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Cube Algebra:
A Generic User-Centric Model and Query Language for OLAP Cubes

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ABSTRACT

The lack of an appropriate conceptual model for data warehouses and OLAP systems has led to the tendency to deploy logical models (for example, star, snowflake, and constellation schemas) for them as conceptual models. ER model extensions, UML extensions, special graphical user interfaces, and dashboards have been proposed as conceptual approaches. However, they introduce their own problems, are somehow complex and difficult to understand, and are not always user-friendly. They also require a high learning curve, and most of them address only structural design, not considering associated operations. Therefore, they are not really an improvement and, in the end, only represent a reflection of the logical model. The essential drawback of offering this system-centric view as a user concept is that knowledge workers are confronted with the full and overwhelming complexity of these systems as well as complicated and user-unfriendly query languages such as SQL OLAP and MDX. In this article, the authors propose a user-centric conceptual model for data warehouses and OLAP systems, called the Cube Algebra. It takes the cube metaphor literally and provides the knowledge worker with high-level cube objects and related concepts. A novel query language leverages well known high-level operations such as roll-up, drill-down, slice, and drill-across. As a result, the logical and physical levels are hidden from the unskilled end user.

Keywords: Conceptual Model, Cube, Data Warehouses, OLAP, Query Language, User-Centric Model

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INTRODUCTION

Nowadays, data warehouses are at the forefront of information technology applications as a way for organizations to effectively use and analyze information for business planning and decision making. Data warehouses are large repositories of analytical and subject-oriented data integrated from several heterogeneous sources over a large period of time. The technique of performing complex analysis over the information stored in the data warehouse is commonly called Online Analytical Processing (OLAP). A review of the evolution of data warehouse technology reveals that research and development has mainly focused on system aspects such as the construction of data warehouses, materialization, indexing, and the implementation of OLAP functionality. This *system-centric* view has led to well-established and commercialized technologies such as relational OLAP (ROLAP), multidimensional OLAP (MOLAP), and hybrid OLAP (HOLAP) at the logical and the physical levels.

However, the unskilled user such as the manager in a consulting company or the analyst in a financial institution is confronted with the problem that the handling of data warehouses and OLAP systems requires expert knowledge due to complicated data warehouse structures and the complexity of OLAP systems and query languages. Two main reasons are responsible for this problem. First, due to the lack of a generic, user-friendly, and comprehensible conceptual data model, data warehouse design is usually performed at the logical level and leads to the exposure of the logical design schemas that are difficult to understand by the unskilled user. In a ROLAP environment, for example, the user is faced with the logical design of relational tables in terms of star, snowflake, or fact constellation schemas. The proposal to alleviate the problem by providing extensions to the Entity-Relationship Model and the Unified Modeling Language, or by offering specific graphical user interfaces or dashboards for data warehouse design is not really convincing since ultimately they represent a reflection and visualization of relational technology concepts and, in addition, reveal their own problems. Second, available OLAP query and analysis languages such as MDX and SQL OLAP operate at the logical level and require the user's deep understanding of the data warehouse structure in order to be able to formulate queries. These languages are quite complex, overwhelm the unskilled user, and are therefore inappropriate as end-user languages.

We conclude that a *generic*, *conceptual*, and *user-centric* data warehouse model that focuses on user requirements is missing and needed. Such a model should fulfill several design criteria. First, it should be located above the logical level. Second, it should abstract from and be independent of the models and technologies (ROLAP, MOLAP, HOLAP) at the logical level. Third, it should be able to cooperate with any of these logical models and technologies. Fourth, it should enable the user to *generically* and *abstractly* represent and query hierarchical multidimensional data. Fifth, it should have an associated query language based exclusively on the conceptual level, thus providing high-level query operations for the user. The goal of this article is to propose and formally describe a conceptual and user-centric data warehouse model and query language that satisfies these design criteria. Surprisingly, the conceptual view this model adopts is not new; on the contrary, it is well known. However, the way and resoluteness in which we offer this concept is novel. Our proposed conceptual model leverages the *cube* view of data warehouses but takes the cube metaphor *literally*. This means that the user's conceptual world is solely the cube that the user can create, manipulate, update, and query. The cube is used as the user concept that completely abstracts from any logical and physical implementation details. Technically, this implies that cubes can be regarded as an *abstract data type* that provides cubes as the only kind of values (objects), offers high-level operations on cubes or between cubes such as *slice*, *dice*, *drill-down*, *roll-up*, and *drill-across* as the only available access methods, and hides any data representation and algorithmic details from the user, who can concentrate on her main interest, namely to analyze large volumes of data. Another characterization is to say that we define a *universal algebra* with cubes as the only sort and a collection of unary and binary operations on cubes. We therefore name our approach *Cube Algebra*. We will show that this algebra develops its full power and expressiveness if it is used as a high-level query language.

The paper is organized as follows. Next section discusses related work and compares available data warehouse models with our Cube Algebra. Then, we describe an application scenario that we use throughout the paper to illustrate important aspects of the Cube Algebra. In the same section, we provide a three-level architecture of a data warehouse and OLAP system that includes our Cube Algebra. We further specify the formal data model supporting the Cube Algebra. The section concludes with a sketch of a data definition language to specify the structure of a cube. Then, we define high-level OLAP cube operations such as *slice* and *drill-across*, and illustrate their use in a number of queries that refer to our application scenario. Finally, the last section draws some conclusions and sketches future work.

RELATED WORK

Several data warehouse (DW) models have been proposed in the literature (see for example the survey in Marcel (1999). Most of these models address the logical level (e.g., Li & Wang, 1996; Cabibbo & Torlone, 1997; Cabibbo & Torlone, 1998, Lehner, 1998). Therefore, they are dependent of specific technologies, for example ROLAP, MOLAP, and HOLAP, which lead to complex and non-user friendly query languages. In this section, we limit ourselves to *conceptual models* and discuss them with respect to our proposal, i.e., we focus on the user support provided by these models. We claim that most of the models aimed at addressing the conceptual level actually rely on structures that are close to the logical level, thus not addressing end-user needs. We call these *system-centric* models, opposite to the *user-centric* approach we present in this paper. Similarly, we also claim that the user should be provided with a query and analysis language that is exclusively based on the conceptual level. Although several proposals in the literature define a set of operators to handle multidimensional data (see for example the survey and the reference algebra in Romero and Abelló (2007), these proposals do not abstract from the logical level and thus, do not provide high-level query operations for the user. Taking the aforementioned discussion into account, we next comment on related work, and present an analysis against our proposal.

We first classify existing models into three classes: (a) conceptual models based on *extensions to the Entity-Relationship* (ER) *Model* (Chen, 1976); (b) conceptual models based on *extensions to the Unified Modeling Language* (UML); (c) models based on a view of data as a cube.

We start with a discussion on models in class (a) (*ER-based* models).

Rizzi (2007) proposed the Dimensional Fact Model (DFM), which uses the typical DW concepts of facts, dimensions, measures, hierarchies, descriptive and cross-dimension attributes; the model also supports shared, incomplete, recursive, and dynamic hierarchies, and notions such as additivity. To represent these concepts, DFM relies on a graphical notation that facilitates the understanding of the conceptual schema, and that is an abstraction of the star schema, in which there is a central fact entity and a graph per dimension to represent the attribute hierarchies. Golfarelli and Rizzi (1998) extend DFM by presenting a methodological framework for DW conceptual modeling, which starts gathering user requirements and carries out the data warehouse design semi-automatically from the operational database schema. In addition to providing an abstraction of the star schema in terms of a central fact entity and several graphs, they also formalize each concept of the DFM and define a language to denote queries according to the DFM to validate the generated schema.

Ravat et al. (2008) study three related issues. First, they propose a conceptual multidimensional model, which is based on the concepts of constellations, dimensions, dimension attributes, hierarchies, facts, measures, and multidimensional table structures (i.e., tabular representations of multidimensional data). Second, they define a set of algebra operators to manipulate multidimensional data, which include: (i) a minimal core of operators to modify analysis precision (i.e., drill-down, roll-up, select), to change analysis criteria (i.e., rotate, add measure, delete measure, push and pull, nest), and to change the multidimensional table presentation (i.e., switch, aggregate); (ii) advanced operators (i.e., frotate, hrotate, order, plot, unselect) that are obtained from the combination of core operators aiming at simplifying complex queries; and (iii) binary operators (set operators), such as union, intersect, and minus, based on the (semi-)compatibility of input tables. Third, they develop a graphical interface based on the multidimensional table structures and the DFM commented above. This interface is supported by a graphic language that encompasses the core algebra operators. Although this work focuses on a user-oriented query language composed of a formalized algebra and a graphical query language, the multidimensional model over which this work is based is very close to the star schema and strongly based on the concept of multidimensional table, i.e., it remains as a system-centric model.

Tryfona et al. (1999) propose the starER model, which combines the star structure with the constructs of the ER Model, in addition to proposing special types of relationships to support attribute hierarchies on dimensions. The starER model encompasses the following main constructors: facts, entities, relationships among entities, and attributes (i.e., properties of entities, relationships, or facts). Further, the starER model provides a graphical notation very close to the ER Model. Along similar lines (i.e., starting from an ER-based data model), Malinowski and Zimányi (2008) introduce a metamodel of hierarchy classification that encompasses from symmetric simple hierarchies until more complex ones, such as non-strict simple hierarchies, asymmetric and generalized simple hierarchies, multiple hierarchies, and parallel hierarchies. They also present a graphical notation for representing these hierarchies, close to a relational representation.

We now move on to discuss models in class (b) (*UML-based* models).

Nguyen et al. (2000) use UML to map their proposed conceptual multidimensional data model to an object-oriented data model. The data model is based on the concepts of dimensions, dimension members, dimension levels, dimension schemas, dimension paths and hierarchies, dimension operators, measures, and data cubes. This model aims at representing natural hierarchical relationships among members within a dimension as well as unbalanced and multiple hierarchies. The authors also define the following cube operators: groupBy, jumping, rollingUp, and drillingDown. However, this model is defined at a level very close to the logical one. Also, this proposal does not introduce a full set of high-level operations such as slice and drill-across.

Abelló et al. (2001) investigate relationships between cubes in an object-oriented framework with navigation operations. Here, the data cube is defined in terms of a set of concepts, such as measures and cells, dimensions and aggregation levels, and facts. An algebra is defined as a set of multidimensional operations, such as base changes, dice, slice, drill-across, roll-up and drill-down. In sequels of this paper (Abelló, Samos, & Saltor, 2002; Abelló, Samos, & Saltor, 2006), the authors propose YAM^2, a multidimensional conceptual object-oriented model for data warehousing and OLAP tools extending the UML, which is defined in terms of its structures, integrity constraints, and query operations. However, in spite of being defined at the conceptual level, YAM^2 relies on star schema-like design, and therefore is not completely independent of logical modeling concepts.

Pardillo et al. (2008) introduce platform-independent conceptual OLAP queries that can be automatically traced to their logical implementation, together with an OLAP algebra at the conceptual level by using the Object Constraint Language (OCL), aimed at allowing end-users to query data warehouses without being aware of logical details. The authors introduce cube manipulation operators (e.g., dimension addition and removal), and operators such as slice, dice, drill-across, multidimensional projection, roll-up and drill-down. Cabot et al. (2009) extend the former work through a conceptual specification of statistical functions using OCL. Finally, Pardillo et al. (2010) further extends OCL for OLAP querying, introducing a code-generation architecture aligned with the model-driven architecture (MDA), to map an extension to the OCL as a set of predefined OLAP operators. The main drawback of this work is that they are based on OCL, which is not user-friendly for data cube manipulation.

Let us now comment on models based on the data cube (i.e., the ones in class (c)).

Tsois et al. (2001) propose a multidimensional aggregation cube (MAC) using the concepts of dimension, dimension level, dimension member, drilling relationship, and dimension path. Although they address the DW modeling problem from the end-user point of view, and describe a set of requirements for the conceptual modeling of real-world OLAP scenarios, the authors do not present a language supporting the model.

Along different lines, some authors formally define the notion of a cube and introduce operations for this. Agrawal et al. (1997) propose a data model whose core features are the symmetric treatment of dimensions and measures, the support of multiple hierarchies along each dimension and the possibility of performing ad-hoc aggregates. They also define a minimal set of algebraic operators that is composed of the following operators: push and pull (to allow symmetric treatment of dimensions and measures), destroy dimension, restriction (slice and dice), join, and associate. This minimal set of operators stays as close to relational algebra as possible and can be translated to SQL through an algebraic application programming interface.

The conceptual multidimensional model proposed by Gyssens and Lakshmanan (1997) focuses on the separation between structural aspects and the content, allowing the definition of a data manipulation language that can express the cube operator. They define an algebra (and an equivalent calculus), which include set operators (like selection, projection, Cartesian product), operators for summarization, and re-structuring operators (fold and unfold).

Vassiliadis (1998) formally defines the concepts of dimensions, hierarchies, and cube to propose a model for multidimensional databases. He also introduces a set of cube operators based on the notion of the base cube, which is used for providing the calculation of the results of cube operations. These operations are: level climbing, packing, function application, projection, navigation, slicing, and dicing. He also provides a mapping of the proposed model to the relational model and to multidimensional arrays.

Datta and Thomas (1999) also defined a data cube model and an associated algebra. The data cube model includes the concepts of data cube, dimensions, dimension attributes, measures and attribute hierarchies, and focuses on the symmetric treatment of dimensions and measures. As for the algebra, the purpose of the work is to provide comprehensive OLAP functionalities, including aggregation (to be applied several times to enable roll-up and drill-down operations and comparisons of aggregate values), transformations to convert dimensions to measures and vice versa and partitioning (grouping of data for aggregating purposes). The algebra has the following operators: restriction, aggregation, Cartesian product, join, union, difference, pull, push and partition.

The proposals in this class, although based on the concept of a cube, do not approach the problem from a conceptual modeling view-point, neither they consider users' needs (i.e., the user-centric view). Further, the proposed operators are not the ones commonly needed by high-level users such as managers.

Table 1 refers to the following core characteristics of each studied DW model: (i) its focus (system-centric vs. user-centric); (ii) if the model supports the data cube metaphor; (iii) if the model provides a cube as an abstract data type; (iv) the level at which the model is defined (conceptual level, logical level, or somewhere in-between them); (v) the model that it is based on; and (vi) if it includes an OLAP algebra or calculus. We can see that almost all proposals defined at the conceptual level are actually system-centric rather than user-centric. On the contrary, the Cube Algebra proposal applies to the conceptual level of a data warehouse architecture, independently from implementation issues. The only user-centric model is the proposal of Tsois et al. (2001). However, this model does not provide a cube as an abstract data type and, more important, it does not propose an OLAP algebra or calculus to support it.

Table 2 compares existing proposals against Cube Algebra, with respect to query language functionalities, namely: (i) if the operations are defined over the cube or over other data objects, at lower levels of abstraction (for example, tables, star schemas); (ii) the complexity of the query language for unskilled end users; (iii) the kind of query language, such as cube-based, relational algebra-based, relational calculus-based, OCL-based and MDX-based, or no language provided at all; and (iv) if the work provides some graphical notation or dashboard to aid in the data warehouse modeling. As we can see in Table 2, no work besides the Cube Algebra that offers a user-friendly query language is the proposal of Abelló et al. (2001), who introduce a query language at the conceptual level. Nevertheless, the query language is complex for unskilled end users. Besides, the operations defined over the cube, such as base changes, generalization, specialization, and derivation, are far from the knowledge of managers and analysts in an OLAP scenario.

Finally, Table 3 details and compares the set of operations that each proposal provides, that is, general functionalities to create, manipulate, and update the cube metaphor, and the set of high-level operations for querying the data cube (roll-up, drill-down, slice, dice, drill-across, pivot). As we can see, only the Cube Algebra encompasses all the operations. We can also see that most of the proposals do not offer operations to create, manipulate, and update the cube.

CUBE DATA MODEL AND THREE-LEVEL DATA ARCHITECTURE

In this section, we present our user-centric cube data model and show how it fits into the landscape of data warehouses. By selecting the application scenario of pollution control, next section informally introduces the main cube concepts that a user should be able to understand. Then, we present a three-level architecture of a data warehouse and OLAP system that integrates our Cube Algebra, and formally define the underlying user-centric data model supporting such algebra. Finally, we sketch a high-level data definition language for data cubes.

Application Scenario: Cubes for Pollution Control

We illustrate the needed user concepts by leveraging an application scenario of pollution control in Belgium. Monitoring stations located at different locations enable the measurement of certain pollutants (such as carbon monoxide, CO). Thus, three aspects or perspectives play a role here for the knowledge worker: (i) the monitoring station where a measurement is captured, (ii) the time when a measurement is taken, and (iii) the kind of pollution that is measured. These aspects form the three dimensions of a cube that are shown on the Figure 1a. In this example, we call the dimensions Station, Time, and Pollution. Dimensions are an essential and

Table 1. Core properties of data warehouse models

Related Work	Focus	Cube Metaphor	Cube as ADT	Model Level	Model Extension	Algebra or Calculus
Abelló et al. (2001)	system-centric	✓	×	conceptual	object-oriented	✓
Abelló et al. (2002) Abelló et al. (2006)	system-centric	×	×	higher than logical, lower than conceptual	UML-based	✓
Agrawal et al. (1997)	system-centric	✓	×	higher than logical, lower than conceptual	not an extension	✓
Cabibbo and Torlone (1997)	system-centric	×	×	logical	not an extension	✓
Cabibbo and Torlone (1998)	system-centric	×	×	logical	not an extension	✓
Cabot et al. (2009)	system-centric	×	×	logical	UML-based	×
Datta and Thomas (1999)	system-centric	✓	×	higher than logical, lower than conceptual	not an extension	✓
Golfarelli and Rizzi (1998)	system-centric	×	×	logical	not an extension	×
Gyssens and Laksshmanan (1997)	system-centric	✓	×	logical	not an extension	✓
Lehner (1998)	system-centric	×	×	logical	not an extension	✓
Li and Wang (1996)	system-centric	✓	×	logical	not an extension	✓
Malinowski and Zimányi (2008)	system-centric	×	×	logical	not an extension	×
Nguyen et al. (2000)	system-centric	✓	×	higher than logical, lower than conceptual	UML-based	✓
Pardillo et al. (2008)	system-centric	✓	×	higher than logical, lower than conceptual	UML-based	✓
Pardillo et al. (2010)	system-centric	✓	×	higher than logical, lower than conceptual	UML-based	✓
Ravat et al. (2008)	system-centric	×	×	higher than logical, lower than conceptual	not an extension	×
Rizzi (2007)	system-centric	✓	×	logical	not an extension	×

continued on following page

Table 1. Continued

Related Work	Focus	Cube Metaphor	Cube as ADT	Model Level	Model Extension	Algebra or Calculus
Tryfona et al. (1999)	system-centric	×	×	conceptual	ER-based	×
Tsois et al. (2001)	user-centric	✓	×	conceptual	not an extension	×
Vassiliadis (1998)	system-centric	✓	×	higher than logical, lower than conceptual	not an extension	✓
Cube Algebra	user-centric	✓	✓	conceptual	not an extension	✓

distinguishing first-class concept in cubes. They are visually or geometrically represented by the lateral faces of the (hyper) cube. A dimension is organized as a containment-like hierarchy. Each hierarchy level represents a different (aggregation) level of detail, as it is later required by the desired analyses. Figure 1b shows the three hierarchical structures of the Station, Time, and Pollution dimensions. Each hierarchical structure is called a *dimension schema*. For example, the dimension schema for Pollution models two hierarchies with Pollutant as their common lowest level, namely the hierarchy consisting of the levels Pollutant, Type, and Group, as well as the hierarchy consisting of the levels Pollutant and Category. The levels above Pollutant allow grouping and are therefore interesting for the knowledge worker. For example, similar pollutants can be grouped under the level Category. Note that there is a unique top level in the dimension schema, denoted All, to which all levels aggregate. Each dimension level includes a finite set of values called *members*. A *dimension instance* comprises all members at all levels of a *dimension hierarchy*. Figure 3 gives an example of a dimension instance with respect to the dimension schema for Pollution shown in Figure 1b. The level Type, for example, contains the three members: T1, T2, and T3.

A combination of members taken from each dimension uniquely defines a cell of a cube and implicitly specifies a *fact* if the cell is not empty. For example, in Figure 1a, at station S2 and quarter Q3, the pollutant P4 was measured since the cell is not empty. A value in a cell (such as 12 in our case) is called a *measure value*. A *measure* represents a numerical property of a fact. In our example, the measure is named concentration. A cube can have several different measures. Each measure comes with an *aggregation function* that can combine several measure values into one.

Data Warehouse Architecture

Figure 2 shows the three-level data warehouse architecture we devise (see Ullman (1988)). At the conceptual level (i.e., the highest abstraction level) of this architecture, there is the cube model described above (independent of how this cube is actually implemented), and the associated cube algebra we introduce later. At the logical level we have the implementation-dependent representation of the data cube. At this level we place the well-known star, snowflake, and constellation schemas (i.e., a ROLAP representation), as well as multidimensional (MOLAP), and hybrid representations (HOLAP). Query languages for these representations are relational query languages such as SQL dialects, and MDX. Note that although MDX works on cubes in the same way as SQL works on tables, it cannot be considered as a language operating at the conceptual level: not only its semantics has never been clearly defined, but the language is far from being user-friendly (as we will show

Table 2. Query language and operations of data warehouse models

Related Work	Operations Defined *Only* Over the Cube Metaphor	Complexity of the Query Language	Kind of Query Language	Graphic Tools or Dashboards
Abelló et al. (2001)	×	complex	conceptual	×
Abelló et al. (2002) Abelló et al. (2006)	×	complex	relational algebra	×
Agrawal et al. (1997)	✓	complex	relational algebra	×
Cabibbo and Torlone (1997)	×	complex	calculus-based	×
Cabibbo and Torlone (1998)	×	complex	relational algebra	✓
Cabot et al. (2009)	×	complex	OCL-based	×
Datta and Thomas (1999)	✓	complex	relational algebra	×
Golfarelli and Rizzi (1998)	×	not available	not available	✓
Gyssens and Lakssh-manan (1997)	×	complex	relational alge-bra/ calculus	×
Lehner (1998)	×	not available	relational algebra	×
Li and Wang (1996)	×	complex	relational algebra	×
Malinowski and Zimányi (2008)	×	not available	not available	✓
Nguyen et al. (2000)	✓	complex	relational calculus	×
Pardillo et al. (2008)	×	complex	OCL-based	×
Pardillo et al. (2010)	✓	complex	relational alge-bra plus OCL	✓
Ravat et al. (2008)	×	user-friendly	relational algebra	✓
Rizzi (2007)	×	not available	not available	✓
Tryfona et al. (1999)	×	not available	not available	✓
Tsois et al. (2001)	×	not available	not available	✓
Vassiliadis (1998)	×	complex	relational calculus	×
Cube Algebra	✓	user-friendly	relational algebra	×

Table 3. Types of operations of data warehouse manipulation languages

Related Work	General Functionalities			Query Cube				
	Create Cube	**Manipulate Cube**	**Update Cube**	**Roll-Up (Drill-Down)**	**Slice**	**Dice**	**Drill-Across**	**Pivot**
Abelló et al. (2001)	×	✓	×	✓	✓	×	✓	×
Abelló et al. (2002) Abelló et al. (2006)	×	×	×	roll-up	×	×	✓	×
Agrawal et al. (1997)	×	×	×	✓	✓	×	×	×
Cabibbo and Torlone (1997)	×	×	×	×	×	×	×	×
Cabibbo and Torlone (1998)	×	×	×	roll-up	×	×	×	×
Cabot et al. (2009)	×	×	×	×	×	×	×	×
Datta and Thomas (1999)	×	×	×	×	✓	×	×	×
Golfarelli and Rizzi (1998)	×	×	×	×	×	×	×	×
Gyssens and Lakshmanan (1997)	×	✓	×	roll-up	✓	×	×	×
Lehner (1998)	×	×	×	✓	✓	×	×	×
Li and Wang (1996)	✓	✓	×	×	×	×	×	×
Malinowski and Zimányi (2005)	×	×	×	×	×	×	×	×
Nguyen et al. (2000)	×	×	×	✓	×	×	×	×
Pardillo et al. (2008)	×	✓	×	✓	✓	×	✓	×
Pardillo et al. (2010)	×	✓	×	✓	✓	×	✓	×
Ravat et al. (2008)	×	×	×	✓	✓	×	✓	✓
Rizzi (2007)	×	×	×	×	×	×	×	×
Tryfona et al (1999)	×	×	×	×	×	×	×	×
Tsois et al. (2001)	×	×	×	×	×	×	×	×
Vassiliadis (1998)	×	×	×	drill-down	✓	×	×	×
Cube Algebra	✓	✓	×	✓	✓	×	✓	✓

later), a key issue in our approach. Finally, at the physical level, we find the different ways of efficiently implementing the data warehouse. For example, for ROLAP implementations multidimensional indices such as variations of R-Trees can be used, as well as bitmap indices. For MOLAP implementations, efficient and proprietary algorithms for implementing sparse matrices are often used.

Figure 1. (a) A three-dimensional cube for AirQuality data having dimensions time, pollution, and station, and a measure concentration (b) dimension hierarchies

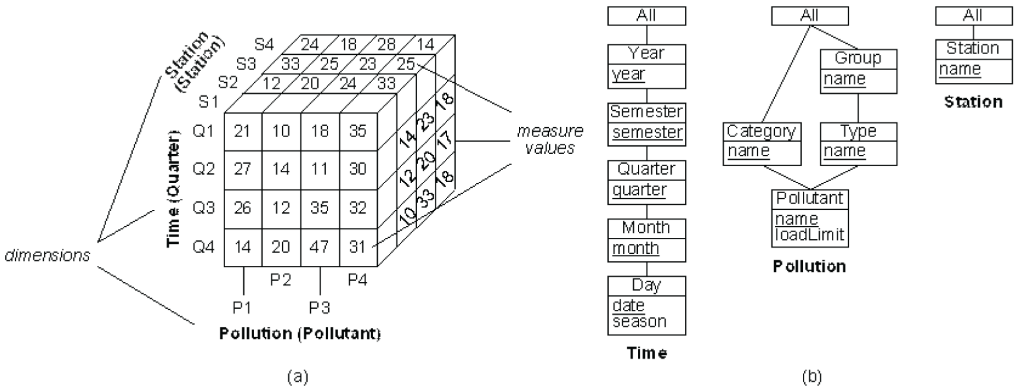


Figure 2. The three-level architecture for multidimensional databases

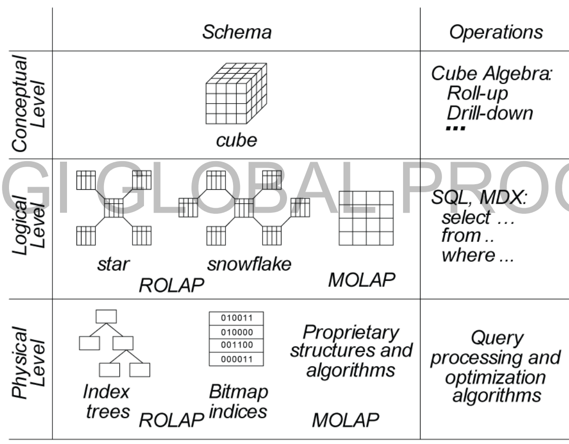
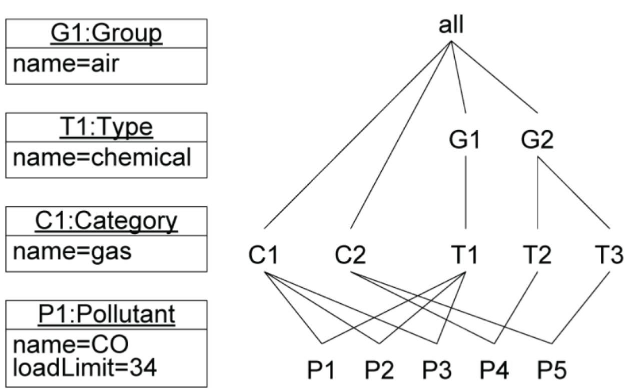


Figure 3. Some instances (left) and roll-up functions (right) of dimension pollution of Figure 1b



Formal Cube Data Model

Our cube-based, user-centric conceptual model is supported by the formal model we introduce next. For simplicity, and without lost of generality, we assume that dimension level names are unique[1]:

Definition 1 (Dimension Schema)

A *dimension schema* is a tuple $\langle nameDS$, L, $\rightarrow\rangle$ where: (a) *nameDS* is the name of the dimension; (b) L is a non-empty finite set of pairs of the form $\langle l, A\rangle$ such that l is a level (there is a distinguished level name denoted *All*, such that $\langle All, \varnothing\rangle \in$ L), and A is a set of attributes describing a level). Each attribute has a domain Dom_a; without loss of generality, we consider that one attribute in A univocally identifies a member in level l; (c) $\rightarrow$ is a partial order on the levels $l \in$ L. This partial order defines a graph, whose nodes are the levels $l \in$ L, and are annotated by attributes in A; (e) The reflexive and transitive closure of $\rightarrow$, denoted $\rightarrow^*$, has a unique bottom level l_b, and a unique top level. The top level is the distinguished level *All*. We denote by $l_i \rightarrow^* l_j$ the fact that there is a path between l_i and l_j. Moreover, all levels l are such that $l_b \rightarrow^* l$, and $l \rightarrow^* All$.

Intuitively, a dimension schema is a directed acyclic graph (DAG). Each node in this graph represents an aggregation level and is annotated with a list of attributes. There is a distinguished level denoted *All* without attributes, and a unique bottom level. All levels are (directly or transitively) reachable from the bottom, and all levels (directly or transitively) reach the level *All*.

Definition 1 could be simplified if we ignore the level attributes, and consider each node in the graph as a single data element. However, we decided to state the formal model in this way to account for the way in which the user operates with the cube in real-world practice, where she defines a dimension level as an aggregation level, and attributes are displayed after aggregation has been performed.

Definition 2 (Dimension Instance)

An instance I of a dimension schema $\langle nameDS$, L, $\rightarrow\rangle$ consists of (a) a finite set of members M_l, for each level l in L, such that each member can be uniquely identified (the level *All* has a unique member *all*); (b) a set of partial functions, denoted as *roll-up functions* (following (Cabibbo & Torlone, 1997)) of the form $Roll\text{-}up_{li}^{lj}$ from the members of level l_i to the members of level l_j, for each pair of levels l_i and l_j in L, such that $l_i \rightarrow l_j$ in $\rightarrow$; (c) a collection of functions $f_l^1, \ldots f_l^k$ mapping members of l to values in the domain of each level attribute $a_1, \ldots, a_k \in A$.

Intuitively, a dimension instance is also a DAG. Associated with an edge (l_i, l_j) in the schema graph such that $l_i \rightarrow l_j$, there is a function from l_i to l_j. This function describes how members in the lower level aggregate to members in the upper level. Thus, a dimension instance is just a collection of such functions. Also note that our model supports multiple hierarchies, meaning that the same DAG models the different aggregation paths from bottom to top.

Example 1 (Dimension Schema and Instance)

In Figure 1b we can see three dimension schemas. Let us consider the one for dimension Pollution. The schema of this dimension is formally defined as follows:

nameDS = Pollution,
L= {⟨ Pollutant,(name, loadLimit)⟩, ⟨Type,(name)⟩,…,⟨Group,(name)⟩},
→ = {Pollutant → Type, Type → Group, Group → All, Pollutant → Category, Category → All }

Note that in Figure 1b the attributes in the dimension levels that are underlined are the identifiers.

The instances for dimension Pollution are depicted in Figure 3, and are of the form:

$M_{Pollutant} = \{P1,\ldots,P5\}$, $M_{Category} = \{C1, C2\}$, $M_{Type} = \{T1, T2, T3\}, \ldots$
$Roll\text{-}up_{Pollutant}^{Type} = \{(P1,T1), (P2,T2), \ldots, (P5,T3)\}\ldots$
$Roll\text{-}up_{Pollutant}^{Category} = \{(P1,C1), (P2,C1), \ldots, (P5,C2)\}$
$Roll\text{-}up_{Category}^{All} = \{(C1,all),(C2,all)\}$
$f_{Pollutant}^{name}(P1) = CO,\ldots,f_{Pollutant}^{name}(P5) = PM.$
$f_{Pollutant}^{loadLimit}(P1) = 34,\ldots,f_{Pollutant}^{loadLimit}(P5) = 44\ldots$
$f_{Category}^{name}(C1) = gas,\ldots,f_{Category}^{name}(C2) = solid.$

Intuitively, the aggregation hierarchy is used as follows: let us suppose that the concentrations of pollutants P2 and P3 measured at station S1 in a given day D1 are 30 and 40 mg/m^3, respectively. If we want to know the average concentration at S1, aggregated by type on that day, according to the instance depicted in Figure 3, P2 and P3 will contribute to the aggregation over type T2. Thus, for D1, S1, T2, we will have a value of 35.

Definition 3 (Cube Schema)

A *cube schema* is a tuple ⟨*nameCS*, D, M⟩ where *nameCS* is the name of the cube, D is a finite set of dimension levels, with $|D| = d$, corresponding to d bottom levels of d dimension schemas, different from each other, and M is a finite set of m attributes called measures. Each measure also has an associated domain.

Definition 4 (Cube Instance)

Consider a cube schema ⟨*nameCS*, D, M⟩; each $l_{bi} \in D$, $i = 1,\ldots,d$, has a set of members. Let us call $Points = \{(c_1,\ldots,c_d) \mid c_i$ is a member of l_{bi}, $i=1,\ldots,d\}$. A *cube instance* C is a partial function $C\colon Points \rightarrow dom(M_1) \times \ldots \times dom(M_m)$ where $M_i \in M$, $i=1,\ldots,m$.

Example 2 (Cube Schema and Instance)

Let us now define a cube denoted AirQuality, composed by dimensions Pollution, Time, and Station, and a measure concentration, as introduced in our application scenario (Figure 1a). The cube has schema ⟨AirQuality, {Pollutant, Day, Station}, {concentration}⟩ with instances of the form $C(P1,T1,S1) = 35,\ldots, C(P5,T3,S4) = 44,\ldots.$

Cube Algebra Definition Language

We now sketch a language, which we denote CADL (standing for Cube Algebra Definition Language). The language aims at providing a description of the conceptual data cube model, and could be the basis of a Cube Definition Language at lower abstraction levels.

In CADL, a cube schema is defined using the keyword CUBE followed by the cube name. Each cube is composed of a set of dimensions, identified with the keyword DIMENSION followed by the dimension name. After defining a dimension, we must list all dimension levels, along with their attributes. A dimension level is defined with the keyword LEVEL, followed by its name. In addition, each level contains a set of associated attributes, defined with the keyword ATTRIBUTES, followed by a list of attribute names and their types. In addition, the optional keyword UNIQUE indicates that the value of the attribute is unique among all the values of such attribute for *all* the level members.

For our running example, we define the AirQuality data cube, along with its dimensions, levels, and attributes in CADL as follows:

CUBE AirQuality {

DIMENSION Time
LEVEL Day ATTRIBUTES {date (date) UNIQUE, season (string)}
LEVEL Month ATTRIBUTES {month (string) UNIQUE}
LEVEL Quarter ATTRIBUTES {quarter (string) UNIQUE}
LEVEL Semester ATTRIBUTES {semester (string) UNIQUE}
LEVEL Year ATTRIBUTES {year (string) UNIQUE}
Day ROLL-UP to Month
Month ROLL-UP to Quarter
Quarter ROLL-UP to Semester
Semester ROLL-UP to Year

DIMENSION Station
LEVEL Station ATTRIBUTES {name (string) UNIQUE}
DIMENSION Road
LEVEL Road ATTRIBUTES {name (string) UNIQUE, length (real)}

DIMENSION Pollution
LEVEL Pollutant ATTRIBUTES {
name (string) UNIQUE,
loadLimit (real)}
LEVEL Category ATTRIBUTES {
name (string) UNIQUE}
LEVEL Type ATTRIBUTES {
name (string) UNIQUE}
LEVEL Group ATTRIBUTES {
name (string) UNIQUE}
Pollutant ROLL-UP to Category
Pollutant ROLL-UP to Type
Type ROLL-UP to Group

DIMENSION Geography
LEVEL District ATTRIBUTES {name (string) UNIQUE, area (real)}
LEVEL Province ATTRIBUTES {name (string) UNIQUE}
District ROLL-UP to Province

MEASURE concentration (real)}

CUBE ALGEBRA QUERY LANGUAGE

In this section we sketch our proposal for a query language that implements the ideas discussed in the preceding sections. We define an algebra, which we denote *Cube Algebra*, such that the user could define her queries just by means of the typical OLAP operators. We first describe how the user would be able to operate in our application scenario, intuitively manipulating the cube using the traditional OLAP operations, e.g., slicing, dicing, rolling-up, drilling down, and pivoting. This basic set of operators can be extended with other ones, useful in real-world OLAP practice. We illustrate this extensibility introducing the *map* operator, which allows changing the values of the measures in a cube, for example, to convert currency units, or perform what-if analysis. Then, we formally define each operator in our algebra. We conclude with some example queries that illustrate the query language.

Application Scenario: OLAP Operations for Pollution Control

We now discuss how the user-centric conceptual model we propose in this paper could be used to analyze data. Let us recall the cube in Figure 1a, containing quarterly values of pollutant concentration at each measuring station, for the year 2011. The end user can operate intuitively over this cube in order to analyze data in different ways. Figures 4 through 9 show a sequence of such operations, which start from the initial cube of Figure 1a. We describe her operations on a step-by-step basis.

The user first wants to compute the sum of concentrations per semester, station, and pollutant, to look for significant differences between these periods, if they exist. For this, the Cube Algebra offers her a *roll-up* operation, which she applies along the Time dimension. The result is shown in Figure 4: the new cube contains two values over the Time dimension, each corresponding to one semester (the original cube contained four values, one for each quarter). The remaining dimensions are not affected. All values in cells corresponding to the same pollutant and station (for example, P1 and S1, respectively), and to quarters Q1 or Q2, contribute to the aggregation to the values in the first semester (S1). We can see in Figure 1a that the concentration of P1 measured at station S1 for the first and second quarters are, respectively, 21 and 27. In Figure 4 we also see that these values are aggregated to 48 in the first semester. Computation of the cells corresponding to the second semester proceeds analogously.

Figure 4. OLAP operation: Roll-up to the semester level

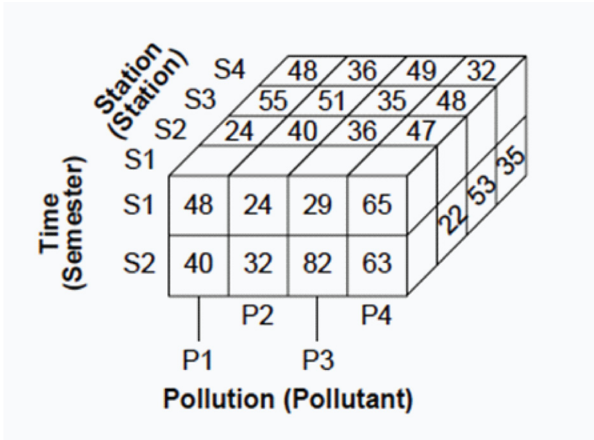
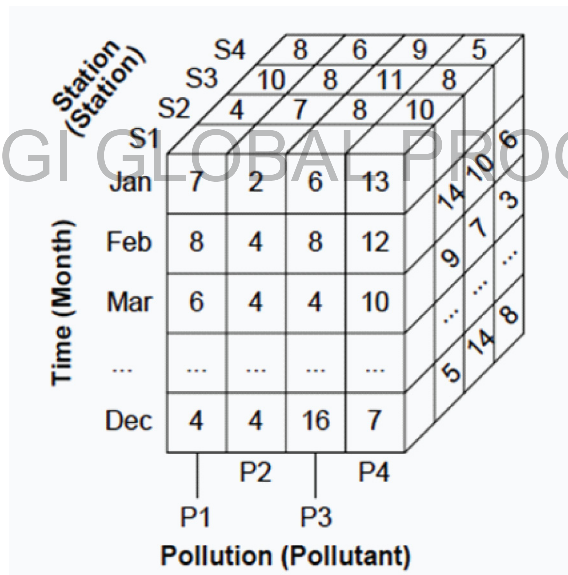


Figure 5. OLAP operation: Drill-down to the month level



Our user then notices that in the second semester the concentration of pollutant P3 at station S1 was unusually high. The Cube Algebra allows her to *drill down* along the Time dimension, to the month level, to find out if this high value is due to a particular month. In this way, she discovers that December presented a much higher concentration of this pollutant than the other months (Figure 5). Note that since she now starts from data aggregated by semester, station, and pollutant, the user needs first to take the cube back to the quarter aggregation level, and then continues drilling-down to the month level.

Figure 6. OLAP operation: Pivot

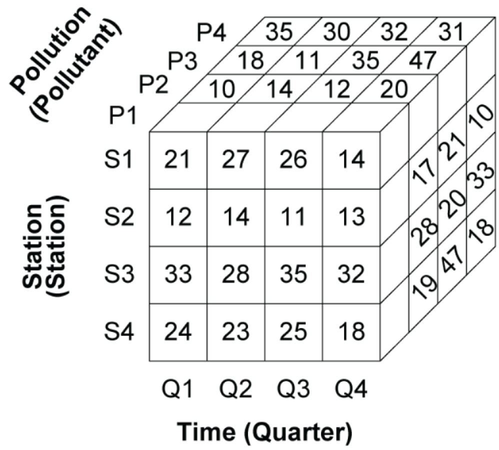
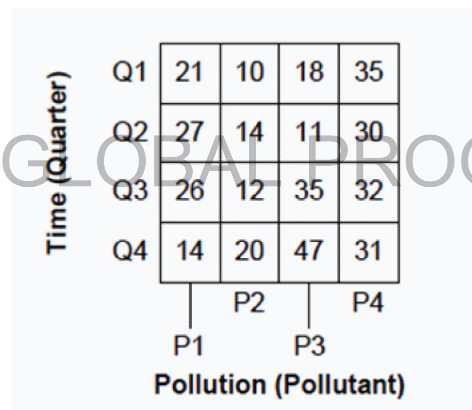


Figure 7. OLAP operation: Slice on station for StationId = 'S1'



Continuing her browsing of the cube, our user now wants to see the cube with the Time dimension on the *x* axis. Therefore, she rotates the axes of the cube without changing granularities. This restructuring operation is called *pivoting* (Figure 6). (Note that she previously *rolled-up* the cube back to the one of Figure 1a).

She then wants to visualize time series of average pollutant concentration by quarter, only for the station S1. For this, she first applies a *dice* operator that selects the sub-cube containing only values for the station S1, and then eliminates the Station dimension, applying a *slice* operation. This is depicted in Figure 7. Here, she obtained a matrix, where each column represents the evolution of the concentration of a pollutant by quarter, i.e., a collection of time series.

Our user also wants to compare, for each pollutant, the concentration values against similar records for 2010. For this, she has a two-dimensional "cube" similar to the one in

Figure 8. OLAP operation: Dice on station = 'S1' or 'S2' and Time.Quarter = 'Q1' or 'Q2'

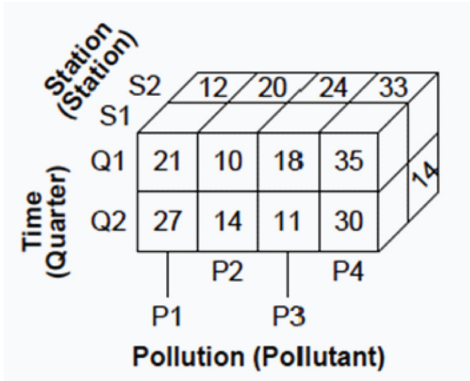


Figure 9. OLAP operation: Map function for defining contamination levels

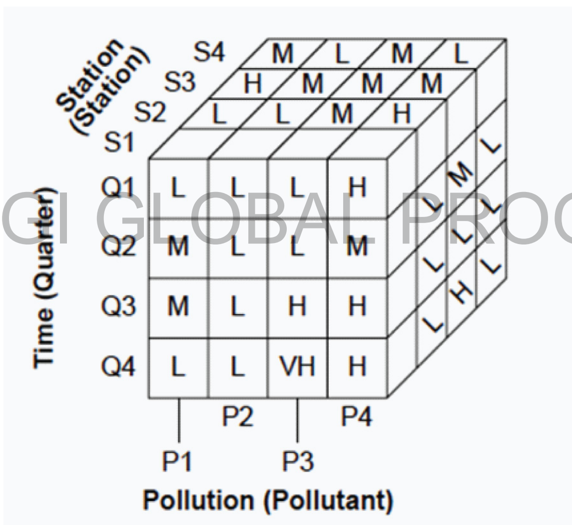


Figure 7, with the average concentrations by quarter and by pollutant, for 2010. She would like to have this information consolidated in a single cube. For this, she knows that the Cube Algebra offers the *drill-across* operator that, given two cubes, builds a new one with the measures of both, making the comparison very easy. In this way, if in 2010, for P4, the average concentration on Q1 was 32, the cell corresponding to (Q1,P4) resulting from a *drill-across*, will contain the pair (35,32).

Next, she wants pollution information corresponding only to stations S1 and S2 in the first two quarters. For this, starting over from the original cube, she produces a sub-cube, using the *dice* operator (Figure 8).

Finally, instead of a cube containing pollution values, she wants to produce a cube containing indicators of pollution classified in four categories: Low (L) for values greater or equal to 20; Moderate (M) for values greater than 20 and less or equal to 30; High (H) for values greater than 30 and less or equal to 45; and Very High (VH), for values greater than 45. For this, she uses the *map* operation shown in Figure 9.

In what follows, to make the examples more interesting, we add a Geography dimension, composed of levels District and Province, with a roll-up relationship defined between them, and a Road dimension, indicating the roads where the stations are located.

Cube Algebra Query Operators

We now formally define the operators of our Cube Algebra in terms of our data model. Even though there are many works describing multidimensional operations (Agrawal, Gupta, & Sarawagi, 1997; Gyssens & Lakshmanan, 1997; Vassiliadis, 1998), curiously none of them describe a common whole set of them in terms of the well-known *slice*, *dice*, *roll-up* (and its inverse, *drill-down*) and *drill-across*, which are the ones that intuitively reflect how an OLAP user manipulates a cube, or combines two cubes. Existing efforts, such as the ones cited above, usually define a subset of these operators, combined with other ones that, although suited to the models proposed by the authors, are, in most of the cases, not intuitive to non-expert users (e.g., push-pull, nest). The same occurs with the many commercial OLAP tools, as we show later taking MDX (the *de facto* standard for OLAP) as an example. We therefore chose to define a small set of the most intuitive and used operators for cube manipulation, which, in addition, can be shown to be orthogonal to each other. That means, no one of them can be expressed as a combination of the others. However, it should not be assumed that this set of operator is minimal in a formal mathematical sense.

This basic set of operators can be extended with many other ones, in order add functionalities to the language. As an example of this, we introduce the *map* operator, which applies the same function to *all* the cells in a data cube, allowing, for example, currency conversion, or more complex operations such as the one illustrated in the example of previous section.

Remark. Although the *pivot* operator was introduced in Figure 6, since it does not modify either the cube schema or the cube instances, and it is just used for (mainly) interactive visualization, we do not include it in the following discussion.

Dice. This operator receives a cube and a Boolean condition φ, and returns another cube containing only the cells that satisfy φ. The *syntax* for this operation is:

DICE(cube_name, φ)

where φ is a Boolean condition over dimension levels and measures.

The *semantics* is the following. *Dice* receives a cube with schema S=⟨*nameCS*, D, M⟩ and instances with the form $C(c_{11},\ldots,c_{1d})=m_1,\ldots, C(c_{m1},\ldots,c_{kj}) = m_r$ (for simplicity let us assume only one measure), and returns a cube with the same schema, and the points $(c_{i1},\ldots,c_{id}, m_i)$ that make the condition φ true. We can consider *dice* analogous to a relational *selection*.

Roll-Up and Drill-Down. The *roll-up* operator aggregates measures according to a dimension hierarchy (using an aggregate function), to obtain measures at a coarser granularity for a given dimension, based on the use of the dimension hierarchy.

The *syntax* for the *roll-up* operation is:

ROLL-UP(cube_name,
Dimension->level, (measure,aggregate_function)*)

The term Dimension->level indicates to which level in a dimension we want the roll-up to be performed. Note that since there can be more than one measure, we must specify an aggregate function for each one of them. In what follows, we assume that if there is only one measure in the cube, for conciseness we only specify the aggregate function.

As for the *semantics*, *roll-up* receives a cube with schema $S = \langle nameCS, \mathrm{D}, \mathrm{M}\rangle$, a level l in a dimension D, such that $l_s \in \mathrm{D}$, $l_s \rightarrow^* l$ in D, and an aggregate function F_{agg}. *Roll-up* returns a cube whose cells are aggregated along D up to the level l. Thus, all values $v_i,\ldots,v_k$ in the original cube, such that $C(c_1,\ldots,c_{li},\ldots,c_d) = v_j$, $j=1,\ldots,k$ (Definition 5) contribute to the aggregation over $Roll\text{-}up(c_{li})_{ls}^{\ l}$.

Example 4 (Roll-up)

Suppose in Example 1 that we have the following coordinates for the cube AirQuality: (P1,T1,35), (P5,T3,44), (P4,T3,22). According to Figure 3:

$Roll\text{-}up(\mathrm{P1})_{Pollutant}^{Category} = \mathrm{C1}$
$Roll\text{-}up(\mathrm{P4})_{Pollutant}^{Category} = \mathrm{C2}$
$Roll\text{-}up(\mathrm{P5})_{Pollutant}^{Category} = \mathrm{C2}$

Then rolling-up from the cube AirQuality to a new cube with schema ⟨AirQCateg, {Category, Time}, {concentration}⟩ yields the instance (C1,T1,35), (C2,T3,66).

Drill-down de-aggregates previously summarized measures and can be considered the inverse of *roll-up*. Following Agrawal et al. (1997), we consider *drill-down* a high-level operation that can be implemented by tracking the (stored) paths followed during user rolling-up. Therefore, we omit its definition.

Slice. Removes a dimension in a cube, i.e., a cube of n-1 dimensions is obtained from a cube of n dimensions. The dimension to remove must contain a unique value in its domain. If the dimension has more than one value, two approaches can be used: apply either a *roll-up* operator for summarizing into a singleton (i.e., *all*) (Agrawal, Gupta, & Sarawagi, 1997), or (prior to slicing) a *dice* operator, to obtain a cube with only one value in the selected dimension.

The *syntax* of this operator is:

SLICE(cube_name, Dimension,
[ROLL-UP{Aggregate_function}])

According to what we explained above, ROLL-UP{Aggregate_function} stands for ROLL-UP(cube_name, Dimension {All,Aggregate_function)}. The former yields a more concise expression. If the roll-up is not included, *slice* will have two arguments, meaning that the dimension instance has already been reduced to a single value. If this is not the case, the operator fails. That is, the operator is analogous to a relational *projection*.

The *semantics* is the following. *Slice* receives a cube with schema $S = \langle nameCS, \mathrm{D}, \mathrm{M}\rangle$, and instances of the form $\{(c_{11},\ldots,c_s,\ldots,c_{1d}, m_1),\ldots, (c_{m1},\ldots,c_s, \ldots,c_{kd}, m_r)\}$ (where c_s is the unique value in l_s), and a dimension name d_s (assume that the level $l_s \in \mathrm{D}$ belongs to dimension d_s). The operator returns a cube with schema $S_1 = \langle nameCS, (\mathrm{D} \setminus l_s), \mathrm{M}\rangle$, where the instances are of the form $(c_{11},\ldots, c_{s-1}, c_{s+1},\ldots,c_{1d}, m_1),\ldots,(c_{m1},\ldots, c_{s-1}, c_{s+1},\ldots,c_{kd}, m_r)$, that is, the same as in the original one, except the coordinate corresponding to l_s.

Drill-Across. Relates information contained in two data cubes having the same dimensions. Thus, measures from different cubes can be compared. According to Kimball and Ross (2002), *drill-across* can only be applied when both cubes have the same schema dimensions and the same instances. Other authors relax this restriction. This is the approach of Abelló et al. (2002), who define two concepts: (a) *Dimension-Dimension Derivation*: Used when two dimensions come from a common concept although their structures differ, for example, because their granularities are not the same. In this case, a roll-up can be applied to make both dimensions consistent. (b) *Dimension-Dimension Association*: Corresponds to the case in which two cubes have different dimensions, but one of them could be defined as the association of several ones. For example in one cube we define latitude and longitude as separated dimensions; in another one we store only one dimension containing the 'point' geometry. A mapping function can solve this problem. Other authors also address this problem, all of them along the same lines (Cabibbo & Torlone, 2004;

Riazati, Thom, & Zhang, 2008). We assume that the operator receives two compatible cubes (i.e., sharing dimensions and instances). The syntax of the operator is:

DRILL-ACROSS(cube_name_1, cube_name_2)

Let us now explain the *semantics*. *Drill-across* receives two cubes with schemas $S_1 = \langle nameCS_1, \mathrm{D}, \mathrm{M}_1 \rangle$ and $S_2 = \langle nameCS_2, \mathrm{D}, \mathrm{M}_2 \rangle$, that means, the dimension levels are the same. The corresponding instances (except for the measures) are the same. The result is a cube with schema $S = \langle nameCS, \mathrm{D}, \mathrm{M}_1 \cup \mathrm{M}_2 \rangle$, with the same instances of the input cubes. In other words, the operator is analogous to a relational *natural join*.

We now formally define the *map* operator, which extends the basic set of operations defined above. We remark that this is only one of the many operators the Cube Algebra could be extended with.

Map. This operator receives a cube and a collection of pairs (m_i, f_i), where m_i is a measure and f_i is a function mapping values in Dom_m to values in the same of another domain (see the example above, where concentration values in the domain of the real numbers are mapped to the domain of strings, by a partitioned function). The operator returns another cube, with the same dimension schema and instances, and with the values in each cell that correspond to the mappings produced by each function f_i. The *syntax* for the *map* operation is:

MAP(cube_name, (measure, function)*)

The *semantics* is the following. *Map* receives a cube with schema $S = \langle nameCS, \mathrm{D}, \mathrm{M} \rangle$ and instances with the form $C(c_{11},\ldots,c_{1d}) = m_1,\ldots, C(c_{m1},\ldots,c_{kj}) = m_r$, (for simplicity we assume only one measure), and returns a cube with the same schema, and instances of the form $C(c_{11},\ldots,c_{1d}) = f(m_1),\ldots, C(c_{m1},\ldots,c_{kj}) = f(m_r)$.

Cube Algebra QL by Example

We now give the flavor of the language by means of examples, and show that Cube Algebra allows an OLAP user to express queries using just the operators she is acquainted to, instead, for example, of complex MDX expressions. We prove our point, showing, for each query, a possible MDX version.

Query 1. For each province and pollutant category, give the average concentration by quarter:

```
c1:= SLICE (AirQuality, Station, ROLL-UP{Avg})
c2:= SLICE (c1, Road, ROLL-UP{Avg})
c3:= ROLL-UP (c2,{Time-> Quarter,
 Pollution-> Category,Geography->Province}, Avg)
```

Remark. The *roll-up* operator can only be applied over one dimension at a time. For simplicity, in the query above, the expression for c3 is shorthand for:

```
c3:= ROLL-UP(c2, Time->Quarter, Avg)}
c4:= ROLL-UP(c3,Geography->Province, Avg)}
c5:= ROLL-UP(c4,Pollution->Category, Avg)}
```

This is the syntax we use in the sequel.

We next show how this query would look in MDX.

Query 1 (MDX Version)

We assume that measure concentration has been associated with the aggregate function Avg. In Cube Algebra, the levels of a dimension are organized as a lattice. On the contrary, in MDX the levels are organized into named linear hierarchies. Thus, when referring to a level we must qualify it with the name of a hierarchy it belongs to (note that there could be more than one). This is the case of the dimension Pollution in the query below.

In MDX, Query 1 reads:

```
SELECT
[Geography].[Hierarchy]
[Province].Members On Axis(0)
[Pollution].[H0].[Category]
Members On Axis(1)
[Time].[Hierarchy].[Quarter]
Members On Axis(2)
FROM AirQuality
WHERE (Station.[All],Road.[All])
```

Compared to the simplicity of the Cube Algebra expression above, the MDX query looks cryptic and unintuitive. In part, this is due to the fact that, as shown in Figure 2, MDX is placed at the logical level, while the Cube Algebra is at the conceptual level. Besides, MDX has not a clearly defined semantics. On the contrary the semantics of each Cube Algebra operator is clear, intuitive, and well known for most OLAP practitioners.

Query 2. Number of districts where, for at least one pollutant, the average load of air pollution in 2011 was larger than the concentration limit:


```
c1:= ROLL-UP (AirQuality,Time->Year,{Avg})
c2:= DICE (c1, Time.Year.year = 2011)
c3:= SLICE (c2, Time,ROLL-UP{Avg})
c4:= SLICE (c3, Station,ROLL-UP{Avg})
c5:= SLICE (c4, Road,ROLL-UP{Avg})
```

We have obtained a cube with dimensions Geography and Pollution containing data only for 2011. Now:

```
c6:= DICE (c5, concentration >= Pollution.
Pollutant.loadLimit)
c7:= SLICE (c6, Pollution,ROLL-UP{Avg})
c8:= ROLL-UP (c7,Geography->All, {Count}})
```

The semantics of the expression:

```
DICE (c4, concentration
>= Pollution.Pollutant.loadLimit)
```

is the following: each cell in c4 is analyzed, and the values of the variables are instantiated with the values of the cell coordinates and measures (i.e., Pollution.Pollutant is instantiated with the identifier of the pollutant corresponding to the cell). Analogously, concentration is obviously instantiated with the value of the measure in the cell.

Query 2 (MDX Version)

The query can be solved in three steps. First, a sub-cube is created for representing the underlying data filtered using the districts satisfying the condition. The use of the TYPED keyword in the Properties function is necessary in order to consider the type of the 'loadLimit' attribute, otherwise MDX would consider it a string, preventing performing the comparison with concentration. For counting elements, in our case the number of different districts, we must define a new measure using the WITH clause over the set of names of the districts. Finally, the sub-cube is dropped:

```
CREATE SUBCUBE [AirQuality] AS
SELECT
Filter(([Pollution].[H0].[Pollutant].
members,[Geography].[Hierarchy].[District].
members,[Time].
[Hierarchy].[Year].[2011]),
[Pollution].[H0].
CurrentMember.
Properties("loadLimit",TYPED) <=[Measures].
[Concentration]) on Axis(0)
FROM AirQuality
WHERE ([Station].[All], [Road].[All])
WITH
SET [DistNames] As
([Geography].[Hierarchy].
[District].members, [Measures].[Concentration])
MEMBER [Measures].[Count Districts] As
COUNT([DistNames])
SELECT [Measures].[Count Districts] On Axis(0)
```

```
FROM [AirQuality]
DROP SUBCUBE [AirQuality]
```

Note, as in Query 1, the difference between the Cube Algebra expression and the MDX one. The latter requires a deep understanding of the MDX syntax, while the former allows the user to focus just on the semantics of the OLAP operators.

Query 3. Build a cube with the dimensions of the original cube, containing only concentrations corresponding to stations located in districts in the province of Limburg, and to polluting agents of 'organic' type such that the station had at least once registered a concentration higher than the limit for the corresponding pollutant.

This query is simply expressed as:

```
c1:= DICE (AirQuality, concentration >=
Pollution.Pollutant.loadLimit AND
Geography.Province.name ='Limburg' AND
Pollution.Category.name = 'organic')
```


Query 3 (MDX Version)

```
SELECT[Pollution].[CategoryName.
[Organic] On Axis(0),
[Time].[Hierarchy].[Day] On Axis(1),
[Station].[Station].[Station] On Axis(2),
[Road].[Road].[Road] On Axis(3),
[Geography].[Hierarchy].
[District] on Axis(4)
FROM AirQuality
WHERE
Filter(([Pollution].[H0].
[Pollutant].members, [Geography].
[Province].
[Limburg]),[Pollution].[H0].
CurrentMember.Properties
("loadLimit",TYPED)<=[Measures].
[Concentration])
```

Even this simple query requires implementation knowledge at the logical level, e.g., the decomposition of the multiple hierarchy into many single ones (in this case, only [H0] is needed).

Query 4. For stations, and pollutants belonging to the 'organic' category, give the maximum concentration by month:

```
c1:= DICE (AirQuality,Pollution.    Category.
name='organic')
c2:= SLICE (c1, Road,ROLL-UP{Max})
c3:= SLICE (c2, Geography,ROLL-UP{Max})
c4:= ROLL-UP (c3, Time->Month, {Max})
```

Query 4 (MDX Version)

```
WITH MEMBER
[Measures].[Maximal] as
Max([Time].[Hierarchy].
currentMember.children, [Measures].
[Concentration])
SELECT[Time].[Hierarchy].[Month.
members On Axis(0),
[Station].[Station].[station].members On Axis(1),
[Pollution].[H0].Pollutant.
members On Axis(2),
[Measures].[Maximal] On Axis(3)
FROM AirQuality
WHERE
([Pollution].[categoryName].
[Organic],[Road].[Road].[All], [Geography].
[Hierarchy].[All])
```

Query 5. Stations located over the part of the E34 road within the Berchem district, with an average content of nitrates in the last quarter of 2011 above the load limit for that pollutant:

```
c1:= DICE (AirQuality, Geography.District
='Berchem' AND Time.Quarter.quarter
='Q4-2011' AND Time.Year.year
= 2011 AND Pollution.Category.name
='Nitrates' AND Road.Road.name = 'E34')
c2:= SLICE (c1, Road)
c3:= SLICE (c2, Geography)
```

Note that in the SLICE operation that generates cubes c2 and c3, we do not use the third argument, since the previous dicing selected unique values for roads and districts. Cube c3 has Station, Time, and Pollution dimensions:

c4:= ROLL-UP (c5, Time-> Quarter, Avg)
c6:= DICE (c5, concentration
>= Pollution.Pollutant.loadLimit)

The following query illustrates the use of the *map* operator. In our running example, assuming that pollutant concentrations are expressed in mg/m^3 (milligrams per cubic meter), we want to express the results in µg/m^3 (micrograms per cubic meter).

Query 5 (MDX Version)

```
SELECT
Filter((([Pollution].[H0].
[Pollutant].members, [Station].[Station][Station].
members), [Time].[Quarter].
[Q4-2011]), [Pollution].[H0].
CurrentMember.Properties
("loadLimit", TYPED) < [Measures].
[Concentration]) on Axis(0)
FROM AirQuality
WHERE ([Road].[Road].[E34],
[Geography].[District].[Berchem,
[Pollution].[typeName].
[Nitrates])
```

Query 6. Average concentration by Station, Time, District, Road, and pollutant Category, expressed in µg/m^3:

c1:= MAP (AirQuality, concentration, Mult(1000))
c2:= ROLL-UP (c1, Pollution-> Category, Avg })

In the expression that generates c1, Mult (1000) is a function that given a value, multiplies it by a constant (in this case, 1000).

Query 6 (MDX Version)

```
WITH MEMBER [Measures].[NewValue] as
 [Measures].[Concentration]*1000
SELECT
[Pollution].[H0].[Category].
members On Axis(0),
[Station].[Station].[Station].
members On Axis(1),
[Geography].[District].
[District].members On Axis(2),
[Road].[Road].[Road].Members
On Axis(3),
[Time].[Hierarchy].[Day].members On Axis(4),
[Measures].[NewValue] On Axis(5)
FROM AirQuality
```

The next query shows the use of the *Drill-across* operator. Let us assume we have a cube denoted Demography with the dimensions Time and Geography described above, and measure population. The instances of the dimensions satisfy the operator's preconditions.

Query 7. Total population and average pollutant concentration by province and year:

c1:= SLICE (AirQuality, Road, ROLL-UP{Sum})
c2:= SLICE (c1, Pollution, ROLL-UP{Max})
c3:= SLICE (c2, Station, ROLL-UP{Max})

Now, the cube c3 contains just the dimensions Time and Geography:

c4:= DRILL-ACROSS (c3, Demography)
c5:= ROLL-UP (c4, Time->Year,
concentration, Avg, population, Sum
c6:= ROLL-UP (c5, Geography-> Province,
concentration, Avg, population, Sum)

The drill-across operator is not directly supported in MDX, since the FROM clause only supports one cube, therefore the only way of adding a measure is to define it at design time.

CONCLUSION AND FURTHER WORK

In this article, we have identified the need for an appropriate conceptual model for data warehouses and OLAP systems. This need stems from the fact that logical models (for example, star, snowflake, and constellation schemas) have been deployed for these systems as conceptual models. But logical models represent a system-centric view of data warehouses and OLAP systems and are ultimately implementation concepts. In this article, we propose a user-centric conceptual model for data warehouses and OLAP systems, called the Cube Algebra. It takes the cube metaphor literally and provides the knowledge worker with high-level cube objects and related high-level concepts. A novel query language leverages high-level operations such as *roll-up*, *slice*, and *drill-across*. An important design criterion is that all aspects of the logical level and the physical level are hidden from the user.

We plan our future research in at least three directions. First, further data definition commands have to be added for updating cube schemas, dimension schemas, and measures. In addition, further data manipulation commands are needed for the insertion, deletion, and update of data into cubes. Second, transformation rules are needed that map the concepts of the Cube Algebra at the conceptual level to corresponding ROLAP, MOLAP, and HOLAP concepts at the logical level. Third, we are interested in adding other data categories such as spatial, spatio-temporal, image, and multimedia data into our Cube Algebra. Questions are here, for example, how the different data categories are integrated and stored, what kind of aggregation operations exist on the different data categories, how the different aggregation operations are defined, and how these operations are implemented.

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ENDNOTES

1 We use this simplification in the formal model to avoid the need of referring to a dimension level as dimension.level, which would make the formal definition too verbose. However, in the algebra defined in next section, we drop this restriction, and qualify level names with dimension names.

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