

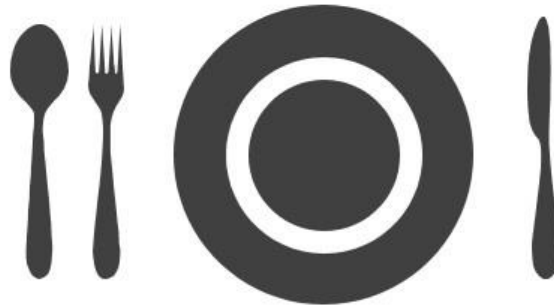


# Advanced Process Mining

*Summer Semester 2020*

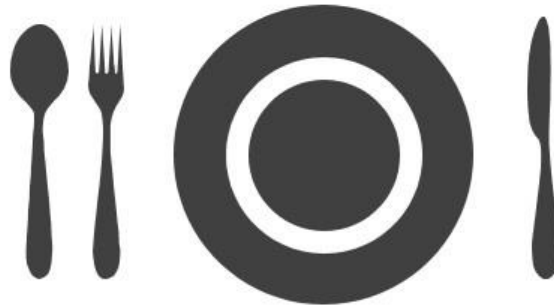
## Lecture XII: Stream Based Process Mining

dr.ir. Sebastiaan J. van Zelst



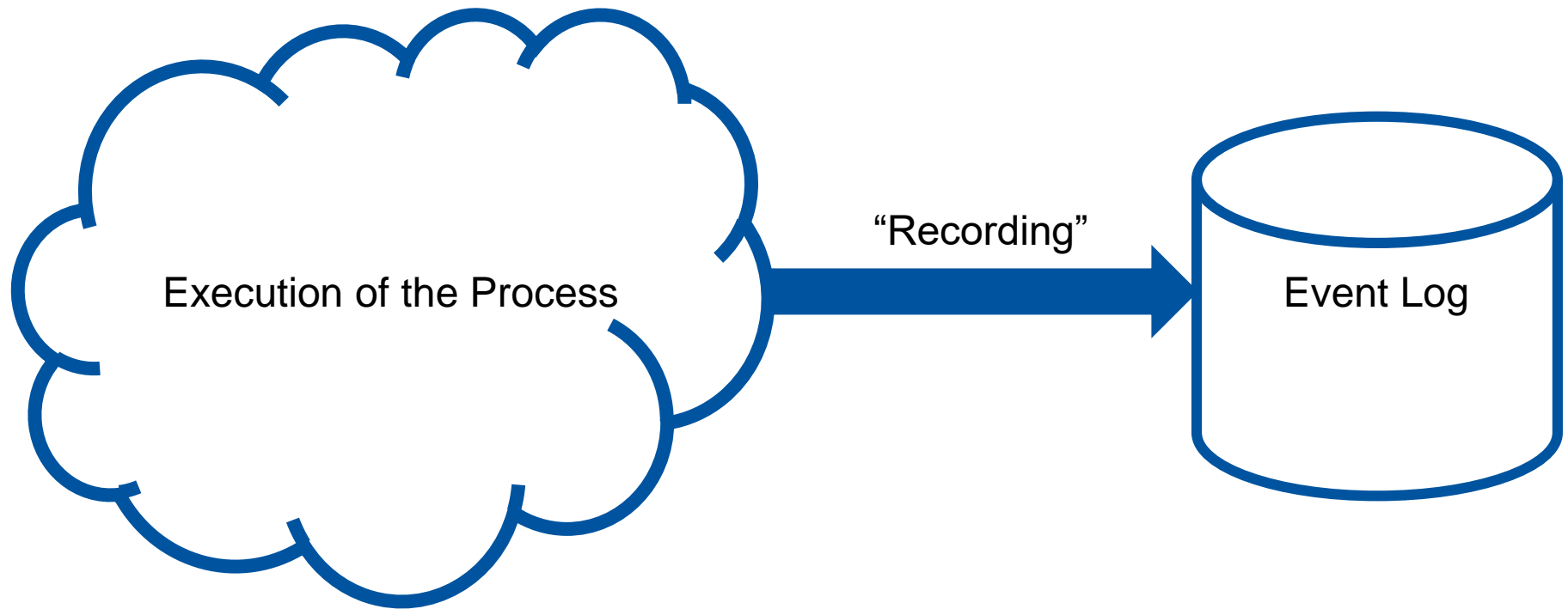
- Motivation
- Streaming Data (Theory)
- Stream Based Discovery
- Online Alignments



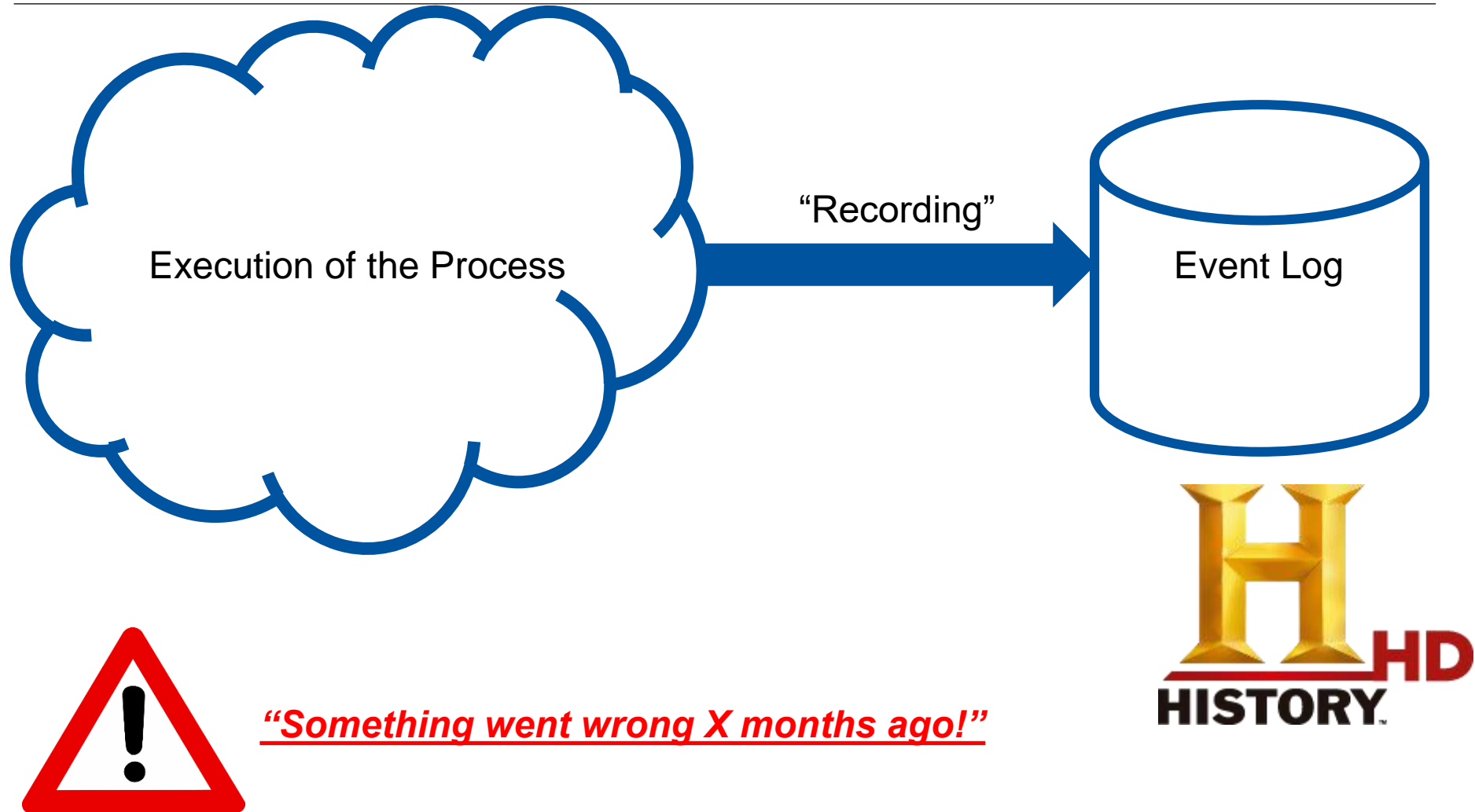


- Motivation
- Streaming Data (Theory)
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- Online Alignments



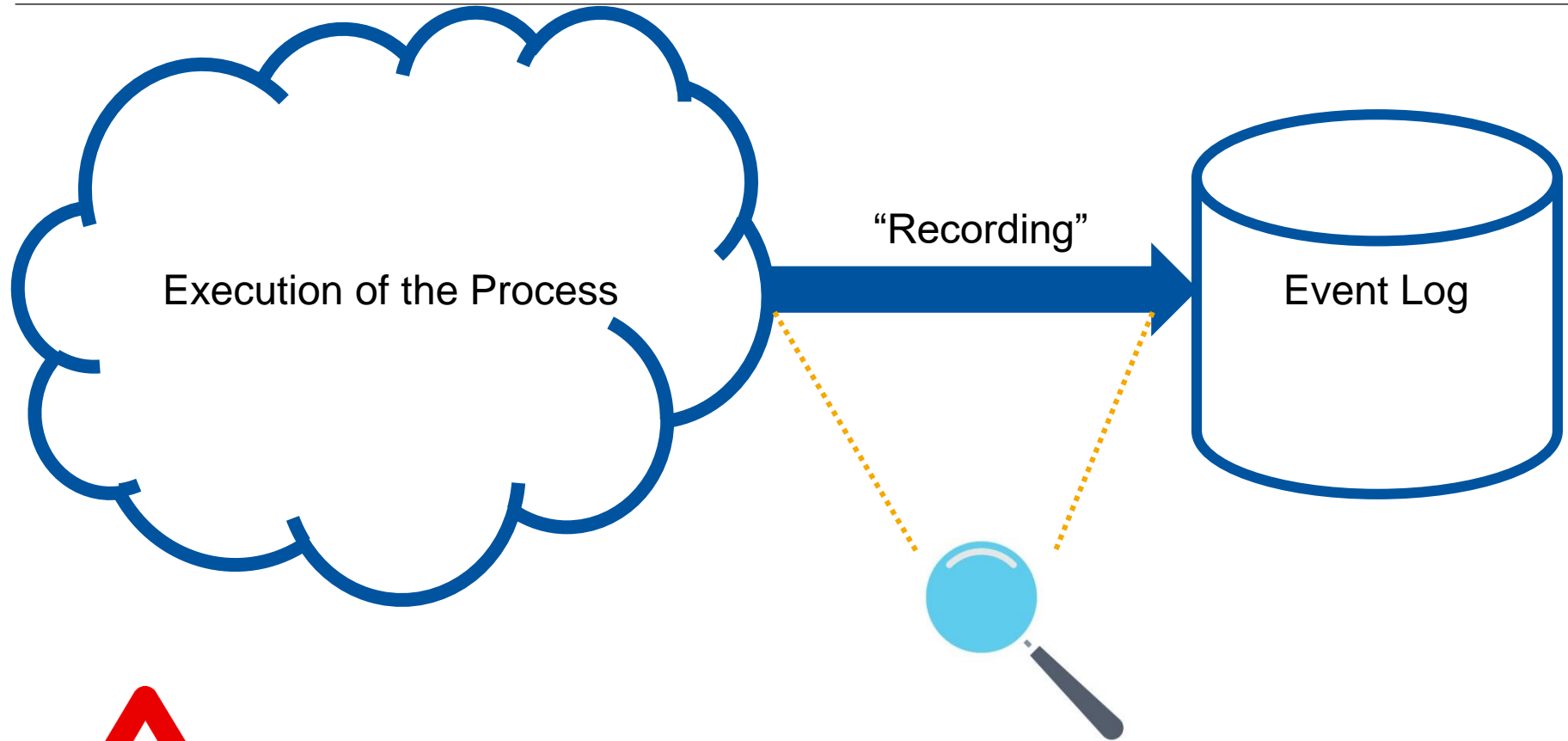


# Streaming Data Motivation



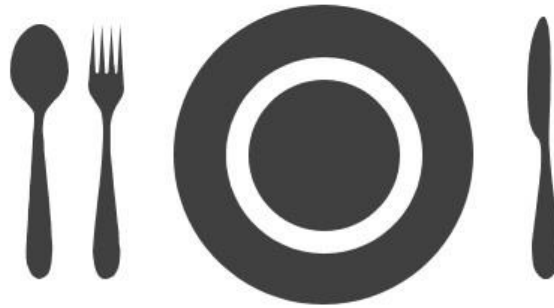
<https://directvlatam-aem65-prod.adobecqms.net/content/dam/public-sites/channels/caribbean/tt/1742.png>  
[https://www.stadt-butzbach.de/wp-content/uploads/warning-sign-30915\\_1280-1.png](https://www.stadt-butzbach.de/wp-content/uploads/warning-sign-30915_1280-1.png)

# Streaming Data Motivation



**“Something went wrong *JUST NOW!*”**

<https://directvlatam-aem65-prod.adobecqms.net/content/dam/public-sites/channels/caribbean/tt/1742.png>  
[https://www.stadt-butzbach.de/wp-content/uploads/warning-sign-30915\\_1280-1.png](https://www.stadt-butzbach.de/wp-content/uploads/warning-sign-30915_1280-1.png)



- Motivation
- Streaming Data (Theory)
- Stream Based Discovery
- Online Alignments





# Counting Numbers

---

- Let's assume, we observe an infinite sequence of numbers





# Counting Numbers

---

- Let's assume, we observe an infinite sequence of numbers

5

# Counting Numbers

---

- Let's assume, we observe an infinite sequence of numbers

37

# Counting Numbers

---

- Let's assume, we observe an infinite sequence of numbers

1,337

# Counting Numbers

---

- Let's assume, we observe an infinite sequence of numbers

1 45,327,243

# Counting Numbers

---

- Let's assume, we observe an infinite sequence of numbers

16,531

# Counting Numbers

---

- Let's assume, we observe an infinite sequence of numbers

■ ■ ■

# Counting Numbers

---

- Let's assume, we observe an infinite sequence of numbers
- **unsigned 'long' values**, i.e.,  $2^{64}$  observable numbers



# Counting Numbers

---

- Let's assume, we observe an infinite sequence of numbers
- **unsigned 'long'** values, i.e.,  $2^{64}$  observable numbers
- maintain a counter per value (again, an unsigned long)

# Counting Numbers

---

1.  $c(x) = 0$  **for each**  $x \in \{0, \dots, 2^{64}\}$
2.  $i = 1$
3. **while** true:
4.      $c(S(i))++$  // S represents 'the stream of numbers'
5.      $i++$

# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- **unsigned 'long' values**, i.e.,  $2^{64}$  observable numbers
- maintain a counter per value (again, an unsigned long)

$2^{64}$  rows

number	value
0	8 byte
1	8 byte
2	8 byte
3	8 byte
4	8 byte
5	8 byte
...	...

# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- **unsigned 'long' values**, i.e.,  $2^{64}$  observable numbers
- maintain a counter per value (again, an unsigned long)

147,573,952,589,676,412,928 bytes

$2^{64}$  rows

0	8 byte
1	8 byte
2	8 byte
3	8 byte
4	8 byte
5	8 byte
...	...

# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- **unsigned 'long'** values, i.e.,  $2^{64}$  observable numbers
- maintain a counter per value (again, an unsigned long)

147,573,952,589,676,412,928 bytes

144,115,188,075,855,872 KB

$2^{64}$  rows

2	8 byte
3	8 byte
4	8 byte
5	8 byte
...	...

# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- **unsigned 'long'** values, i.e.,  $2^{64}$  observable numbers
- maintain a counter per value (again, an unsigned long)

147,573,952,589,676,412,928 bytes

144,115,188,075,855,872 KB

140,737,488,355,328 MB

— ...

3	8 byte
4	8 byte
5	8 byte
...	...

# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- **unsigned 'long'** values, i.e.,  $2^{64}$  observable numbers
- maintain a counter per value (again, an unsigned long)

147,573,952,589,676,412,928 bytes

144,115,188,075,855,872 KB

140,737,488,355,328 MB

137,438,953,472 GB

...	...
5	8 byte
...	...



# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- **unsigned 'long'** values, i.e.,  $2^{64}$  observable numbers
- maintain a counter per value (again, an unsigned long)

147,573,952,589,676,412,928 bytes

144,115,188,075,855,872 KB

140,737,488,355,328 MB

137,438,953,472 GB

134,217,728 TB



# Counting Numbers

- Let's assume, we observe an infinite stream of
- **unsigned 'long' values**, i.e.,  $2^{32}$  possible values (unsigned long)
- maintain a counter per value

147,573,952.56

14'

Can we use less memory?  
(significantly)

bytes

35,872 KB

38,355,328 MB

37,438,953,472 GB

134,217,728 TB



# Counting Numbers

---

- Let's assume, we observe an infinite sequence of numbers
- **unsigned 'long'** values, i.e.,  $2^{64}$  observable numbers
- Let  $h : \mathbb{N} \rightarrow \{0, \dots, n\}$  here  $n$  is much smaller than  $|\mathbb{N}|$ 
  - i.e. **hash** function

# Counting Numbers

---

- Let's assume, we observe an infinite sequence of numbers
- **unsigned 'long'** values, i.e.,  $2^{64}$  observable numbers
- Let  $h : \mathbb{N} \rightarrow \{0, \dots, n\}$  here  $n$  is much smaller than  $|\mathbb{N}|$
- Consider  $n = 4$ , and  $h(i) = i \bmod 5$

# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- **unsigned 'long' values**, i.e.,  $2^{64}$  observable numbers
- Let  $h : \mathbb{N} \rightarrow \{0, \dots, n\}$  here  $n$  is much smaller than  $|\mathbb{N}|$
- Consider  $n = 4$ , and  $h(i) = i \bmod 5$
- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, ...



# Counting Numbers

---

- Let's assume, we observe an infinite sequence of numbers
- **unsigned 'long'** values, i.e.,  $2^{64}$  observable numbers
- Let  $h : \mathbb{N} \rightarrow \{0, \dots, n\}$  here  $n$  is much smaller than  $|\mathbb{N}|$
- Take  $k$  hash functions of size  $n$ , i.e.,  $h_1, h_2, \dots, h_k$

# Streaming Data

## Counting Numbers

hash ranges

	1	2	3	4	5	6	7	8	...	n
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
...	0	0	0	0	0	0	0	0	0	0
k	0	0	0	0	0	0	0	0	0	0



# Streaming Data

## Counting Numbers

- Receive 1<sup>st</sup> number: 5

hash ranges

	1	2	3	4	5	6	7	8	...	n
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
...	0	0	0	0	0	0	0	0	0	0
k	0	0	0	0	0	0	0	0	0	0

# Counting Numbers

- Receive 1<sup>st</sup> number: 5
- Calculate  $h_1(5)$

hash ranges

	1	2	3	4	5	6	7	8	...	n
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
...	0	0	0	0	0	0	0	0	0	0
k	0	0	0	0	0	0	0	0	0	0

# Counting Numbers

- Receive 1<sup>st</sup> number: 5
- Calculate  $h_1(5) = 4$

hash ranges

	1	2	3	4	5	6	7	8	...	n
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
...	0	0	0	0	0	0	0	0	0	0
k	0	0	0	0	0	0	0	0	0	0

# Counting Numbers

- Receive 1<sup>st</sup> number: 5
- Calculate  $h_1(5) = 4$

hash ranges

	1	2	3	4	5	6	7	8	...	n
1	0	0	0	1	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
...	0	0	0	0	0	0	0	0	0	0
k	0	0	0	0	0	0	0	0	0	0

# Counting Numbers

- Receive 1<sup>st</sup> number: 5
- Calculate  $h_2(5) = 2$

hash ranges

	1	2	3	4	5	6	7	8	...	n
1	0	0	0	1	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
...	0	0	0	0	0	0	0	0	0	0
k	0	0	0	0	0	0	0	0	0	0

# Counting Numbers

- Receive 1<sup>st</sup> number: 5
- Calculate  $h_2(5) = 2$

hash ranges

hash functions

	1	2	3	4	5	6	7	8	...	n
1	0	0	0	1	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
...	0	0	0	0	0	0	0	0	0	0
k	0	0	0	0	0	0	0	0	0	0

# Counting Numbers

- Receive 1<sup>st</sup> number: 5
- Calculate  $h_3(5) = 7$

hash ranges

	1	2	3	4	5	6	7	8	...	n
1	0	0	0	1	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	1	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
...	0	0	0	0	0	0	0	0	0	0
k	0	0	0	0	0	0	0	0	0	0



# Counting Numbers

- Receive 1<sup>st</sup> number: 5
- Calculate the hash value of each hash function

hash ranges

	1	2	3	4	5	6	7	8	...	n
1	0	0	0	1	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	1	0	0	0
4	0	0	1	0	0	0	0	0	0	0
5	0	0	0	1	0	0	0	0	0	0
...	0	0	0	0	0	0	0	0	1	0
k	0	0	0	0	0	0	0	1	0	0

# Counting Numbers

- Receive 2<sup>nd</sup> number: 37
- Calculate the hash value of each hash function

hash ranges

	1	2	3	4	5	6	7	8	...	n
1	1	0	0	1	0	0	0	0	0	0
2	0	1	1	0	0	0	0	0	0	0
3	0	0	0	0	0	1	1	0	0	0
4	0	0	1	1	0	0	0	0	0	0
5	0	0	0	1	0	0	1	0	0	0
...	0	1	0	0	0	0	0	0	1	0
k	1	0	0	0	0	0	0	1	0	0

# Counting Numbers

- Receive 3<sup>d</sup> number: 1,337
- Calculate the hash value of each hash function

hash ranges

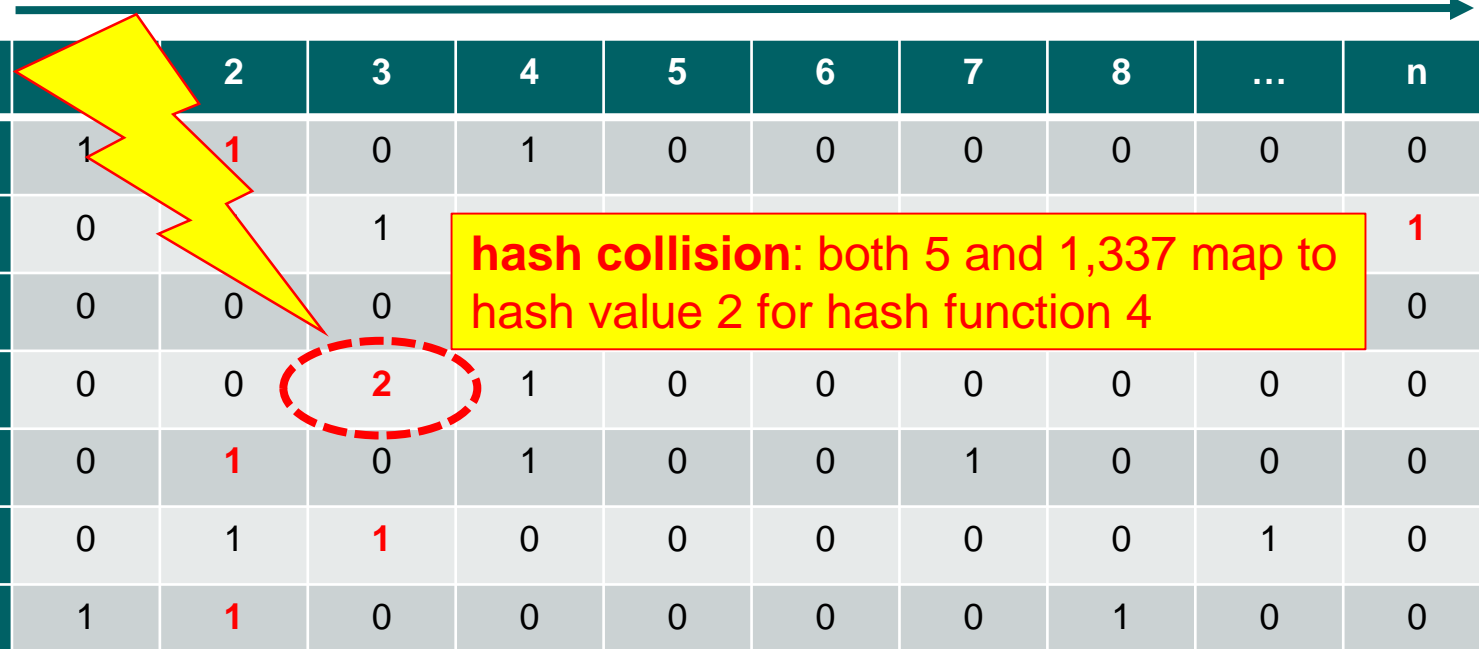
	1	2	3	4	5	6	7	8	...	n
1	1	1	0	1	0	0	0	0	0	0
2	0	1	1	0	0	0	0	0	0	1
3	0	0	0	0	1	1	1	0	0	0
4	0	0	2	1	0	0	0	0	0	0
5	0	1	0	1	0	0	1	0	0	0
...	0	1	1	0	0	0	0	0	1	0
k	1	1	0	0	0	0	0	1	0	0

# Streaming Data

## Counting Numbers

- Receive 3<sup>d</sup> number: 1,337
- Calculate the hash value of each hash function

hash ranges



		2	3	4	5	6	7	8	...	n
hash functions	1	1	0	1	0	0	0	0	0	0
2	0		1							1
3	0	0	0							0
4	0	0	2	1	0	0	0	0	0	0
5	0	1	0	1	0	0	1	0	0	0
...	0	1	1	0	0	0	0	0	1	0
k	1	1	0	0	0	0	0	1	0	0

# Counting Numbers

- Query: How many times did we observe number 5?
- Calculate the hash value of each hash function

hash ranges

	1	2	3	4	5	6	7	8	...	n
1	1	1	0	1	0	0	0	0	0	0
2	0	1	1	0	0	0	0	0	0	1
3	0	0	0	0	1	1	1	0	0	0
4	0	0	2	1	0	0	0	0	0	0
5	0	1	0	1	0	0	1	0	0	0
...	0	1	1	0	0	0	0	0	1	0
k	1	1	0	0	0	0	0	1	0	0

# Counting Numbers

- Query: How many times did we observe number 5?
- Calculate the hash value of each hash function

hash ranges

	1	2	3	4	5	6	7	8	...	n
1	1	1	0	1	0	0	0	0	0	0
2	0	1	1	0	0	0	0	0	0	1
3	0	0	0	0	1	1	1	0	0	0
4	0	0	2	1	0	0	0	0	0	0
5	0	1	0	1	0	0	1	0	0	0
...	0	1	1	0	0	0	0	0	1	0
k	1	1	0	0	0	0	0	1	0	0

# Counting Numbers

- Query: How many times did we observe  $r$ ?
- Calculate the hash value of each hash

hash ranges

hash functions

What value to pick?

	1	2	3	...	7	8	...	n
1	1	1	0	0	0	0	0	0
2	0	1	0	0	0	0	0	1
3	0	0	0	1	1	1	0	0
4	0	0	1	0	0	0	0	0
5	0	1	0	1	0	0	1	0
...	0	1	1	0	0	0	0	1
k	1	1	0	0	0	0	1	0

# Counting Numbers

---

- Query: How many times did we observe number 5?
- Calculate the hash value of each hash function
- Every value in each cell can be seen as follows:

number of occurrences of 5

number of occurrences of others



# Counting Numbers

- Query: How many times did we observe number 5?
- Calculate the hash value of each hash function
- Every value in each cell can be seen as follows:



- Looking at all cells for value 5, we see:



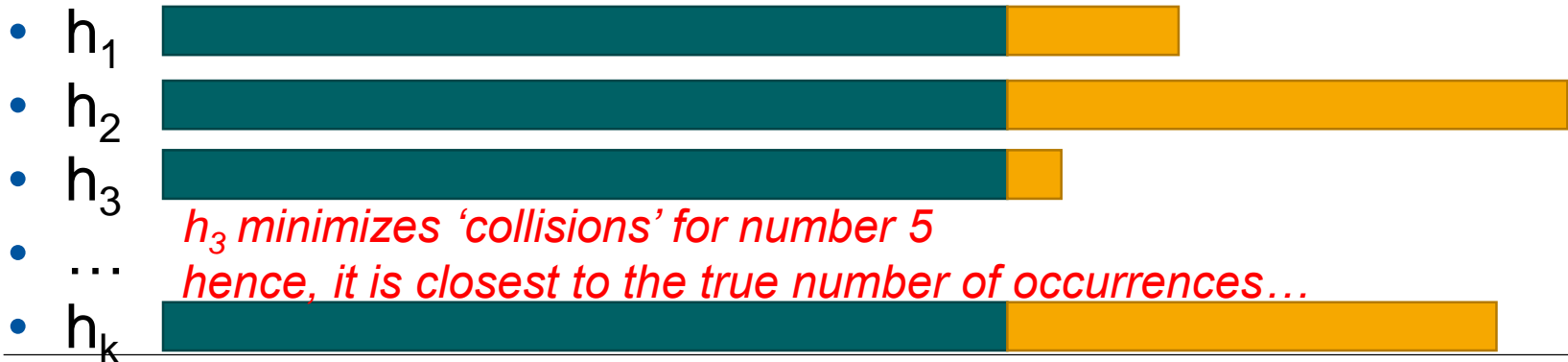
# Counting Numbers

- Query: How many times did we observe number 5?
- Calculate the hash value of each hash function
- Every value in each cell can be seen as follows:

number of occurrences of 5

number of occurrences of others

- Looking at all cells for value 5, we see:



*$h_3$  minimizes 'collisions' for number 5*

*hence, it is closest to the true number of occurrences...*

# Counting Numbers

---

- *Count-Min-Sketch:*
- Memory:  $O(kn)$

# Counting Numbers

---

- *Count-Min-Sketch:*
- Memory:  $O(kn)$
- Let  $N$  denote the number of elements seen so far
- Let  $\epsilon, \delta \in [0,1]$
- If we let:  $k = \lceil \ln \left( \frac{1}{\delta} \right) \rceil$  and  $n = \lceil \frac{e}{\epsilon} \rceil$  then:
- $\hat{x}_i \leq x_i + \epsilon N$  with probability  $1 - \delta$

# Counting Numbers

---

- Let's assume, we observe an infinite sequence of numbers
- What are the  $k$  most frequent numbers observed?

# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- What are the  $k$  most frequent numbers observed?

**Algorithm 3.1:** FREQUENT( $k$ )

```

 $n \leftarrow 0; T \leftarrow \emptyset;$ 
for each  $i$  :
    do {
         $n \leftarrow n + 1;$ 
        if  $i \in T$ 
            then  $c_i \leftarrow c_i + 1;$ 
            else if  $|T| < k - 1$ 
                then {
                     $T \leftarrow T \cup \{i\};$ 
                     $c_i \leftarrow 1;$ 
                }
            else for all  $j \in T$ 
                do {
                     $c_j \leftarrow c_j - 1;$ 
                    if  $c_j = 0$ 
                        then  $T \leftarrow T \setminus \{j\};$ 
                }
    }

```

# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- What are the  $k$  most frequent numbers observed?
- Observe: 5

num	val
5	1

## Algorithm 3.1: FREQUENT( $k$ )

```

 $n \leftarrow 0; T \leftarrow \emptyset;$ 
for each  $i$  :
     $n \leftarrow n + 1;$ 
    if  $i \in T$ 
        then  $c_i \leftarrow c_i + 1;$ 
        else if  $|T| < k - 1$ 
            then  $\begin{cases} T \leftarrow T \cup \{i\}; \\ c_i \leftarrow 1; \end{cases}$ 
        else for all  $j \in T$ 
            do  $\begin{cases} c_j \leftarrow c_j - 1; \\ \text{if } c_j = 0 \\ \text{then } T \leftarrow T \setminus \{j\}; \end{cases}$ 

```

# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- What are the  $k$  most frequent numbers observed?
- Observe: 13

num	val
5	1
13	1

## Algorithm 3.1: FREQUENT( $k$ )

```

 $n \leftarrow 0; T \leftarrow \emptyset;$ 
for each  $i$  :
     $n \leftarrow n + 1;$ 
    if  $i \in T$ 
        then  $c_i \leftarrow c_i + 1;$ 
        else if  $|T| < k - 1$ 
            then  $\begin{cases} T \leftarrow T \cup \{i\}; \\ c_i \leftarrow 1; \end{cases}$ 
        else for all  $j \in T$ 
            do  $\begin{cases} c_j \leftarrow c_j - 1; \\ \text{if } c_j = 0 \\ \text{then } T \leftarrow T \setminus \{j\}; \end{cases}$ 

```



# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- What are the  $k$  most frequent numbers observed?
- Observe: 1337

num	val
5	1
13	1
1337	1

## Algorithm 3.1: FREQUENT( $k$ )

```

 $n \leftarrow 0; T \leftarrow \emptyset;$ 
for each  $i$  :
     $n \leftarrow n + 1;$ 
    if  $i \in T$ 
        then  $c_i \leftarrow c_i + 1;$ 
        else if  $|T| < k - 1$ 
            then  $\begin{cases} T \leftarrow T \cup \{i\}; \\ c_i \leftarrow 1; \end{cases}$ 
        else for all  $j \in T$ 
            do  $\begin{cases} c_j \leftarrow c_j - 1; \\ \text{if } c_j = 0 \\ \text{then } T \leftarrow T \setminus \{j\}; \end{cases}$ 

```

# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- What are the  $k$  most frequent numbers observed?
- Observe: ...

num	val
5	1
13	1
1337	1
...	...
...	...
...	...

**Algorithm 3.1:** FREQUENT( $k$ )

```

 $n \leftarrow 0; T \leftarrow \emptyset;$ 
for each  $i$  :
     $n \leftarrow n + 1;$ 
    if  $i \in T$ 
        then  $c_i \leftarrow c_i + 1;$ 
        else if  $|T| < k - 1$ 
            then  $\begin{cases} T \leftarrow T \cup \{i\}; \\ c_i \leftarrow 1; \end{cases}$ 
        else for all  $j \in T$ 
            do  $\begin{cases} c_j \leftarrow c_j - 1; \\ \text{if } c_j = 0 \\ \text{then } T \leftarrow T \setminus \{j\}; \end{cases}$ 

```

# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- What are the  $k$  most frequent numbers observed?
- Observe: 8

num	val
5	2
13	5
1337	1
...	...
...	...
...	...
8	1

## Algorithm 3.1: FREQUENT( $k$ )

```

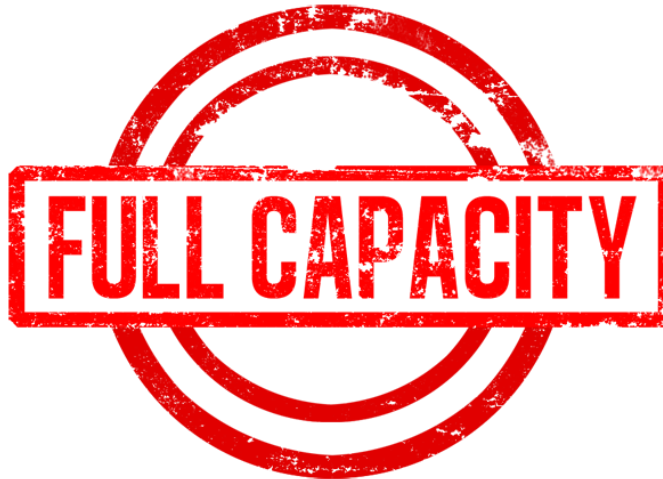
 $n \leftarrow 0; T \leftarrow \emptyset;$ 
for each  $i$  :
     $n \leftarrow n + 1;$ 
    if  $i \in T$ 
        then  $c_i \leftarrow c_i + 1;$ 
        else if  $|T| < k - 1$ 
            then  $\begin{cases} T \leftarrow T \cup \{i\}; \\ c_i \leftarrow 1; \end{cases}$ 
        else for all  $j \in T$ 
            do  $\begin{cases} c_j \leftarrow c_j - 1; \\ \text{if } c_j = 0 \\ \text{then } T \leftarrow T \setminus \{j\}; \end{cases}$ 

```

# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- What are the  $k$  most frequent numbers observed?
- Observe: 175

num	val
5	2
13	5
1337	1
...	...
...	...
...	...
8	1



## Algorithm 3.1: FREQUENT( $k$ )

```

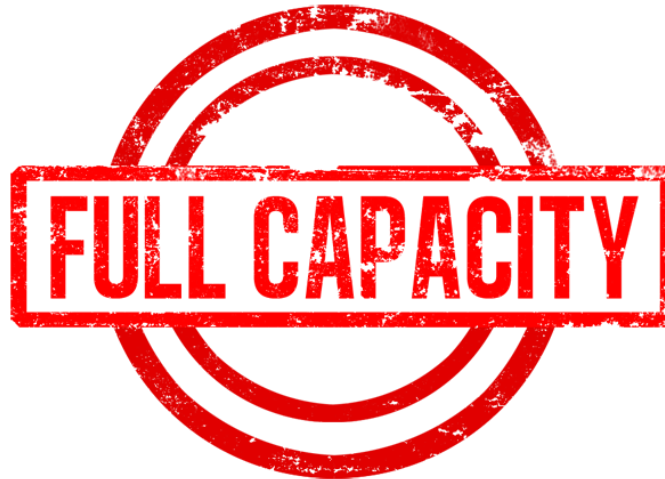
 $n \leftarrow 0; T \leftarrow \emptyset;$ 
for each  $i$  :
  do {
     $n \leftarrow n + 1;$ 
    if  $i \in T$ 
      then  $c_i \leftarrow c_i + 1;$ 
    else if  $|T| < k - 1$ 
      then  $\begin{cases} T \leftarrow T \cup \{i\}; \\ c_i \leftarrow 1; \end{cases}$ 
    else for all  $j \in T$ 
      do {
         $c_j \leftarrow c_j - 1;$ 
        if  $c_j = 0$ 
          then  $T \leftarrow T \setminus \{j\};$ 
      }
  }
```

# Streaming Data

## Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- What are the  $k$  most frequent numbers observed?
- Observe: 175

num	val
5	1
13	4
1337	0
...	...
...	...
...	...
8	0



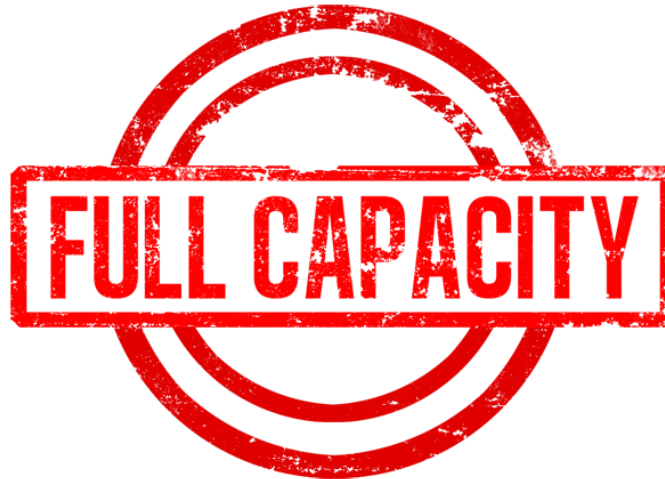
**Algorithm 3.1:** FREQUENT( $k$ )

```
 $n \leftarrow 0; T \leftarrow \emptyset;$   
for each  $i$  :  
  do  $\left\{ \begin{array}{l} n \leftarrow n + 1; \\ \text{if } i \in T \\ \quad \text{then } c_i \leftarrow c_i + 1; \\ \quad \text{else if } |T| < k - 1 \\ \quad \quad \text{then } \left\{ \begin{array}{l} T \leftarrow T \cup \{i\}; \\ c_i \leftarrow 1; \end{array} \right. \\ \quad \text{else for all } j \in T \\ \quad \quad \text{do } \left\{ \begin{array}{l} c_j \leftarrow c_j - 1; \\ \text{if } c_j = 0 \\ \quad \text{then } T \leftarrow T \setminus \{j\}; \end{array} \right. \end{array} \right.$ 
```

# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- What are the  $k$  most frequent numbers observed?
- Observe: 175

num	val
5	1
13	4
1337	0
...	...
...	...
...	...
0	0



## Algorithm 3.1: FREQUENT( $k$ )

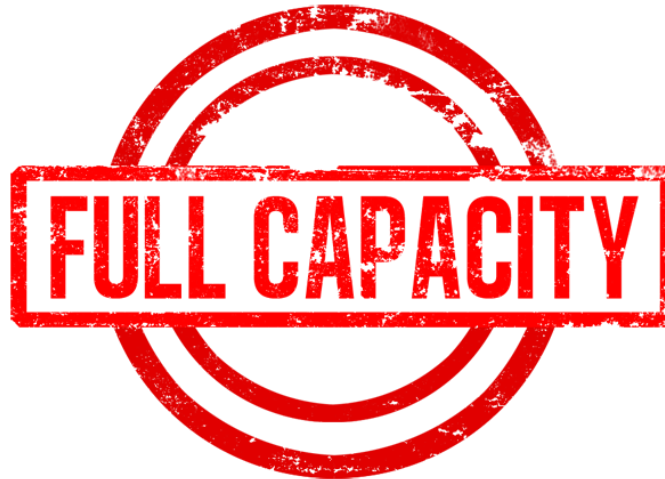
```

 $n \leftarrow 0; T \leftarrow \emptyset;$ 
for each  $i$  :
  do {
     $n \leftarrow n + 1;$ 
    if  $i \in T$ 
      then  $c_i \leftarrow c_i + 1;$ 
    else if  $|T| < k - 1$ 
      then {
         $T \leftarrow T \cup \{i\};$ 
         $c_i \leftarrow 1;$ 
      }
    else for all  $j \in T$ 
      do {
         $c_j \leftarrow c_j - 1;$ 
        if  $c_j = 0$ 
          then  $T \leftarrow T \setminus \{j\};$ 
      }
  }
```

# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- What are the  $k$  most frequent numbers observed?
- Observe: 175

num	val
5	1
13	4
...	...
...	...
...	...
...	...
...	...
...	...



## Algorithm 3.1: FREQUENT( $k$ )

```

 $n \leftarrow 0; T \leftarrow \emptyset;$ 
for each  $i$  :
  do {
     $n \leftarrow n + 1;$ 
    if  $i \in T$ 
      then  $c_i \leftarrow c_i + 1;$ 
    else if  $|T| < k - 1$ 
      then  $\begin{cases} T \leftarrow T \cup \{i\}; \\ c_i \leftarrow 1; \end{cases}$ 
    else for all  $j \in T$ 
      do {
         $c_j \leftarrow c_j - 1;$ 
        if  $c_j = 0$ 
          then  $T \leftarrow T \setminus \{j\};$ 
      }
  }
```

# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- What are the  $k$  most frequent numbers observed?

## Algorithm 3.1: FREQUENT( $k$ )

```

 $n \leftarrow 0; T \leftarrow \emptyset;$ 
for each  $i$  :
   $n \leftarrow n + 1;$ 
  if  $i \in T$ 
    then  $c_i \leftarrow c_i + 1;$ 
  else if  $|T| < k - 1$ 
    then  $\begin{cases} T \leftarrow T \cup \{i\}; \\ c_i \leftarrow 1; \end{cases}$ 
  else for all  $j \in T$ 
    do  $\begin{cases} c_j \leftarrow c_j - 1; \\ \text{if } c_j = 0 \\ \text{then } T \leftarrow T \setminus \{j\}; \end{cases}$ 

```

## Algorithm 3.3: SPACESAVING( $k$ )

```

 $n \leftarrow 0;$ 
 $T \leftarrow \emptyset;$ 
for each  $i$  :
   $n \leftarrow n + 1;$ 
  if  $i \in T$ 
    then  $c_i \leftarrow c_i + 1;$ 
  else if  $|T| < k$ 
    then  $\begin{cases} T \leftarrow T \cup \{i\}; \\ c_i \leftarrow 1; \end{cases}$ 
  else  $\begin{cases} j \leftarrow \arg \min_{j \in T} c_j; \\ c_i \leftarrow c_j + 1; \\ T \leftarrow T \cup \{i\} \setminus \{j\}; \end{cases}$ 

```



# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- What are the  $k$  most frequent numbers observed?
- Observe: 8

num	val
5	2
13	5
1337	1
...	...
...	...
...	...
8	1

## Algorithm 3.3: SPACESAVING( $k$ )

```

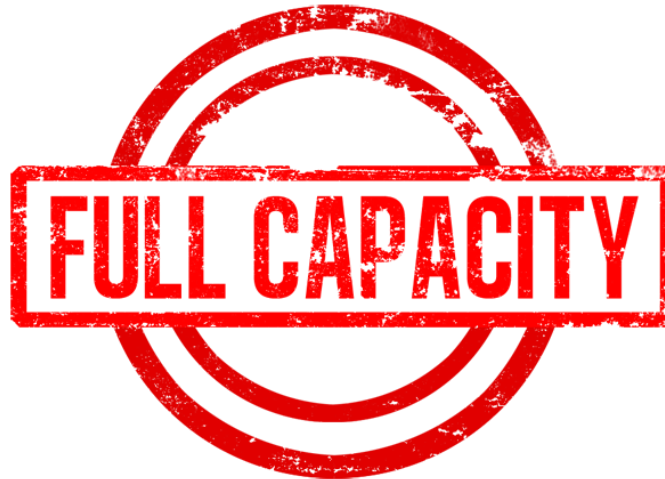
 $n \leftarrow 0;$ 
 $T \leftarrow \emptyset;$ 
for each  $i$  :
    do  $\left\{ \begin{array}{l} n \leftarrow n + 1; \\ \text{if } i \in T \\ \text{then } c_i \leftarrow c_i + 1; \\ \text{else if } |T| < k \\ \text{then } \left\{ \begin{array}{l} T \leftarrow T \cup \{i\}; \\ c_i \leftarrow 1; \end{array} \right. \\ \text{else } \left\{ \begin{array}{l} j \leftarrow \arg \min_{j \in T} c_j; \\ c_i \leftarrow c_j + 1; \\ T \leftarrow T \cup \{i\} \setminus \{j\}; \end{array} \right. \end{array} \right.$ 

```

# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- What are the  $k$  most frequent numbers observed?
- Observe: 175

num	val
5	2
13	5
1337	1
...	...
...	...
...	...
8	1



**Algorithm 3.3:** SPACESAVING( $k$ )

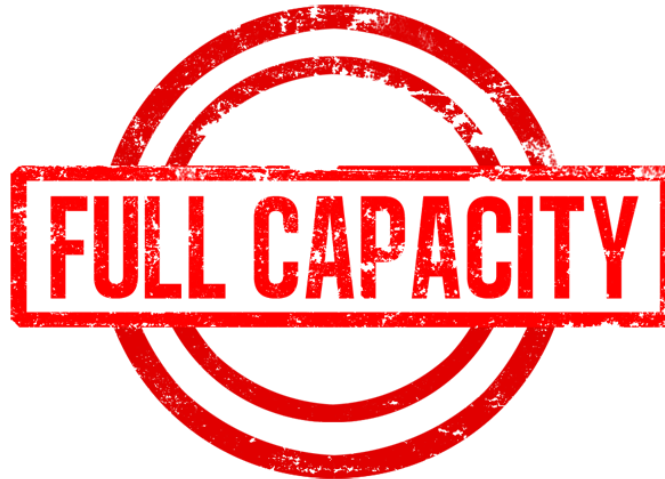
```

 $n \leftarrow 0;$ 
 $T \leftarrow \emptyset;$ 
for each  $i$  :
  do {
     $n \leftarrow n + 1;$ 
    if  $i \in T$ 
      then  $c_i \leftarrow c_i + 1;$ 
    else if  $|T| < k$ 
      then {
         $T \leftarrow T \cup \{i\};$ 
         $c_i \leftarrow 1;$ 
      }
    else {
       $j \leftarrow \arg \min_{j \in T} c_j;$ 
       $c_i \leftarrow c_j + 1;$ 
       $T \leftarrow T \cup \{i\} \setminus \{j\};$ 
    }
  }
```

# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- What are the  $k$  most frequent numbers observed?
- Observe: 175

num	val
5	2
13	5
<b>1337</b>	<b>1</b>
...	...
...	...
...	...
8	1



## Algorithm 3.3: SPACESAVING( $k$ )

```

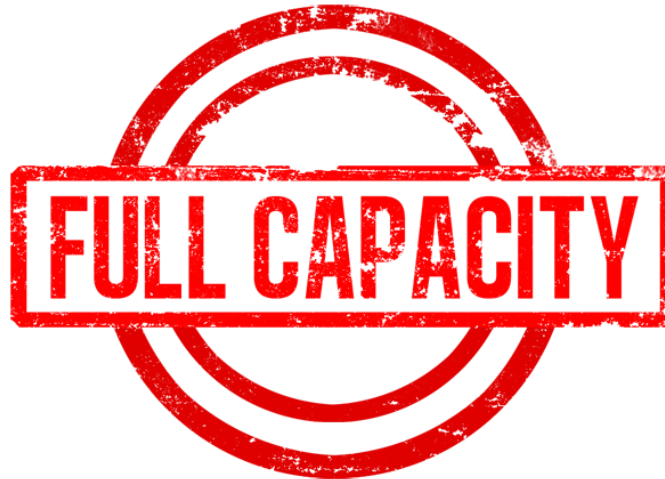
 $n \leftarrow 0;$ 
 $T \leftarrow \emptyset;$ 
for each  $i$  :
  do {
     $n \leftarrow n + 1;$ 
    if  $i \in T$ 
      then  $c_i \leftarrow c_i + 1;$ 
    else if  $|T| < k$ 
      then {
         $T \leftarrow T \cup \{i\};$ 
         $c_i \leftarrow 1;$ 
      }
    else {
       $j \leftarrow \arg \min_{j \in T} c_j;$ 
       $c_i \leftarrow c_j + 1;$ 
       $T \leftarrow T \cup \{i\} \setminus \{j\};$ 
    }
  }
```

# Streaming Data

## Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- What are the  $k$  most frequent numbers observed?
- Observe: 175

num	val
5	2
13	5
175	2
...	...
...	...
...	...
8	1



**Algorithm 3.3:** SPACESAVING( $k$ )

```
 $n \leftarrow 0;$   
 $T \leftarrow \emptyset;$   
for each  $i$  :  
do {  
   $n \leftarrow n + 1;$   
  if  $i \in T$   
  then  $c_i \leftarrow c_i + 1;$   
  else if  $|T| < k$   
  then {  $T \leftarrow T \cup \{i\};$   
         $c_i \leftarrow 1;$   
  }  
  else {  $j \leftarrow \arg \min_{j \in T} c_j;$   
         $c_i \leftarrow c_j + 1;$   
         $T \leftarrow T \cup \{i\} \setminus \{j\};$   
  }
```

# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- What are the  $k$  most frequent numbers observed?

**Algorithm 3.2:** LOSSYCOUNTING( $k$ )

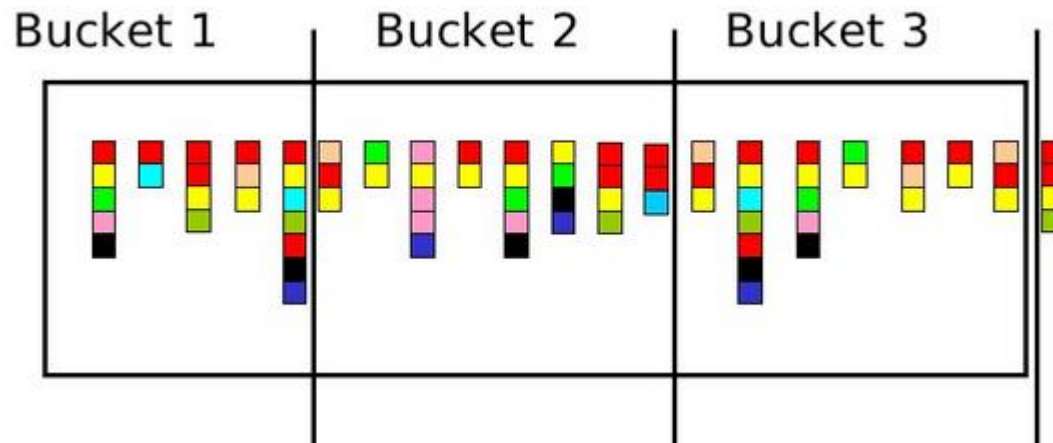
```

 $n \leftarrow 0; \Delta \leftarrow 0; T \leftarrow \emptyset;$ 
for each  $i$  :
  do  $\left\{ \begin{array}{l} n \leftarrow n + 1; \\ \text{if } i \in T \\ \text{then } c_i \leftarrow c_i + 1; \\ \text{else } \left\{ \begin{array}{l} T \leftarrow T \cup \{i\}; \\ c_j \leftarrow 1 + \Delta; \end{array} \right. \\ \text{if } \lfloor \frac{n}{k} \rfloor \neq \Delta \\ \text{then } \left\{ \begin{array}{l} \Delta \leftarrow n/k; \\ \text{for all } j \in T \\ \text{do if } c_j < \Delta \\ \text{then } T \leftarrow T \setminus \{j\} \end{array} \right. \end{array} \right.$ 

```

# Counting Numbers

- Let's assume, we observe an infinite sequence of numbers
- What are the  $k$  most frequent numbers observed?



# General Idea/Requirements

---

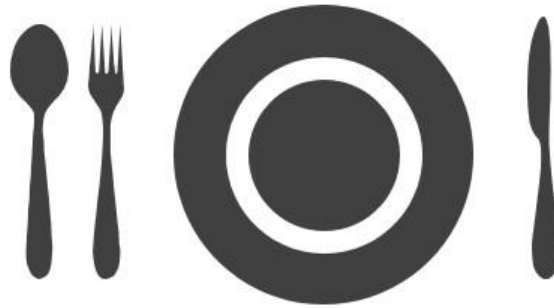
- Let's assume, we observe an infinite sequence of objects
- Any answer we can provide, typically, is an approximate answer.

# General Idea/Requirements

---

- Let's assume, we observe an infinite sequence of objects
- Any answer we can provide, typically, is an approximate answer.
- Typically:
  - Result is  $1 \pm \epsilon$  from true value
  - With probability  $1 - \delta$
- Memory: (for example)  $O(\frac{1}{\epsilon^2} \log(\frac{1}{\delta}))$
- Ideally:  $O(\log^k(N))$  (polylog in number of observed items)





- Motivation
- Streaming Data (Theory)
- Stream Based Discovery
- Online Alignments



# Stream-Based Process Discovery

---

- Let's assume, we observe an infinite sequence of **events**
- Events describe:
  - Case-Identifier
  - Activity
  - Payload

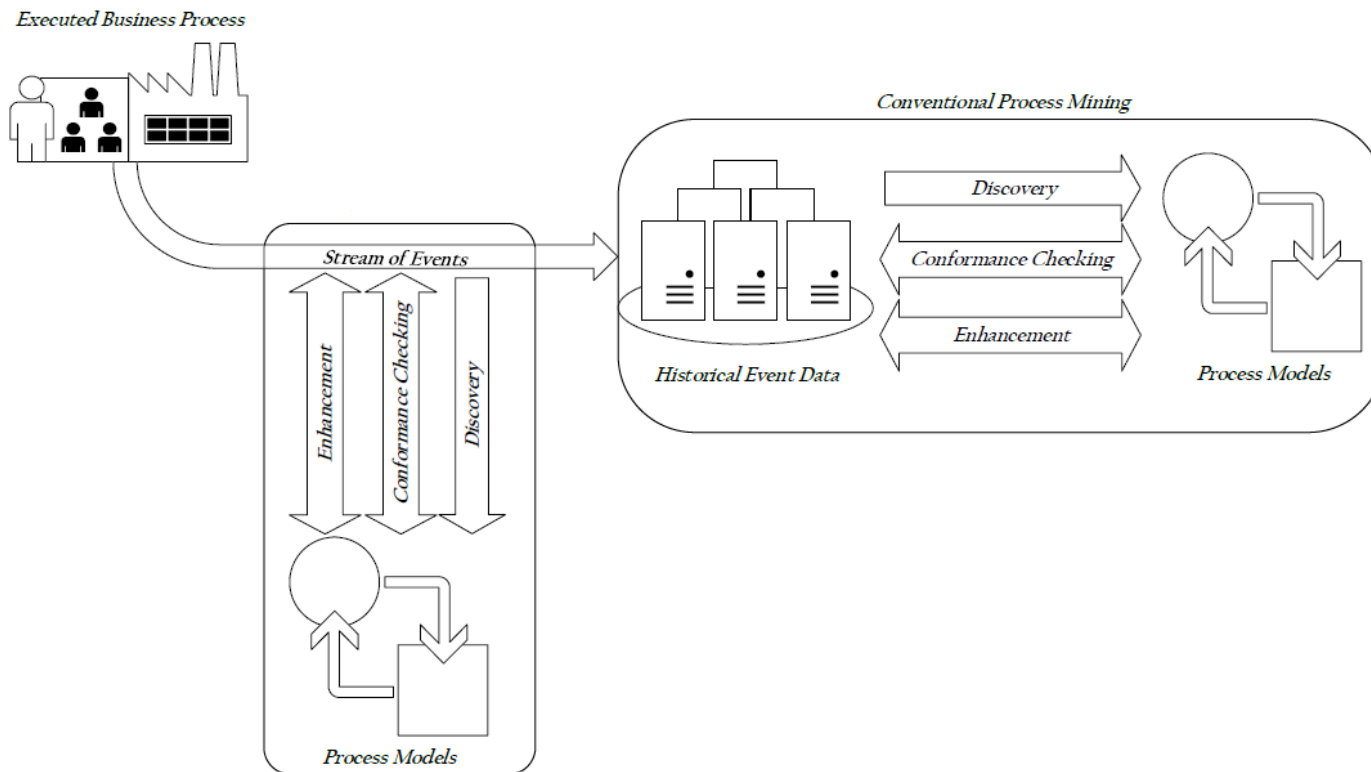
# Stream-Based Process Discovery

---

- Let's assume, we observe an infinite sequence of **events**
- Events describe:
  - Case-Identifier
  - Activity
  - Payload
- $e=(c,a,...)$

# Stream-Based Process Discovery

- Let's assume, we observe an infinite sequence of **events**



# Stream-Based Process Discovery

---

- Let's assume, we observe an infinite sequence of **events**
- Typical 'streaming' analysis:
- How many times does an event occur?
- What are the most frequent events?

# Stream-Based Process Discovery

---

- Let's assume, we observe an infinite sequence of **events**
- Typical 'streaming' analysis:
- How many times does an event occur?
  - Once, every event is **unique**
- What are the most frequent events?
  - Every event is equally frequent, every event is **unique**

# Stream-Based Process Discovery

---

- Let's assume, we observe an infinite sequence of **events**
- Typical 'streaming' analysis:
- How many times does a **case** occur?
- What are the most frequent **cases**?

# Stream-Based Process Discovery

---

- Let's assume, we observe an infinite sequence of **events**
- Typical 'streaming' analysis:
- How many times does a **case** occur?
  - Trivial adoption of standard streaming algorithms...
- What are the most frequent **cases**?
  - Trivial adoption of standard streaming algorithms...



# Stream-Based Process Discovery

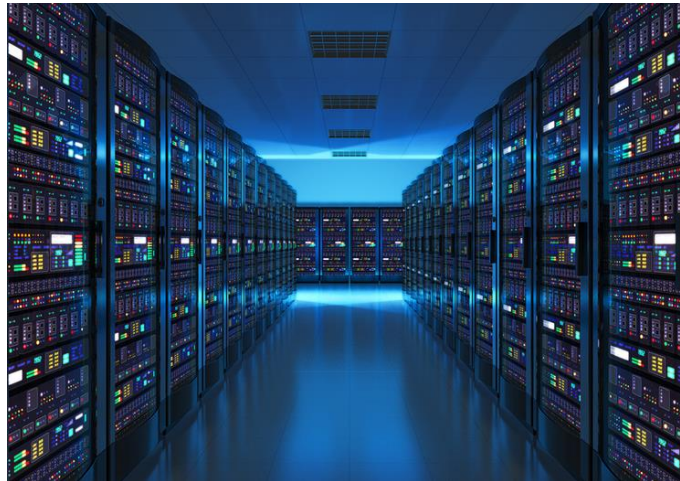
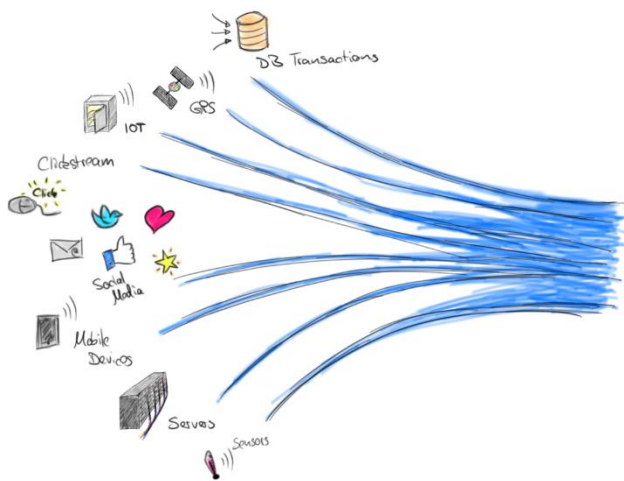
---

- Let's assume, we observe an infinite sequence of **events**
- Typical 'streaming' analysis:
- How many times does a certain event occur?
  - Trivial adoption of standard streaming algorithms...
- What are the most frequent **cases**?
  - Trivial adoption of standard streaming algorithms...

Nice statistics, but, it does not give us any process models!

# Stream-Based Process Discovery

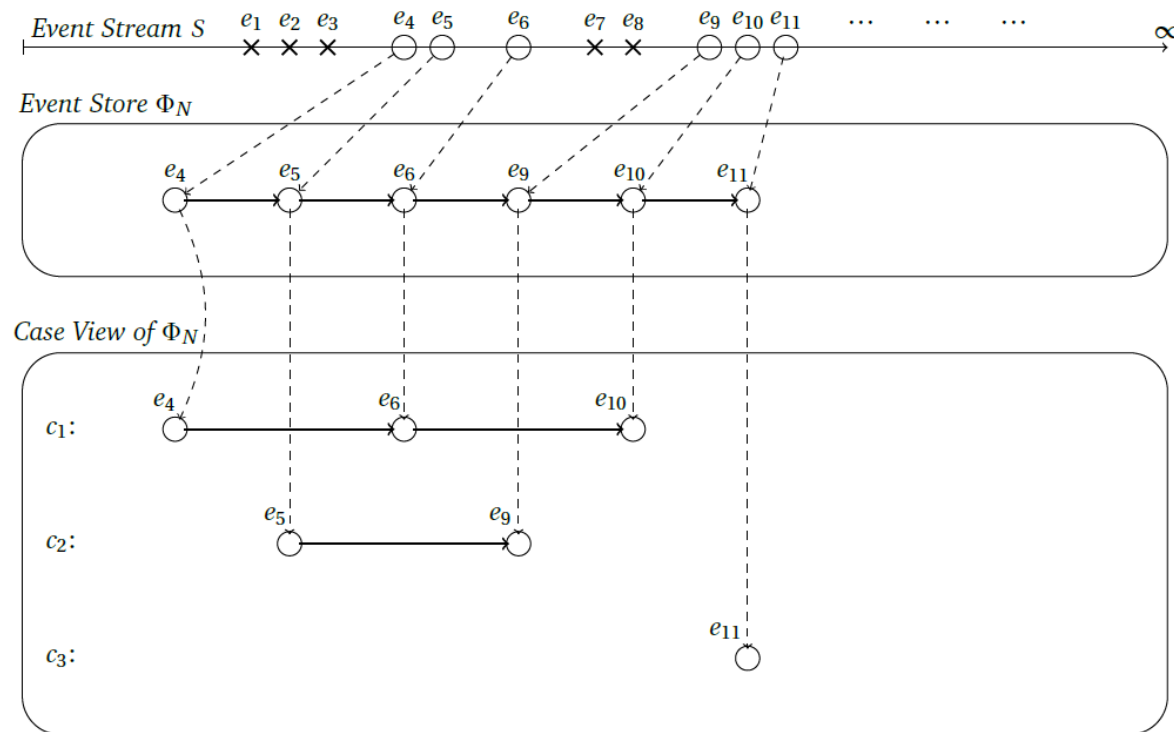
- Let's assume, we observe an infinite sequence of **events**



- We need smart techniques to store events, **temporarily**, and (from time to time) discover a model

# Stream-Based Process Discovery

- Let's assume, we observe an infinite sequence of **events**



# Stream-Based Process Discovery

- Let's assume, we observe an infinite sequence of **events**
- Using a “sliding window”



# Stream-Based Process Discovery

- Let's assume, we observe an infinite sequence of **events**
- Using a “sliding window”



# Stream-Based Process Discovery

- Let's assume, we observe an infinite sequence of **events**
- Using a “sliding window”



# Stream-Based Process Discovery

---

- Let's assume, we observe an infinite sequence of **events**
1.  $a \leftarrow$  array of length  $k$
  2.  $i, j \leftarrow 0$
  3. **while** true:
  4.      $a[i \bmod k] \leftarrow S(i)$
  5.      $i++$
  6.     **if**  $i \geq k$ :
  7.          $j \leftarrow (j+1) \bmod k$

# Stream-Based Process Discovery

- Let's assume, we observe an infinite sequence of **events**

1.  $a \leftarrow$  array of length  $k$

2.  $i, j \leftarrow 0$

3. **while** true:

4.  $a[i \bmod k] \leftarrow S(i)$

5.  $i++$

6. **if**  $i \geq k$ :

7.  $j \leftarrow (j+1) \bmod k$

## Element on stream



# Stream-Based Process Discovery

- Let's assume, we observe an infinite sequence of **events**

1.  $a \leftarrow$  array of length  $k$

2.  $i, j \leftarrow 0$

3. **while** true:

4.      $a[i \bmod k] \leftarrow S(i)$

5.      $i++$

6.     **if**  $i \geq k$ :

7.          $j \leftarrow (j+1) \bmod k$

**Index of 'oldest' item**

# Stream-Based Process Discovery

---

- Let's assume, we observe an infinite sequence of **events**
1.  $a \leftarrow$  array of length  $k$
  2.  $i, j \leftarrow 0$
  3. **while** true:
  4.      $a[i \bmod k] \leftarrow S(i)$
  5.      $i++$
  6.     **if**  $i \geq k$ :
  7.          $j \leftarrow (j+1) \bmod k$
- $a[j]$  = oldest element

# Stream-Based Process Discovery

---

- Let's assume, we observe an infinite sequence of **events**
- 1.  $a \leftarrow$  array of length  $k$
- 2.  $i, j \leftarrow 0$
- 3. **while** true:
- 4.      $a[i \bmod k] \leftarrow S(i)$
- 5.      $i++$
- 6.     **if**  $i \geq k$ :
- 7.          $j \leftarrow (j+1) \bmod k$
- $a[j-1]$  = newest element (or  $|a|-1$  if  $j=0$ )

# Stream-Based Process Discovery

---

- Let's assume, we observe an infinite sequence of **events**
- 1.  $a \leftarrow$  array of length  $k$
- 2.  $i, j \leftarrow 0$
- 3. **while** true:
- 4.      $a[i \bmod k] \leftarrow S(i)$
- 5.      $i++$
- 6.     **if**  $i \geq k$ :
- 7.          $j \leftarrow (j+1) \bmod k$
- $[a[j], a[j+1], \dots, a[|a|-1], a[0], \dots, a[j-1]] = \text{window}$

# Stream-Based Process Discovery

---

- Let's assume, we observe an infinite sequence of **events**
- Store events in a sliding window
- (projection onto cases is easy)
  - Either when we need it, or, continuously...

# Stream-Based Process Discovery

---

- Let's assume, we observe an infinite sequence of **events**
- Store events in a sliding window
- (projection onto cases is easy)
  - Either when we need it, or, continuously...
- Give to any algorithm of choice

# Stream-Based Process Discovery

---

- Let's assume, we observe an infinite sequence of **events**
- Store events in a sliding window
- (projection onto cases is easy)
  - Either when we need it, or, continuously...
- Let's focus on **Inductive Miner (Alpha / Heuristics...)**

# Stream-Based Process Discovery

---

- Let's assume, we observe an infinite sequence of **events**
- Store events in a sliding window
- (projection onto cases is easy)
  - Either when we need it, or, continuously...
- Let's focus on **Inductive Miner (Alpha / Heuristics...)**
- $a > b$ ,  $b > c$ , ...



# Stream-Based Process Discovery

---



- Idea:
  - For each case, store the last received activity

# Stream-Based Process Discovery



- Idea:
  - For each case, store the last received activity
  - Upon receiving a new event, assess that activity, deduce dfg and update last received activity

# Stream-Based Process Discovery



case	act
$c_1$	a

pair	count

# Stream-Based Process Discovery



- $e_2 = (c_2, a)$

case	act
$c_1$	a
$c_2$	a

pair	count

# Stream-Based Process Discovery



- $e_3 = (c_1, b)$

case	act
$c_1$	a
$c_2$	a

pair	count

# Stream-Based Process Discovery



- $e_3 = (c_1, b)$

case	act
$c_1$	a
$c_2$	a

pair	count

# Stream-Based Process Discovery



- $e_3 = (c_1, b)$

case	act
$c_1$	b
$c_2$	a

pair	count
(a,b)	1

# Stream-Based Process Discovery



- $e_4 = (c_1, c)$

case	act
$c_1$	b
$c_2$	a

pair	count
(a,b)	1



# Stream-Based Process Discovery



- $e_4 = (c_1, c)$

case	act
$c_1$	b
$c_2$	a

pair	count
(a,b)	1

# Stream-Based Process Discovery



- $e_4 = (c_1, c)$

case	act
$c_1$	c
$c_2$	a

pair	count
(a,b)	1
(b,c)	1

# Stream-Based Process Discovery



- $e_5 = (c_2, c)$

case	act
$c_1$	c
$c_2$	a

pair	count
(a,b)	1
(b,c)	1

# Stream-Based Process Discovery



- $e_5 = (c_2, c)$

case	act
$c_1$	c
$c_2$	a

pair	count
(a,b)	1
(b,c)	1

# Stream-Based Process Discovery



- $e_5 = (c_2, c)$

case	act
$c_1$	c
$c_2$	c

pair	count
(a,b)	1
(b,c)	1
(a,c)	1

# Stream-Based Process Discovery

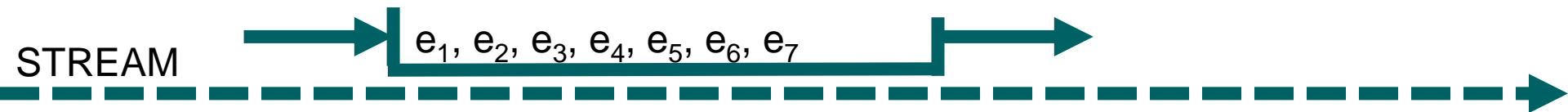


- $e_6 = (c_3, a)$

case	act
$c_1$	c
$c_2$	c
$c_3$	a

pair	count
(a,b)	1
(b,c)	1
(a,c)	1

# Stream-Based Process Discovery



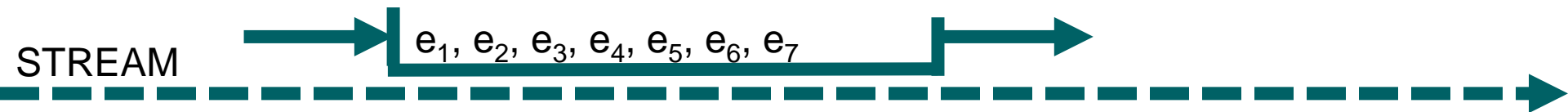
- $e_7 = (c_3, b)$

case	act
$c_1$	c
$c_2$	c
$c_3$	b



pair	count
(a,b)	2
(b,c)	1
(a,c)	1

# Stream-Based Process Discovery



- $e_7 = (c_3, b)$

case	act
$c_1$	c
$c_2$	c
$c_3$	b



pair	count
------	-------

**Algorithm 3.1:** FREQUENT( $k$ )

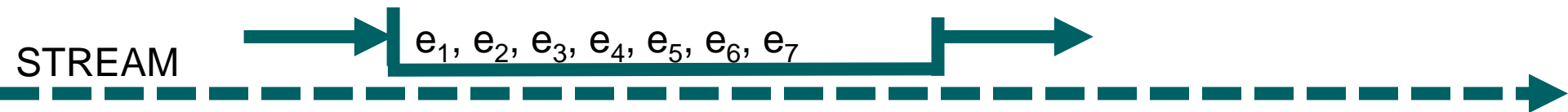
```

 $n \leftarrow 0; T \leftarrow \emptyset;$ 
for each  $i$  :
     $n \leftarrow n + 1;$ 
    if  $i \in T$ 
        then  $c_i \leftarrow c_i + 1;$ 
    else if  $|T| < k - 1$ 
        then  $\begin{cases} T \leftarrow T \cup \{i\}; \\ c_i \leftarrow 1; \end{cases}$ 
    else for all  $j \in T$ 
        do  $\begin{cases} c_j \leftarrow c_j - 1; \\ \text{if } c_j = 0 \\ \text{then } T \leftarrow T \setminus \{j\}; \end{cases}$ 

```



# Stream-Based Process Discovery

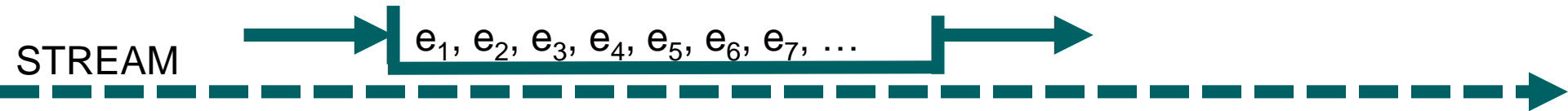


- $e_7 = (c_3, b)$

case	act
$c_1$	c
$c_2$	c
$c_3$	b

pair	count
(a,b)	2
(b,c)	1
(a,c)	1

# Stream-Based Process Discovery

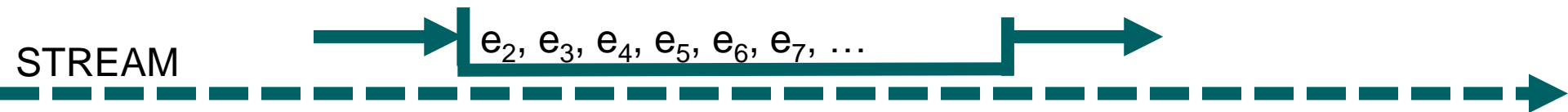


- $e_7 = (c_3, b)$

case	act	win
$c_1$	c	3
$c_2$	c	2
$c_3$	b	2
...	...	...
...	...	...
...	...	...
...	...	...

pair	count
(a,b)	2
(b,c)	1
(a,c)	1

# Stream-Based Process Discovery

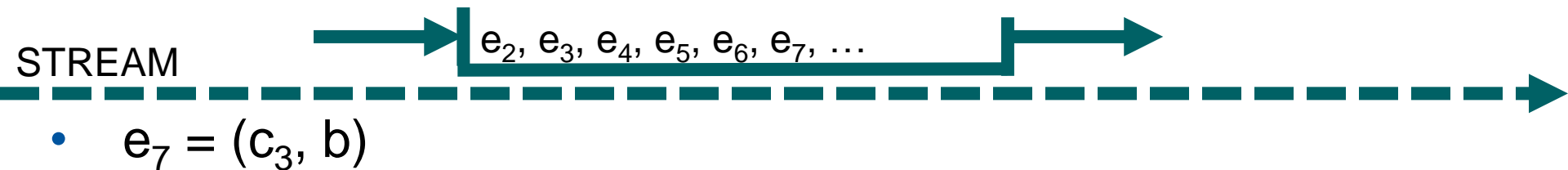


- $e_7 = (c_3, b)$

case	act	win
$c_1$	c	3 2
$c_2$	c	2
$c_3$	b	2
...	...	...
...	...	...
...	...	...
...	...	...

pair	count
(a,b)	2
(b,c)	1
(a,c)	1

# Stream-Based Process Discovery

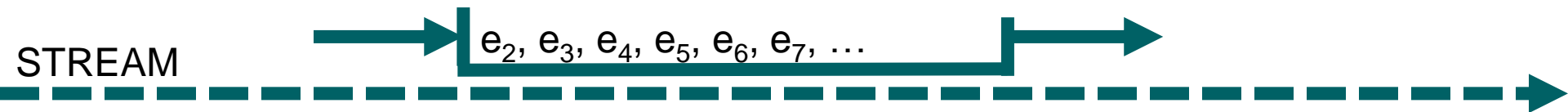


case	act	win
$c_1$	c	3 2
$c_2$	c	2
$c_3$	b	2
...	...	...
...	...	...
...	...	...
...	...	...

## Decoupled

pair	count
(a,b)	2
(b,c)	1
(a,c)	1

# Stream-Based Process Discovery



- $e_7 = (c_3, b)$

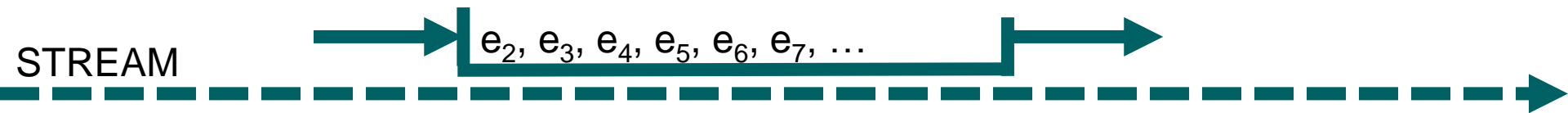
case	act	win
------	-----	-----

## Inductive Miner (DFG Only Option)

...	...	...
...	...	...
...	...	...

pair	count
(a,b)	2
(b,c)	1
(a,c)	1

# Stream-Based Process Discovery

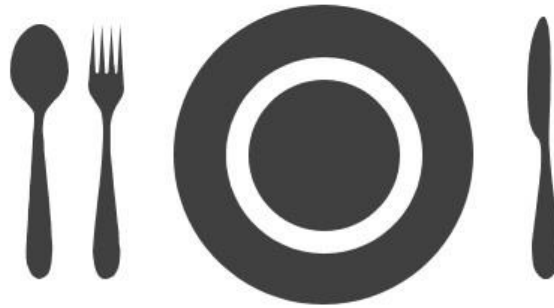


- $e_7 = (c_3, b)$

case	act	win
$c_1$		
$c_2$		
$c_3$	b	2
...	...	...
...	...	...
...	...	...
...	...	...

## Challenges?

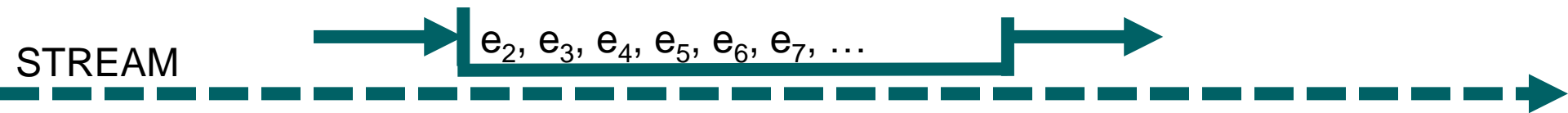
pair	count
(a,b)	2
(b,c)	1
(a,c)	1



- Motivation
- Streaming Data (Theory)
- Stream Based Discovery
- Online Alignments



# Stream-Based Conformance Checking

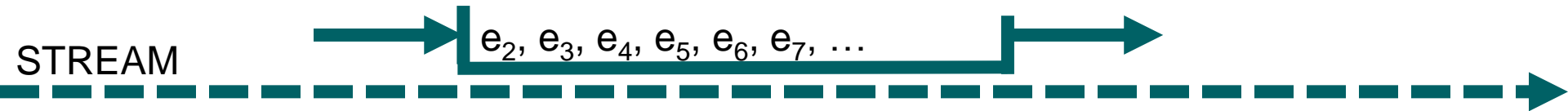


- ... we don't care how we store stuff, we assume you (smartly) store (currently active) traces:

case	act
$c_1$	$\langle a, b, c \rangle$
$c_2$	$\langle a, c \rangle$
$c_3$	$\langle a, b \rangle$
...	...
...	...
...	...
...	...



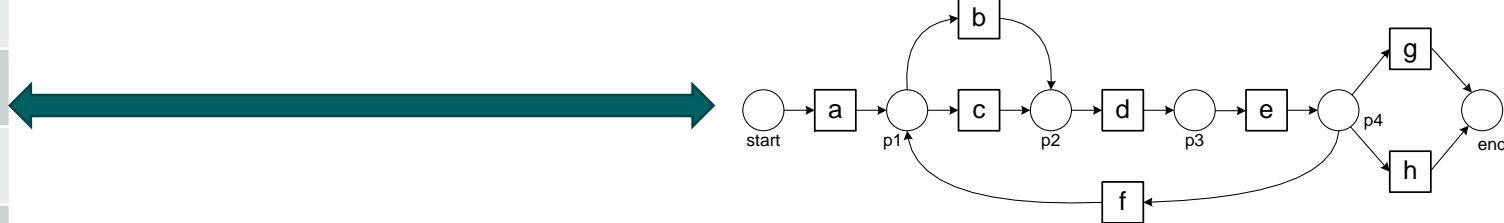
# Stream-Based Conformance Checking



- ... we don't care how we store stuff, we assume you (smartly) store (currently active) traces:

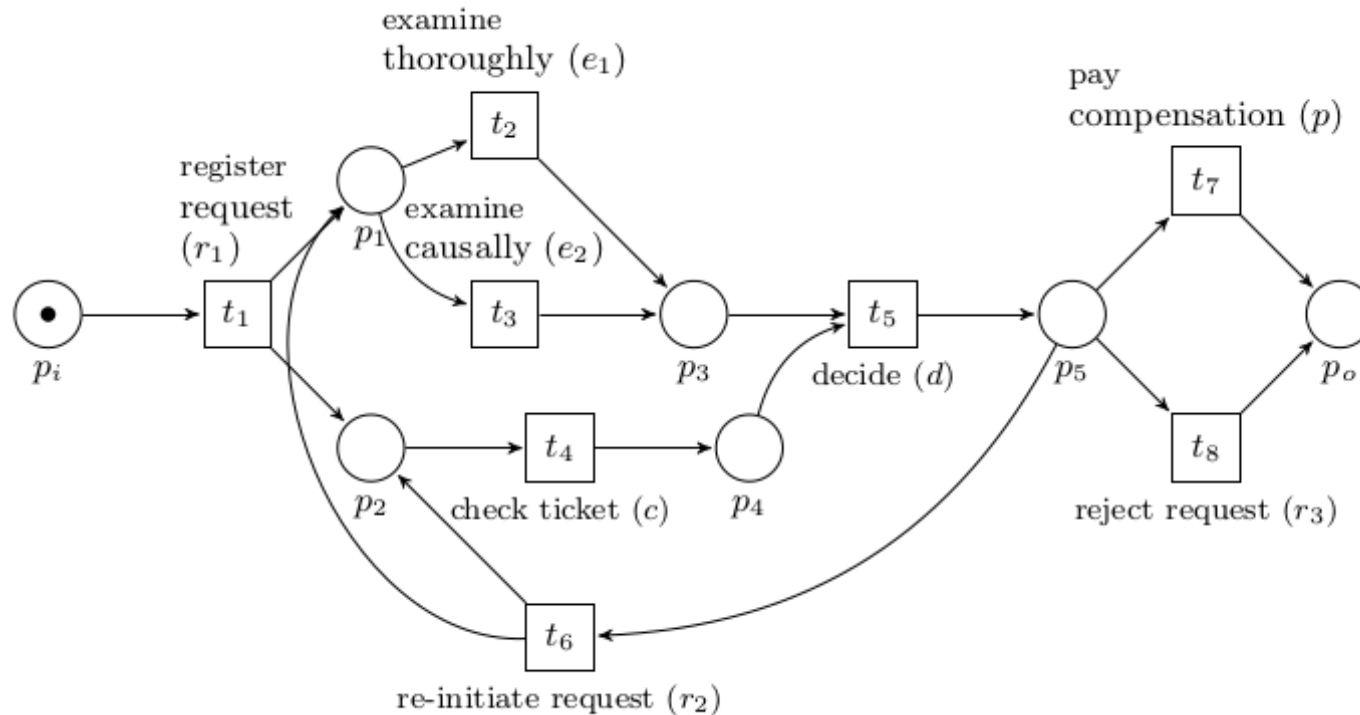
case	act
$c_1$	$\langle a, b, c \rangle$
$c_2$	$\langle a, c \rangle$
$c_3$	$\langle a, b \rangle$
...	...
...	...
...	...
...	...

## Conformance?

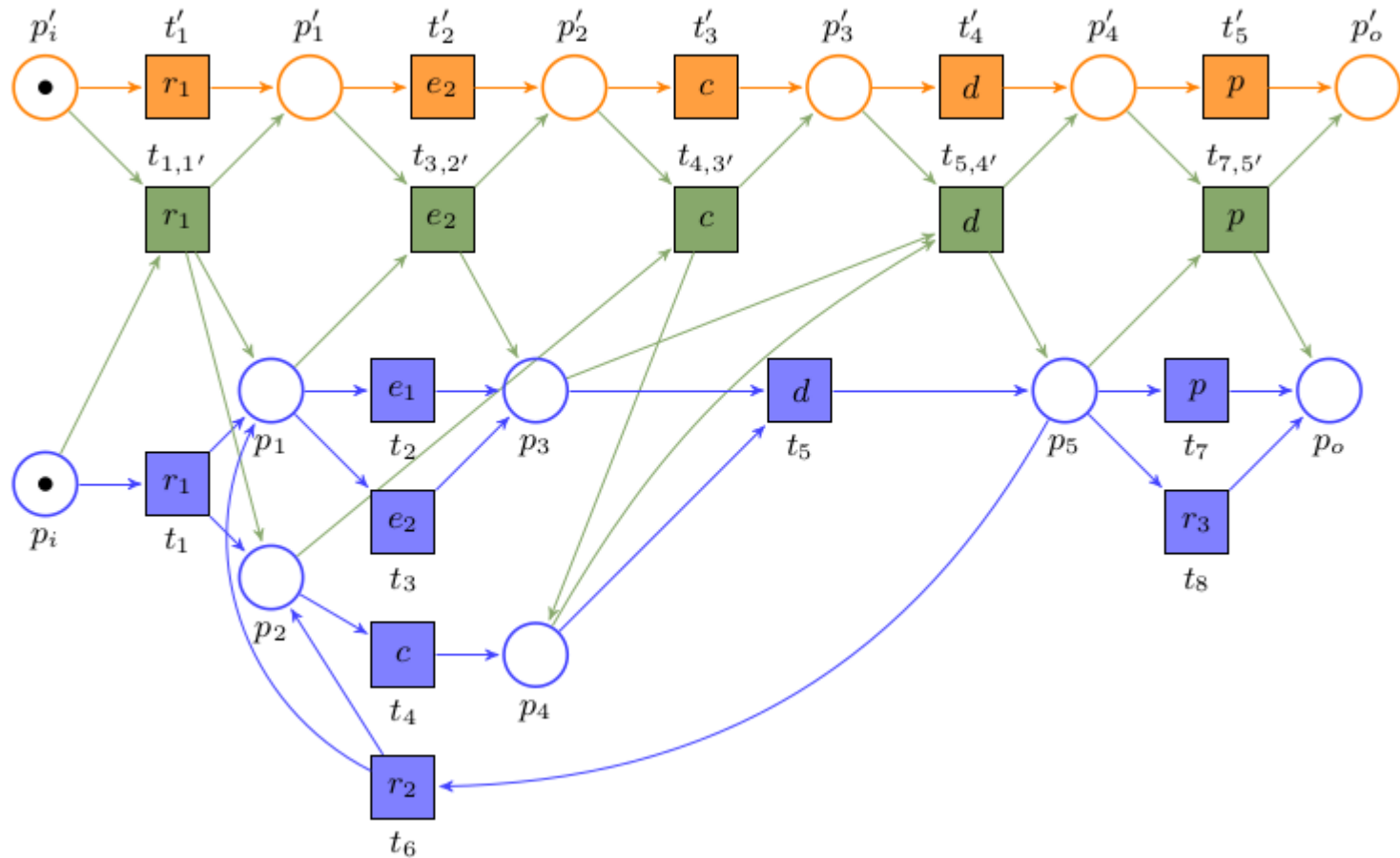


# Stream-Based Conformance Checking

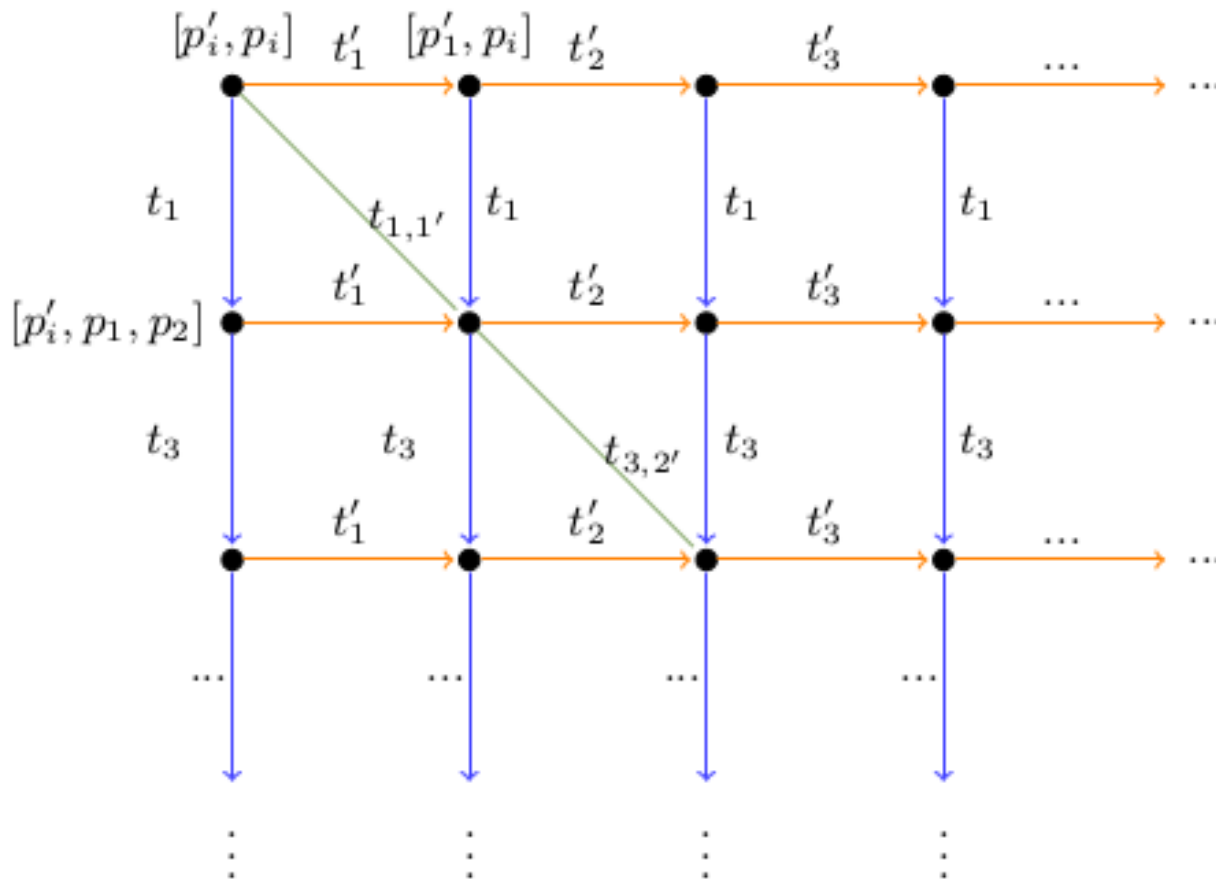
- $\langle r1, e2, c, d, p \rangle$



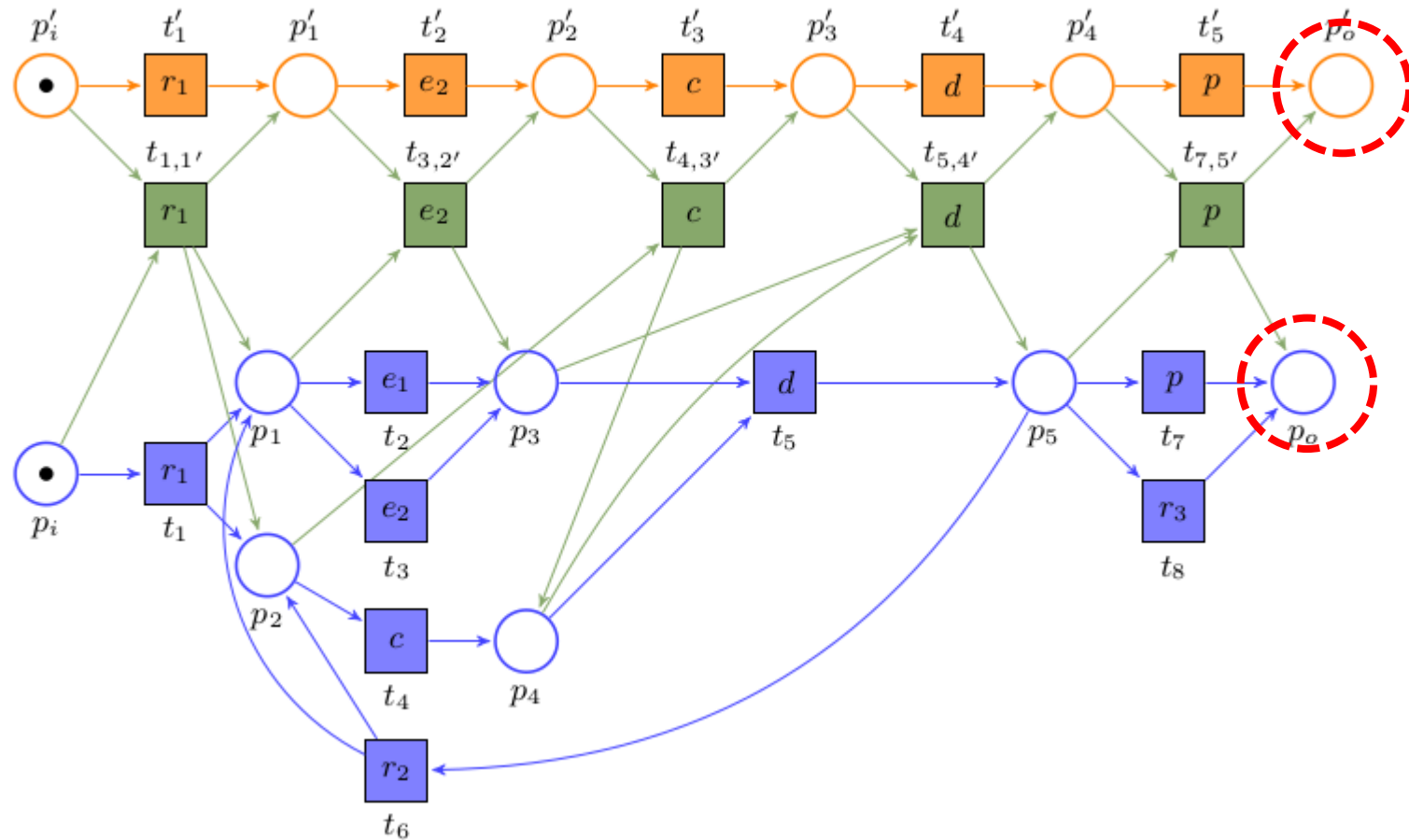
# Stream-Based Conformance Checking



# Stream-Based Conformance Checking

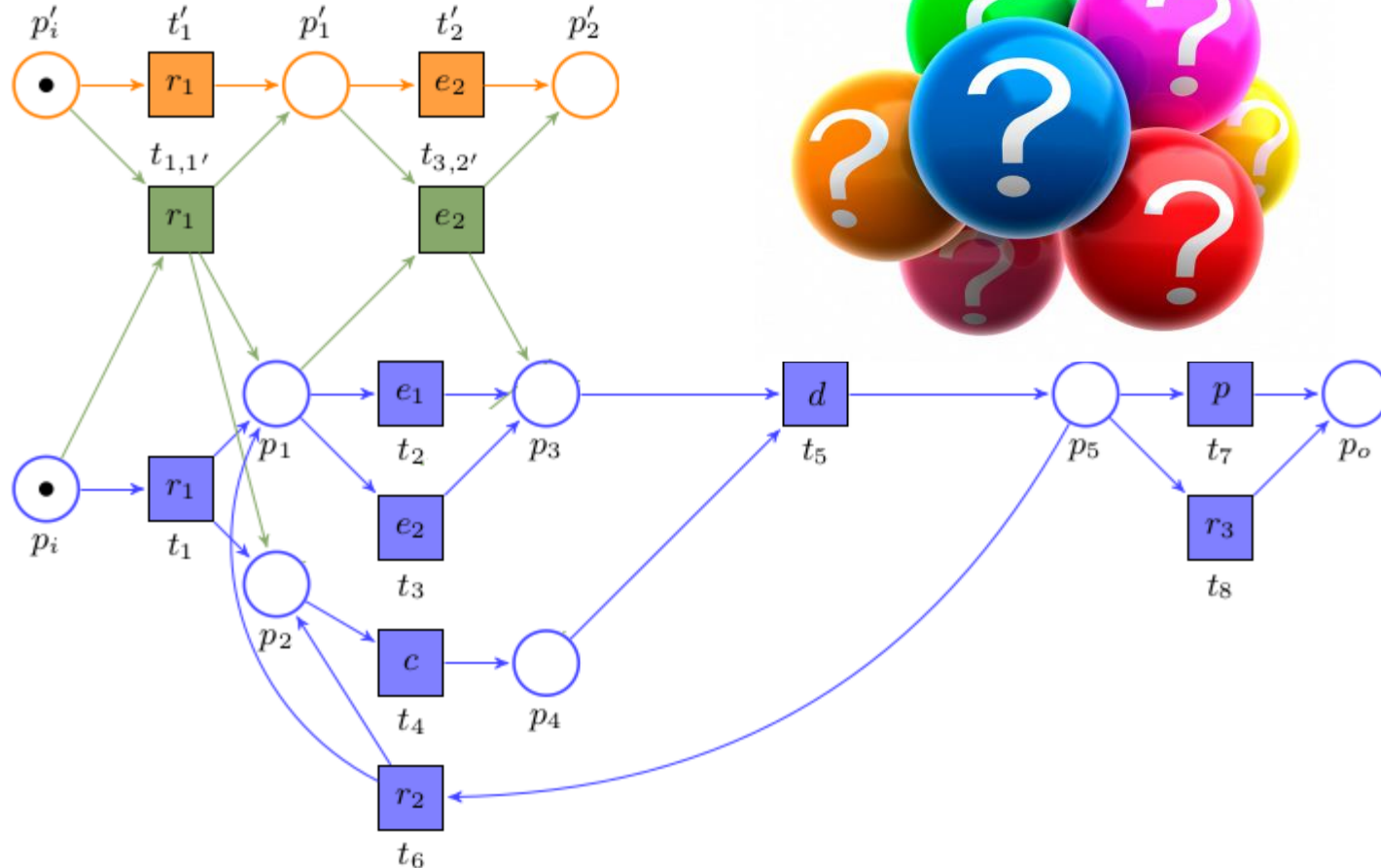


# Stream-Based Conformance Checking



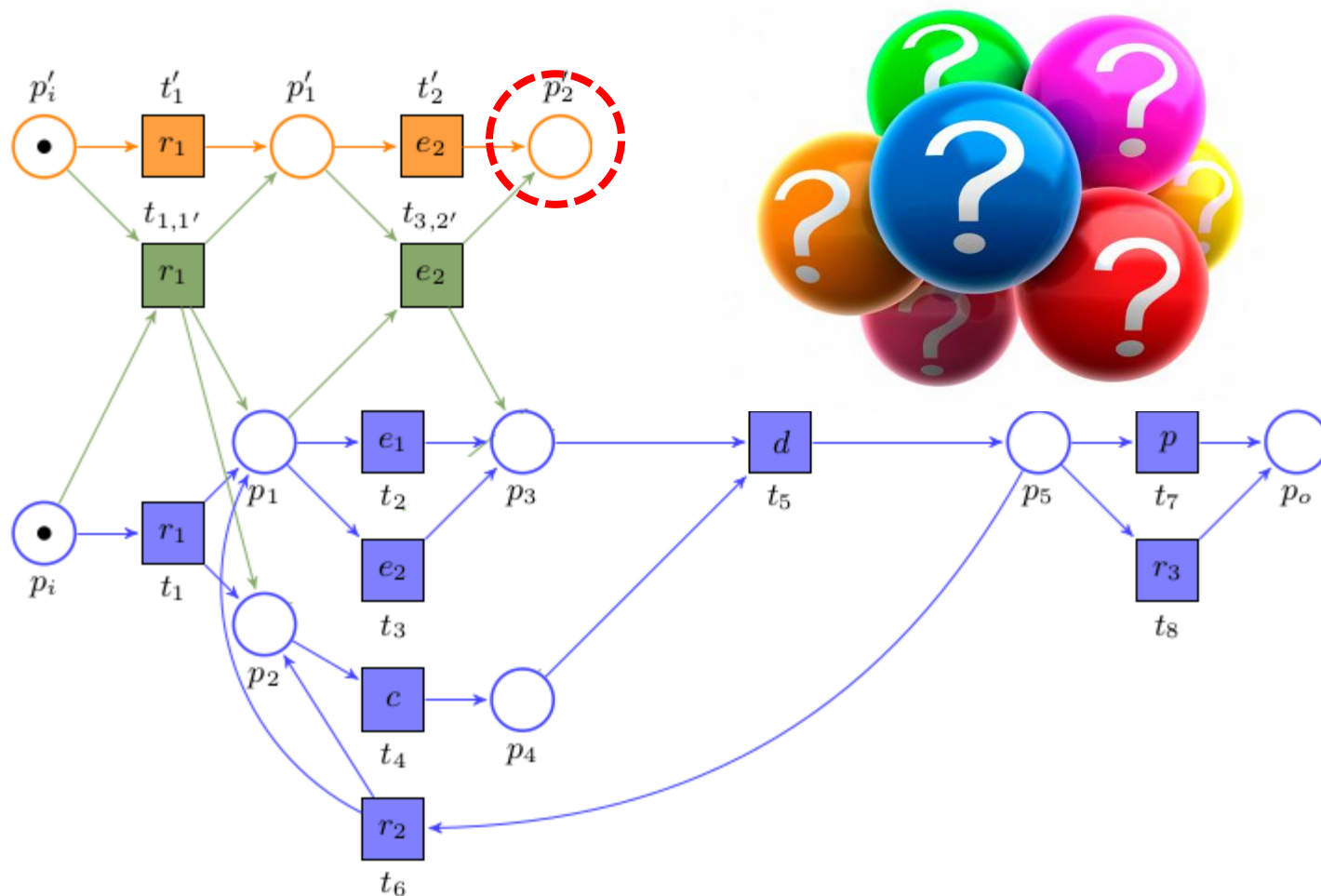
# Stream-Based Conformance Checking

- $\langle r_1, e_2, \rangle \rightarrow$  all we have seen so far



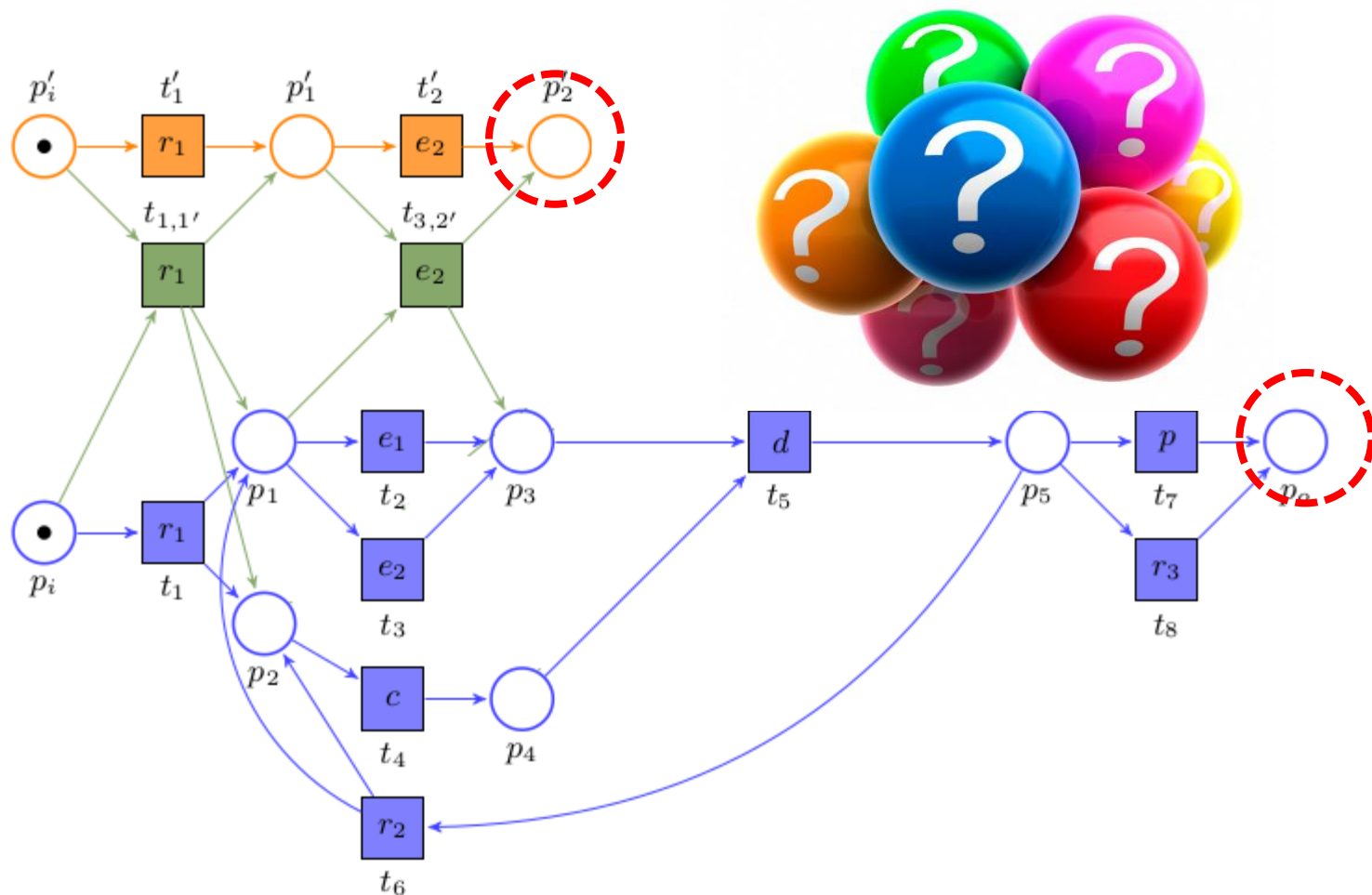
# Stream-Based Conformance Checking

- $\langle r_1, e_2, \rangle \rightarrow$  all we have seen so far



# Stream-Based Conformance Checking

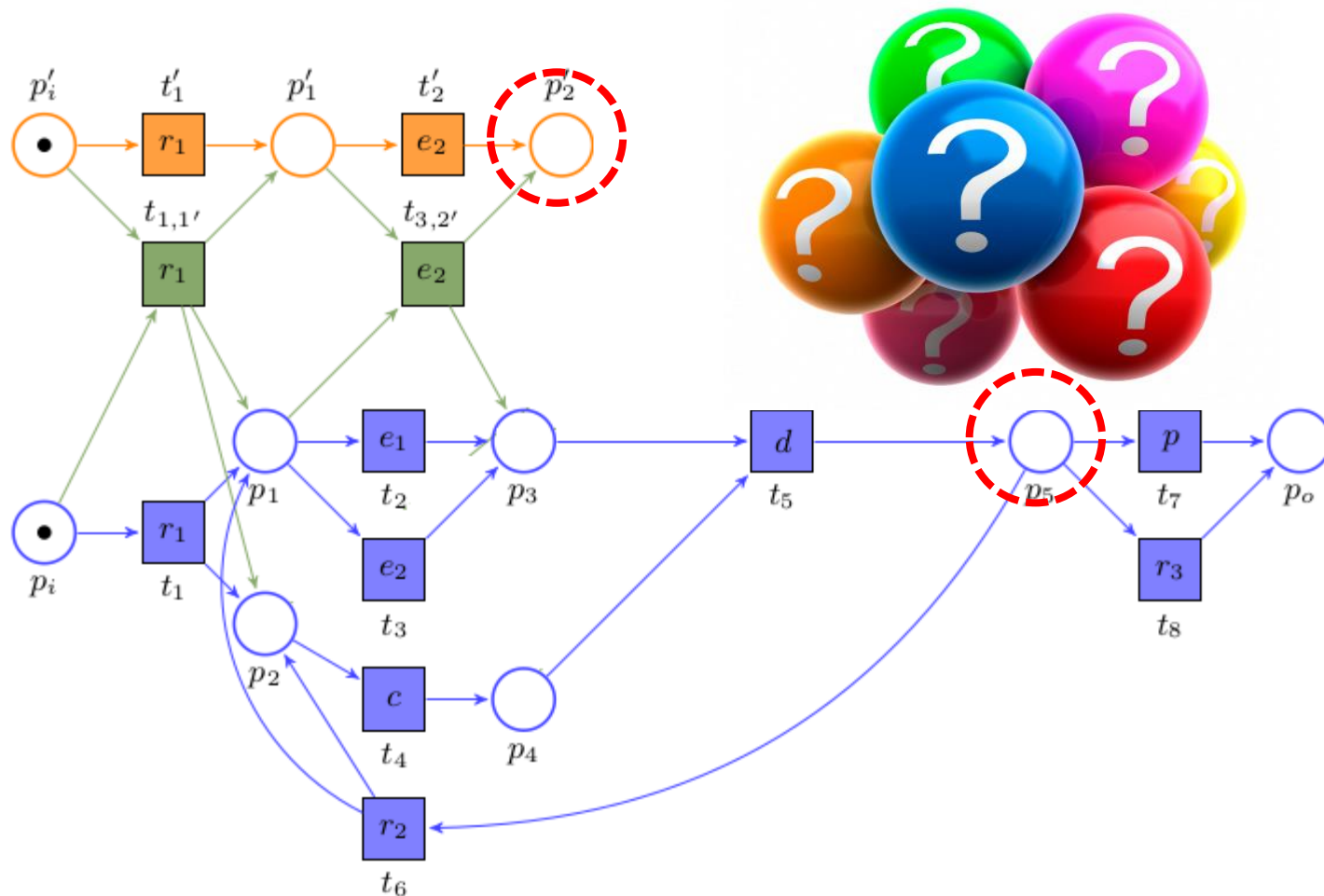
- $\langle r_1, e_2, \rangle \rightarrow$  all we have seen so far





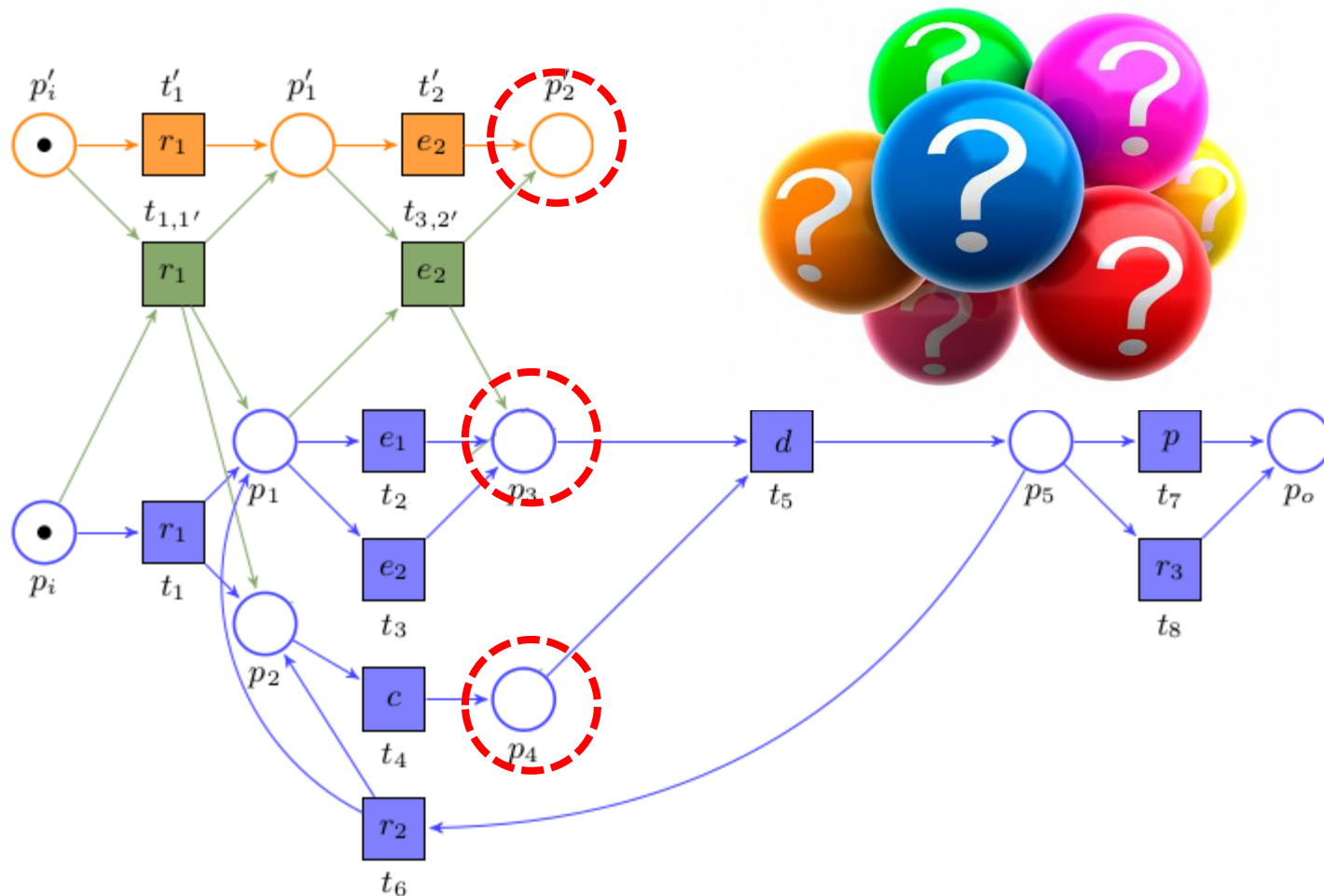
# Stream-Based Conformance Checking

- $\langle r_1, e_2, \rangle \rightarrow$  all we have seen so far



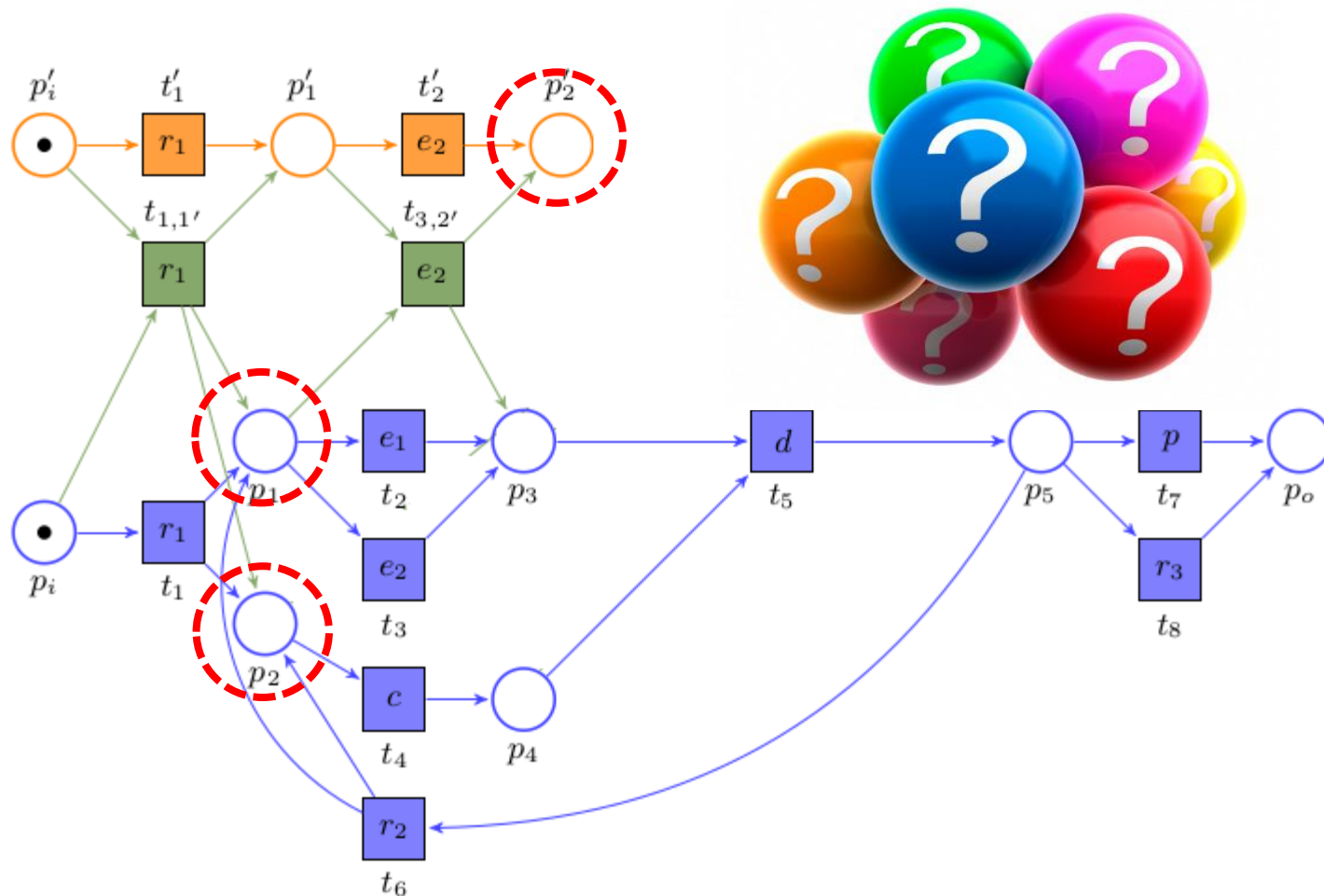
# Stream-Based Conformance Checking

- $\langle r_1, e_2, \rangle \rightarrow$  all we have seen so far

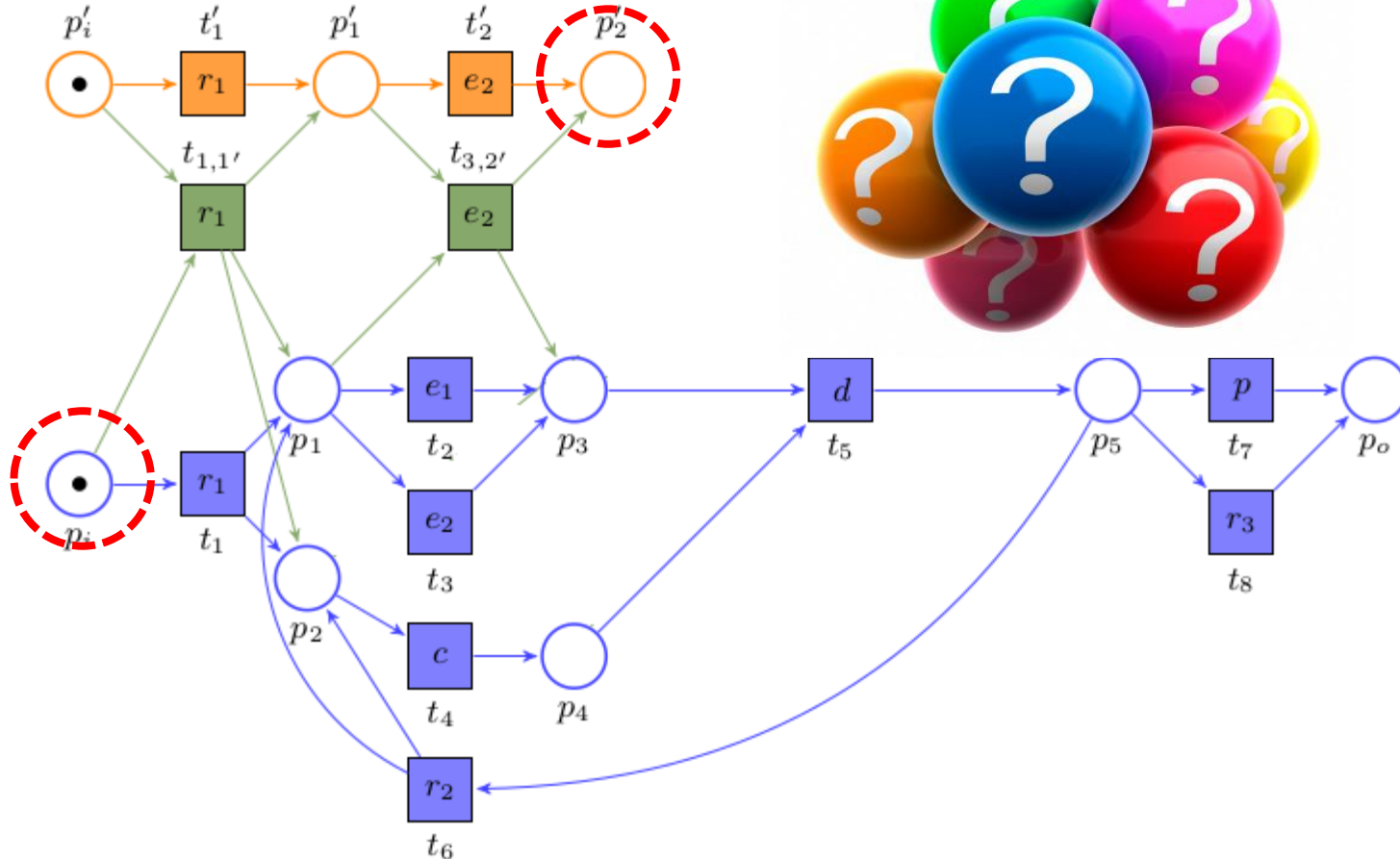


# Stream-Based Conformance Checking

- $\langle r_1, e_2, \rangle \rightarrow$  all we have seen so far



- 



# Stream-Based Conformance Checking

---

- Prefix-Alignment
  - Explain all events in the trace
  - Finish in a marking that still enables us to reach the final marking

# Stream-Based Conformance Checking

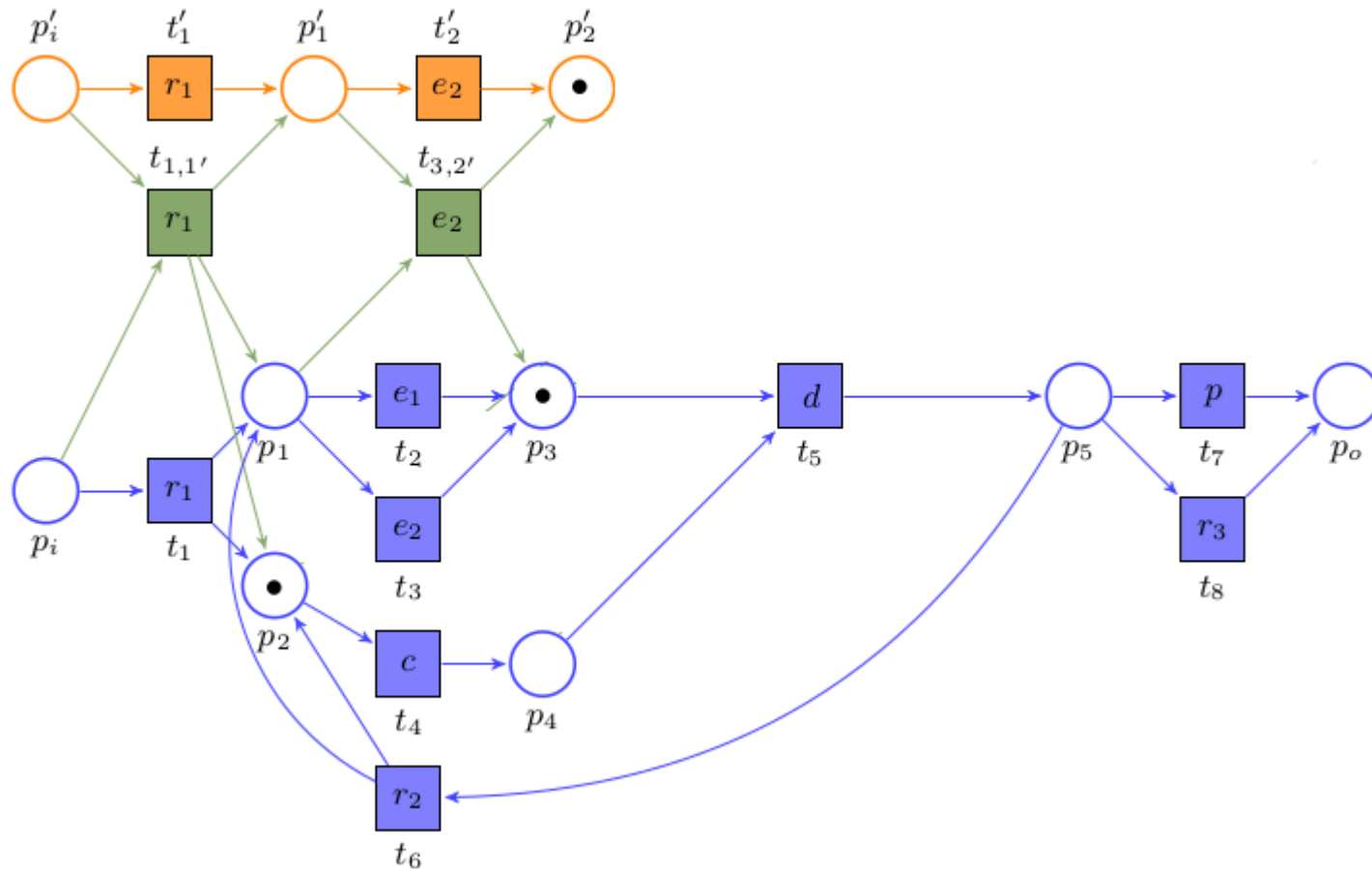
---

- Prefix-Alignment
  - Explain all events in the trace
  - Finish in a marking that still enables us to reach the final marking
  
- ... to fix req #2: We assume that the model is sound

# Stream-Based Conformance Checking

## Approximation Scheme

- $\langle t_{1,1'}, t_{3,2'} \rangle$  **Current Alignment**

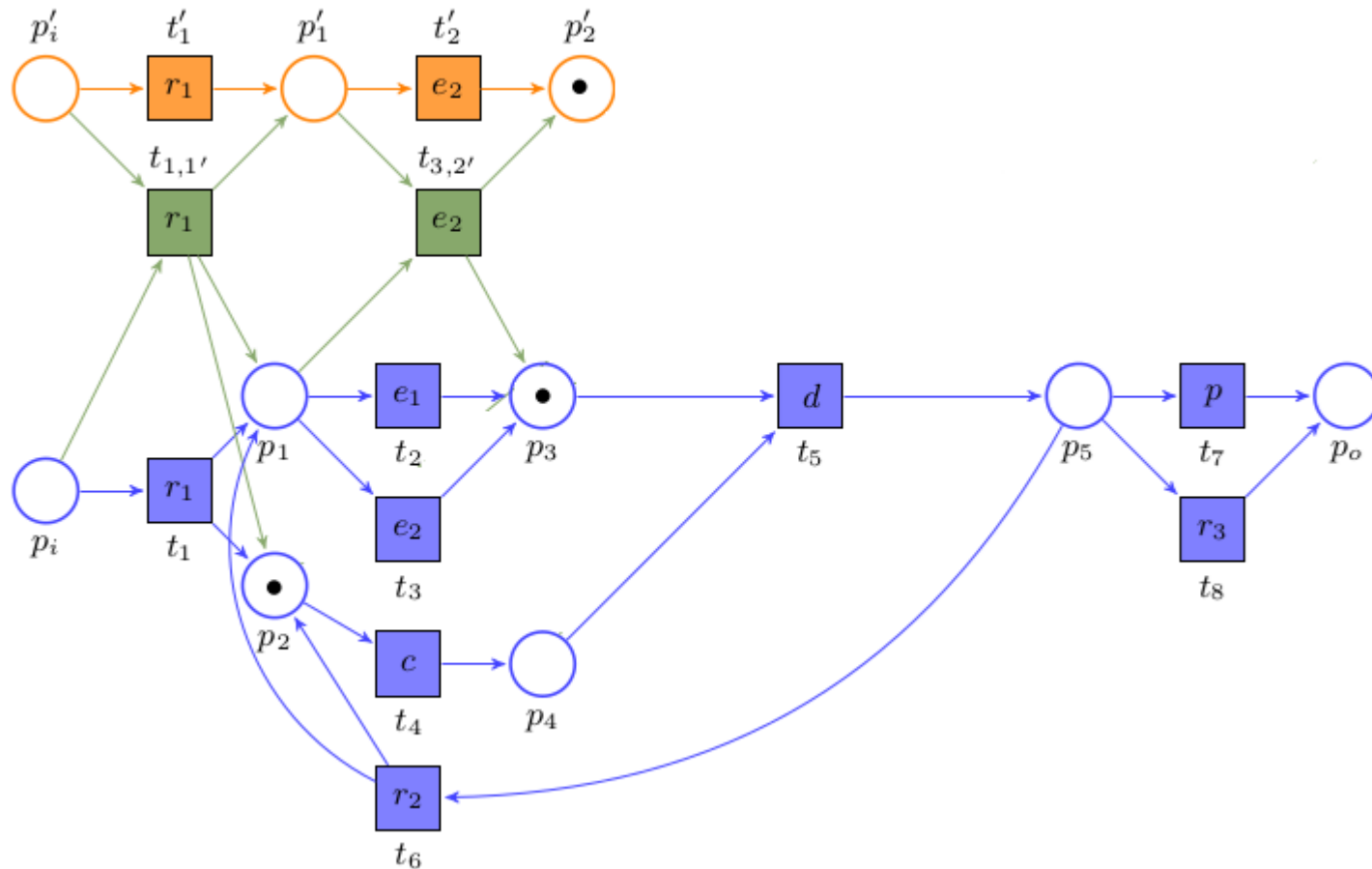


# Stream-Based Conformance Checking

## Approximation Scheme

- $\langle t_{1,1'}, t_{3,2'} \rangle$

receive: activity **c**



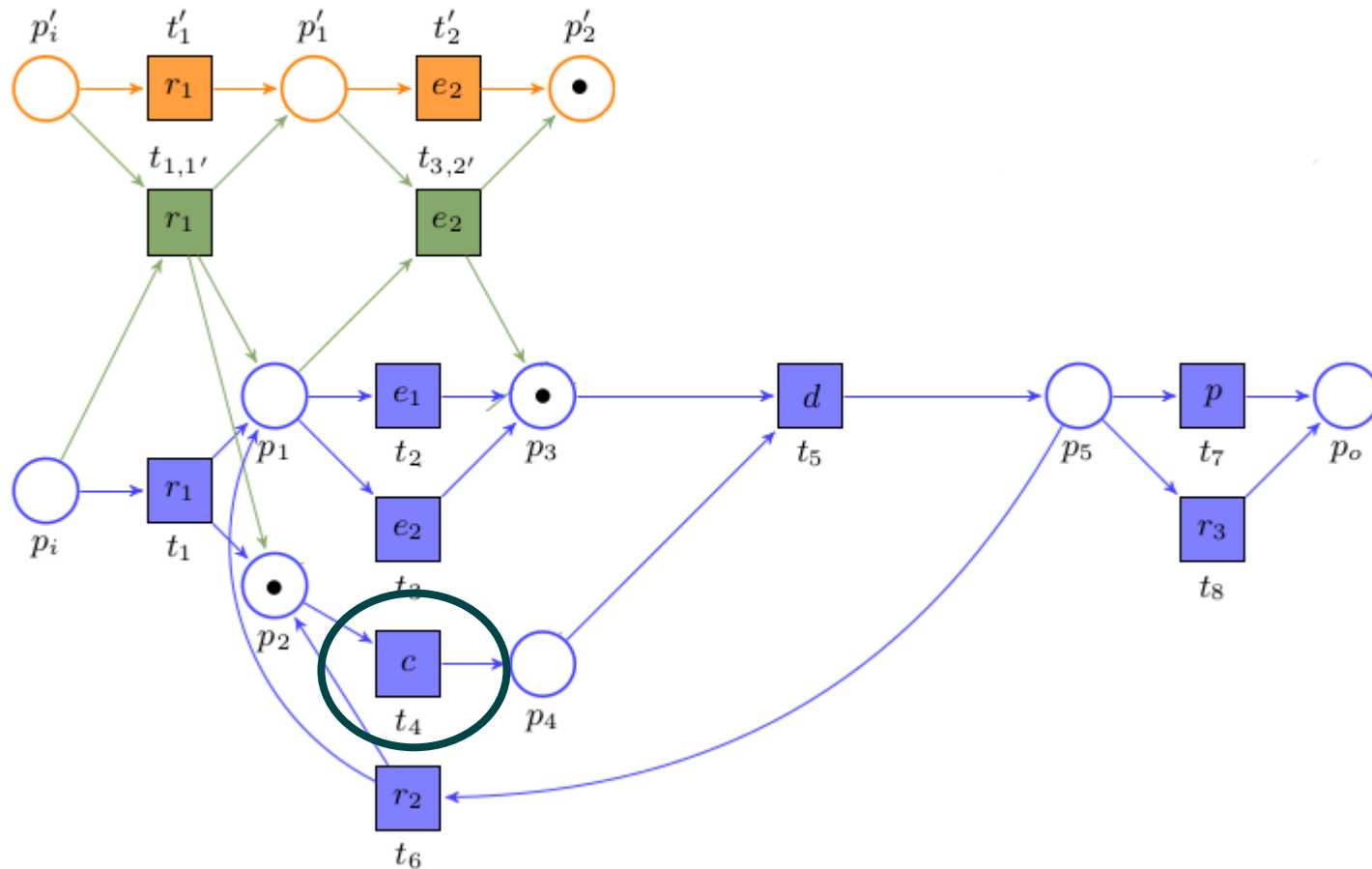


# Stream-Based Conformance Checking

## Approximation Scheme

- $\langle t_{1,1'}, t_{3,2'} \rangle$

receive: activity **C**

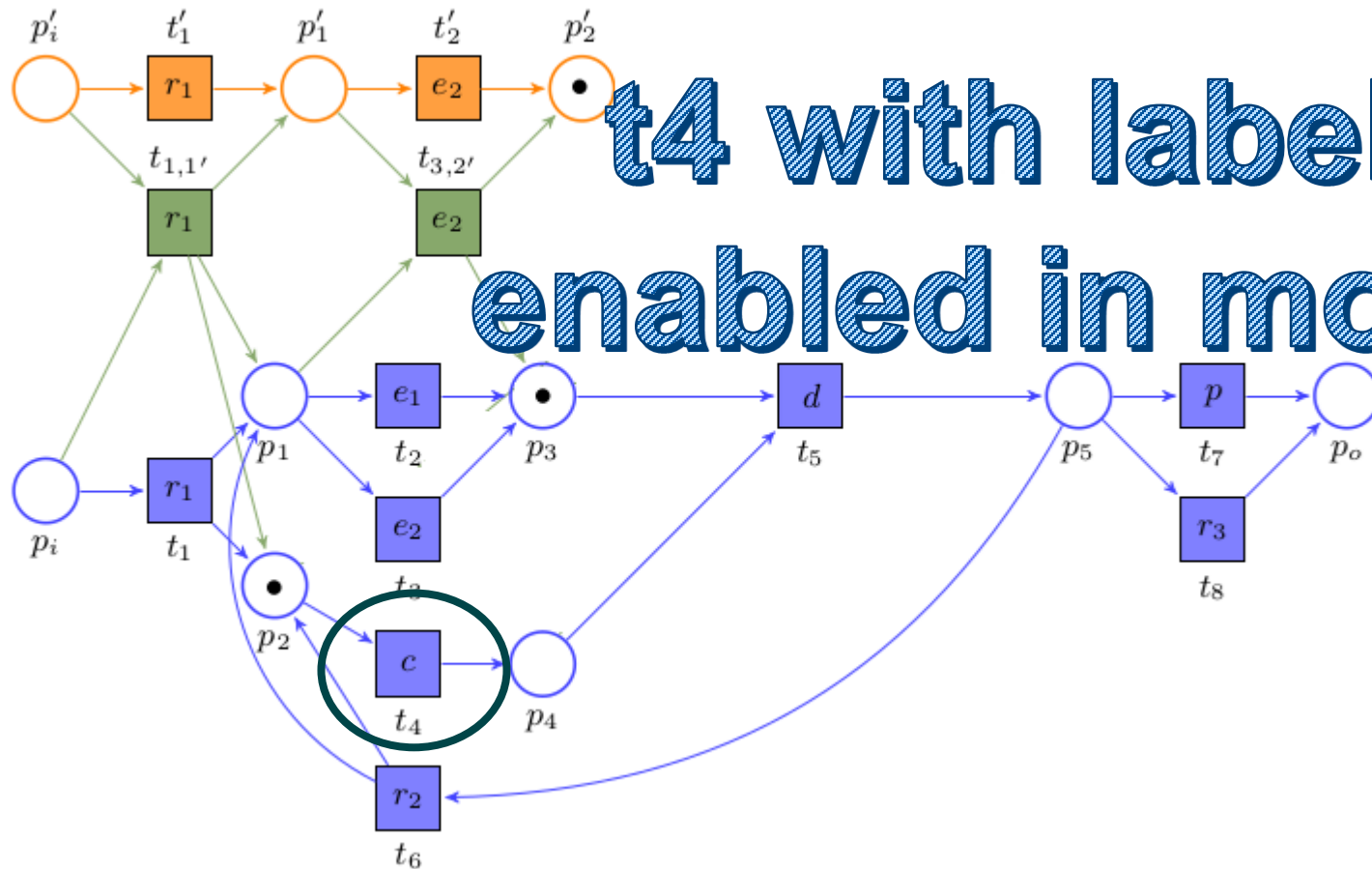


# Stream-Based Conformance Checking

## Approximation Scheme

- $\langle t_{1,1'}, t_{3,2'} \rangle$

receive: activity **c**



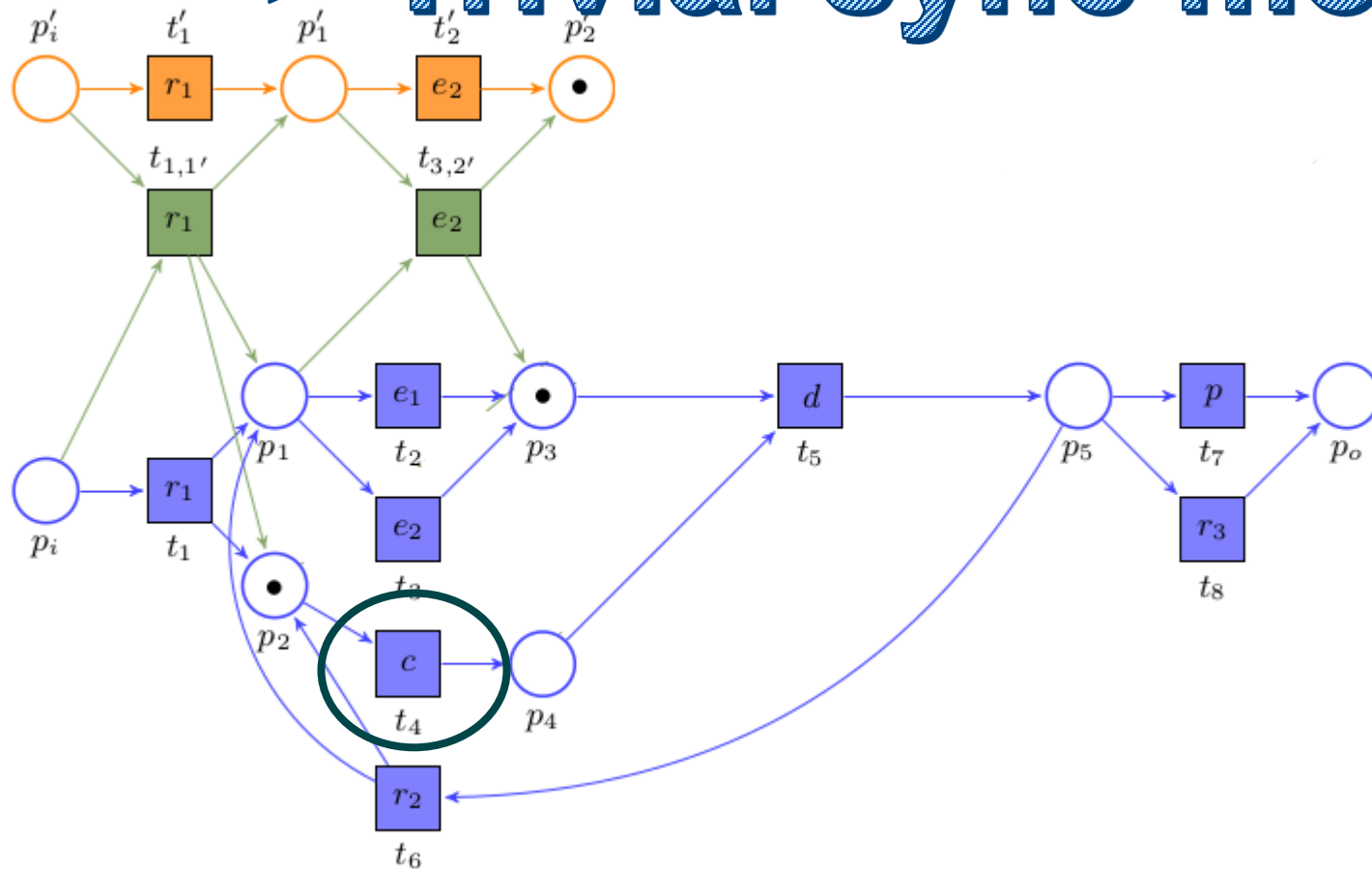
# Stream-Based Conformance Checking

## Approximation Scheme

- $\langle t_{1,1'}, t_{3,2'} \rangle$

receive: activity **C**

$\Rightarrow$  Trivial sync move

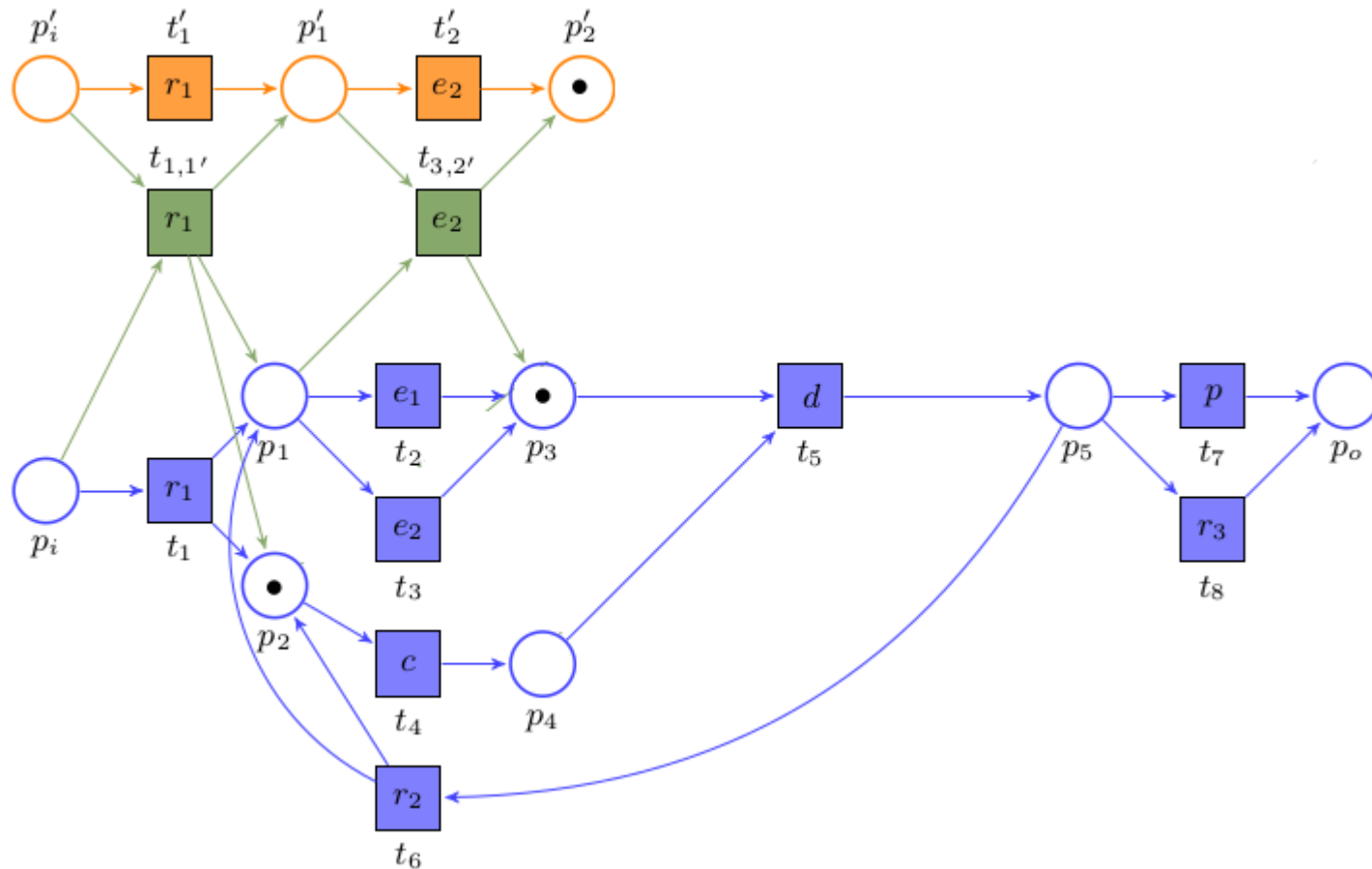


# Stream-Based Conformance Checking

## Approximation Scheme

- $\langle t_{1,1'}, t_{3,2'} \rangle$

receive: activity **f**

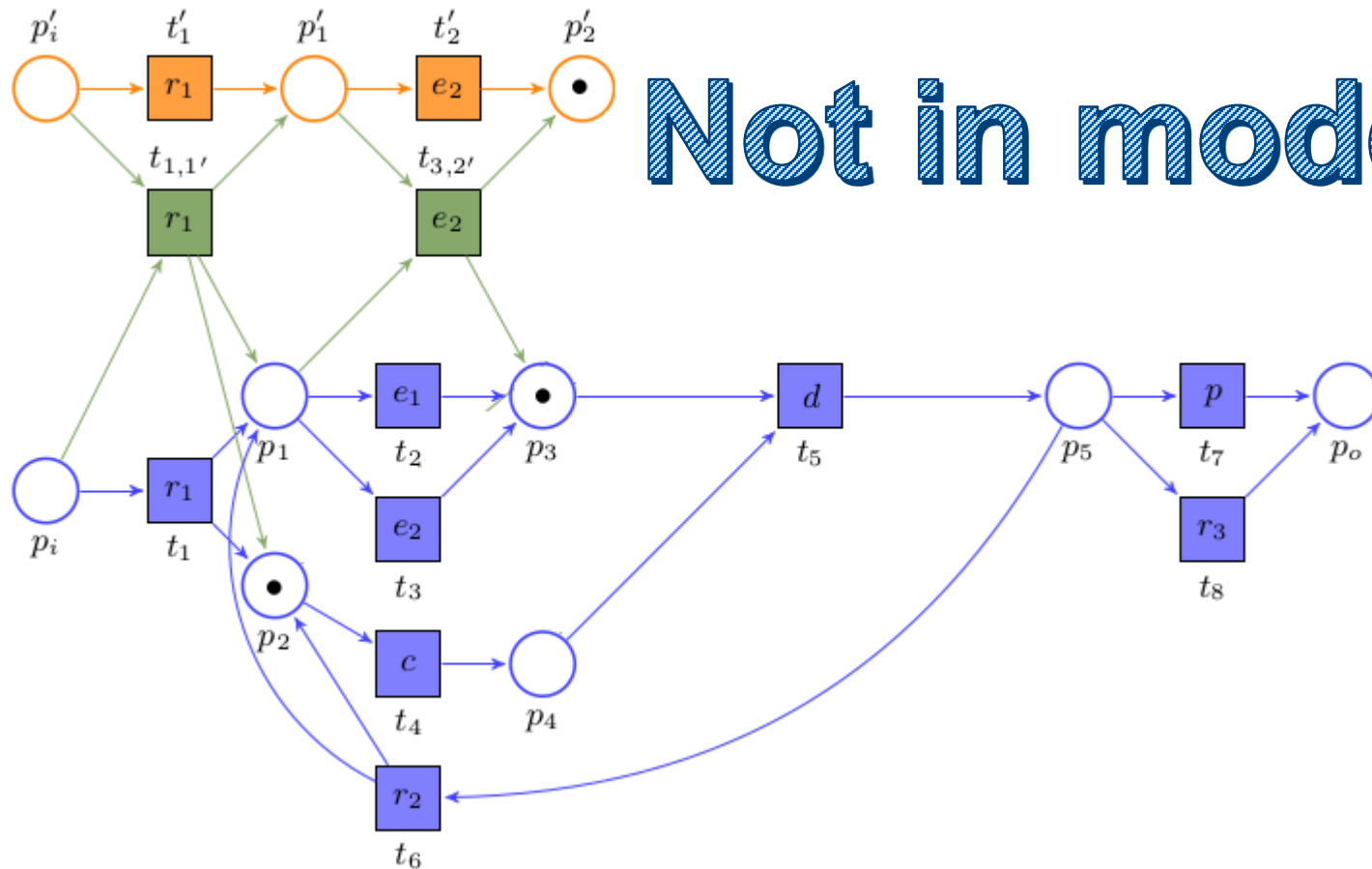


# Stream-Based Conformance Checking

## Approximation Scheme

- $\langle t_{1,1'}, t_{3,2'} \rangle$

receive: activity **f**



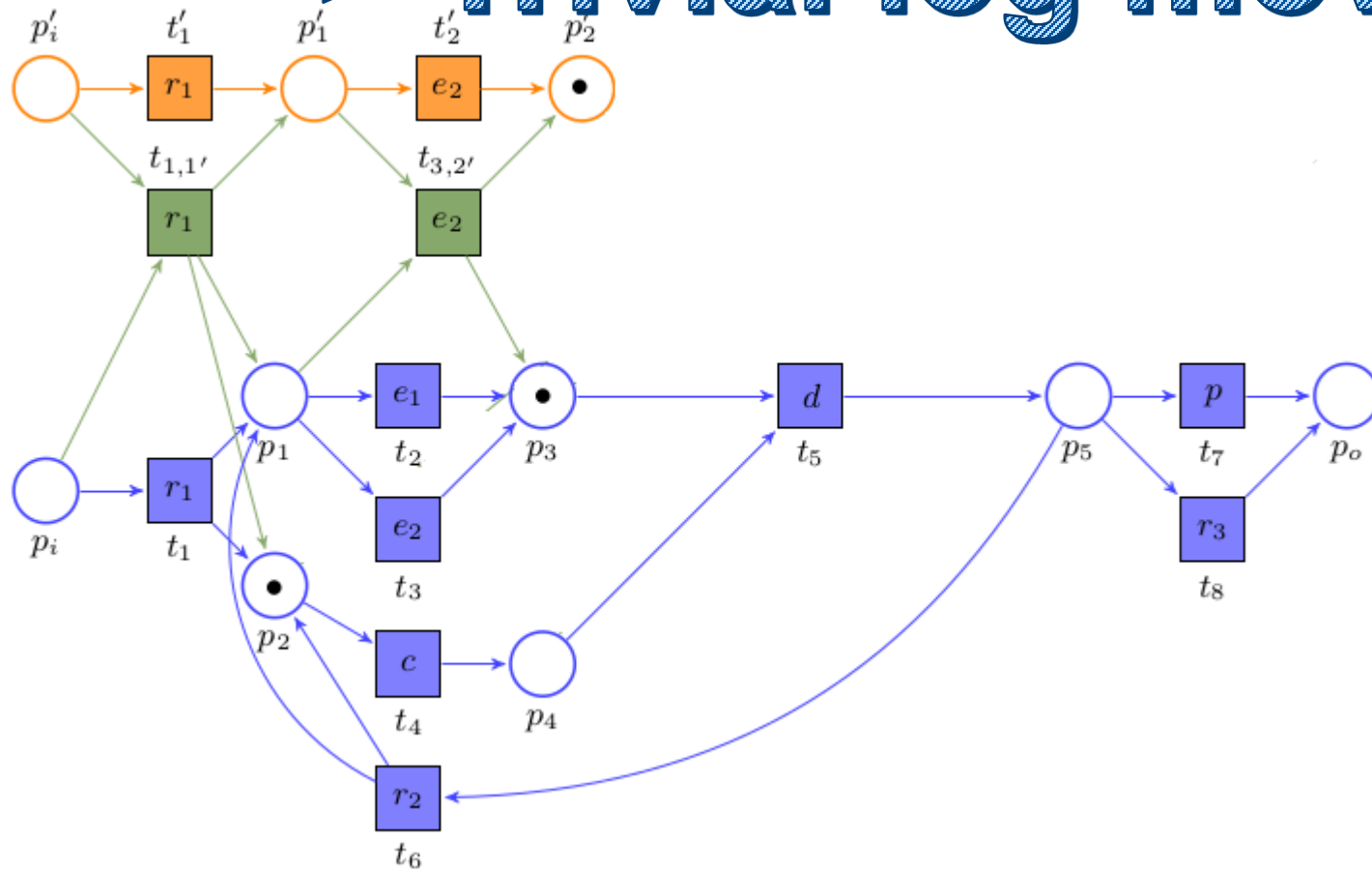
# Stream-Based Conformance Checking

## Approximation Scheme

- $\langle t_{1,1'}, t_{3,2'} \rangle$

receive: activity **f**

$\Rightarrow$  Trivial log move

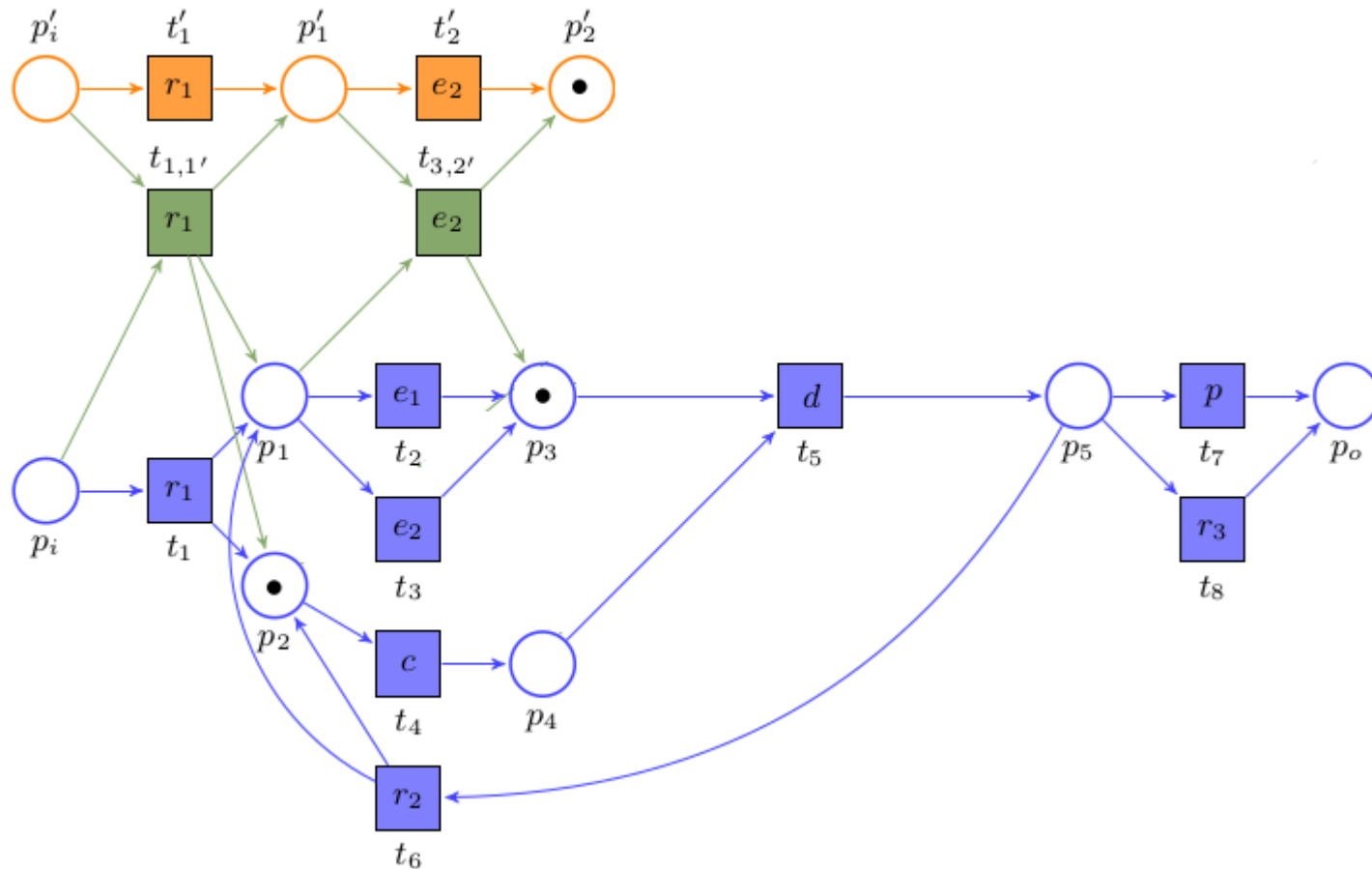


# Stream-Based Conformance Checking

## Approximation Scheme

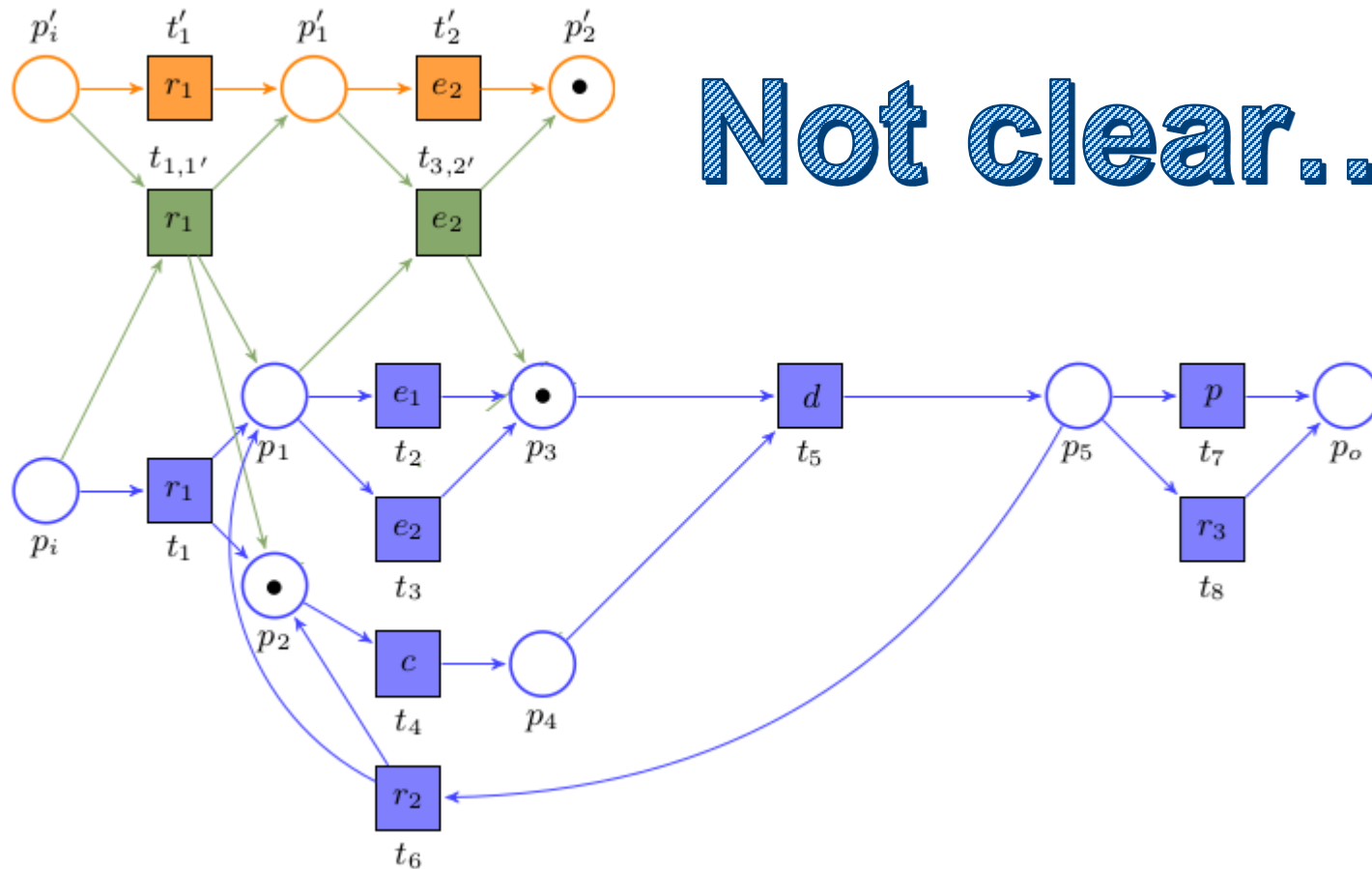
- $\langle t_{1,1'}, t_{3,2'} \rangle$

receive: activity  $e_1$



- $\langle t_{1,1'}, t_{3,2'} \rangle$

# Not clear...





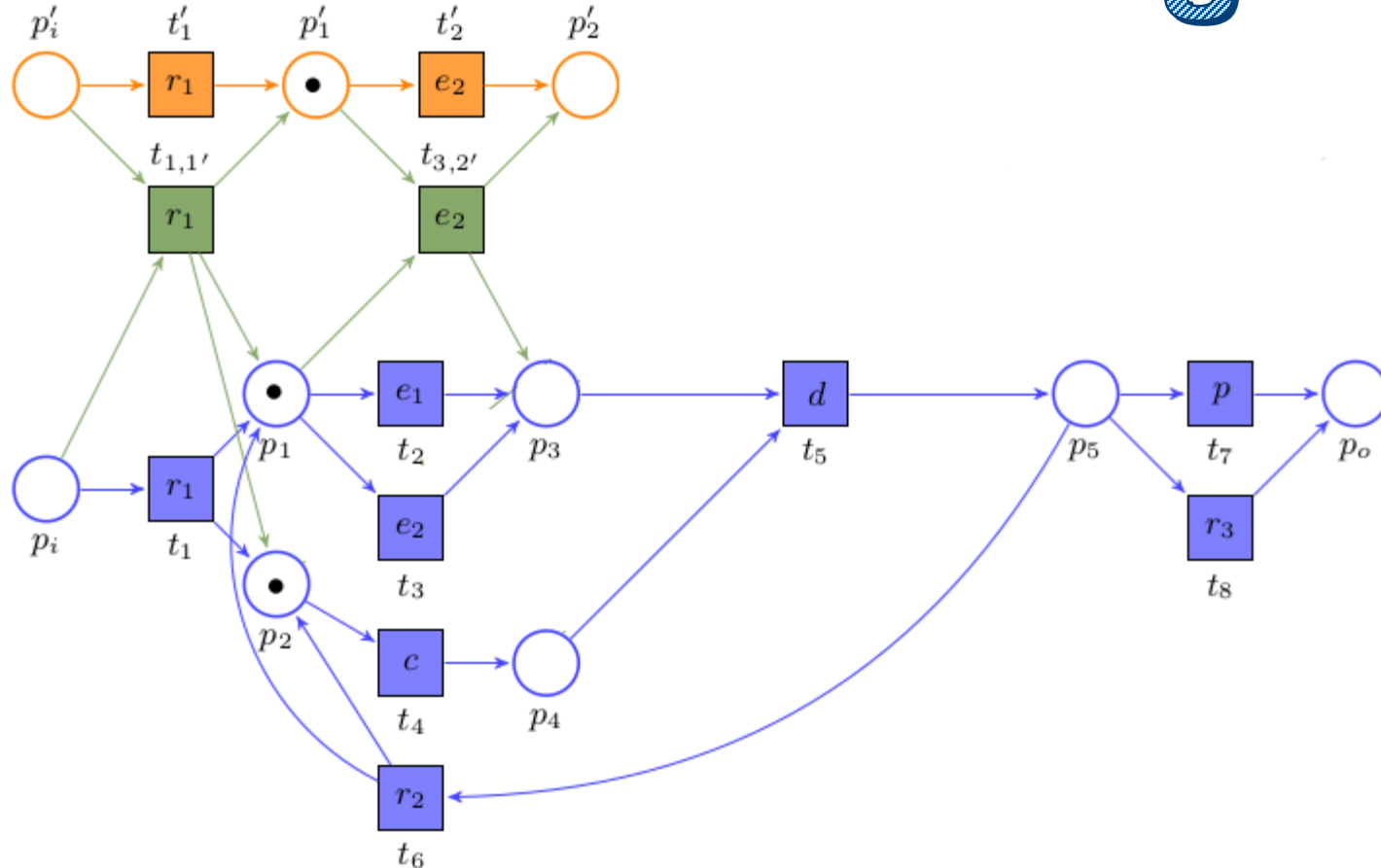
# Stream-Based Conformance Checking

## Approximation Scheme

- $\langle t_{1,1'}, t_{3,2'} \rangle$

receive: activity  $e_1$

# Revert alignment



# Stream-Based Conformance Checking

## Approximation Scheme

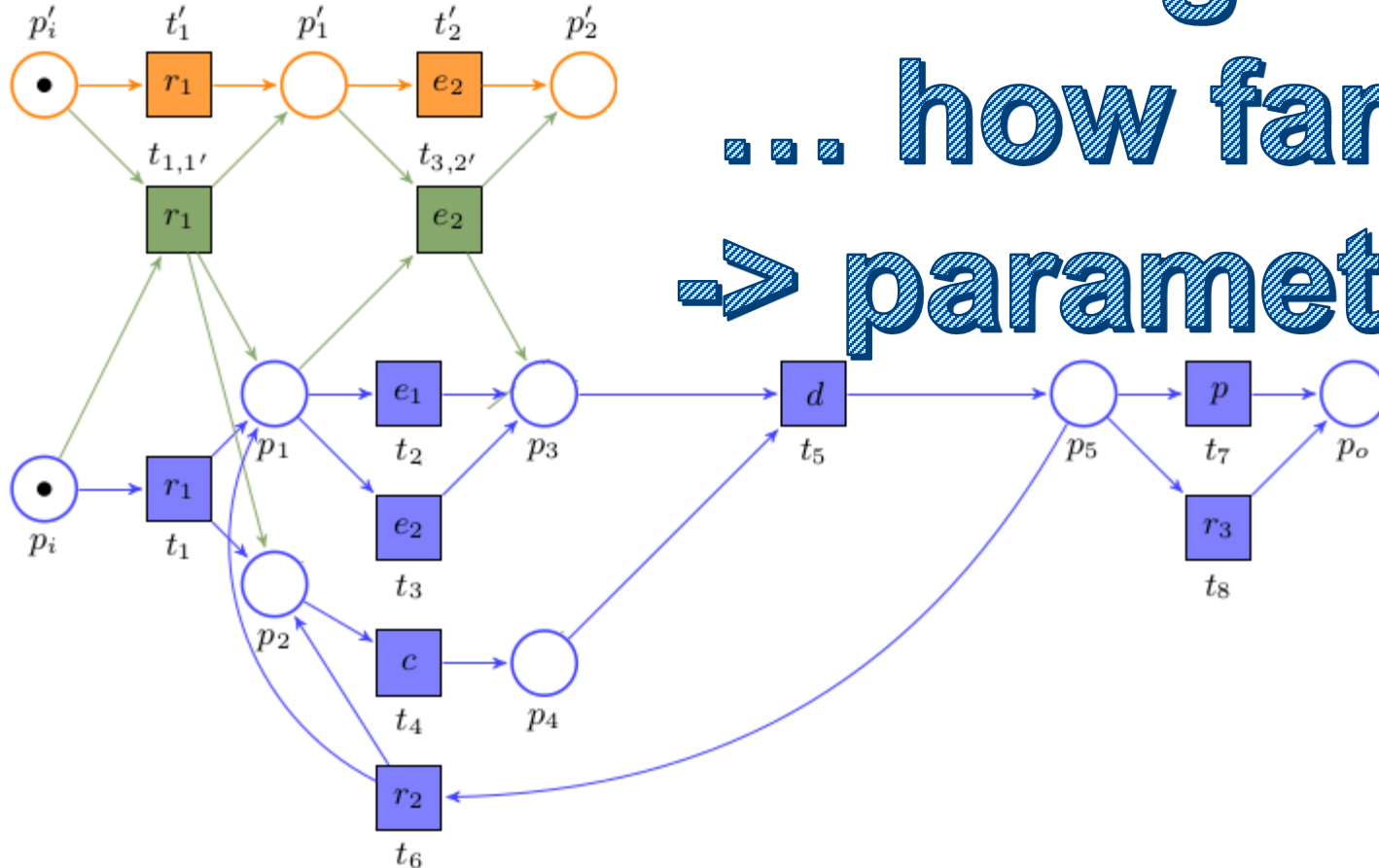
- $\langle t_{1,1'}, t_{3,2'} \rangle$

receive: activity  $e_1$

# Revert alignment

## ... how far?

## => parameter



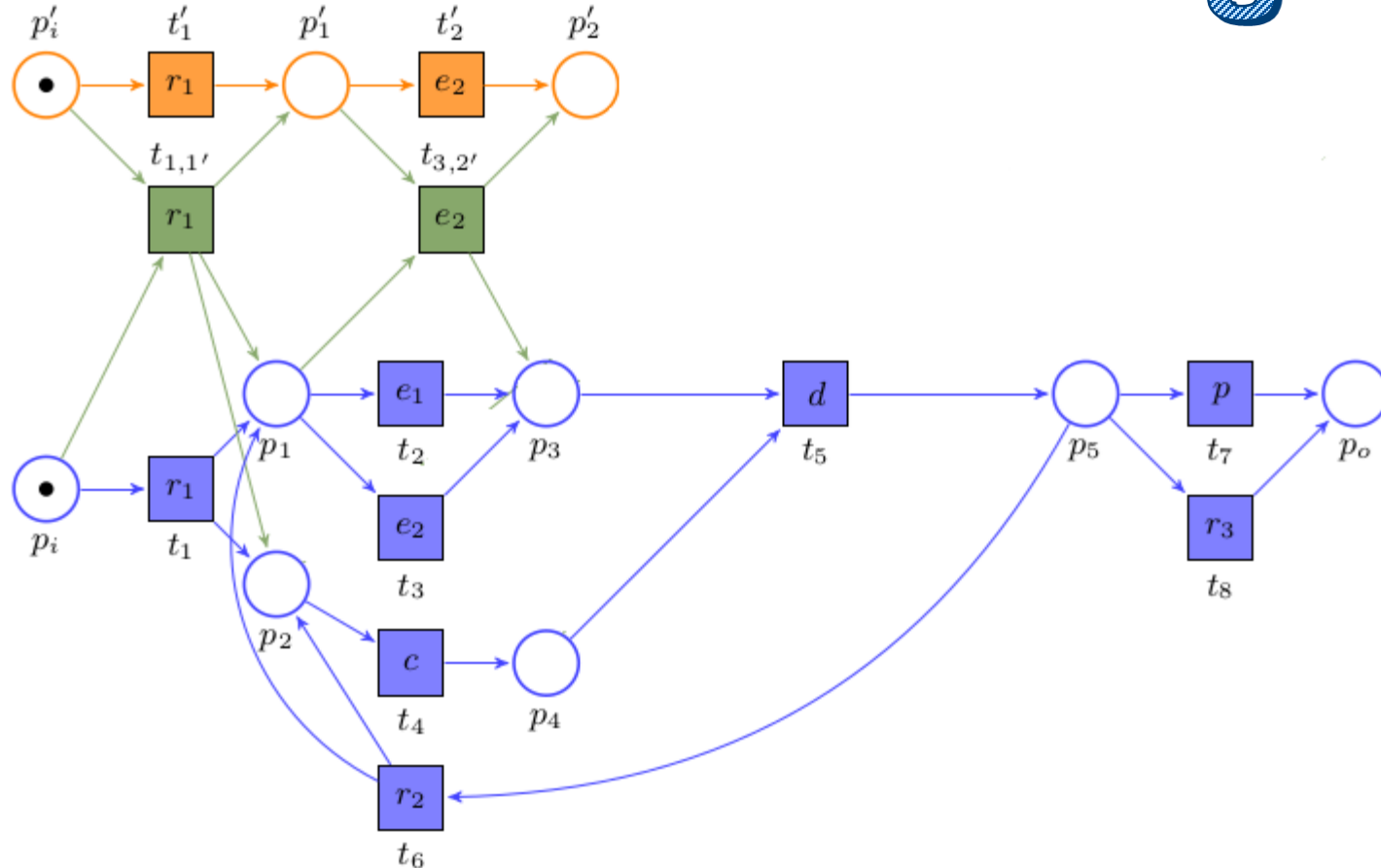
# Stream-Based Conformance Checking

## Approximation Scheme

- $\langle t_{1,1'}, t_{3,2'} \rangle$

receive: activity  $e_1$

# Re-align



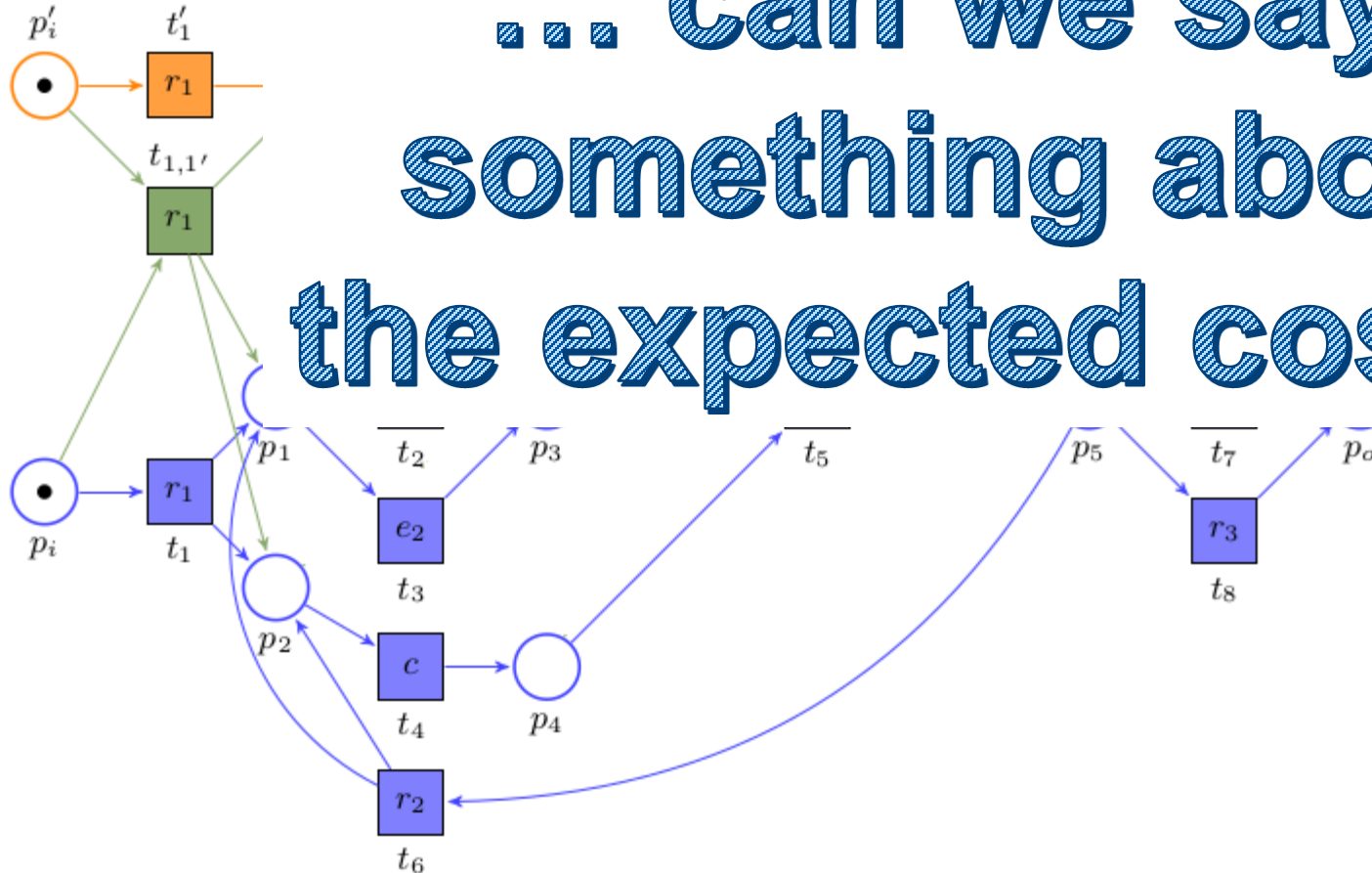
# Stream-Based Conformance Checking

## Approximation Scheme

- $\langle t_{1,1'}, t_{3,2'} \rangle$

receive: activity  $e_1$

... can we say  
something about  
the expected costs?



# Stream-Based Conformance Checking

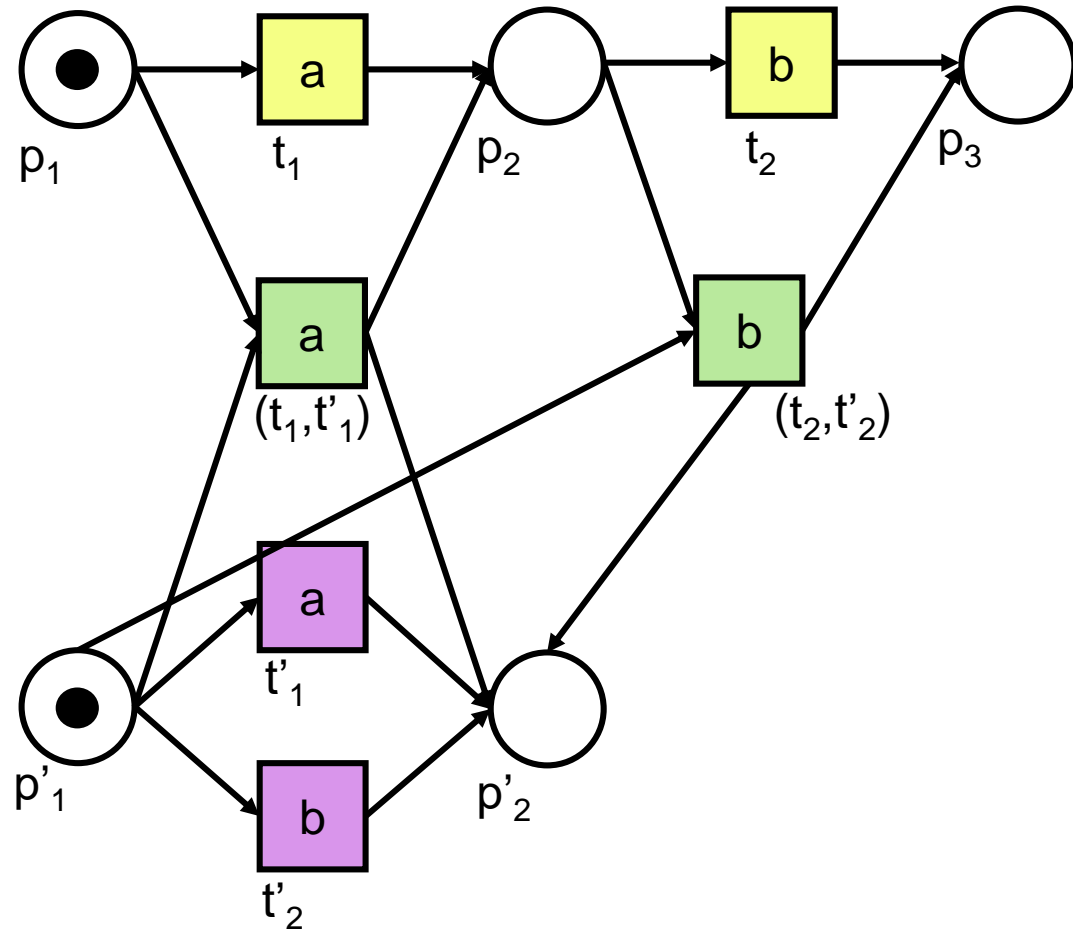
---

## Approximation Scheme

$$\min(\vec{c}^\top \vec{x} \mid \vec{m}' = \vec{m} + \mathbf{A}^\top \vec{x})$$

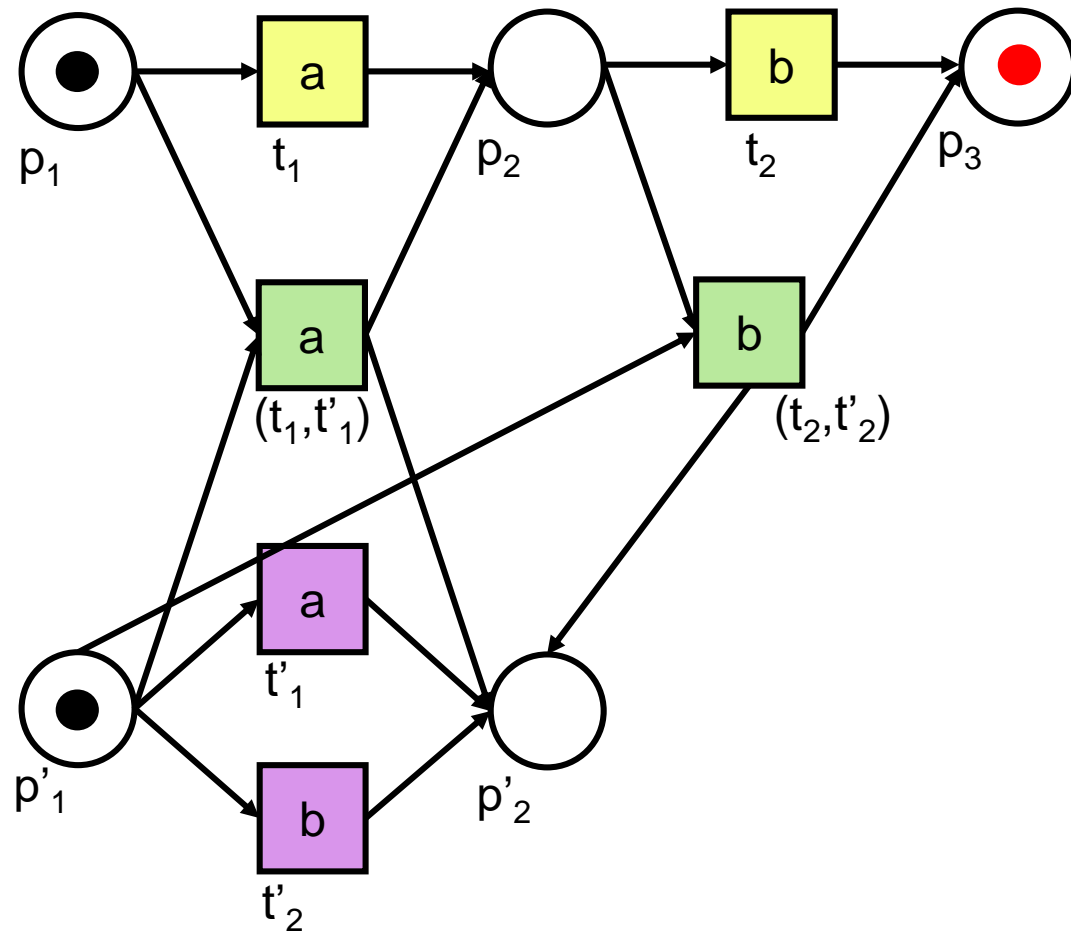
- What to do when we do not know  $m'$  exactly???

# Stream-Based Conformance Checking

$$\begin{bmatrix} t_1 \\ t_2 \\ (t_1, t'_1) \\ (t_2, t'_2) \\ t'_1 \\ t'_2 \end{bmatrix}$$


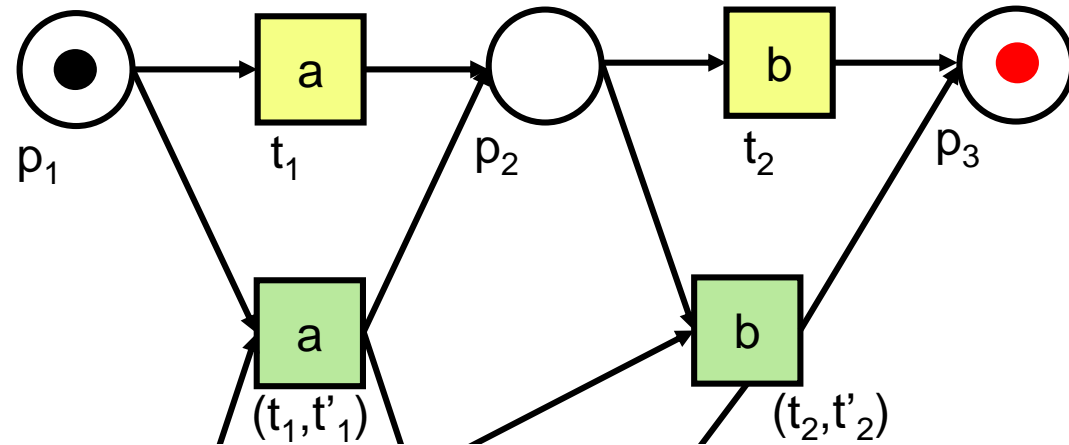
# Limiting the Exploration Area

$p_1: 0$   
 $p_2: 0$   
 $p_3: 1$   
 $p'_1: 0$   
 $p'_2: 0$

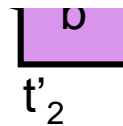


# Limiting the Exploration Area

$p_1: 0$   
 $p_2: 0$   
 $p_3: 1$   
 $p'_1: 0$   
 $p'_2: 0$



We *\*at least\** want a token  
in  $p_3$





# Stream-Based Conformance Checking

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- More recently ‘discovered’
  - (MSc thesis Daniel Schuster @ PADS RWTH)
- We can simply continue the search in conventional A\*
  - Recalculate the Heuristic for each state in the Open Set when a new event arrives

# Stream-Based Conformance Checking

---

- More recently ‘discovered’
  - (MSc thesis Daniel Schuster @ PADS RWTH)
- We can simply continue the search in conventional A\*
- Guarantees Optimality!!!
- Comparative with revert windows of 5-10...

# Stream-Based Conformance Checking

---

- More recently ‘discovered’
  - (MSc thesis Daniel Schuster @ PADS RWTH)
- When models are flower-like models:
  - Use approximation scheme with small windows
  - Often leads to optimal-ish results

# Stream-Based Conformance Checking

---

- More recently ‘discovered’
  - (MSc thesis Daniel Schuster @ PADS RWTH)
- When models are flower-like models:
  - Use approximation scheme with small windows
  - Often leads to optimal-ish results
- When models contain choices
  - The Optimal scheme starts to beat the approximation scheme

