12}\,l - 5.83987 \times 10^8\,l^2 - 6.84697600000 \times 10^6\,l^3 \\ 2.83558 \times 10^{12} + 3.76013 \times 10^9\,l + 2.24235520000 \times 10^7\,l^2 \\ -6.54705 \times 10^9 - 4.25656320000 \times 10^7\,l \\ 3.59792640000 \times 10^7 \end{pmatrix}$$

0.6×1: 103.388

∞

```
Module[{boundariesDist, polyDistEqns, specialTestValues, polyDist, cflDist},
    With[{
        row0X = (l - dist (n - 1)) / 2,
        row0Y = s - r,
        row1X = r,
        (* row1Y *)
        row2X = l / 2,
        (* row2Y *)
        (* row3X *)
        row3Y = r,
        centreOffsetX = dist (n - 2) / 2
    },
        boundariesDist = {dist > 0, row1Y > 0, row1Y < s, row2Y > 0, row2Y < s, row3X > 0};
        polyDistEqns = {
            (row1Y - row0Y)^2 + (row1X - row0X)^2 == dist^2,
            (row3Y - row1Y)^2 + (row3X - row1X)^2 == dist^2,
            (row1Y - row2Y)^2 + (row1X + centreOffsetX - row2X)^2 == dist^2,
            (row3Y - row2Y)^2 + (row3X + centreOffsetX - row2X)^2 == dist^2
        };
    ];
    specialTestValues = Flatten[{(LandscapePoint6 /. {w → l, h → s}),
    n → 4, r → 103.38797850731751591628004827224490933}];
    Print[Solve[Join[boundariesDist, polyDistEqns] /. specialTestValues,
   {dist, row1Y, row2Y, row3X}, Reals]];
    polyDist = Factor[GroebnerBasis[polyDistEqns, {dist},
      {row1Y, row2Y, row3X}, MonomialOrder → EliminationOrder][[1]]];
    Print[Solve[0 == polyDist /. specialTestValues, dist, Reals]];
    Print[polyDist];
    cflDist = Factor /@ Simplify /@
    CoefficientList[If[SameQ[Head[polyDist], Times], polyDist[[-1]], polyDist], dist];
    Print[cflDist // MatrixForm];
    Print[cflDist /. specialTestValues // MatrixForm];
    Print[Min[Select[(dist / r /. specialTestValues) /.
      Solve[0 == polyDist /. specialTestValues, {dist}, Reals], # ∈ Reals && # > 0 &]]];
    StringJoin @@ Riffle[StringJoin /@ Transpose[
      {Map[StringTrim[# // PostScriptForm] &, (cflDist /. {s → S, l → L, r → R, n → N})],
       Table["   % Coeff " <> ToString[i], {i, 0, Length[cflDist] - 1}]}], "\r\n"]
]
```

Solve::ratnz : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. ≫

```
{{dist → 206.776, row1Y → 308.774, row2Y → 226.807, row3X → 127.321}}

{{dist → 0}, {dist → 0}, {dist → 206.776}, {dist → 317.799}, {dist → 636.309}, {dist → 727.081}}
```

$$\text{dist}^2$$

$$\begin{aligned}
&(144\,\text{dist}^6\,l^2 + 744\,\text{dist}^5\,l^3 + 1393\,\text{dist}^4\,l^4 + 1188\,\text{dist}^3\,l^5 + 510\,\text{dist}^2\,l^6 + 108\,\text{dist}\,l^7 + 9\,l^8 - 288\,\text{dist}^7\,l\,n - \\
&2232\,\text{dist}^6\,l^2\,n - 5572\,\text{dist}^5\,l^3\,n - 5940\,\text{dist}^4\,l^4\,n - 3060\,\text{dist}^3\,l^5\,n - 756\,\text{dist}^2\,l^6\,n - 72\,\text{dist}\,l^7\,n + \\
&144\,\text{dist}^8\,n^2 + 2232\,\text{dist}^7\,l\,n^2 + 8358\,\text{dist}^6\,l^2\,n^2 + 11\,880\,\text{dist}^5\,l^3\,n^2 + 7650\,\text{dist}^4\,l^4\,n^2 + 2268\,\text{dist}^3\,l^5\,n^2 + \\
&252\,\text{dist}^2\,l^6\,n^2 - 744\,\text{dist}^8\,n^3 - 5572\,\text{dist}^7\,l\,n^3 - 11\,880\,\text{dist}^6\,l^2\,n^3 - 10\,200\,\text{dist}^5\,l^3\,n^3 - 3780\,\text{dist}^4\,l^4\,n^3 - \\
&504\,\text{dist}^3\,l^5\,n^3 + 1393\,\text{dist}^8\,n^4 + 5940\,\text{dist}^7\,l\,n^4 + 7650\,\text{dist}^6\,l^2\,n^4 + 3780\,\text{dist}^5\,l^3\,n^4 + 630\,\text{dist}^4\,l^4\,n^4 - \\
&1188\,\text{dist}^8\,n^5 - 3060\,\text{dist}^7\,l\,n^5 - 2268\,\text{dist}^6\,l^2\,n^5 - 504\,\text{dist}^5\,l^3\,n^5 + 510\,\text{dist}^8\,n^6 + 756\,\text{dist}^7\,l\,n^6 + \\
&252\,\text{dist}^6\,l^2\,n^6 - 108\,\text{dist}^8\,n^7 - 72\,\text{dist}^7\,l\,n^7 + 9\,\text{dist}^8\,n^8 - 576\,\text{dist}^6\,l\,r - 4464\,\text{dist}^5\,l^2\,r - 11\,144\,\text{dist}^4\,l^3\,r - \\
&11\,880\,\text{dist}^3\,l^4\,r - 6120\,\text{dist}^2\,l^5\,r - 1512\,\text{dist}\,l^6\,r - 144\,l^7\,r + 576\,\text{dist}^7\,n\,r + 8928\,\text{dist}^6\,l\,n\,r + \\
&33\,432\,\text{dist}^5\,l^2\,n\,r + 47\,520\,\text{dist}^4\,l^3\,n\,r + 30\,600\,\text{dist}^3\,l^4\,n\,r + 9072\,\text{dist}^2\,l^5\,n\,r + 1008\,\text{dist}\,l^6\,n\,r - \\
&4464\,\text{dist}^7\,n^2\,r - 33\,432\,\text{dist}^6\,l\,n^2\,r - 71\,280\,\text{dist}^5\,l^2\,n^2\,r - 61\,200\,\text{dist}^4\,l^3\,n^2\,r - 22\,680\,\text{dist}^3\,l^4\,n^2\,r - \\
&3024\,\text{dist}^2\,l^5\,n^2\,r + 11\,144\,\text{dist}^7\,n^3\,r + 47\,520\,\text{dist}^6\,l\,n^3\,r + 61\,200\,\text{dist}^5\,l^2\,n^3\,r + 30\,240\,\text{dist}^4\,l^3\,n^3\,r + \\
&5040\,\text{dist}^3\,l^4\,n^3\,r - 11\,880\,\text{dist}^7\,n^4\,r - 30\,600\,\text{dist}^6\,l\,n^4\,r - 22\,680\,\text{dist}^5\,l^2\,n^4\,r - 5040\,\text{dist}^4\,l^3\,n^4\,r + \\
&6120\,\text{dist}^7\,n^5\,r + 9072\,\text{dist}^6\,l\,n^5\,r + 3024\,\text{dist}^5\,l^2\,n^5\,r - 1512\,\text{dist}^7\,n^6\,r - 1008\,\text{dist}^6\,l\,n^6\,r + 144\,\text{dist}^7\,n^7\,r + \\
&576\,\text{dist}^6\,r^2 + 13\,536\,\text{dist}^5\,l\,r^2 + 26\,904\,\text{dist}^4\,l^2\,r^2 + 34\,848\,\text{dist}^3\,l^3\,r^2 + 24\,104\,\text{dist}^2\,l^4\,r^2 + 7728\,\text{dist}\,l^5\,r^2 + \\
&912\,l^6\,r^2 - 13\,536\,\text{dist}^6\,n\,r^2 - 53\,808\,\text{dist}^5\,l\,n\,r^2 - 104\,544\,\text{dist}^4\,l^2\,n\,r^2 - 96\,416\,\text{dist}^3\,l^3\,n\,r^2 - \\
&38\,640\,\text{dist}^2\,l^4\,n\,r^2 - 5472\,\text{dist}\,l^5\,n\,r^2 + 26\,904\,\text{dist}^6\,n^2\,r^2 + 104\,544\,\text{dist}^5\,l\,n^2\,r^2 + 144\,624\,\text{dist}^4\,l^2\,n^2\,r^2 + \\
&77\,280\,\text{dist}^3\,l^3\,n^2\,r^2 + 13\,680\,\text{dist}^2\,l^4\,n^2\,r^2 - 34\,848\,\text{dist}^6\,n^3\,r^2 - 96\,416\,\text{dist}^5\,l\,n^3\,r^2 - 77\,280\,\text{dist}^4\,l^2\,n^3\,r^2 - \\
&18\,240\,\text{dist}^3\,l^3\,n^3\,r^2 + 24\,104\,\text{dist}^6\,n^4\,r^2 + 38\,640\,\text{dist}^5\,l\,n^4\,r^2 + 13\,680\,\text{dist}^4\,l^2\,n^4\,r^2 - 7728\,\text{dist}^6\,n^5\,r^2 - \\
&5472\,\text{dist}^5\,l\,n^5\,r^2 + 912\,\text{dist}^6\,n^6\,r^2 - 15\,168\,\text{dist}^5\,r^3 - 18\,464\,\text{dist}^4\,l\,r^3 - 19\,008\,\text{dist}^3\,l^2\,r^3 -
\end{aligned}$$

$$29\,632\,\mathrm{dist}^2\,l^3\,r^3 - 16\,800\,\mathrm{dist}\,l^4\,r^3 - 2880\,l^5\,r^3 + 18\,464\,\mathrm{dist}^5\,n\,r^3 + 38\,016\,\mathrm{dist}^4\,l\,n\,r^3 + 88\,896\,\mathrm{dist}^3\,l^2\,n\,r^3 + 67\,200\,\mathrm{dist}^2\,l^3\,n\,r^3 + 14\,400\,\mathrm{dist}\,l^4\,n\,r^3 - 19\,008\,\mathrm{dist}^5\,n^2\,r^3 - 88\,896\,\mathrm{dist}^4\,l\,n^2\,r^3 - 100\,800\,\mathrm{dist}^3\,l^2\,n^2\,r^3 - 28\,800\,\mathrm{dist}^2\,l^3\,n^2\,r^3 + 29\,632\,\mathrm{dist}^5\,n^3\,r^3 + 67\,200\,\mathrm{dist}^4\,l\,n^3\,r^3 + 28\,800\,\mathrm{dist}^3\,l^2\,n^3\,r^3 - 16\,800\,\mathrm{dist}^5\,n^4\,r^3 - 14\,400\,\mathrm{dist}^4\,l\,n^4\,r^3 + 2880\,\mathrm{dist}^5\,n^5\,r^3 + 33\,040\,\mathrm{dist}^4\,r^4 - 50\,880\,\mathrm{dist}^3\,l\,r^4 - 23\,776\,\mathrm{dist}^2\,l^2\,r^4 + 15\,936\,\mathrm{dist}\,l^3\,r^4 + 6112\,l^4\,r^4 + 50\,880\,\mathrm{dist}^4\,n\,r^4 + 47\,552\,\mathrm{dist}^3\,l\,n\,r^4 - 47\,808\,\mathrm{dist}^2\,l^2\,n\,r^4 - 24\,448\,\mathrm{dist}\,l^3\,n\,r^4 - 23\,776\,\mathrm{dist}^4\,n^2\,r^4 + 47\,808\,\mathrm{dist}^3\,l\,n^2\,r^4 + 36\,672\,\mathrm{dist}^2\,l^2\,n^2\,r^4 - 15\,936\,\mathrm{dist}^4\,n^3\,r^4 - 24\,448\,\mathrm{dist}^3\,l\,n^3\,r^4 + 6112\,\mathrm{dist}^4\,n^4\,r^4 + 51\,072\,\mathrm{dist}^3\,r^5 + 71\,040\,\mathrm{dist}^2\,l\,r^5 - 20\,352\,\mathrm{dist}\,l^2\,r^5 - 15\,104\,l^3\,r^5 - 71\,040\,\mathrm{dist}^3\,n\,r^5 + 40\,704\,\mathrm{dist}^2\,l\,n\,r^5 + 45\,312\,\mathrm{dist}\,l^2\,n\,r^5 - 20\,352\,\mathrm{dist}^3\,n^2\,r^5 - 45\,312\,\mathrm{dist}^2\,l\,n^2\,r^5 + 15\,104\,\mathrm{dist}^3\,n^3\,r^5 - 130\,688\,\mathrm{dist}^2\,r^6 + 51\,456\,\mathrm{dist}\,l\,r^6 + 44\,288\,l^2\,r^6 - 51\,456\,\mathrm{dist}^2\,n\,r^6 - 88\,576\,\mathrm{dist}\,l\,n\,r^6 + 44\,288\,\mathrm{dist}^2\,n^2\,r^6 - 44\,544\,\mathrm{dist}\,r^7 - 80\,896\,l\,r^7 + 80\,896\,\mathrm{dist}\,n\,r^7 + 123\,136\,r^8 - 4608\,\mathrm{dist}^5\,l\,r\,s + 6528\,\mathrm{dist}^4\,l^2\,r\,s + 12\,672\,\mathrm{dist}^3\,l^3\,r\,s + 6496\,\mathrm{dist}^2\,l^4\,r\,s + 1344\,\mathrm{dist}\,l^5\,r\,s + 96\,l^6\,r\,s + 4608\,\mathrm{dist}^6\,n\,r\,s - 13\,056\,\mathrm{dist}^5\,l\,n\,r\,s - 38\,016\,\mathrm{dist}^4\,l^2\,n\,r\,s - 25\,984\,\mathrm{dist}^3\,l^3\,n\,r\,s - 6720\,\mathrm{dist}^2\,l^4\,n\,r\,s - 576\,\mathrm{dist}\,l^5\,n\,r\,s + 6528\,\mathrm{dist}^6\,n^2\,r\,s + 38\,016\,\mathrm{dist}^5\,l\,n^2\,r\,s + 38\,976\,\mathrm{dist}^4\,l^2\,n^2\,r\,s + 13\,440\,\mathrm{dist}^3\,l^3\,n^2\,r\,s + 1440\,\mathrm{dist}^2\,l^4\,n^2\,r\,s - 12\,672\,\mathrm{dist}^6\,n^3\,r\,s - 25\,984\,\mathrm{dist}^5\,l\,n^3\,r\,s - 13\,440\,\mathrm{dist}^4\,l^2\,n^3\,r\,s - 1920\,\mathrm{dist}^3\,l^3\,n^3\,r\,s + 6496\,\mathrm{dist}^6\,n^4\,r\,s + 6720\,\mathrm{dist}^5\,l\,n^4\,r\,s + 1440\,\mathrm{dist}^4\,l^2\,n^4\,r\,s - 1344\,\mathrm{dist}^6\,n^5\,r\,s - 576\,\mathrm{dist}^5\,l\,n^5\,r\,s + 96\,\mathrm{dist}^6\,n^6\,r\,s + 9216\,\mathrm{dist}^5\,r^2\,s - 26\,112\,\mathrm{dist}^4\,l\,r^2\,s - 76\,032\,\mathrm{dist}^3\,l^2\,r^2\,s - 51\,968\,\mathrm{dist}^2\,l^3\,r^2\,s - 13\,440\,\mathrm{dist}\,l^4\,r^2\,s - 1152\,l^5\,r^2\,s + 26\,112\,\mathrm{dist}^5\,n\,r^2\,s + 152\,064\,\mathrm{dist}^4\,l\,n\,r^2\,s + 155\,904\,\mathrm{dist}^3\,l^2\,n\,r^2\,s + 53\,760\,\mathrm{dist}^2\,l^3\,n\,r^2\,s + 5760\,\mathrm{dist}\,l^4\,n\,r^2\,s - 76\,032\,\mathrm{dist}^5\,n^2\,r^2\,s - 155\,904\,\mathrm{dist}^4\,l\,n^2\,r^2\,s - 80\,640\,\mathrm{dist}^3\,l^2\,n^2\,r^2\,s - 11\,520\,\mathrm{dist}^2\,l^3\,n^2\,r^2\,s + 51\,968\,\mathrm{dist}^5\,n^3\,r^2\,s + 53\,760\,\mathrm{dist}^4\,l\,n^3\,r^2\,s + 11\,520\,\mathrm{dist}^3\,l^2\,n^3\,r^2\,s - 13\,440\,\mathrm{dist}^5\,n^4\,r^2\,s - 5760\,\mathrm{dist}^4\,l\,n^4\,r^2\,s + 1152\,\mathrm{dist}^5\,n^5\,r^2\,s - 47\,616\,\mathrm{dist}^4\,r^3\,s + 139\,776\,\mathrm{dist}^3\,l\,r^3\,s + 136\,448\,\mathrm{dist}^2\,l^2\,r^3\,s + 35\,328\,\mathrm{dist}\,l^3\,r^3\,s + 2176\,l^4\,r^3\,s - 139\,776\,\mathrm{dist}^4\,n\,r^3\,s - 272\,896\,\mathrm{dist}^3\,l\,n\,r^3\,s - 105\,984\,\mathrm{dist}^2\,l^2\,n\,r^3\,s - 8704\,\mathrm{dist}\,l^3\,n\,r^3\,s + 136\,448\,\mathrm{dist}^4\,n^2\,r^3\,s + 105\,984\,\mathrm{dist}^3\,l\,n^2\,r^3\,s + 13\,056\,\mathrm{dist}^2\,l^2\,n^2\,r^3\,s - 35\,328\,\mathrm{dist}^4\,n^3\,r^3\,s - 8704\,\mathrm{dist}^3\,l\,n^3\,r^3\,s + 2176\,\mathrm{dist}^4\,n^4\,r^3\,s - 76\,800\,\mathrm{dist}^3\,r^4\,s - 130\,048\,\mathrm{dist}^2\,l\,r^4\,s + 3072\,\mathrm{dist}\,l^2\,r^4\,s + 13\,312\,l^3\,r^4\,s + 130\,048\,\mathrm{dist}^3\,n\,r^4\,s - 6144\,\mathrm{dist}^2\,l\,n\,r^4\,s - 39\,936\,\mathrm{dist}\,l^2\,n\,r^4\,s + 3072\,\mathrm{dist}^3\,n^2\,r^4\,s + 39\,936\,\mathrm{dist}^2\,l\,n^2\,r^4\,s - 13\,312\,\mathrm{dist}^3\,n^3\,r^4\,s + 321\,024\,\mathrm{dist}^2\,r^5\,s - 113\,664\,\mathrm{dist}\,l\,r^5\,s - 87\,552\,l^2\,r^5\,s + 113\,664\,\mathrm{dist}^2\,n\,r^5\,s + 175\,104\,\mathrm{dist}\,l\,n\,r^5\,s - 87\,552\,\mathrm{dist}^2\,n^2\,r^5\,s + 104\,448\,\mathrm{dist}\,r^6\,s + 194\,560\,l\,r^6\,s - 194\,560\,\mathrm{dist}\,n\,r^6\,s - 411\,648\,r^7\,s + 1152\,\mathrm{dist}^5\,l\,s^2 - 1632\,\mathrm{dist}^4\,l^2\,s^2 - 3168\,\mathrm{dist}^3\,l^3\,s^2 - 1624\,\mathrm{dist}^2\,l^4\,s^2 - 336\,\mathrm{dist}\,l^5\,s^2 - 24\,l^6\,s^2 - 1152\,\mathrm{dist}^6\,n\,s^2 + 3264\,\mathrm{dist}^5\,l\,n\,s^2 + 9504\,\mathrm{dist}^4\,l^2\,n\,s^2 + 6496\,\mathrm{dist}^3\,l^3\,n\,s^2 + 1680\,\mathrm{dist}^2\,l^4\,n\,s^2 + 144\,\mathrm{dist}\,l^5\,n\,s^2 - 1632\,\mathrm{dist}^6\,n^2\,s^2 - 9504\,\mathrm{dist}^5\,l\,n^2\,s^2 - 9744\,\mathrm{dist}^4\,l^2\,n^2\,s^2 - 3360\,\mathrm{dist}^3\,l^3\,n^2\,s^2 - 360\,\mathrm{dist}^2\,l^4\,n^2\,s^2 + 3168\,\mathrm{dist}^6\,n^3\,s^2 + 6496\,\mathrm{dist}^5\,l\,n^3\,s^2 + 3360\,\mathrm{dist}^4\,l^2\,n^3\,s^2 + 480\,\mathrm{dist}^3\,l^3\,n^3\,s^2 - 1624\,\mathrm{dist}^6\,n^4\,s^2 - 1680\,\mathrm{dist}^5\,l\,n^4\,s^2 - 360\,\mathrm{dist}^4\,l^2\,n^4\,s^2 + 336\,\mathrm{dist}^6\,n^5\,s^2 + 144\,\mathrm{dist}^5\,l\,n^5\,s^2 - 24\,\mathrm{dist}^6\,n^6\,s^2 - 2304\,\mathrm{dist}^5\,r\,s^2 + 6528\,\mathrm{dist}^4\,l\,r\,s^2 + 19\,008\,\mathrm{dist}^3\,l^2\,r\,s^2 + 12\,992\,\mathrm{dist}^2\,l^3\,r\,s^2 + 3360\,\mathrm{dist}\,l^4\,r\,s^2 + 288\,l^5\,r\,s^2 - 6528\,\mathrm{dist}^5\,n\,r\,s^2 - 38\,016\,\mathrm{dist}^4\,l\,n\,r\,s^2 - 38\,976\,\mathrm{dist}^3\,l^2\,n\,r\,s^2 - 13\,440\,\mathrm{dist}^2\,l^3\,n\,r\,s^2 - 1440\,\mathrm{dist}\,l^4\,n\,r\,s^2 + 19\,008\,\mathrm{dist}^5\,n^2\,r\,s^2 + 38\,976\,\mathrm{dist}^4\,l\,n^2\,r\,s^2 + 20\,160\,\mathrm{dist}^3\,l^2\,n^2\,r\,s^2 + 2880\,\mathrm{dist}^2\,l^3\,n^2\,r\,s^2 - 12\,992\,\mathrm{dist}^5\,n^3\,r\,s^2 - 13\,440\,\mathrm{dist}^4\,l\,n^3\,r\,s^2 - 2880\,\mathrm{dist}^3\,l^2\,n^3\,r\,s^2 + 3360\,\mathrm{dist}^5\,n^4\,r\,s^2 + 1440\,\mathrm{dist}^4\,l\,n^4\,r\,s^2 - 288\,\mathrm{dist}^5\,n^5\,r\,s^2 + 48\,768\,\mathrm{dist}^4\,r^2\,s^2 - 28\,800\,\mathrm{dist}^3\,l\,r^2\,s^2 - 24\,384\,\mathrm{dist}^2\,l^2\,r^2\,s^2 + 384\,\mathrm{dist}\,l^3\,r^2\,s^2 + 1248\,l^4\,r^2\,s^2 + 28\,800\,\mathrm{dist}^4\,n\,r^2\,s^2 + 48\,768\,\mathrm{dist}^3\,l\,n\,r^2\,s^2 - 1152\,\mathrm{dist}^2\,l^2\,n\,r^2\,s^2 - 4992\,\mathrm{dist}\,l^3\,n\,r^2\,s^2 - 24\,384\,\mathrm{dist}^4\,n^2\,r^2\,s^2 + 1152\,\mathrm{dist}^3\,l\,n^2\,r^2\,s^2 + 7488\,\mathrm{dist}^2\,l^2\,n^2\,r^2\,s^2 - 384\,\mathrm{dist}^4\,n^3\,r^2\,s^2 - 4992\,\mathrm{dist}^3\,l\,n^3\,r^2\,s^2 + 1248\,\mathrm{dist}^4\,n^4\,r^2\,s^2 + 6912\,\mathrm{dist}^3\,r^3\,s^2 - 6400\,\mathrm{dist}^2\,l\,r^3\,s^2 - 56\,064\,\mathrm{dist}\,l^2\,r^3\,s^2 - 17\,664\,l^3\,r^3\,s^2 + 6400\,\mathrm{dist}^3\,n\,r^3\,s^2 + 112\,128\,\mathrm{dist}^2\,l\,n\,r^3\,s^2 + 52\,992\,\mathrm{dist}\,l^2\,n\,r^3\,s^2 - 56\,064\,\mathrm{dist}^3\,n^2\,r^3\,s^2 - 52\,992\,\mathrm{dist}^2\,l\,n^2\,r^3\,s^2 + 17\,664\,\mathrm{dist}^3\,n^3\,r^3\,s^2 - 336\,256\,\mathrm{dist}^2\,r^4\,s^2 + 139\,008\,\mathrm{dist}\,l\,r^4\,s^2 + 89\,472\,l^2\,r^4\,s^2 - 139\,008\,\mathrm{dist}^2\,n\,r^4\,s^2 - 178\,944\,\mathrm{dist}\,l\,n\,r^4\,s^2 + 89\,472\,\mathrm{dist}^2\,n^2\,r^4\,s^2 - 99\,840\,\mathrm{dist}\,r^5\,s^2 - 204\,288\,l\,r^5\,s^2 + 204\,288\,\mathrm{dist}\,n\,r^5\,s^2 + 623\,104\,r^6\,s^2 - 18\,432\,\mathrm{dist}^4\,r\,s^3 - 3072\,\mathrm{dist}^3\,l\,r\,s^3 - 4864\,\mathrm{dist}^2\,l^2\,r\,s^3 - 4608\,\mathrm{dist}\,l^3\,r\,s^3 - 896\,l^4\,r\,s^3 + 3072\,\mathrm{dist}^4\,n\,r\,s^3 + 9728\,\mathrm{dist}^3\,l\,n\,r\,s^3 + 13\,824\,\mathrm{dist}^2\,l^2\,n\,r\,s^3 + 3584\,\mathrm{dist}\,l^3\,n\,r\,s^3 - 4864\,\mathrm{dist}^4\,n^2\,r\,s^3 - 13\,824\,\mathrm{dist}^3\,l\,n^2\,r\,s^3 - 5376\,\mathrm{dist}^2\,l^2\,n^2\,r\,s^3 + 4608\,\mathrm{dist}^4\,n^3\,r\,s^3 + 3584\,\mathrm{dist}^3\,l\,n^3\,r\,s^3 - 896\,\mathrm{dist}^4\,n^4\,r\,s^3 + 6144\,\mathrm{dist}^3\,r^2\,s^3 + 19\,456\,\mathrm{dist}^2\,l\,r^2\,s^3 + 27\,648\,\mathrm{dist}\,l^2\,r^2\,s^3 + 7168\,l^3\,r^2\,s^3 - 19\,456\,\mathrm{dist}^3\,n\,r^2\,s^3 - 55\,296\,\mathrm{dist}^2\,l\,n\,r^2\,s^3 - 21\,504\,\mathrm{dist}\,l^2\,n\,r^2\,s^3 + 27\,648\,\mathrm{dist}^3\,n^2\,r^2\,s^3 + 21\,504\,\mathrm{dist}^2\,l\,n^2\,r^2\,s^3 - 7168\,\mathrm{dist}^3\,n^3\,r^2\,s^3 + 226\,304\,\mathrm{dist}^2\,r^3\,s^3 - 55\,296\,\mathrm{dist}\,l\,r^3\,s^3 - 41\,984\,l^2\,r^3\,s^3 + 55\,296\,\mathrm{dist}^2\,n\,r^3\,s^3 + 83\,968\,\mathrm{dist}\,l\,n\,r^3\,s^3 - 41\,984\,\mathrm{dist}^2\,n^2\,r^3\,s^3 + 36\,864\,\mathrm{dist}\,r^4\,s^3 + 110\,592\,l\,r^4\,s^3 - 110\,592\,\mathrm{dist}\,n\,r^4\,s^3 - 555\,008\,r^5\,s^3 + 2304\,\mathrm{dist}^4\,s^4 + 384\,\mathrm{dist}^3\,l\,s^4 + 608\,\mathrm{dist}^2\,l^2\,s^4 + 576\,\mathrm{dist}\,l^3\,s^4 + 112\,l^4\,s^4 - 384\,\mathrm{dist}^4\,n\,s^4 - 1216\,\mathrm{dist}^3\,l\,n\,s^4 - 1728\,\mathrm{dist}^2\,l^2\,n\,s^4 - 448\,\mathrm{dist}\,l^3\,n\,s^4 + 608\,\mathrm{dist}^4\,n^2\,s^4 + 1728\,\mathrm{dist}^3\,l\,n^2\,s^4 + 672\,\mathrm{dist}^2\,l^2\,n^2\,s^4 - 576\,\mathrm{dist}^4\,n^3\,s^4 - 448\,\mathrm{dist}^3\,l\,n^3\,s^4 + 112\,\mathrm{dist}^4\,n^4\,s^4 - 768\,\mathrm{dist}^3\,r\,s^4 - 2432\,\mathrm{dist}^2\,l\,r\,s^4 - 3456\,\mathrm{dist}\,l^2\,r\,s^4 - 896\,l^3\,r\,s^4 + 2432\,\mathrm{dist}^3\,n\,r\,s^4 + 6912\,\mathrm{dist}^2\,l\,n\,r\,s^4 + 2688\,\mathrm{dist}\,l^2\,n\,r\,s^4 - 3456\,\mathrm{dist}^3\,n^2\,r\,s^4 - 2688\,\mathrm{dist}^2\,l\,n^2\,r\,s^4 + 896\,\mathrm{dist}^3\,n^3\,r\,s^4 - 89\,728\,\mathrm{dist}^2\,r^2\,s^4 + 6912\,\mathrm{dist}\,l\,r^2\,s^4 + 10\,368\,l^2\,r^2\,s^4 - 6912\,\mathrm{dist}^2\,n\,r^2\,s^4 - 20\,736\,\mathrm{dist}\,l\,n\,r^2\,s^4 + 10\,368\,\mathrm{dist}^2\,n^2\,r^2\,s^4 - 4608\,\mathrm{dist}\,r^3\,s^4 - 34\,304\,l\,r^3\,s^4 + 34\,304\,\mathrm{dist}\,n\,r^3\,s^4 + 319\,232\,r^4\,s^4 + 18\,432\,\mathrm{dist}^2\,r\,s^5 - 1536\,l^2\,r\,s^5 + 3072\,\mathrm{dist}\,l\,n\,r\,s^5 - 1536\,\mathrm{dist}^2\,n^2\,r\,s^5 + 6144\,l\,r^2\,s^5 - 6144\,\mathrm{dist}\,n\,r^2\,s^5 - 120\,832\,r^3\,s^5 - 1536\,\mathrm{dist}^2\,s^6 + 128\,l^2\,s^6 - 256\,\mathrm{dist}\,l\,n\,s^6 + 128\,\mathrm{dist}^2\,n^2\,s^6 - 512\,l\,r\,s^6 + 512\,\mathrm{dist}\,n\,r\,s^6 + 29\,184\,r^2\,s^6 - 4096\,r\,s^7 + 256\,s^8)$$

$$2\left(255\,l^6-378\,l^6\,n+126\,l^6\,n^2-3060\,l^5\,r+4536\,l^5\,n\,r-1512\,l^5\,n^2\,r+12\,052\,l^4\,r^2-19\,320\,l^4\,n\,r^2+6840\,l^4\,n^2\,r^2-1\right.$$

$$\left.1393\,l^4-5940\,l^4\,n+7650\,l^4\,n^2-3780\,l^4\,n^3+630\,l^4\right.$$

$$\begin{pmatrix} 1.99042\times10^{24} \\ -4.14262\times10^{22} \\ 4.06295\times10^{20} \\ -2.3262\times10^{18} \\ 8.05964\times10^{15} \\ -1.68915\times10^{13} \\ 2.09445\times10^{10} \\ -1.42146\times10^{7} \\ 4096 \end{pmatrix}$$

2.

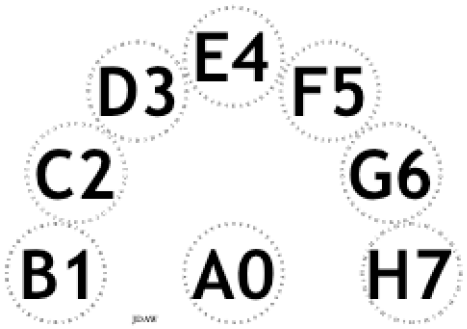
```
L 8 R mul sub L mul 4 2 R mul 4 S mul add R mul S dup mul sub mul add L mul 16 R mul
  2 R mul 4 S mul sub R mul S dup mul add mul add L mul 16 13 R mul 28 S mul
  sub R mul 23 S dup mul mul add R mul 8 S dup dup mul mul mul sub R mul S dup
  mul dup mul add mul add 9 L mul 72 R mul sub L mul 12 22 R mul 4 S mul sub R
  mul S dup mul add mul add L mul 48 R mul 10 R mul 4 S mul sub R mul S dup mul
  add mul sub L mul 16 37 R mul 44 S mul sub R mul 27 S dup mul mul add R mul 8
  S dup dup mul mul mul sub R mul S dup mul dup mul add mul add mul  % Coeff 0

-4 L R 2 mul sub mul 9 2 N mul 3 sub mul L mul 108 2 N mul 3 sub mul R mul sub L mul 12
  78 N mul 107 sub R mul 4 3 N mul 7 sub mul S mul add R mul 3 N mul 7 sub S dup
  mul mul sub mul add L mul 96 R mul 18 N mul 17 sub R mul 4 3 N mul 7 sub mul S
  mul add R mul 3 N mul 7 sub S dup mul mul sub mul sub L mul 16 166 N mul 45 sub R
  mul 8 27 N add mul S mul sub R mul 6 19 N mul 15 sub mul S dup mul mul add R mul
  8 7 N mul 9 sub mul S dup dup mul mul mul sub R mul 7 N mul 9 sub S dup mul dup
  mul mul add mul add L mul 64 R mul 94 N mul 57 sub R mul 8 19 N mul 15 sub mul S
  mul sub R mul 6 25 N mul 29 sub mul S dup mul mul add R mul 8 7 N mul 9 sub mul S
  dup dup mul mul mul sub R mul 7 N mul 9 sub S dup mul dup mul mul add mul sub L
  mul 64 158 N mul 87 sub R mul 4 95 N mul 51 sub mul S mul sub R mul 3 133 N mul
  65 sub mul S dup mul mul add R mul 72 3 N mul 1 sub mul S dup dup mul mul mul sub
  R mul 67 N mul 9 sub S dup mul dup mul mul add R mul 12 N mul S dup dup mul dup
  mul mul mul sub R mul N S dup mul dup dup mul mul mul add mul add mul  % Coeff 1

2 3 42 N mul 126 sub N mul 85 add mul L mul 36 42 N mul 126 sub N mul 85 add mul R
  mul sub L mul 4 45 38 R mul 4 S mul add R mul S dup mul sub mul N mul 210 23 R
  mul 4 S mul add R mul S dup mul sub mul sub N mul 3013 R mul 812 S mul add R
  mul 203 S dup mul mul sub add mul add L mul 32 R mul 45 10 R mul 4 S mul add R
  mul S dup mul sub mul N mul 210 5 R mul S sub mul R S add mul sub N mul 463 R
  mul 812 S mul add R mul 203 S dup mul mul sub add mul sub L mul 16 1146 N mul
  1494 sub N mul 743 sub R mul 8 51 N mul 414 sub N mul 533 add mul S mul add
  R mul 6 39 N mul 6 sub N mul 127 sub mul S dup mul mul add R mul 8 21 N mul
  54 sub N mul 19 add mul S dup dup mul mul mul sub R mul 21 N mul 54 sub N mul
  19 add S dup mul dup mul mul add mul add L mul 64 R mul 3 118 N mul 106 sub
  N mul 185 sub mul R mul 8 39 N mul 6 sub N mul 127 sub mul S mul sub R mul 2
  207 N mul 438 sub N mul 25 add mul S dup mul mul add R mul 8 21 N mul 54 sub
  N mul 19 add mul S dup dup mul mul mul sub R mul 21 N mul 54 sub N mul 19 add
  S dup mul dup mul mul add mul sub L mul 64 346 N mul 402 sub N mul 1021 sub R
  mul 12 57 N mul 74 sub N mul 209 sub mul S mul sub R mul 699 N mul 1086 sub
  N mul 2627 sub S dup mul mul add R mul 8 41 N mul 54 sub N mul 221 sub mul
  S dup dup mul mul mul sub R mul 81 N mul 54 sub N mul 701 sub S dup mul dup
  mul mul add R mul 12 N dup mul 12 sub mul S dup dup mul dup mul mul mul sub R
  mul N dup mul 12 sub S dup mul dup dup mul mul mul add mul add mul  % Coeff 2

-4 L R 2 mul sub mul 9 2 N mul 3 sub mul 7 N mul 21 sub N mul 11 add mul L mul 72
  2 N mul 3 sub mul 7 N mul 21 sub N mul 11 add mul R mul sub L mul 8 3 106 R
```

```
  mul 20 S mul add R mul 5 S dup mul mul sub mul N mul 21 61 R mul 20 S mul add
  R mul 5 S dup mul mul sub mul sub N mul 1483 R mul 812 S mul add R mul 203 S
  dup mul mul sub add N mul 99 5 R mul S sub mul R S add mul sub mul add L mul
  32 R mul 3 22 R mul 20 S mul add R mul 5 S dup mul mul sub mul N mul 21 7 R
  mul 20 S mul add R mul 5 S dup mul mul sub mul sub N mul -47 R mul 812 S mul
  add R mul 203 S dup mul mul sub add N mul 99 R 4 S mul sub R mul S dup mul add
  mul add mul sub L mul 16 118 N mul 159 sub N mul 555 sub N mul 399 add R mul
  8 13 N mul 3 sub N mul 127 sub N mul 75 add mul S mul sub R mul 2 69 N mul
  219 sub N mul 25 add N mul 27 add mul S dup mul mul add R mul 8 7 N mul 27
  sub N mul 19 add N mul 6 sub mul S dup dup mul mul mul sub R mul 7 N mul 27
  sub N mul 19 add N mul 6 sub S dup mul dup mul mul add mul add mul  % Coeff 3

630 N mul 3780 sub N mul 7650 add N mul 5940 sub N mul 1393 add L mul 8 630 N mul
  3780 sub N mul 7650 add N mul 5940 sub N mul 1393 add mul R mul sub L mul 24
  15 38 R mul 4 S mul add R mul S dup mul sub mul N mul 140 23 R mul 4 S mul
  add R mul S dup mul sub mul sub N mul 2 3013 R mul 812 S mul add R mul 203 S
  dup mul mul sub mul add N mul 396 11 R mul 4 S mul add R mul S dup mul sub
  mul sub N mul 1121 R mul 272 S mul add R mul 68 S dup mul mul sub add mul add
  L mul 32 R mul 45 10 R mul 4 S mul add R mul S dup mul sub mul N mul 420 5 R
  mul S sub mul R S add mul sub N mul 6 463 R mul 812 S mul add R mul 203 S dup
  mul mul sub mul add N mul 1188 R 4 S mul add R mul S dup mul sub mul sub N
  mul 577 R mul 816 S mul add R mul 204 S dup mul mul sub add mul sub L mul 16
  382 R mul 136 S mul add R mul 78 S dup mul mul add R mul 56 S dup dup mul mul
  mul sub R mul 7 S dup mul dup mul mul add N mul 12 83 R mul 184 S mul add R
  mul 2 S dup mul mul add R mul 24 S dup dup mul mul mul sub R mul 3 S dup mul
  dup mul mul add mul sub N mul 2 743 R mul 4264 S mul sub R mul 762 S dup mul
  mul add R mul 152 S dup dup mul mul mul add R mul 19 S dup mul dup mul mul
  sub mul sub N mul 12 265 R mul 728 S mul sub R mul 150 S dup mul mul add R
  mul 16 S dup dup mul mul mul add R mul 2 S dup mul dup mul mul sub mul add N
  mul 2065 R mul 2976 S mul sub R mul 3048 S dup mul mul add R mul 1152 S dup
  dup mul mul mul sub R mul 144 S dup mul dup mul mul add add mul add  % Coeff 4

-4 L R 2 mul sub mul 18 7 L mul 28 R mul sub L mul 2 10 R mul 4 S mul add R mul
  S dup mul sub mul add mul N mul 105 3 L mul R 2 mul sub S 2 mul sub mul 3 L
  mul R 10 mul sub S 2 mul add mul sub N mul 2 1275 L mul 5100 R mul sub L mul
  4 463 R mul 812 S mul add R mul 203 S dup mul mul sub mul add mul add N mul
  594 5 L mul 20 R mul sub L mul 4 R 4 S mul add R mul S dup mul sub mul add
  mul sub N mul 1393 L mul 5572 R mul sub L mul 4 577 R mul 816 S mul add R mul
  204 S dup mul mul sub mul add add N mul 6 31 L mul 124 R mul sub L mul 4 79
  R mul 48 S mul sub R mul 12 S dup mul mul add mul add mul sub mul  % Coeff 5

2 6 21 L mul 84 R mul sub L mul 2 38 R mul 4 S mul add R mul S dup mul sub mul add mul N
  mul 42 27 L mul 108 R mul sub L mul 4 23 R mul 4 S mul add R mul S dup mul sub mul
  add mul sub N mul 3825 L mul 15300 R mul sub L mul 4 3013 R mul 812 S mul add R
  mul 203 S dup mul mul sub mul add add N mul 396 15 L mul 60 R mul sub L mul 4 11
  R mul 4 S mul add R mul S dup mul sub mul add mul sub N mul 3 1393 L mul 5572 R
  mul sub L mul 4 1121 R mul 272 S mul add R mul 68 S dup mul mul sub mul add mul
  add N mul 36 31 L mul 124 R mul sub L mul 4 47 R mul 16 S mul sub R mul 4 S dup
  mul mul add mul add mul sub N mul 72 L R 2 mul sub dup mul mul add mul  % Coeff 6

-4 N 3 sub mul N mul 2 N mul 3 sub mul 3 N mul 9 sub N mul
  2 add mul 3 N mul 9 sub N mul 4 add mul L R 2 mul sub mul  % Coeff 7

N 3 sub dup mul N dup mul mul 3 N mul 9 sub N mul 4 add dup mul mul  % Coeff 8
```

# Arch

Arch

```
Print[ D3 E4 F5
       C2    G6
       B1 A0 H7 ];
```




```
(* http://www.jdawiseman.com/papers/placemat/placemat.html
  #PermittedPackingStyles_Arch_PostsAndLintel *)

(* No analytic solution. ☹ *)

(* But r bounded above by h/2 and by w/4, and if one central glass by w/6,
and if two or three by w/8. Central glass(es) also cause
 bounds in h that vary according to whether odd or even on edge. *)

(* http://www.jdawiseman.com/2015/20151128_GroebnerBasis_slow.nb *)
```