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FEUP FACULDADE DE ENGENHARIA UNIVERSIDADE DO PORTO

Masters in Engineering and Data Science

ADSS - Analytical Decision Support Systems Report

Aircraft Landings Scheduling

GRUPO B3

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Introduction

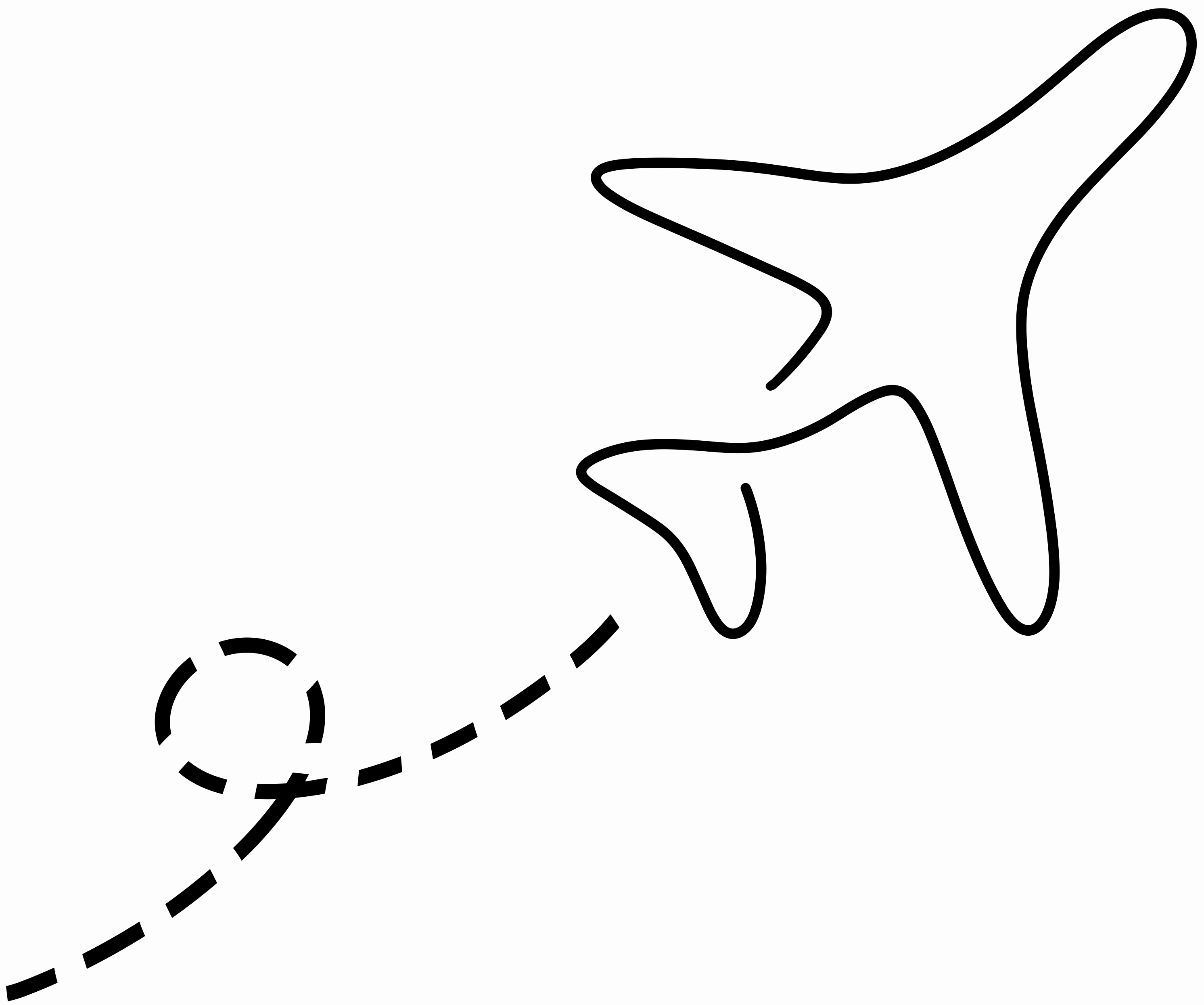
Introduction

- Aircraft landing scheduling is crucial for efficient airport operations;

- Single and Multiple runways scenarios;

- Mathematical Programming (MP) and Constraint Programming (CP) to optimize landing schedules and handle complex constraints;

- The project evaluates the effectiveness of these methods in improving airport efficiency and handling real-world complexities.



Problem Formulations

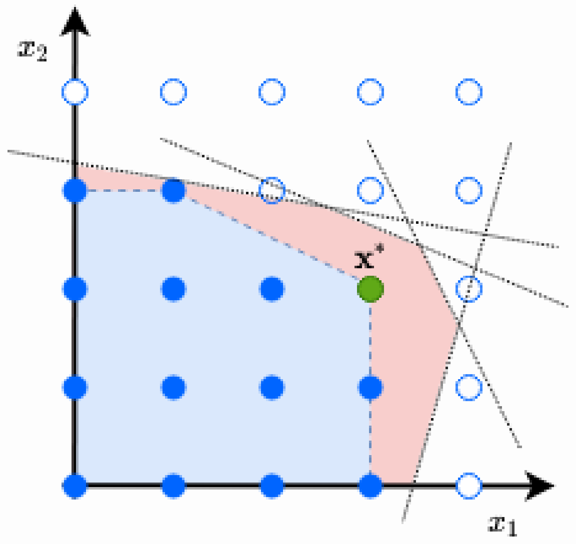
MIP - Parameters & Decision Variables

Sets and Indices

- i, j: Indices for aircraft ($i, j = 1, \ldots, P$).
- r: Index for runways ($r = 1, \ldots, R$).
- U: the set of pairs (i, j) of planes for which we are uncertain whether plane i lands before plane j.
- V: the set of pairs (i, j) of planes for which i definitely lands before j (but for which the separation constraint is not automatically satisfied).
- W: the set of pairs (i, j) of planes for which i definitely lands before j (and for which the separation constraint is automatically satisfied).

Parameters

- P: Number of planes.
- R: Number of runways.
- E_i: Earliest allowable landing time for aircraft i.
- L_i: Latest allowable landing time for aircraft i.
- T_i: Target (preferred) landing time for aircraft i.
- g_i: Penalty cost per unit of time for landing before T_i.
- h_i: Penalty cost per unit of time for landing after T_i.
- S_{ij}: Minimum separation time between the landings of aircraft i and j (if i lands before j).



Decision Variables

- x_i: Actual landing time of aircraft i.
- α_i: Deviation from T_i if aircraft i lands earlier ($\alpha_i \geq 0$).
- β_i: Deviation from T_i if aircraft i lands later ($\beta_i \geq 0$).
- δ_{ij}: Binary variable, $\delta_{ij} = 1$ if aircraft i lands before j, 0 otherwise.
- z_{ij}: Binary variable, $z_{ij} = 1$ if aircraft i and j land on the same runway, 0 otherwise.
- y_{ir}: Binary variable, $y_{ir} = 1$ if aircraft i is assigned to runway r, 0 otherwise.

MIP - Mathematical Formulation

Objective Function: $$\text{Minimize} \quad \sum_{i=1}^{P} (g_i \alpha_i + h_i \beta_i)$$

Sets:

$$W = [(i, j) | L_i < E_j \text{ and } L_i + S_{ij} \leqslant E_j$$
$$i = 1, \ldots, P; j = 1, \ldots, P; i \neq j].$$

$$V = [(i, j) | L_i < E_j \text{ and } L_i + S_{ij} > E_j$$
$$i = 1, \ldots, P; j = 1, \ldots, P; i \neq j].$$

$$U = [(i, j) | i = 1, \ldots, P; j = 1, \ldots, P; i \neq j;$$
$$E_j \leqslant E_i \leqslant L_j \text{ or } E_j \leqslant L_i \leqslant L_j$$
$$\text{or } E_i \leqslant E_j \leqslant L_i \text{ or } E_i \leqslant L_j \leqslant L_i].$$

Constraints:

Time Window Constraints: $$E_i \leq x_i \leq L_i, \quad \forall i.$$

Deviation Definition: $$x_i = T_i - \alpha_i + \beta_i, \quad \forall i$$
$$\alpha_i \geq T_i - x_i, \quad 0 \leq \alpha_i \leq T_i - E_i, \quad \beta_i \geq x_i - T_i, \quad 0 \leq \beta_i \leq L_i - T_i, \quad \forall i$$

Separation Constraints:
$$\delta_{ij} = 1 \quad \forall (i, j) \in W \cup V.$$
$$x_j \geqslant x_i + S_{ij} \quad \forall (i, j) \in V,$$
$$x_j \geqslant x_i + S_{ij}\delta_{ij} - (L_i - E_j)\delta_{ji} \quad \forall (i, j) \in U.$$

Variable Domains: $$\alpha_i, \beta_i \geq 0, \quad x_i \geq 0, \quad z_{ij} \in \{0,1\}, \quad y_{ir} \in \{0,1\}, \quad \forall i, j, r.$$

Landing Order: $$\delta_{ij} + \delta_{ji} = 1, \quad \forall (i, j), \quad j > i$$

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MIP - Extension for Multiple Runways

New Decision Variables:

$$z_{ij} = \begin{cases} 1 & \text{if planes } i \text{ and } j \text{ land on the same runway} \\ & (i = 1, \dots, P;\ j = 1, \dots, P;\ i \neq j) \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ir} = \begin{cases} 1 & \text{if plane } i\ (i = 1, \dots, P) \text{ lands on} \\ & \text{runway } r (r = 1, \dots, R) \\ 0 & \text{otherwise} \end{cases}$$

New Constraints:

$$\sum_{r=1}^{R} y_{ir} = 1, \quad \forall i.$$

$$z_{ij} = z_{ji} \quad i = 1, \dots, P;\ j = 1, \dots, P;\ j > i$$

$$z_{ij} \geqslant y_{ir} + y_{jr} - 1 \quad i = 1, \dots, P;\ j = 1, \dots, P;$$

$$j > i;\ r = 1, \dots, R.$$

$$x_j \geqslant x_i + S_{ij}\delta_{ij} - (L_i - E_j)\delta_{ji} \quad \forall (i, j) \in U.$$ **replaced by** $$x_j \geq x_i + S_{ij}z_{ij} - (L_i + S_{ij} - E_j)\delta_{ji}, \quad \forall (i,j) \in U.$$

CP - Parameters & Decision Variables

Sets and Indices

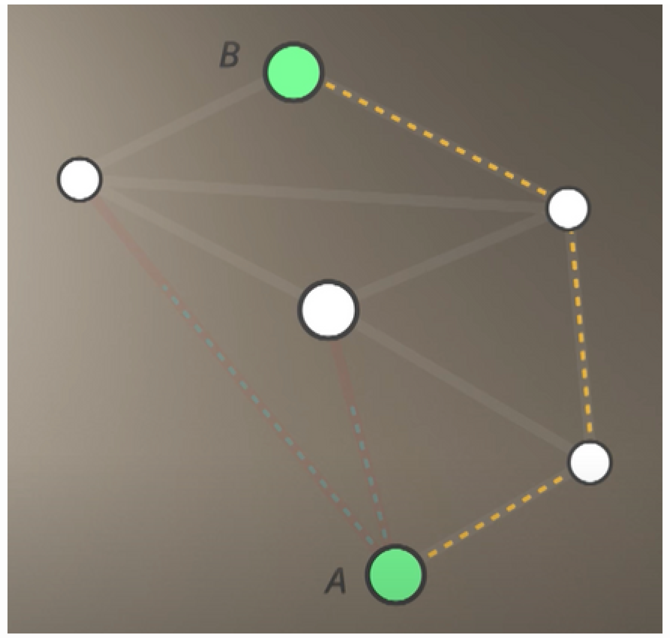
- i, j: Indices for aircraft ($i, j = 1, \dots, P$).

Parameters

- P: Number of planes.
- R: Number of runways.
- E_i: Earliest allowable landing time for aircraft i.
- L_i: Latest allowable landing time for aircraft i.
- T_i: Target (preferred) landing time for aircraft i.
- g_i: Penalty cost per unit of time for landing before T_i.
- h_i: Penalty cost per unit of time for landing after T_i.
- S_{ij}: Minimum separation time between the landings of aircraft i and j (if i lands before j).

Decision Variables

- position_i: The position of aircraft i in the landing order ($\text{position}_i \in \{1, \dots, P\}$).
- x_i: Actual landing time of aircraft i.
- e_i: Earliness of aircraft i ($e_i \geq 0$).
- l_i: Lateness of aircraft i ($l_i \geq 0$).
- before_{ij}: Binary variable indicating if aircraft i lands before j ($\text{before}_{ij} \in \{0, 1\}$).
- runway_i: Runway assigned to aircraft i ($\text{runway}_i \in \{1, \dots, R\}$).
- same_runway_{ij}: Binary variable indicating if aircraft i and j share the same runway ($\text{same\_runway}_{ij} \in \{0, 1\}$).



CP - Mathematical Formulation

Objective Function:

$$\text{Minimize} \quad \sum_{i=1}^{P} (g_i e_i + h_i l_i)$$

Constraints:

Time Window Constraints:

$$E_i \leq x_i \leq L_i, \quad \forall i.$$

Deviation Definition:

$$e_i \geq T_i - x_i, \quad l_i \geq x_i - T_i, \quad \forall i.$$

$$e_i, l_i \geq 0, \quad \forall i.$$

Landing Order:

$$\text{position}_i \neq \text{position}_j, \quad \forall i \neq j.$$

Separation Constraints:

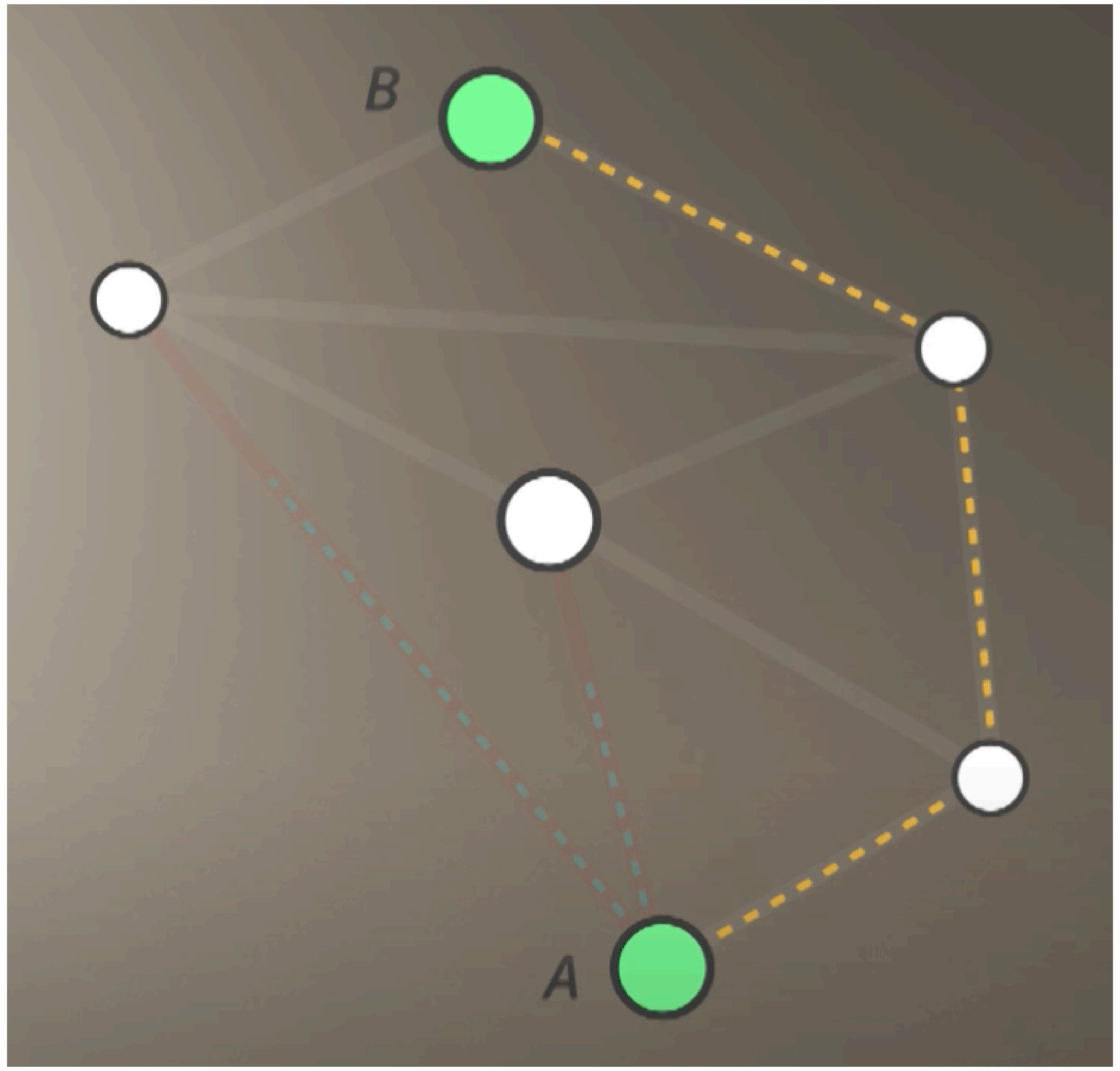
$$\text{if position}_i < \text{position}_j, \quad x_j \geq x_i + S_{ij}.$$

$$\text{if position}_j < \text{position}_i, \quad x_i \geq x_j + S_{ji}.$$

Binary Variable:

$$\text{before}_{ij} = 1 \implies x_j \geq x_i + S_{ij}.$$

$$\text{before}_{ij} = 0 \implies x_i \geq x_j + S_{ji}.$$



CP - Extension for Multiple Runways

New Variables:

$$\text{runway}_i \in \{1, \dots, R\}, \quad \forall i.$$

$$\text{same\_runway}_{ij} = \begin{cases} 1 & \text{if runway}_i = \text{runway}_j, \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{same\_runway}_{ij} = 1 \iff \text{runway}_i = \text{runway}_j, \quad \forall i < j.$$

New Constraints:

Conditional Separation Based on Runway:

$$x_j \geq x_i + S_{ij} \quad \text{only if } (\text{before}_{ij} = 1) \text{ and } (\text{same\_runway}_{ij} = 1).$$

$$x_i \geq x_j + S_{ji} \quad \text{only if } (\text{before}_{ij} = 0) \text{ and } (\text{same\_runway}_{ij} = 1).$$

Search Strategies

Variable, Branching and Value Strategies

Variable Strategies

Branching strategies dictate how the search tree is explored.

Branching Strategies

Variable selection strategies help determine which variables should be assigned values first during the search process.

Value Strategies

Value selection strategies decide which values should be assigned to a selected variable.

Hint

- **Guidance for Solver:** Hints guide the solver by fixing a variable or directing the exploration process;

- **Runway Allocation:** In our case, the hint fixes the allocation of one runway at the start, narrowing the search space;

- **Optimization of Search:** The use of hints helps optimize the search

Performance Metrics

Performance Metrics - MIP

In MIP, performance metrics assess solver efficiency, solution quality, and computational requirements to ensure effective model design

- **Execution Time:** Time taken by the solver to find an optimal solution or declare infeasibility;
- **Number of Variables:** The total number of decision variables;
- **Number of Constraints:** Defines the complexity by the number of rules the solution must satisfy;
- **Total Penalty:** Objective function value that represents cost, profit, or performance indicators.

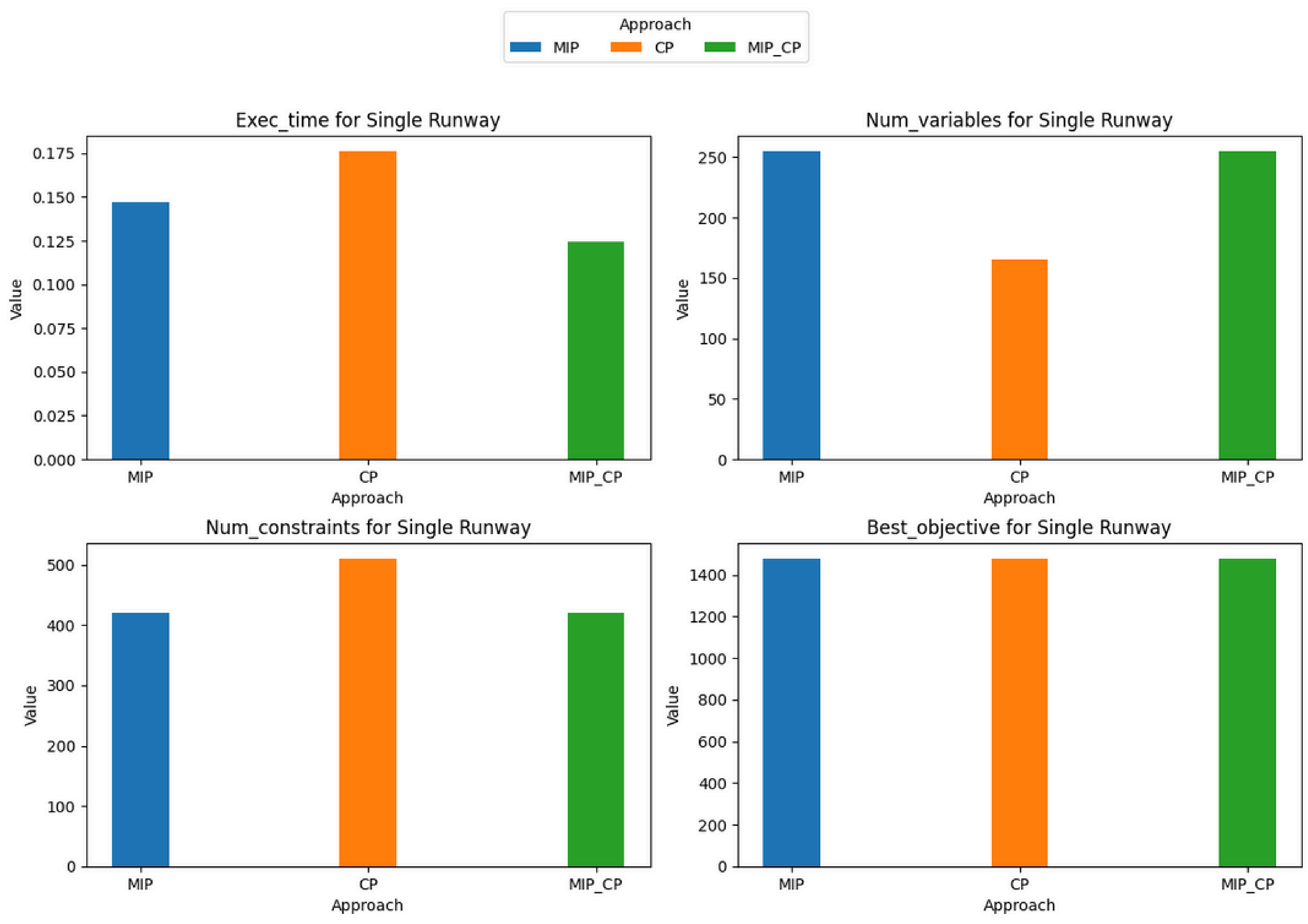
Performance Metrics - CP

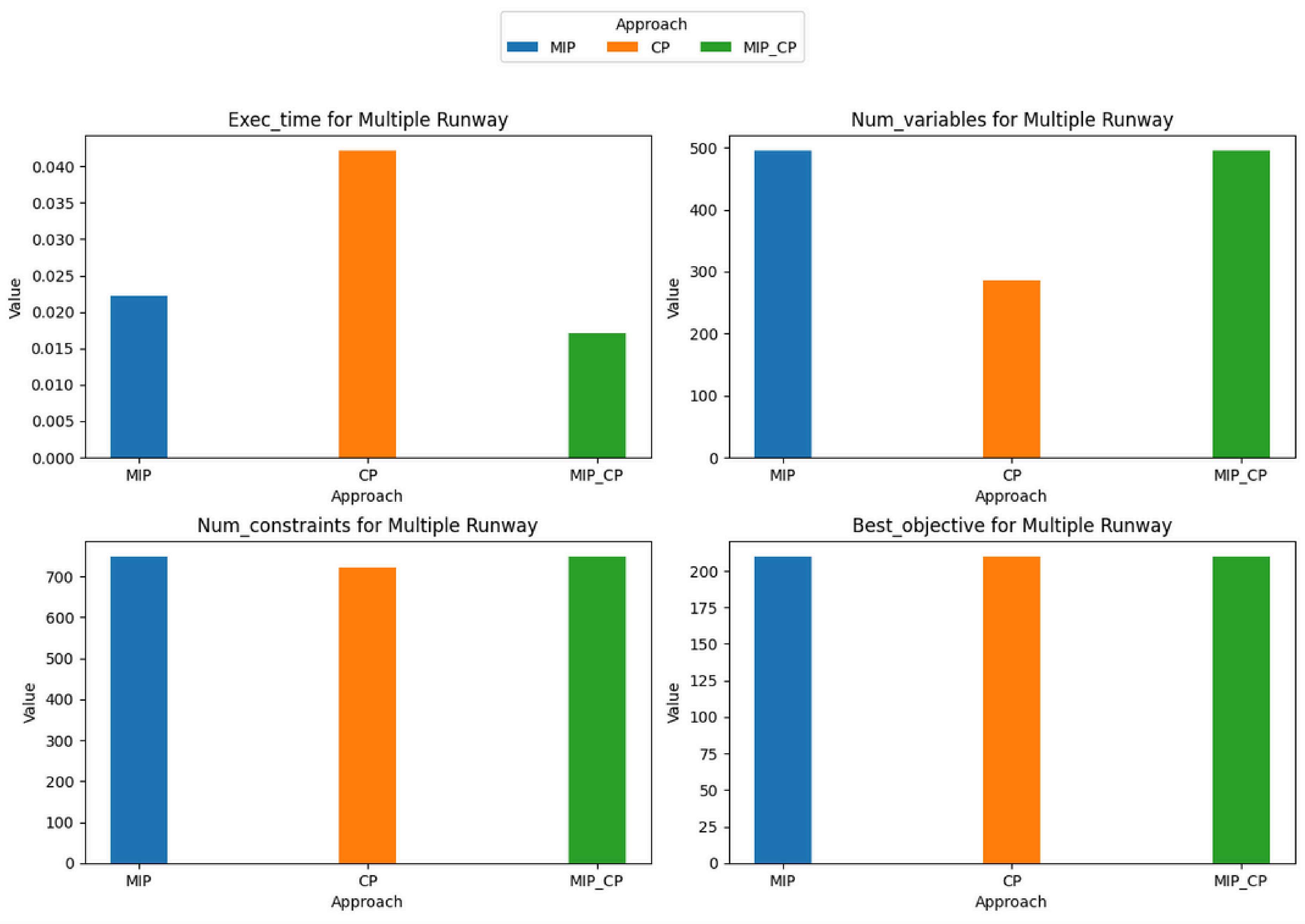
In CP, performance metrics evaluate the solver’s efficiency, solution quality, and the handling of combinatorial problems, highlighting the effectiveness of constraint handling and search strategies

- **Execution Time:** Time taken to solve the problem;
- **Solution Status:** Indicates if an optimal solution was found;
- **Number of Conflicts:** Tracks conflicts when partial solutions violate constraints;
- **Number of Branches:** Total branches explored in the search tree;
- **Best Objective Bound:** The best bound achieved during search;

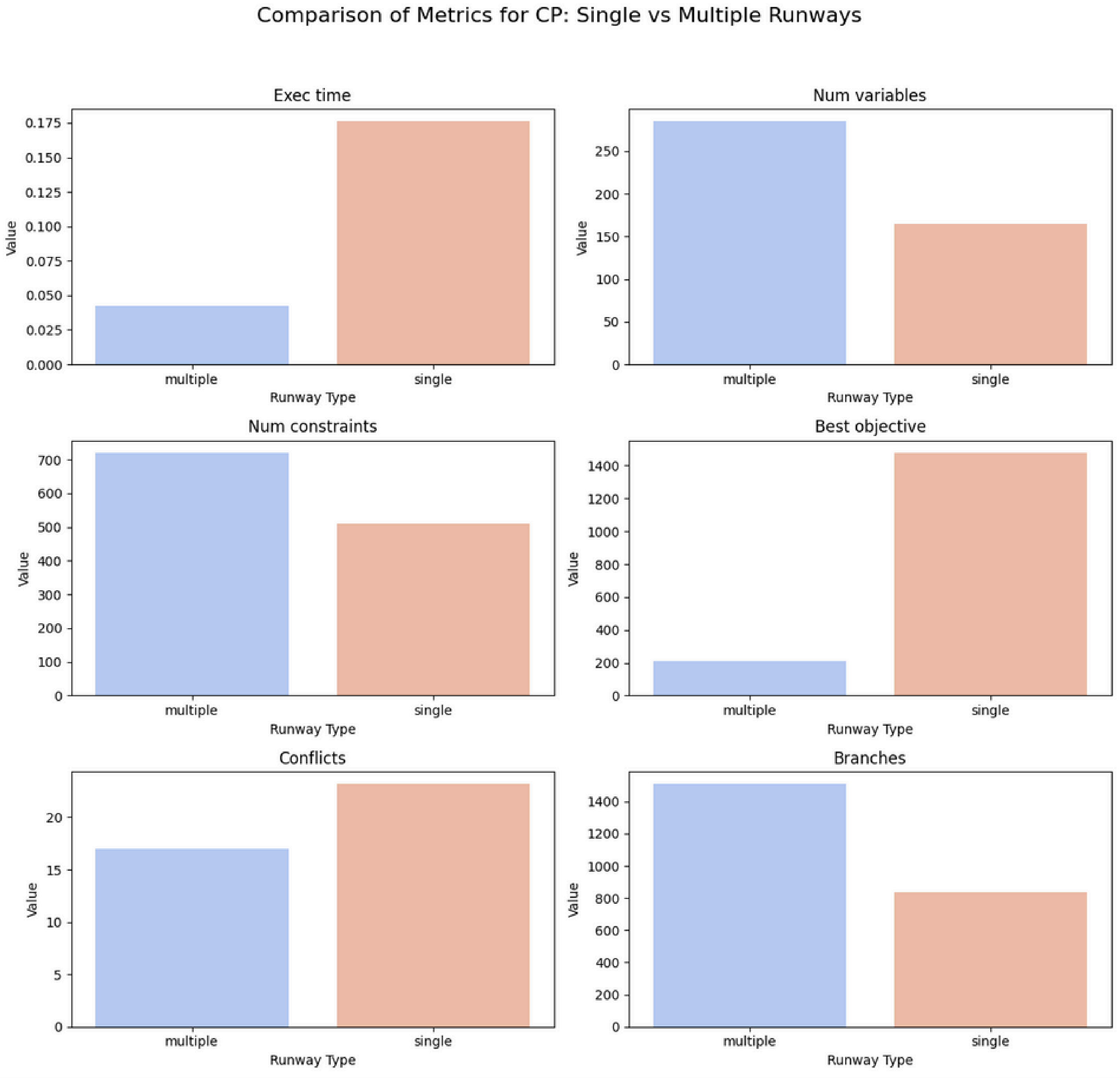
Result Analysis

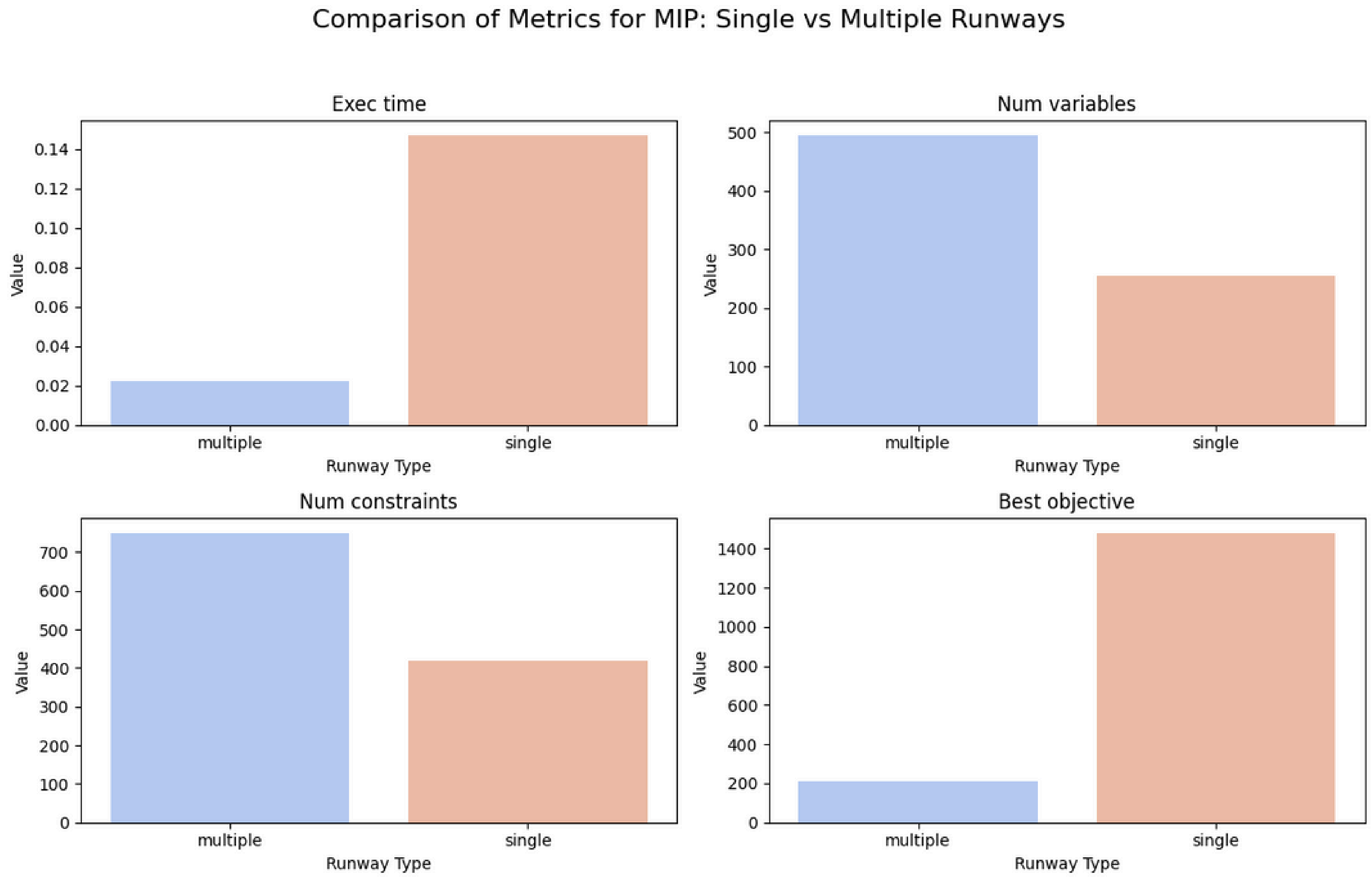
Comparison between different Approaches



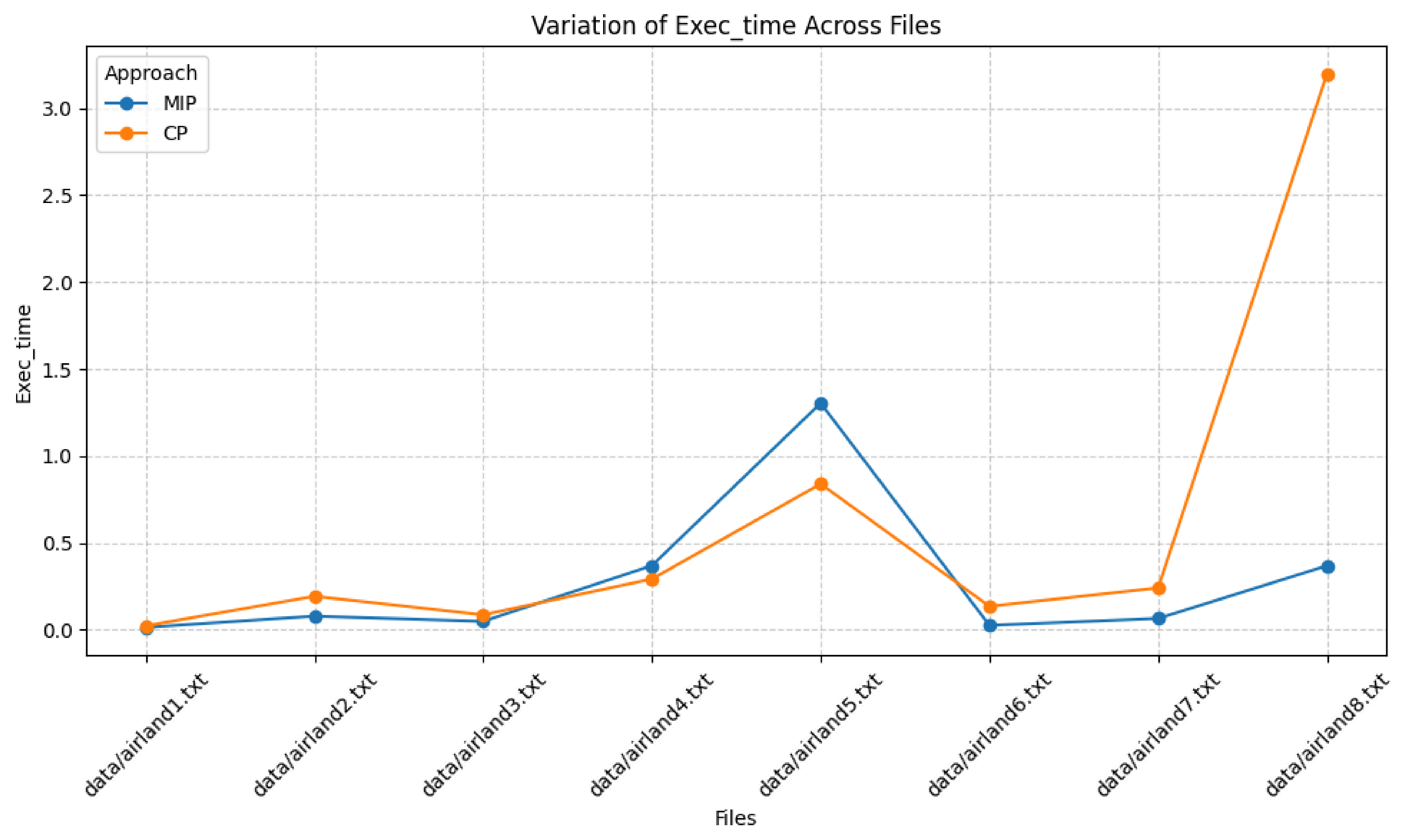


Single vs Multiple Runways





Variation of Execution Time



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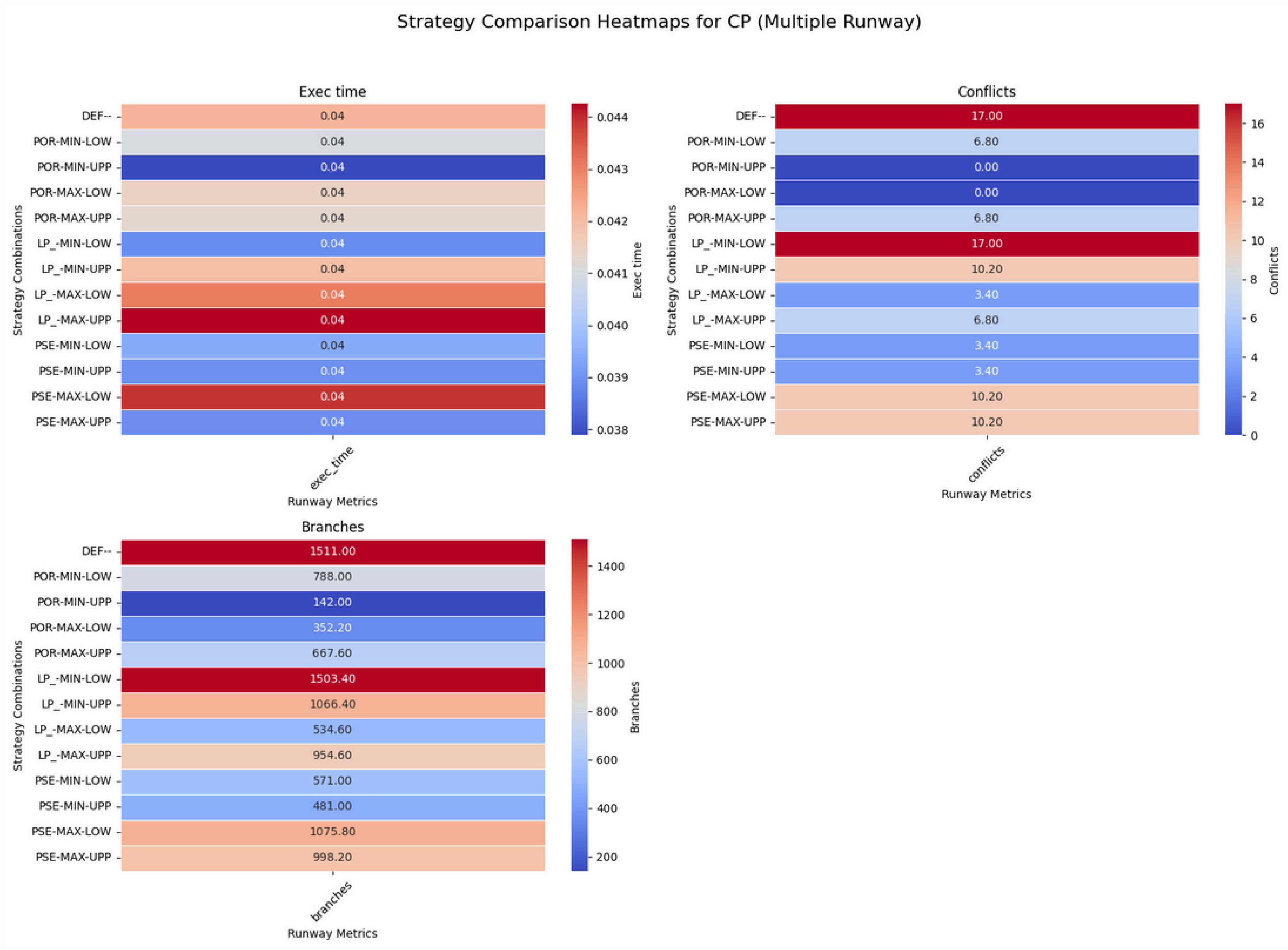
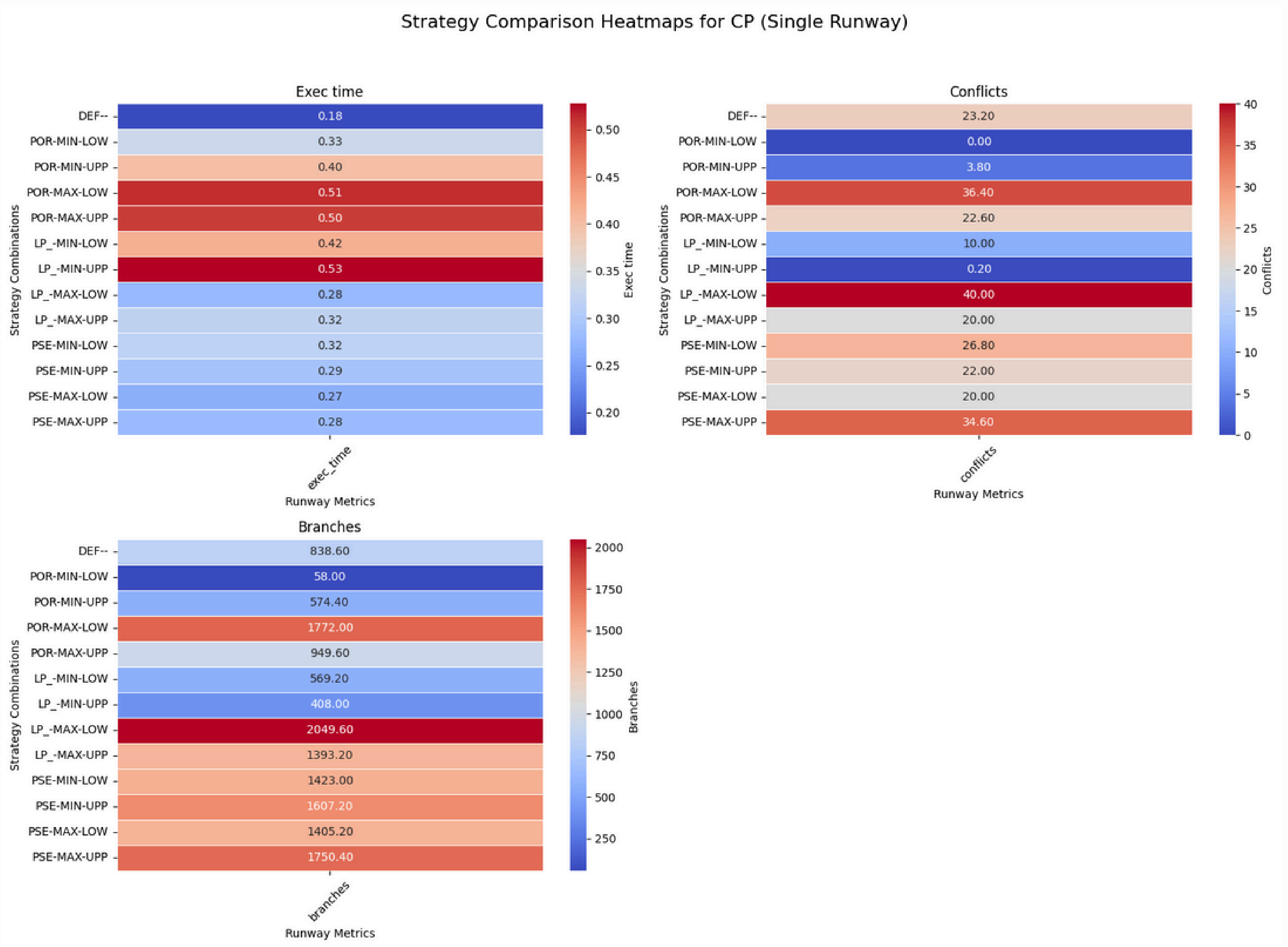
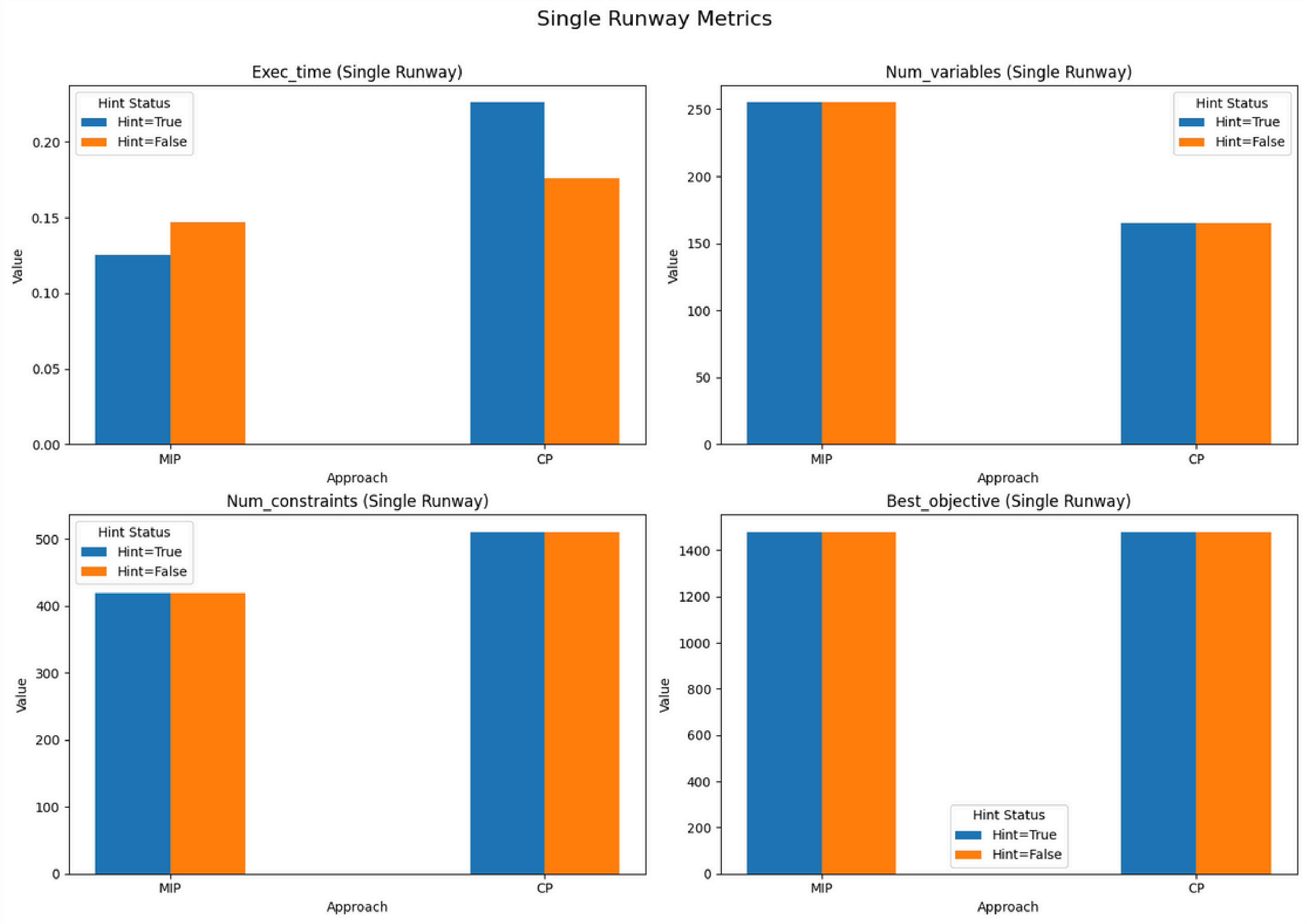
Strategies Combination

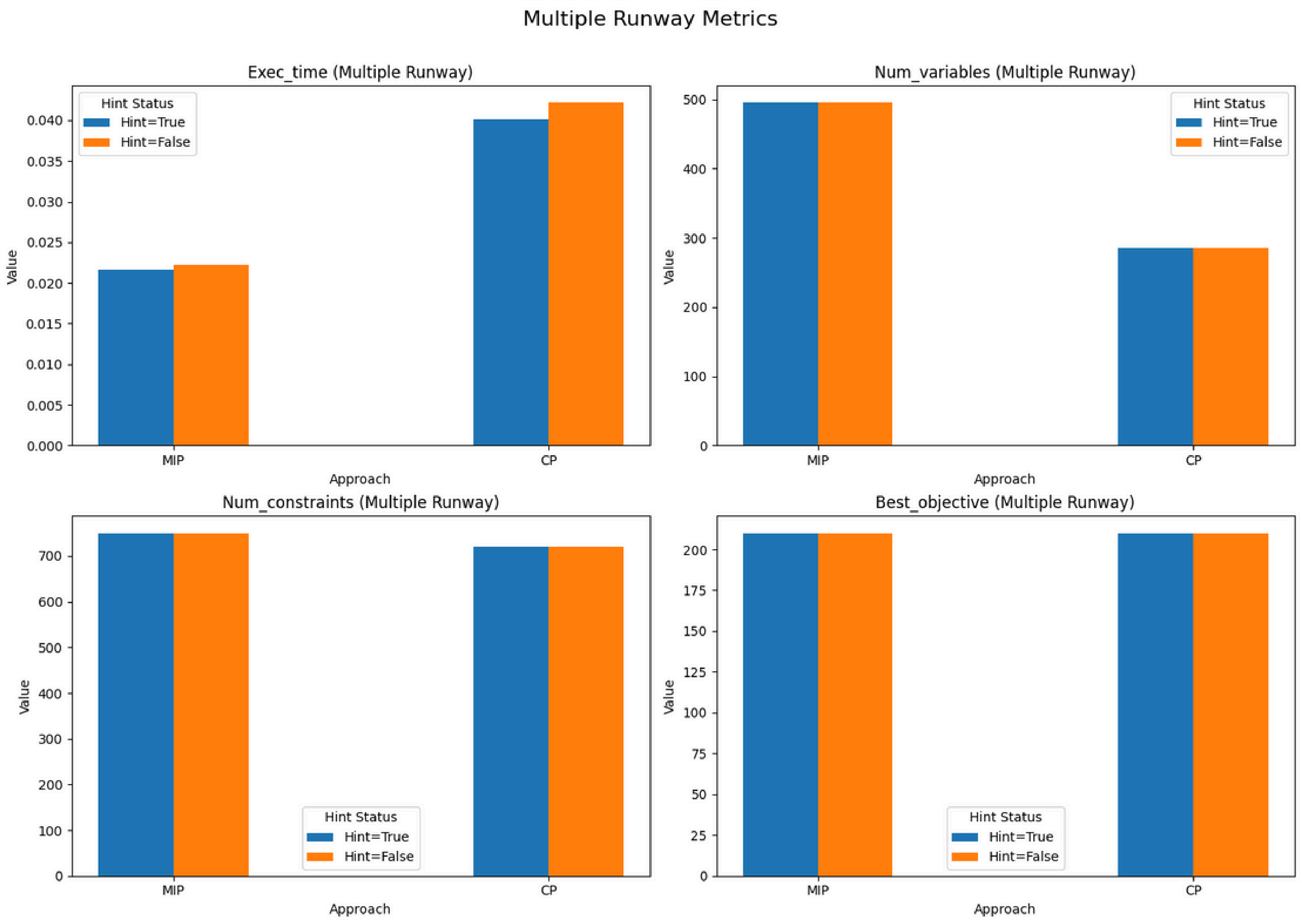




Hint Impact



Single Runway Metrics
Exec_time (Single Runway)
Num_variables (Single Runway)
Num_constraints (Single Runway)
Best_objective (Single Runway)
Hint Status
Hint=True
Hint=False
Value
Approach
MIP
CP



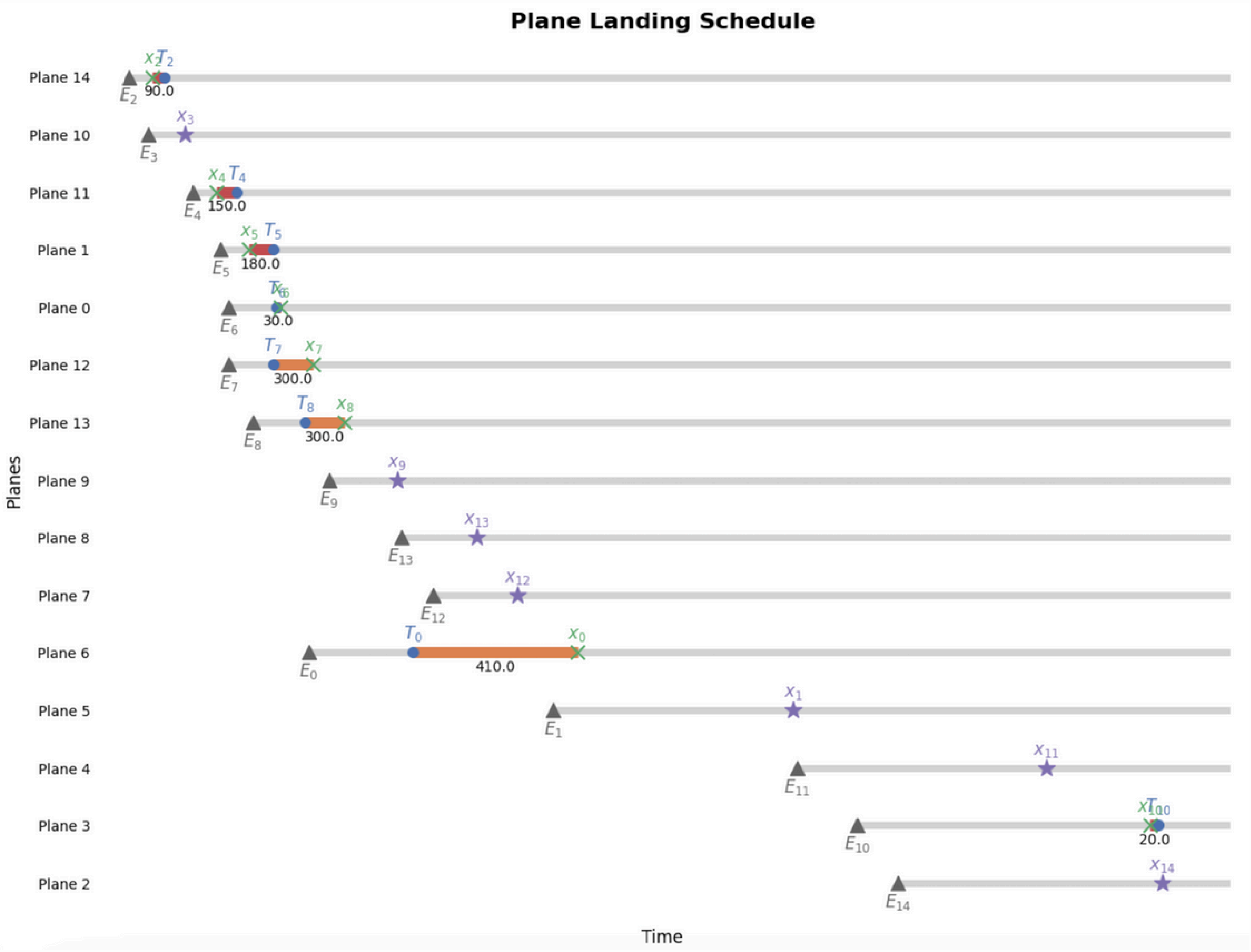
Multiple Runway Metrics
Exec_time (Multiple Runway)
Num_variables (Multiple Runway)
Num_constraints (Multiple Runway)
Best_objective (Multiple Runway)
Hint Status
Hint=True
Hint=False
Value
Approach
MIP
CP

Solution Visualization

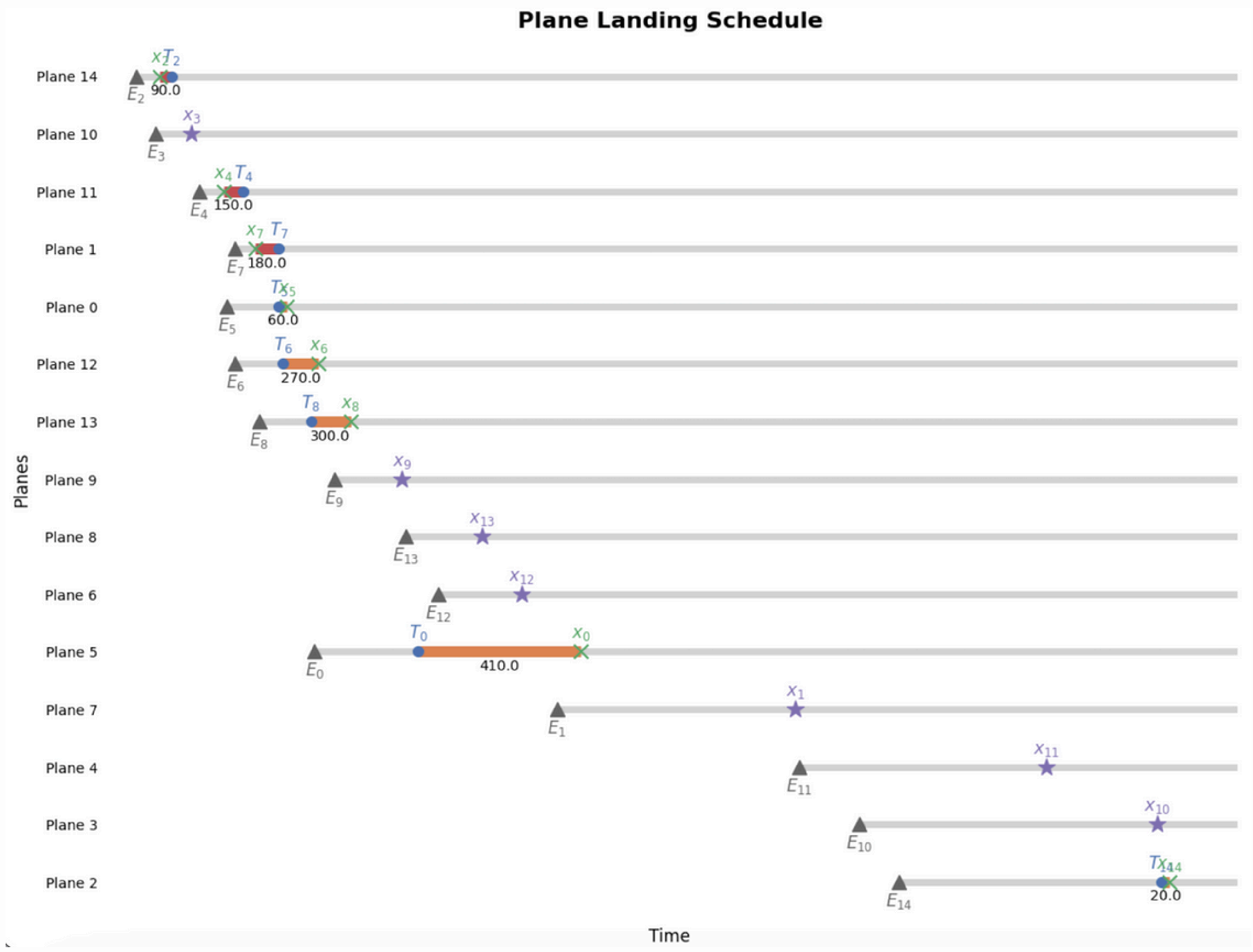
Legend
- Available Time Range
- Earliest Time (E_i)
- Latest Time (L_i)
- Target Landing Time (T_i)
- Optimal Landing Time (x_i)
- Optimal = Target ($x_i = T_i$)
- Early Deviation (α_i)
- Late Deviation (β_i)

Penalty values are shown below the deviation.

CP



MIP



Conclusion

Conclusion

- MIP, CP, and hybrid MIP_CP approaches were evaluated to solve the Aircraft Landing Scheduling Problem;
- MIP proved efficient for simpler problems, offering faster execution times;
- CP handled complex constraints effectively but required more computational resources;
- The hybrid MIP_CP approach combined MIP’s speed with CP’s flexibility, achieving balanced results;
- In multiple-runway scenarios, CP excelled due to increased flexibility, reducing conflicts and execution times;
- Strategies like branching and hints significantly improved solver efficiency by narrowing the search space;
- The choice of method depends on the problem’s complexity, with CP suited for intricate challenges, MIP for simpler cases, and hybrid methods offering versatility.

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