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STRUCTURAL CONTROL IN A GRADED MANPOWER SYSTEM*†

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Control is exercised on the parameters of a Markov chain model for a graded manpower system, and the effects on the structure of the system are examined. Questions of attainability and maintainability of structures are discussed for both fixed size and expanding systems. Examples are presented to illustrate the theory.

1. Introduction

This work is concerned with the control of the grade structure in a manpower system. The initial and required structures will be given and we shall investigate whether, with the given model, the system can attain and then maintain the latter structure. The general model we shall adopt was first introduced by Gani [3] and Young and Almond [4]. It is a Markov chain model which assumes that changes take place at integral points in time. This is appropriate, for example, in organizations which have an annual change in structure.

Control can be exercised through three main aspects of the system. The least desirable of these is the rate of loss from the system; throughout our work this rate will be assumed constant and no redundancies will occur. Our attention will thus be focussed on the following other aspects:

(a) The proportions recruited into each grade;
(b) The promotion and demotion rates.

The major part of the theory will be developed for the model in which the rates in (b) remain constant. This involves the least disruption of the system and is the most desirable. Bartholomew [1] used this model to examine problems in the context of university faculties, and considered various strategies for approaching the goal structure. At the end of the paper we briefly discuss the wider control offered by two further models. The first of these assumes that the demotion rates are fixed at zero, and control is exercised on the rates of recruitment and promotion. The second model has the three parameters in (a) and (b) amenable to control, and has the advantage of allowing an immediate change to any goal structure.

In the first section where the system is of fixed size we present a mathematical analysis of the control mechanism. We describe geometrically the set of structures which can be attained in n steps from a *given* structure, and then describe the set which is n-step attainable from *at least one* structure. A relevant limit set is introduced. We discuss the process of selection of suitable recruitment vectors, and examine how this selection may be facilitated.

The concept of n-step attainability leads us to consider structures and sets of structures that are maintainable. This area of the work and its continuation in the next section is an extension of a recent paper of Forbes [2].

The latter part of the first section is concerned with the numerical application of the above results. We present a table to illustrate the step-by-step changes arising when

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† This paper is an abridged version of a much longer paper containing a detailed analysis of the problems. The longer edition is obtainable from the business office of TIMS.

the existing structure is replaced by a new one, and include figures to illustrate the geometry of the sets of structures arising from the analysis.

The object of the second section is to examine the control problem in the context of expanding systems. We present relationships between the sets introduced earlier under different expansion rates, and show by example that when a system expands from one size to another the choice of the rate of expansion is crucial to the problems of this paper.

2. A Fixed Size System: A Mathematical Analysis

The Markov Chain Model

The system under consideration has k grades, and will be maintained at a constant size throughout. Changes occur at discrete points in time. The model will be treated deterministically, so that the proportions arising in the sequel are either exact or expected values.

Let the proportion who move from grade i to grade j at any point be denoted by p_{ij}. Such a move will be referred to as a promotion if $i \leqq j$ and a demotion otherwise. The proportions p_{ii} refer to those who remain in grade i. Let w_i denote the proportion who leave the system at any point from grade i. Next let $p_i(n)$ be the proportion of new recruits who enter grade i at step n. Finally, let $x_i(n)$ represent the proportion of members of the system who are in grade i at step n. The initial structure is that existing at step zero.

The above model implies that the promotion rates and rates of leaving are time homogeneous, but that the recruitment rates are amenable to control. We have the following equations relating the proportions

$$\sum_{j=1}^{k} p_{ij} + w_i = 1 \quad \text{for} \quad i = 1, 2, \cdots, k, \tag{1}$$

$$\boldsymbol{x}(n+1) = \boldsymbol{x}(n)\boldsymbol{P} + \boldsymbol{x}(n)\boldsymbol{w}'\boldsymbol{p}(n+1), \tag{2}$$

where

$$\boldsymbol{x}(n) = (x_1(n), \cdots, x_k(n)),$$
$$\boldsymbol{w} = (w_1, \cdots, w_k),$$
$$\boldsymbol{p}(n) = (p_1(n), \cdots, p_k(n)),$$

and

$$\boldsymbol{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1k} \\ p_{21} & p_{22} & \cdots & p_{2k} \\ \vdots & \vdots & & \vdots \\ p_{k1} & p_{k2} & \cdots & p_{kk} \end{bmatrix}.$$

It is easy to see that (2) yields the alternative equation

$$\boldsymbol{x}(n) = \boldsymbol{x}(0)\prod_{m=1}^{n}(\boldsymbol{P} + \boldsymbol{w}'\boldsymbol{p}(m)).$$

Attainability

The problem to be considered here may be stated as follows: given a present structure $\boldsymbol{y}$ and required structure $\boldsymbol{x}$ of the system, can we within the confines of the model under consideration find an integer n such that $\boldsymbol{x}$ is attainable in n steps from $\boldsymbol{y}$. This would necessitate the discovery of a sequence $\boldsymbol{p}(1), \cdots, \boldsymbol{p}(n)$ of recruitment vectors such that

$$\boldsymbol{x} = \boldsymbol{y}\prod_{m=1}^{n}(\boldsymbol{P} + \boldsymbol{w}'\boldsymbol{p}(m)).$$

The following set becomes relevant.

The set $A_n(y)$. Let $A_n(\boldsymbol{y})$ denote the set of structures attainable in n steps (or n-step attainable) from the structure $\boldsymbol{y}$. It will be convenient to consider $A_n(\boldsymbol{y})$ as a set of points in Euclidean k-space. The set introduced next is of importance in this direction.

The set X. Let $X = \{\boldsymbol{x} \mid \boldsymbol{x} \geqq \boldsymbol{0}, \boldsymbol{x}\cdot\boldsymbol{1}' = 1\}$. X represents all possible structures in the system, and is the convex hull of the points $\boldsymbol{e}_i$, $i = 1, \cdots, k$, where $\boldsymbol{e}_i$ is the row vector with 1 in the ith entry and zero elsewhere. It is often useful to decide whether or not a structure is attainable from anywhere at all before proceeding to a particular initial structure. We are then interested in the following set.

The set A_n. Let A_n denote the set of points attainable in n steps from at least one point in X.

Clearly $A_n = \bigcup_{y\epsilon X} A_n(\boldsymbol{y})$, $A_0 = X$. If a goal structure $\boldsymbol{x}$ does not belong to A_n then no matter what the existing structure happens to be $\boldsymbol{x}$ is unattainable in n steps. It is finally convenient to introduce the following set.

The set $B_1(x)$. Let $B_1(\boldsymbol{x})$ denote the set of points from which $\boldsymbol{x}$ may be attained in one step. We are now in a position to proceed with the analysis, and we first present the geometrical description of the sets $A_n(\boldsymbol{y})$ and A_n.

Theorem 1. $A_n(\boldsymbol{y})$ is the convex hull of the points

$$\{\boldsymbol{y} \textstyle\prod_{\nu=1}^{n} (\boldsymbol{P} + \boldsymbol{w}'\boldsymbol{e}_{j_\nu})\}, \qquad j_\nu = 1, \cdots, k.$$

Theorem 2. A_n is the convex hull of the points

$$\{\boldsymbol{e}_i \textstyle\prod_{\nu=1}^{n} (\boldsymbol{P} + \boldsymbol{w}'\boldsymbol{e}_{j_\nu})\}, \qquad i, j_\nu = 1, \cdots, k.$$

There exists the following inclusion relationship between the sets A_n.

Theorem 3. $A_{n-1} \supseteq A_n$, $n = 1, 2, \cdots$.

Corollary. $A_n \to A = \bigcap_{n=1}^{\infty} A_n$ as $n \to \infty$.

The members of A are characterized by being attainable in an infinity of steps. The limit set is thus as a whole of little practical importance. It does, however, contain various useful subsets and we shall meet some of these later.

Recruitment Vector Selection Process

Once it is established that a required structure $\boldsymbol{x}$ belongs to $A_n(\boldsymbol{y})$ we must then proceed to determine a sequence of n satisfactory recruitment vectors. In other words, we have to find a sequence $\boldsymbol{z}_1, \cdots, \boldsymbol{z}_{n-1}$ of structures satisfying

$$\boldsymbol{z}_i \in A_1(\boldsymbol{z}_{i-1}) \quad \text{or} \quad \boldsymbol{z}_i \geqq \boldsymbol{z}_{i-1}\boldsymbol{P},$$

$i = 1, \cdots, n$, $\boldsymbol{z}_0 = \boldsymbol{y}$, $\boldsymbol{z}_n = \boldsymbol{x}$. The sequence of recruitment vectors is then given by

$$\boldsymbol{p}(i) = \frac{\boldsymbol{z}_i - \boldsymbol{z}_{i-1}\boldsymbol{P}}{\boldsymbol{z}_{i-1}\boldsymbol{w}'}$$

Suitable structures are obtained by selecting $\boldsymbol{z}_{i-1}$ to be any member of the set $A_{i-1}(\boldsymbol{y}) \cap B_1(\boldsymbol{z}_i)$, starting with the choice of $\boldsymbol{z}_{n-1}$. Note that

$$B_1(\boldsymbol{x}) = \{\boldsymbol{y} \mid \boldsymbol{x} \geqq \boldsymbol{y}\boldsymbol{P}, \boldsymbol{y} \in X\}.$$

At the University of Kent, in Canterbury England, linear programming techniques are currently being applied to select the recruitment policy that is optimal in some sense.

Maintainability

If the structure required in an organization is attainable it is clearly desirable that there should exist a policy for maintaining this structure. This is unfortunately not the case in our model, and we have to consider the question of maintainability in some approximate sense. We introduce the following set.

The set S_n. Let S_n denote the set of structures that are maintainable at intervals of n steps. That is,

$$\boldsymbol{x} \in S_n \Leftrightarrow \boldsymbol{x} \in A_n(\boldsymbol{x}).$$

The concept of n-step maintainability leads to the consideration of maintainable sets of points as described in the next result.

Theorem 4. Let $\boldsymbol{x} \in S_n$, and suppose that $\boldsymbol{z}_1, \cdots, \boldsymbol{z}_{n-1}$ are intermediary points between $\boldsymbol{x}$ and its reappearance. Then $\boldsymbol{z}_i \in S_n$, $i = 1, \cdots, n - 1$.

The above implies that the n structures $(\boldsymbol{x}, \boldsymbol{z}_1, \cdots, \boldsymbol{z}_{n-1})$ form a maintainable set. Thus although it is not always possible to maintain $\boldsymbol{x}$ we can in some cases specify a set of n structures containing $\boldsymbol{x}$ that is maintainable.

Our next theorem concerns the relationship between the sets $A_n(\boldsymbol{y})$ when $\boldsymbol{y} \in S_n$, and contrasts sharply with Theorem 3.

Theorem 5. If $\boldsymbol{y} \in S_m$, then $A_n(\boldsymbol{y}) \subseteq A_{n+m}(\boldsymbol{y})$, $n = 1, 2, \cdots$

$$S_1 \subseteq \textstyle\bigcup_{n=1}^{\infty} A_n(\boldsymbol{y}) \subseteq A.$$

Corollary. If $\boldsymbol{y} \in S_1$, then $A_n(\boldsymbol{y})$ is an increasing sequence of sets.

The final result in this chapter relates the sets S_n.

Theorem 6. $S_n \subseteq S_{nm}$, $n, m = 1, 2, \cdots$; $\bigcup_{n=1}^{\infty} S_n \subseteq A$.

Corollary. If $\boldsymbol{x} \notin A_n$ for *some* n, then $\boldsymbol{x} \notin S_n$ for *any* n.

The above corollary is a useful test of non-maintainability.

Applications

In this chapter we shall illustrate how the preceding theorems may be used to devise a suitable recruitment policy. The particular model used will be one of those examined by Bartholomew, and will serve as a useful comparison between his strategies and the "minimum number of steps" strategy under consideration. It is emphasized that the model presents only an artificial picture of a real manpower system, but it and the examples that follow have been chosen to exaggerate the difficulties that occur in the control problem.

Let the system have the following historic promotion and wastage rates.

$$\boldsymbol{P} = \begin{bmatrix} 0.5 & 0.4 & 0 \\ 0 & 0.6 & 0.3 \\ 0 & 0 & 0.8 \end{bmatrix}; \qquad \boldsymbol{w} = (0.1, 0.1, 0.2).$$

Example 1. The goal structure $\boldsymbol{x} = (0.27, 0.23, 0.50)$. This Structure is 4-step attainable and 3-step maintainable.

Since $(0, 0, 1)\,\boldsymbol{P} = (0, 0, 0.8) \leqq (0, 0, 1)$, the initial structure is maintainable and $A_n(0, 0, 1)$ is an increasing sequence of sets (Theorem 5, Corollary). It can be seen from Figure 1 that $(0.27, 0.23, 0.50)$ belongs to $A_4(\boldsymbol{y})$ and not any previous set, and from Figure 2 that the structure belongs to $A_3(\boldsymbol{x})$ but not $A_1(\boldsymbol{x})$ and $A_2(\boldsymbol{x})$.

Table 1 presents the recruitment policy. The effect of the recruitment vector (1, 0, 0) at Step 1 is to take y to a vertex of $A_1(y)$. At the next step the recruitment vector (1, 0, 0) takes this vertex to a vertex of $A_2(y)$, and so on. Note that we now have a maintainable set of three structures.

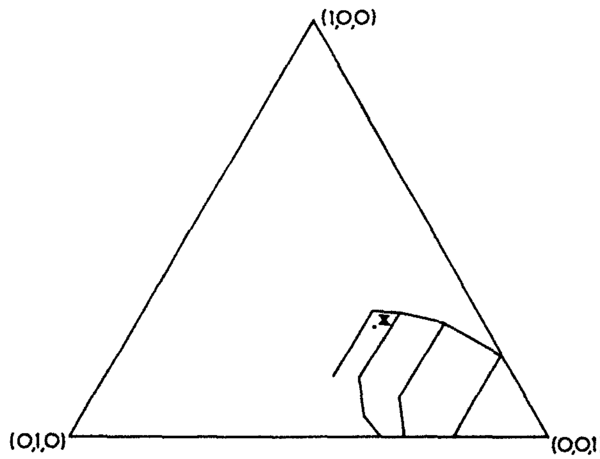

FIGURE 1. $A_1 - A_3\,(0, 0, 1)$ and the relevant part of $A_4\,(0, 0, 1)$. x = (0.27, 0.23, 0.50).

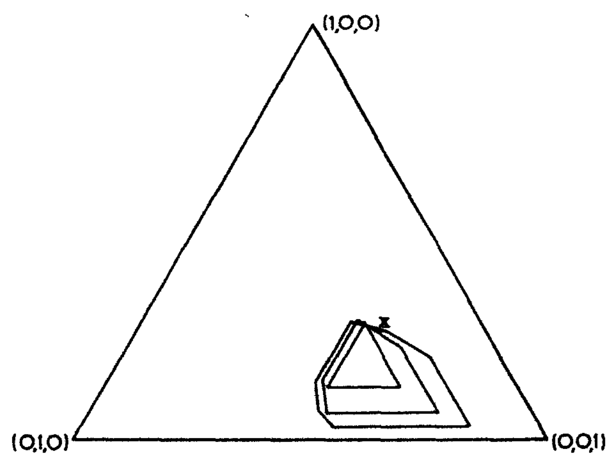

FIGURE 2. $A_1 - A_3(x)$; x = (0.27, 0.23, 0.50).

TABLE 1

Recruitment policy for goal (0.27, 0.23, 0.50)

Step	Structure	Recruitment vector
1	(0, 0, 1)	(1, 0, 0)
2	(0.2, 0, 0.8)	(1, 0, 0)
3	(0.28, 0.08, 0.64)	(1, 0, 0)
4	(0.304, 0.160, 0.536)	(0.768, 0.081, 0.151)
5	(0.27, 0.23, 0.50)	(0.44, 0, 0.56)
6	(0.201, 0.246, 0.553)	(0.853, 0, 0.147)
7	(0.233, 0.228, 0.539)	(0.997, 0, 0.003)
	(0.27, 0.23, 0.50)	

3. An Expanding System: A Mathematical Analysis

The control model to be considered here is the natural extension of the previous model to expanding systems. Let $N(n)$ denote the (given) size of the system at step n. Then we have the following equation.

$$N(n+1)\boldsymbol{x}(n+1) = N(n)\boldsymbol{x}(n)\boldsymbol{P} + (N(n)\boldsymbol{x}(n)\boldsymbol{w}' + M(n+1))\boldsymbol{p}(n+1), \tag{3}$$

where

$$M(n+1) = N(n+1) - N(n).$$

We shall assume that the system expands or contracts at a constant rate. Thus $N(n) = \rho N(n+1)$, for some constant $\rho > 0$. Writing

$$M(n+1) = (1-\rho)N(n+1) = \delta N(n+1),$$

(3) becomes

$$\begin{aligned}\boldsymbol{x}(n+1) &= \rho\boldsymbol{x}(n)\boldsymbol{P} + (\rho\boldsymbol{x}(n)\boldsymbol{w}' + \delta)\boldsymbol{p}(n+1)\\ &= \rho\boldsymbol{x}(n)(\boldsymbol{P} + \boldsymbol{w}'\boldsymbol{p}(n+1)) + \delta\boldsymbol{p}(n+1).\end{aligned}$$

Note, that since $\boldsymbol{x}(n+1) \geqq \rho\boldsymbol{x}(n)\boldsymbol{P}$, we have

$$\rho\boldsymbol{x}(n)\boldsymbol{w}' + \delta \geqq 0. \tag{4}$$

(4) is particularly important when we are considering contracting systems and $\delta < 0$. For given ρ, it imposes a restriction on the choice of the structure $\boldsymbol{x}(n)$; for given $\boldsymbol{x}(n)$ (for example, the initial structure), it presents an upper bound for ρ. In the sequel it will be assumed, in fact, that

$$\rho\boldsymbol{e}_i\boldsymbol{w}' + \delta \geqq 0 \quad \text{for} \quad i = 1, \cdots, k, \tag{5}$$

or $\rho \leqq 1/(1 - w^*)$, where $w^* = \min_i w_i$. This amounts to assuming that (4) is satisfied for all possible structures $\boldsymbol{x}(n)$.

Attainability

Let the system have a present structure $\boldsymbol{y}$ (and size $N(0)$). It is required to move to a new structure $\boldsymbol{x}$ and size N^*. If the expansion (positive or negative) has to follow some predetermined form in say n steps, we would be given the sequence of $(n+1)$ sizes $N(0), N(1), \cdots, N(n-1), N^*$ involved in the change. If there is no restriction of the above type we can determine the most appropriate expansion rate for the given situation.

The sets of structures introduced in the last section have natural counterparts in the present setting. We shall denote these by $A_n(\boldsymbol{y}; \rho)$ and $A_n(\rho)$. It is clearly useful to have some comparison between the sets under different expansion rates. Information concerning a structure in, for example, a fixed size system ($\rho = 1$) could then be used as a preliminary guide to the same structure during a period of expansion. In this direction we have the following results.

Theorem 7. If $\rho_1 \leqq \rho_2$, then

$$A_n(\boldsymbol{y}; \rho_1) \supseteq A_n(\boldsymbol{y}; \rho_2).$$

Theorem 8. If $\rho_1 \leqq \rho_2$, then

$$A_n(\rho_1) \supseteq A_n(\rho_2).$$

Maintainability

Let us now assume that the initial structure is to be maintained as far as possible during and after the period of expansion. $A_n(y; \rho)$ is independent of the existing size of the system and the concept of n-step maintainability continues to be applicable in the present case. We may thus introduce the set $S_n(\rho)$ corresponding to its namesake in the previous section. The value of n will, of course, have to relate to the number of steps involved in the expansion process. When the expansion is complete, the theory for fixed size systems becomes relevant.

The following result on maintainability is an immediate corollary to Theorem 7.

Theorem 9. If $\rho_1 \leqq \rho_2$, then

$$S_n(\rho_1) \supseteq S_n(\rho_2).$$

Thus, from Theorem 9, if a structure is n-step maintainable before a period of positive expansion ($\rho < 1$) it remains n-step maintainable during this period.

Applications

Our example will once more refer to the three grade manpower system described in § 2.2. From (5) the data impose the restriction

$$\rho \leqq 1/(1 - 0.1) = 10/9$$

on the expansion rate.

EXAMPLE 2. The present structure of the system is $y = (0, 0, 1)$, and a period of expansion is to take place in which the size of the system is to change from $N(0)$ to $(8/7)^4 N(0)$. It is decided that this expansion may be achieved in one, two or four steps. The problem is to determine the structures available immediately following the expansion.

The set of structures attainable is given by $A_4(y; 7/8) \cup A_2(y; (7/8)^2) \cup A_1(y; (7/8)^4)$, and is illustrated in Figure 3. The individual sets should be compared with those of Figure 1; for example

$$A_2(y; (7/8)^2) \supset A_2(y; 1) \ (= A_2(y))$$

(c.f. Theorem 7).

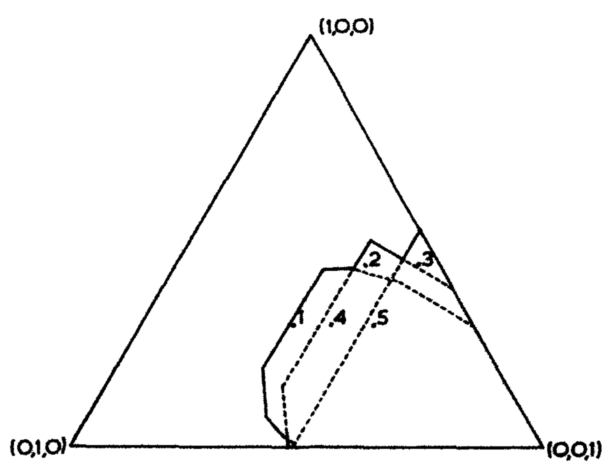

FIGURE 3. $A_4(y; 7/8) \cup A_2(y; (7/8)^2) \cup A_1(y; (7/8)^4)$ (Dotted lines are the boundaries of the individual sets not forming part of the boundary of the union.) $y = (0, 0, 1)$. 1. (0.3, 0.38, 0.32) 2. (0.44, 0.16, 0.4) 3. (0.44, 0.05, 0.51) 4. (0.3, 0.3, 0.4) 5. (0.3, 0.21, 0.49).

We list several possible goal structures and use the analysis to give details of their attainability properties. It is seen that in some cases the goal structure dictates the rate of expansion of the system while in others the rate is immaterial.

Goal structure	Attainability
(0.3, 0.38, 0.32)	4-step attainable
(0.44, 0.16, 0.4)	2-step attainable
(0.44, 0.05, 0.51)	1-step attainable
(0.3, 0.3, 0.4)	2 and 4-step attainable
(0.3, 0.21, 0.49)	1,2 and 4-step attainable

4. Other Control Models

More extensive control on a manpower system can clearly be obtained by influencing the promotion and demotion rates as well as the recruitment distribution. We briefly indicate here results that can be obtained in this direction.

Model β. For this model promotion rates become time-dependent, while demotion rates are fixed at zero. Thus, for $i \leqq j$, $p_{ij}(n)$ will denote the proportion who move from grade i to grade j at step n. $p_{ij} = p_{ij}(n) \equiv 0$ for $i > j$. All other aspects remain as for the previous model.

Model γ. Promotion *and* demotion rates are now time-dependent.

We have the following theorems (in which the superfix refers the set to the particular model).

Theorem 10. $A_n^{(\beta)}(\boldsymbol{y})$ is the convex hull of the points

$$\{\boldsymbol{y} \textstyle\prod_{t=1}^{n} (\boldsymbol{E}_{1,j_{1t}} + \cdots + \boldsymbol{E}_{k,j_{kt}} + \boldsymbol{w}'\boldsymbol{e}_{j_t})\}, \qquad j_{\nu t} = \nu, \cdots, k, j_t = 1, \cdots, k,$$

where $\boldsymbol{E}_{ij}$ is the $k \times k$ matrix with (i, j)th entry $1 - w_i$ and zero elsewhere.

Theorem 11. For any $\boldsymbol{y} \in X$, $A_n^{(\beta)}(\boldsymbol{y})$ increases to X as $n \to \infty$.

Theorem 12. $A_n^{(\beta)} = A_n^{(\gamma)}(\boldsymbol{y}) = A_n^{(\gamma)} = S_n^{(\beta)} = S_n^{(\gamma)} = X$.

5. Conclusion

When a management requires a change of structure in its organization there are three major courses open to it. Differences in the courses lie in the amount of control exercised on the parameters of the system. If a new structure is required immediately, control is in general forced onto the recruitment, promotion and demotion rates. It is possible under these conditions to move from *any* given structure to *any* goal structure in one step. Since demotions rarely occur, a policy for such a drastic change could be totally unacceptable. If, however, demotion rates are held at zero we are still able to guarantee the eventual attainability of any structure. In a lot of cases the number of steps required to reach the goal may be prohibitive, and we are then forced back to the complete control situation. On the other hand, if the number of steps is reasonable, we have a control policy far more satisfactory than one enforcing demotions against the run of normal policy.

Even control of promotion rates can create problems; if we finally disallow this aspect, we are left with the recruitment vector as the single parameter of the model. This is certainly the most desirable of all the options open to management, but it severely diminishes the set of structures which may be attained. Now, in fact, there are structures which cannot be attained from *any* given structure let alone the one in question.

Once a structure is found to be attainable, the question of its maintainability arises. Every structure is maintainable if all the parameters can be controlled. This result holds good when demotions rates are held at zero and control is exercised on the recruitment and promotion rates. In the above cases care must be taken since the policy adopted will be a permanent one. While, for example, it may have been possible earlier to disallow promotions from some grade at a particular step it is clearly impracticable to adopt this strategy as the norm.

With control on recruitment only, we have to reconsider the problem of maintainability. The set of maintainable structures is now quite small, but we can extend the concept to maintainable collections of structures. Thus, for instance, it may be possible to maintain a pair of structures in rotation although each of them is not maintainable in the strict sense.

When the system undergoes a period of expansion or contraction at a constant rate, two major conclusions can be drawn. Firstly, the attainability of a structure is critically dependent on the rate of expansion adopted for the system. Thus a structure which is attainable following an n-step expansion process may cease to be attainable if the same total expansion is completed in m-steps ($m \neq n$). The above statement holds good whether $m > n$ or $m < n$, so there can be no general rule in this direction.

Secondly, n-step attainability under one expansion rate implies the same attainability under a faster rate. (Note that this does not contradict the previous paragraph since we were then considering expansions involving differing numbers of steps.) We are able to conclude now, for example, that a structure (or set of structures) which is maintainable during a period of contraction remains maintainable when the system returns to a fixed size.

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