

# CGraph documentation

Bruno Kim Medeiros Cesar

May 27, 2013

## Abstract

## Contents

<b>1</b>	<b>sorting</b>	<b>3</b>
<b>2</b>	<b>stat</b>	<b>4</b>
<b>3</b>	<b>list</b>	<b>5</b>
<b>4</b>	<b>set</b>	<b>6</b>
<b>5</b>	<b>graph</b>	<b>7</b>
<b>6</b>	<b>graph_metric</b>	<b>8</b>
6.1	Constants . . . . .	8
6.1.1	GRAPH_METRIC_TOLERANCE . . . . .	8
6.1.2	GRAPH_METRIC_MAX_ITERATIONS . . . . .	8
6.2	Component identification and extraction . . . . .	8
6.2.1	graph_undirected_components . . . . .	8
6.2.2	graph_directed_components . . . . .	8
6.2.3	graph_num_components . . . . .	9
6.2.4	graph_components . . . . .	9
6.2.5	graph_components . . . . .	9
6.3	Degree metrics . . . . .	9
6.3.1	graph_degree . . . . .	9
6.3.2	graph_directed_degree . . . . .	10
6.4	Clustering metrics . . . . .	10
6.4.1	graph_clustering . . . . .	10
6.4.2	graph_num_triplets . . . . .	10
6.4.3	graph_transitivity . . . . .	10
6.5	Geodesic distance metrics . . . . .	11
6.5.1	Definitions . . . . .	11
6.5.2	graph_geodesic_distance . . . . .	11
6.5.3	graph_geodesic_vertex . . . . .	11
6.5.4	graph_geodesic_all . . . . .	11
6.5.5	graph_geodesic_distribution . . . . .	11
6.6	Centrality measures . . . . .	11

6.6.1	<code>graph_betweenness</code>	11
6.6.2	<code>graph_eigenvector</code>	11
6.6.3	<code>graph_pagerank</code>	11
6.6.4	<code>graph_kcore</code>	11
6.7	Correlation measures	11
6.7.1	<code>graph_degree_matrix</code>	11
6.7.2	<code>graph_neighbor_degree_vertex</code>	11
6.7.3	<code>graph_neighbor_degree_all</code>	11
6.7.4	<code>graph_knn</code>	11
6.7.5	<code>graph_assortativity</code>	11
<b>7</b>	<b><code>graph_layout</code></b>	<b>12</b>
7.1	Types	12
7.1.1	<code>coord_t</code>	12
7.1.2	<code>box_t</code>	12
7.1.3	<code>color_t</code>	12
7.1.4	<code>circle_style_t</code>	12
7.1.5	<code>path_style_t</code>	12
7.2	Layout	13
7.2.1	<code>graph_layout_random</code>	13
7.2.2	<code>graph_layout_random_wout_overlap</code>	13
7.2.3	<code>graph_layout_circle</code>	13
7.2.4	<code>graph_layout_circle_edges</code>	13
7.2.5	<code>graph_layout_degree</code>	14
7.3	Printing	14
7.3.1	<code>graph_print_svg</code>	14
7.3.2	<code>graph_print_svg_one_style</code>	14
7.3.3	<code>graph_print_svg_some_styles</code>	15
<b>8</b>	<b><code>graph_model</code></b>	<b>16</b>
8.1	Graph creation	16
8.1.1	<code>new_clique</code>	16
8.1.2	<code>new_erdos_renyi</code>	16
8.1.3	<code>new_watts_strogatz</code>	16
8.1.4	<code>new_barabasi_albert</code>	17
<b>9</b>	<b><code>graph_propagation</code></b>	<b>18</b>

## 1 sorting

2 stat

**3** list

4 set

## 5 graph

## 6 graph\_metric

### 6.1 Constants

These constants are hard-coded to protect some numeric processes of hanging. They can be redefined during compilation, passing a flag such as  
-DGRAPH\_METRIC\_TOLERANCE=1E-3.

#### 6.1.1 GRAPH\_METRIC\_TOLERANCE

Error tolerance for numeric methods.

#### 6.1.2 GRAPH\_METRIC\_MAX\_ITERATIONS

Maximum number of iterations for numeric methods.

### 6.2 Component identification and extraction

#### 6.2.1 graph\_undirected\_components

Label vertices' components treating edges as undirected.

**Preconditions** `label` must have dimension  $n$ .

**Postconditions** `label[i]` is the component ID of vertex  $v_i$ .

**Return** Number of components

For directed graphs, considers adjacencies as incidences. Labels start from 0 and are sequential with step 1. Component IDs are not ordered according to size.

#### 6.2.2 graph\_directed\_components

Label vertices' components treating edges as directed. NOT IMPLEMENTED YET.

**Preconditions** `label` must have dimension  $n$ .

**Postconditions** `label[i]` is the component ID of vertex  $v_i$ .

**Return** Number of components

For undirected graphs, simply call `graph_undirected_components`. For directed graphs, two vertices  $v_i$  and  $v_j$  are in the same component if and only if

$$\begin{aligned}d(v_i, v_j) &\neq \infty \\d(v_j, v_i) &\neq \infty\end{aligned}$$

where  $d(u, v)$  is the geodesic distance between them. In other words, they are in the same component if they are mutually reachable.

Labels start from 0 and are sequential with step 1. Component IDs are not ordered according to size.



### 6.2.3 `graph_num_components`

Extract number of components from label vector.

#### **Preconditions**

$n > 0$   
`label` must have dimension  $n$ .  
`label` must contain sequential IDs starting from 0.

**Return** Number of components

### 6.2.4 `graph_components`

Map components to vertices from label vector.

#### **Preconditions**

$n > 0$   
`label` must have dimension  $n$ .  
`label` must contain sequential IDs starting from 0.  
`comp` must have size `num_comp` and all sets should be already initialized.  
`graph_num_components(g) == num_comp`

#### **Postconditions**

If  $v_i$  is in component  $c_j$ , then  
`label[i] == j` and  
`set_contains(comp[j], i)` is true.

**Return** Number of components

### 6.2.5 `graph_components`

Creates a new graph from `g`'s largest component.

The guarantee of vertices' order ID is the same as `graph_subset`. If two or more components have the same maximum size, one will be chosen in an undefined way.

**Return** A new graph isomorphic to `g`'s largest component.

#### **Memory deallocation**

```
graph_t *largest = graph_components(g);  
delete_graph(largest);
```

## 6.3 Degree metrics

### 6.3.1 `graph_degree`

List all vertices' degrees.

**Preconditions** `degree` must have dimension  $n$ .

**Postconditions** `degree[i]` is the degree of vertex  $v_i$ .

The degree of a directed graph's vertex is defined as the sum of incoming and outgoing edges.

### 6.3.2 graph\_directed\_degree

List all vertices' incoming and outgoing degrees.

#### Preconditions

`g` must be directed. `in_degree` must have dimension  $n$ . `out_degree` must have dimension  $n$ .

#### Postconditions

`in_degree[i]` is the number of incoming edges to vertex  $v_i$ . `out_degree[i]` is the number of outgoing edges from vertex  $v_i$ .

## 6.4 Clustering metrics

### 6.4.1 graph\_clustering

List all vertices' local clustering.

#### Preconditions

`g` must be undirected.  
`clustering` must have dimension  $n$ .

**Postconditions** `clustering[i]` is the local clustering coefficient of vertex  $v_i$ .

The local clustering coefficient is only defined for undirected graphs, and gives the ratio of edges between a vertex' neighbors and all possible edges.

Formally,

$$C_i = \frac{e_i}{\binom{k_i}{2}} = \frac{2e_i}{k_i(k_i - 1)}$$

where

$C_i$  is the local clustering coefficient of vertex  $v_i$ .

$e_i$  is the number of edges between  $v_i$ 's neighbors.

$k_i$  is the degree of  $v_i$ .

If a vertex  $v_i$  has 0 or 1 adjacents,  $C_i = 0$  by definition.

### 6.4.2 graph\_num\_triplets

Counts number of triplets and triangles ( $6 * \text{number of closed triplets}$ ).

### 6.4.3 graph\_transitivity

Compute the ratio between number of triangles and number of triplets.

## **6.5 Geodesic distance metrics**

### **6.5.1 Definitions**

**6.5.2** `graph_geodesic_distance`

**6.5.3** `graph_geodesic_vertex`

**6.5.4** `graph_geodesic_all`

**6.5.5** `graph_geodesic_distribution`

## **6.6 Centrality measures**

**6.6.1** `graph_betweenness`

**6.6.2** `graph_eigenvector`

**6.6.3** `graph_pagerank`

**6.6.4** `graph_kcore`

## **6.7 Correlation measures**

**6.7.1** `graph_degree_matrix`

**6.7.2** `graph_neighbor_degree_vertex`

**6.7.3** `graph_neighbor_degree_all`

**6.7.4** `graph_knn`

**6.7.5** `graph_assortativity`

## 7 graph\_layout

### 7.1 Types

#### 7.1.1 coord\_t

Euclidean coordinates in 2D.

#### 7.1.2 box\_t

Box (rectangle) definition in 2D, given by its SW and NE vertices in a positively oriented world frame, such as the screen. Images may have a negatively oriented frame, with  $y$  pointing down. It is necessary that `box.sw.y < box.ne.y` and `box.sw.x < box.ne.x`.

#### 7.1.3 color\_t

Array with 4 colors between 0 and 255, inclusive: red ( $R$ ), green ( $G$ ), blue ( $B$ ) and alpha ( $A$ ).  $A = 0$  means totally transparent, and  $A = 255$  means totally opaque.

#### 7.1.4 circle\_style\_t

SVG circle style.

`radius` Circle radius in pixels.

`width` Stroke width in pixels. This is added to the radius for total size.

`fill` Color of the fill.

`stroke` Color of the stroke.

#### 7.1.5 path\_style\_t

SVG path style.

`type` Path type.

`from`, `to` Path origin and destination.

`control` Control point

`width` Stroke width in pixels.

`color` Stroke color.

For `style.type == GRAPH_STRAIGHT`, draws a straight line from origin to destination.

For `style.type == GRAPH_PARABOLA`, draws a parabola from origin to destination using the control point.

For `style.type == GRAPH_CIRCULAR`, draws the arc of a circle from origin to destination using the control point as the circle center.

## 7.2 Layout

### 7.2.1 `graph_layout_random`

Place points uniformly inside specified box.

#### Preconditions

`box` must be a valid box.

`p` must have dimension  $n$ .

**Postconditions** `p[i]` is a random coordinate inside `box`.

### 7.2.2 `graph_layout_random_wout_overlap`

Place points with specified radius uniformly avoiding overlap with probability  $t$ .

#### Preconditions

`radius` must be positive.

$t$  must be a valid probability ( $0 \leq t \leq 1$ ).

`p` must have dimension  $n$ .

**Postconditions** `p[i]` is a random coordinate.

The algorithm determines a box with size  $l$  such that, if  $n$  points with radius  $r$  are thrown within it, will not have any collision with probability  $t$ . The formula is derived in Math Exchange.

$$l = \frac{nr}{2} \sqrt{\frac{2\pi}{-\log(1-t)}}$$

### 7.2.3 `graph_layout_circle`

Place points with specified radius in a circle without overlap.

#### Preconditions

`radius` must be positive.

`p` must have dimension  $n$ .

**Postconditions** `p[i]` is a coordinate in a circle.

**Return value** Circle bounding box size.

Points are positioned sequentially in a circle, starting from the rightmost and following in counterclockwise order.

### 7.2.4 `graph_layout_circle_edges`

Fill edge style for a circular layout.

#### Preconditions

`size` must be the circle bounding box size.

`width` must be positive.

`color` must be a valid color.

`es` must have dimension  $m$ .

`edge_style` must have dimension 2.

**Postconditions**

`es[i]` is one of the styles `CIRCULAR` or `PARABOLA`.  
`edge_style[0]` is the `CIRCULAR` style.  
`edge_style[1]` is the `PARABOLA` style.

This function maps `es` to a circular or parabolic style, where an edge is circular if its endpoints are adjacent in a circle, and parabolic otherwise.

**7.2.5 graph\_layout\_degree**

Place points in concentric shells, with highest degrees near the center.

**Preconditions**

`radius` must be positive.  
`p` must have dimension  $n$ .

**Postconditions** `p[i]` is a coordinate.

Each shell is attached to a degree value; the inner shell contains elements of the highest degree, and the outer shell contains elements with the lowest degree. In each shell, elements are placed equally apart.

**7.3 Printing**

Printing functions accept optional `width` and `height` parameters in pixels. They won't be considered if they are negative or zero.

**7.3.1 graph\_print\_svg**

Prints graph as SVG to file, using vertex coordinates given in `p` and with a style for each point and edge.

**Preconditions**

`p` must have dimension  $n$ . `point_style` must have dimension  $n$ . `edge_style` must have dimension  $m$ .

**Postconditions** `filename` is a valid SVG file.

Edges are ordered according to vertices' order. In undirected graphs, an edge  $E_{ij}$  is considered only if  $i < j$ . In directed graphs, mutual edges will superimpose if `edge_style.type == GRAPH_STRAIGHT`.

**7.3.2 graph\_print\_svg\_one\_style**

Prints graph as SVG to file, using vertex coordinates given in `p` and with a single style for all points and edges.

**Preconditions**

`p` must have dimension  $n$ .

**Postconditions** `filename` is a valid SVG file.

The edge style type is ignored, using only `GRAPH_STRAIGHT`.

### 7.3.3 graph\_print\_svg\_some\_styles

Prints graph as SVG to file, using vertex coordinates given in `p` and with a number of styles given. The mapping vertex→style is given in `ps`, and the mapping edge→style is given in `es`.

#### Preconditions

- `p` must have dimension  $n$ .
- `ps` must have dimension  $n$ .
- `es` must have dimension  $m$ .
- `point_style` must have dimension `num_point_style`.
- `edge_style` must have dimension `num_edge_style`.

**Postconditions** `filename` is a valid SVG file.

This function tries to avoid extensive memory utilization one just some styles are desired. If vertex  $v_i$  should have style  $S_j$ , then `ps[i] = j`. Ditto for edges.

Edge order is based on vertices order. In undirected edges, edge  $E_{ij}$  is considered only if  $i < j$ .

## 8 graph\_model

### 8.1 Graph creation

These functions creates new graphs, whose memory should be managed by the caller.

The reentrant versions `new_erdos_renyi_r`, `new_watts_strogatz_r` and `new_barabasi_albert_r` accept a state argument that will be used to call `rand_r` for pseudo-random number generation. Two calls with the same state argument yield the same graph and same final state, allowing reproducibility.

#### 8.1.1 new\_clique

Creates a complete network with  $n$  vertices.

**Preconditions**  $n > 0$

**Return value** An undirected, unweighted complete graph, or NULL in case of memory exhaustion.

It should be noticed that the data structure is inefficient to represent large dense graphs, so it is recommended to check for memory exhaustion upon return.

#### 8.1.2 new\_erdos\_renyi

Creates a random network with  $n$  vertices and average degree  $k$ .

**Preconditions**

$$n > 0$$

$$0 < k < n$$

**Return value** An undirected, unweighted random graph.

There is no guarantee that the network will be connected. The size and characteristic of the largest component follow different regimes depending on  $k$ :

Regime	Size	Loop
$k < 1$	$\log n$	No loop
$k = 1$	$n^{2/3}$	No loop
$k > 1$	$\alpha n$	Some loops
$k > \log n$	$n$	Many loops

#### 8.1.3 new\_watts\_strogatz

Creates a small-world network with  $n$  vertices and average degree  $k$ , with rewiring probability  $\beta$ .

**Preconditions**

$$n > 0$$

$k$  is even

$$0 < k < n$$

$\beta$  is a valid probability ( $0 \leq \beta \leq 1$ )

**Return value** An undirected, unweighted small-world graph.



#### 8.1.4 `new_barabasi_albert`

Creates a scale-free network with  $n$  vertices and average degree  $k$ .

**Preconditions**

$$n > 0$$

$$0 < k < n$$

**Return value** An undirected, unweighted scale-free graph.

## 9 graph\_propagation