Practical exam: Computational Geometry and Pattern Recognition

Duration: 1h30 / 20 points

All documents allowed, NO internet connection (except for ecampus documents).

1 Fractal dimension

In fractal geometry, the Minkowski–Bouligand dimension, also known as Minkowski dimension or box-counting dimension, is a way of determining the fractal dimension of a set S in a Euclidean space $\mathbb{R}^n$.
Suppose that $N(\epsilon)$ is the number of boxes of side length ϵ required to cover the **border of the set**. Then the box-counting dimension is defined as:

$$\dim_{\text{box}}(S) := \lim_{\varepsilon \to 0} \frac{\log N(\varepsilon)}{\log(1/\varepsilon)}.$$

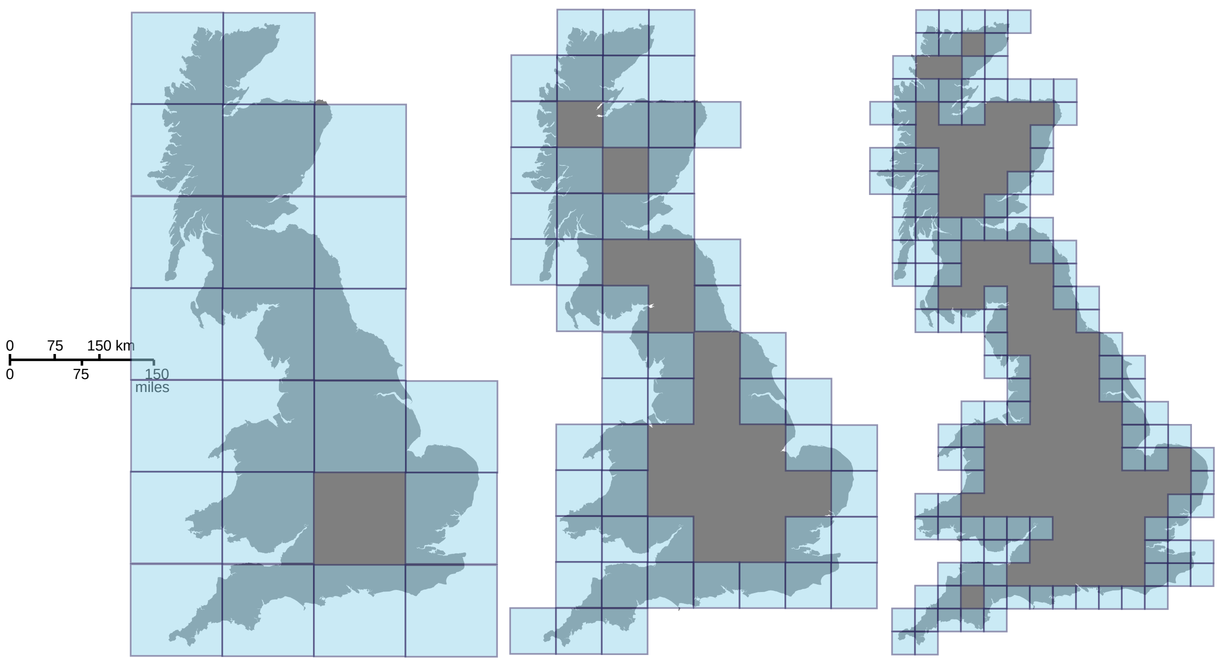

Figure 1: Grid decomposition for box counting.

1.1 Boxcount method

For a 2D binary array representing the set S, code a function boxcount that will return the sum of all values of the grid area (square).

```matlab
function B = boxcount(A, epsilon)
    % code here
```

```python
def boxcount(A, epsilon):
    ### code here
    return B
```

For example, if A is an array ones 16×16 ones,

$$A = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix}$$

$$\text{boxcount}(A, 6) = \begin{pmatrix} 36 & 36 & 24 \\ 36 & 36 & 24 \\ 24 & 24 & 16 \end{pmatrix}$$

$$\text{boxcount}(A, 5) = \begin{pmatrix} 25 & 25 & 25 & 5 \\ 25 & 25 & 25 & 5 \\ 25 & 25 & 25 & 5 \\ 5 & 5 & 5 & 1 \end{pmatrix}$$

1.2 Fractal dimension

Code a function fractal_dimension that takes a binary image as input and returns the fractal dimension. In order to do that:

- code a function $N(B, epsilon)$ that gives the number of grid areas that touch the border of B (see Fig.1).
- evaluate this number for different scales ϵ (these must be taken as powers of 2).
- represent a in plot the values $\log(N)$ as a function of the scales $\log(\epsilon)$ (see Fig.2).

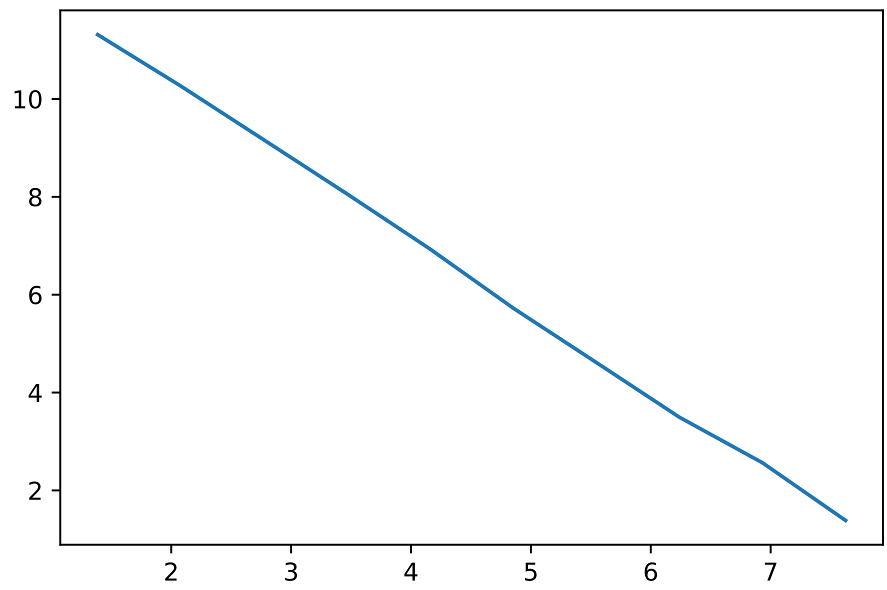

Figure 2: $\log(N)$ as a function of the scales $\log(\epsilon)$

- find an approximation of the fractal dimension.

1.3 Sierpinski triangle and numerical application

The Sierpinski triangle Fig.3 is a famous fractal figure.

- Give the theoretical value of its Haussdorf dimension (fractal dimension)
- Compute the fractal dimension with your code. As a hint, you should find a value around 1.6.

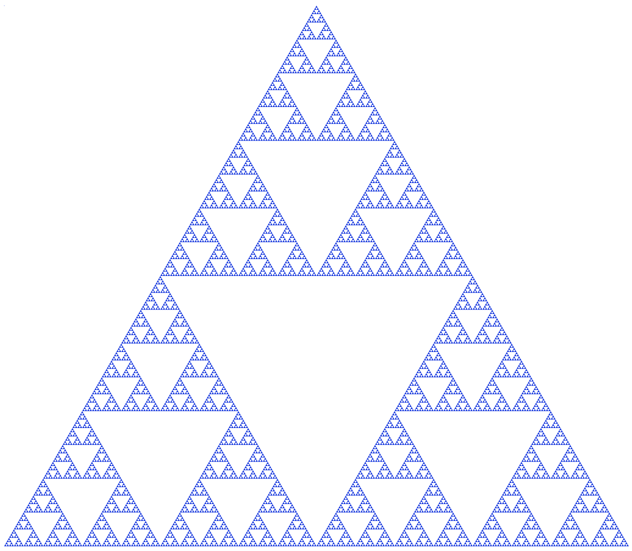

Figure 3: Sierpinski triangle.