

$$j = 1, 2, 3$$

$$k = 1, 2$$

$$a_1 = 1, a_2 = 2, a_3 = 3$$

$$b_1 = 4 \text{ e } b_2 = 5$$

$$H = \left\{ \begin{array}{c} a_1+b_1 \\ 5 \\ c_1 \end{array}, \begin{array}{c} a_1+b_2 \\ 6 \\ c_2 \end{array}, \begin{array}{c} a_2+b_1 \\ 7 \\ c_3 \end{array}, \begin{array}{c} a_2+b_2 \\ 8 \\ c_4 \end{array} \right\}$$

$a_1+b_1 = a_2+b_1$ $a_1+b_2 = a_2+b_2$ a_3+b_2

$a_2+b_2 = a_3+b_1$

$$\# H = 4$$

$$G_1 = E_1 \cap F_1, \quad G_2 = (E_1 \cap F_2) \cup (\underline{E_2 \cap F_1}), \quad G_3 = (E_2 \cap F_2) \cup (E_3 \cap F_1)$$

\uparrow
 $h=1$

$$G_4 = E_3 \cap F_2$$

$$\mu(G_2) = \mu(E_1 \cap F_2) + \mu(E_2 \cap F_1)$$

$$\varphi + \psi = \sum_{i=1}^3 a_i \chi_{E_i} + \sum_{j=1}^2 b_j \chi_{F_j} = \sum_{h=1}^4 c_h \chi_{G_h}$$

$$\int \varphi + \psi = \sum_{h=1}^4 c_h \mu(G_h) = c_1 \mu(G_1) + c_2 \mu(G_2) + c_3 \mu(G_3) + c_4 \mu(G_4)$$

$$= c_1 \mu(E_1 \cap F_1) + c_2 \mu(E_1 \cap F_2) + c_2 \mu(E_2 \cap F_1) + c_3 \mu(E_2 \cap F_2)$$

$$+ c_3 \mu(E_3 \cap F_1) + c_4 \mu(E_3 \cap F_2)$$

$$= \underline{(a_1+b_1) \mu(E_1 \cap F_1)} + \underline{(a_1+b_2) \mu(E_1 \cap F_2)} + \underline{(a_2+b_1) \mu(E_2 \cap F_1)}$$

$$+ \underline{(a_2+b_2) \mu(E_2 \cap F_2)} + \underline{(a_3+b_1) \mu(E_3 \cap F_1)} + \underline{(a_3+b_2) \mu(E_3 \cap F_2)}$$

$$= \sum_{j=1}^2 (a_1+b_j) \mu(E_1 \cap F_j) + \sum_{j=1}^2 (a_2+b_j) \mu(E_2 \cap F_j)$$

$\nearrow i=1$ $\nearrow i=2$

$$+ \sum_{j=1}^2 (a_3+b_j) \mu(E_3 \cap F_j)$$

$\hookrightarrow i=3$

$$= \sum_{i=1}^3 \sum_{j=1}^2 (a_i+b_j) \mu(E_i \cap F_j)$$