Circuit Theory and Electronics Fundamentals

Department of Electrical and Computer Engineering, Técnico, University of Lisbon

Second Laboratory Report

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Contents

1 Introduction

The objective of this laboratory assignment is to study a circuit containing a sinusoidal voltage source v_s connected to seven resistors (R_1 to R_7), a capacitor C, a dependent voltage source (current-controlled) and a dependent current source (voltage-controlled). The circuit can be seen in Figure 1.

The nodes are designated with numbers as seen in the Figure 1 and the node voltages will be represented with their respective numbers (ex. V_3 represents the voltage in node number 3). The characteristic equations of the dependent sources can also be seen in the Figure 1.

The sinusoidal voltage source follws the equation:

$$v_s(t) = V_s u(-t) + sin(2\pi f t)u(t) \tag{1}$$

where

$$u(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases} \tag{2}$$

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

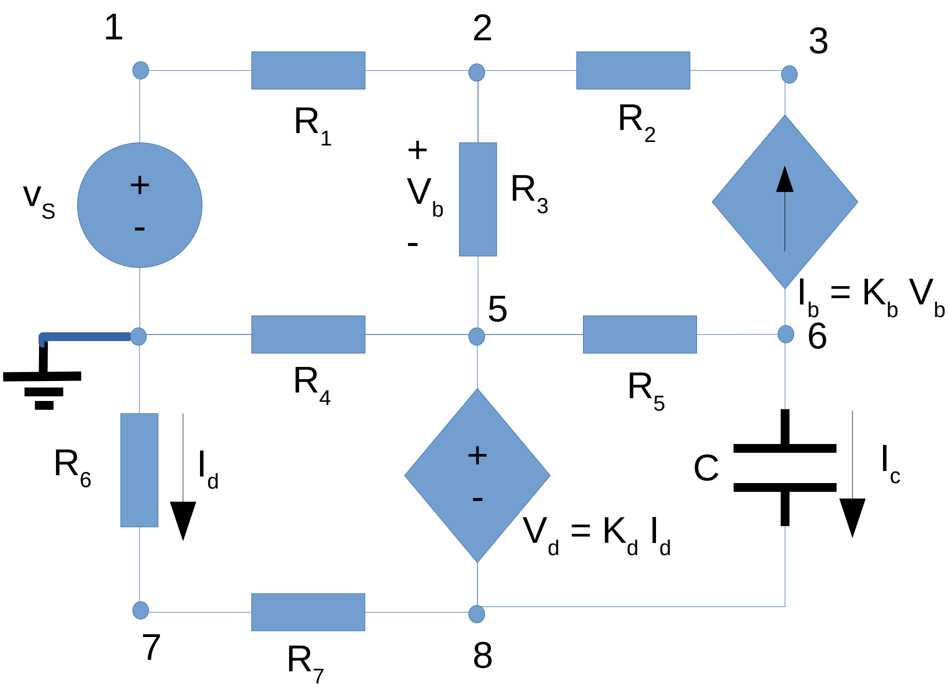

Figure 1: Voltage driven serial RC circuit.

2 Theoretical Analysis

In this section we will analyse the circuit shown in Figure 1 theoretically using tools like the Octave and Python, given that the last gives us the values needed for the analysis of the circuit, as seen in the Table 1.

Name	Values
R1	1.04001336091 kOhm
R2	2.04372276851 kOhm
R3	3.11359737601 kOhm
R4	4.17085404861 kOhm
R5	3.02859283303 kOhm
R6	2.070545767 kOhm
R7	1.01835949725 kOhm
Vs	5.20102702949 V
C	1.00460501759 uF
Kb	7.19043597753 mA
Kd	8.06397385506 kOhm

Table 1: Values given by the Python script using the number 95803 as input.

2.1 Nodal Method for $t < 0$

In the first point the values of the voltages and currents in all branches of the cicruit for t<0 are calculated using the nodal method and using the values given by the Python script.

Since we are working in $t < 0$, $u(t) = 0$ and $u(-t) = 1$ and $v(s) = V_s$

The equations used to obtain the various results are:

- Node 1:

$$V_1 = V_s \tag{3}$$

- Node 2:

$$V_2(-\frac{1}{R_1} - \frac{1}{R_2} - \frac{1}{R_3}) + V_3\frac{1}{R_2} + V_5\frac{1}{R_3} = -\frac{V_s}{R_1} \tag{4}$$

- Node 3:

$$V_2(K_b + \frac{1}{R_2}) + V_3(-\frac{1}{R_2}) + V_5(-K_b) = 0 \tag{5}$$

- Node 6:

$$V_2(-K_b) + V_5(\frac{1}{R_5} + K_b) + V_6(-\frac{1}{R_5}) = 0 \tag{6}$$

- Node 7:

$$V_7(-\frac{1}{R_6} - \frac{1}{R_7}) + V_8\frac{1}{R_7} = 0 \tag{7}$$

- Supernode 5 and 8:

$$V_2(-\frac{1}{R_3}) + V_5(-\frac{1}{R_3} - \frac{1}{R_4} - \frac{1}{R_5}) + V_6\frac{1}{R_5} + V_7\frac{1}{R_7} + V_8(-\frac{1}{R_7}) = 0 \tag{8}$$

- Additional equation from the dependent voltage source:

$$V_5 + V_7 \frac{K_d}{R_6} - V_8 = 0 \tag{9}$$

The system that uses the previous equations and that solves the problem is the following:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R_1} - \frac{1}{R_2} - \frac{1}{R_3} & \frac{1}{R_2} & \frac{1}{R_3} & 0 & 0 & 0 \\ 0 & K_b + \frac{1}{R_2} & -\frac{1}{R_2} & -K_b & 0 & 0 & 0 \\ 0 & -K_b & 0 & \frac{1}{R_5} + K_b & -\frac{1}{R_5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{R_6} - \frac{1}{R_7} & \frac{1}{R_7} \\ 0 & -\frac{1}{R_3} & 0 & -\frac{1}{R_3} - \frac{1}{R_4} - \frac{1}{R_5} & \frac{1}{R_5} & \frac{1}{R_7} & -\frac{1}{R_7} \\ 0 & 0 & 0 & 1 & 0 & \frac{K_d}{R_6} & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} V_s \\ -\frac{V_s}{R_1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

Using Octave to solve the matrix system, the results obtained are shown in Table 2:

Name	**Value [V or mA]**
V_1	5.201027
V_2	4.998410
V_3	4.581633
V_5	5.026771
V_6	5.644394
V_7	-2.092064
V_8	-3.121006
I_b	-0.203930
$R_1[i]$	0.194822
$R_2[i]$	0.203930
$R_3[i]$	0.009109
$R_4[i]$	1.205214
$R_5[i]$	0.203930
$R_6[i]$	1.010392
$R_7[i]$	1.010392

Table 2: Results of theoretical operating point analysis for t<0. A variable that starts with V is of type voltage and is expressed in Volt (V). A variable that has $[i]$ is of type current and is expressed in miliampere (mA).

After calculating the nodes voltages we are able to obtain the currents flowing through each component using the following equations:

$$I_b = K_b(V_2 - V_5) \tag{11}$$

$$R_1[i] = \frac{(V_1 - V_2)}{R_1} \tag{12}$$

$$R_2[i] = \frac{(V_2 - V_3)}{R_2} \tag{13}$$

$$R_3[i] = \frac{(V_5 - V_2)}{R_3} \tag{14}$$

$$R_4[i] = \frac{V_5}{R_4} \tag{15}$$

$$R_5[i] = \frac{(V_6 - V_5)}{R_5} \tag{16}$$

$$R_6[i] = \frac{-V_7}{R_6} \tag{17}$$

$$R_7[i] = \frac{V_7 - V_8}{R_7} \tag{18}$$

The results of these equations can be seen in Table 2.

2.2 Equivalent resistance and time constant

In this section we analyse the circuit for t = 0, so with v_s = 0 and V_1 = 0. To obtain this, the capacitor in the circuit is replaced with:

$$V_x = V_6(t<0) - V_8(t<0), \tag{19}$$

where $V_6(t<0)$ and $V_8(t<0)$ have the values obtain previously.

With dependent sources in a circuit like the one analysed, we can't turn off all sources to compute the equivalent resistance as seen from the capacitor terminals. So we need to obtain the equivalent current, flowing through the capacitor, I_x, and the equivalent voltage, V_x, which we know already from equation above.

To discover the values of the voltages in the nodes for $t = 0$, we compute a similar matrix from the previous section, with the only change being that V_1(t=0) is now 0. The equations to solve the problem are, with the voltages values being for t = 0:

- Node 1:

$$V_1 = V_s \tag{20}$$

- Node 2:

$$V_2(-\frac{1}{R_1} - \frac{1}{R_2} - \frac{1}{R_3}) + V_3\frac{1}{R_2} + V_5\frac{1}{R_3} = -\frac{V_s}{R_1} \tag{21}$$

- Node 3:

$$V_2(K_b + \frac{1}{R_2}) + V_3(-\frac{1}{R_2}) + V_5(-K_b) = 0 \tag{22}$$

- Node 6:

$$V_2(-K_b) + V_5(\frac{1}{R_5} + K_b) - V_6\frac{1}{R_5} - I_x = 0 \tag{23}$$

- Node 7:

$$V_7(-\frac{1}{R_6} - \frac{1}{R_7}) + V_8\frac{1}{R_7} = 0 \tag{24}$$

- Supernode 5 and 8:

$$V_2(\frac{1}{R_3}) + V_5(-\frac{1}{R_3} - \frac{1}{R_4} - \frac{1}{R_5}) + V_6\frac{1}{R_5} + V_7\frac{1}{R_7} + V_8(-\frac{1}{R_7}) + I_x = 0 \tag{25}$$

- Additional equation from the dependent voltage source:

$$V_5 + V_7 \frac{K_d}{R_6} - V_8 = 0 \tag{26}$$

The new equation, that relates voltages of nodes 6 and 8, which are now connected by a voltage source V_x is:

$$V_6(t=0) - V_8(t=0) = V_x \tag{27}$$

These equations are translated through the following system of matrix to obtain the node voltages and the current I_x:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R_1} - \frac{1}{R_2} - \frac{1}{R_3} & \frac{1}{R_2} & \frac{1}{R_3} & 0 & 0 & 0 & 0 \\ 0 & K_b + \frac{1}{R_2} & -\frac{1}{R_2} & -K_b & 0 & 0 & 0 & 0 \\ 0 & -K_b & 0 & \frac{1}{R_5} + K_b & -\frac{1}{R_5} & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{R_6} - \frac{1}{R_7} & \frac{1}{R_7} & 0 \\ 0 & \frac{1}{R_3} & 0 & -\frac{1}{R_3} - \frac{1}{R_4} - \frac{1}{R_5} & \frac{1}{R_5} & \frac{1}{R_7} & -\frac{1}{R_7} & 1 \\ 0 & 0 & 0 & 1 & 0 & \frac{K_d}{R_6} & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ I_x \end{bmatrix} = \begin{bmatrix} V_s \\ -\frac{V_s}{R_1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ Vx \end{bmatrix} \tag{28}$$

The solution of this system was obtained using Octave and the results are in Table 3:

Name	Value [V or mA or kOhm]
V_1	0.000000
V_2	0.000000
V_3	-0.000000
V_5	0.000000
V_6	8.765400
V_7	0.000000
V_8	0.000000
I_b	0.000000
$R_1[i]$	0.000000
$R_2[i]$	-0.000000
$R_3[i]$	0.000000
$R_4[i]$	0.000000
$R_5[i]$	2.894215
$R_6[i]$	-0.000000
$R_7[i]$	0.000000
I_x	-2.894215
R_{eq}	-3.028593
tau	-0.003043

Table 3: Results of theoretical operating point analysis for $t = 0$. A variable that starts with V is of type voltage and is expressed in Volt (V). I_b, I_x and the variables that have $[i]$ are of type current and are expressed in miliampere (mA). R_{eq} is of type resistance and is expressed in kOhm.

And with these voltage values and the following equations we can compute the values to the currents in the various components:

$$I_b = K_b(V_2 - V_5) \tag{29}$$

$$R_1[i] = \frac{V_1 - V_2}{R_1} \tag{30}$$

$$R_2[i] = \frac{(V_3 - V_2)}{R_2} \tag{31}$$

$$R_3[i] = \frac{(V_2 - V_5)}{R_3} \tag{32}$$

$$R_4[i] = \frac{V_5}{R_4} \tag{33}$$

$$R_5[i] = \frac{(V_5 - V_6)}{R_5} \tag{34}$$

$$R_6[i] = \frac{-V_7}{R_6} \tag{35}$$

$$R_7[i] = \frac{V_7 - V_8}{R_7} \tag{36}$$

Finally we are able to compute the results for R_{eq} (equivalent resistance) and τ (time constant) using the following equations:

$$R_{eq} = \frac{V_x}{I_x} \tag{37}$$

$$\tau = R_{eq}C. \tag{38}$$

The values obtained using these equations are also shown in Table 3.

2.3 Natural solution for $V_6(t)$

In this section, we are given the task to find and compute the natural solution for $V_6(t)$: $V_{6n}(t)$

For that we use the following equation:

$$v_{6n}(t) = V_x \cdot e^{-\frac{t}{\tau}} \tag{39}$$

Where τ is the time constant previously determined and V_x is the constant of the natural solution formula obtained through the boundary conditions.

Plotting this equation we obtain the graphic of the Figure 2.

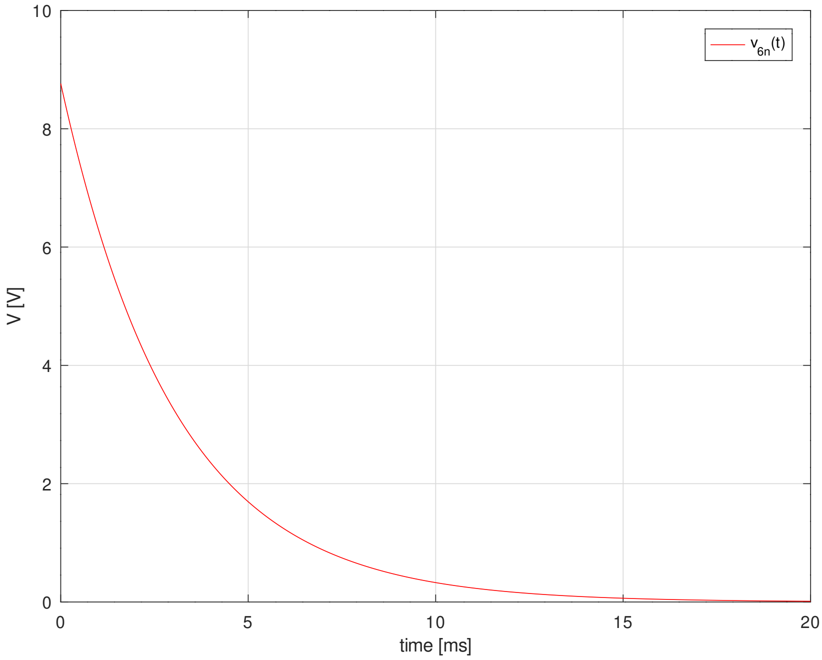

Figure 2: Natural response for V_6 as a function of time in [0,20] ms

2.4 Forced solution for $V_6(t)$

Here we are asked to compute and find the forced solution for $V_6(t)$: $V_{6f}(t)$

For this, we have to compute first the complex amplitudes of the voltages in each node, using the nodal method, but replacing the capacitor with its impedance, Z_C. Also, a phasor voltage source $\tilde{V}_S = j$ with magnitude $V_S = 1$ was used.

Hence, the only equations that are different from those written in subsection 2.1 are the ones referring to node 6 and supernode 5,8.

In the capacitor, we have:

$$Z_C = \frac{1}{jwc}, \tag{40}$$

where $w = 2\pi f$ and f is the given frequency, $f = 1000Hz$. And:

$$V_C = I_C Z_C. \tag{41}$$

Because of these changes, the equation of node 6 is now:

$$\frac{\tilde{V}_5 - \tilde{V}_6}{R_5} - K_b(\tilde{V}_2 - \tilde{V}_5) - \frac{\tilde{V}_6 - \tilde{V}_8}{Z_c} = 0 \tag{42}$$

and the equation of supernode 5,8 is:

$$\frac{\tilde{V}_2 - \tilde{V}_5}{R_3} + \frac{\tilde{V}_7 - \tilde{V}_8}{R_7} + \frac{\tilde{V}_6 - \tilde{V}_8}{Z_C} - \frac{\tilde{V}_5}{R_4} - \frac{\tilde{V}_5 - \tilde{V}_6}{R_5} = 0 \tag{43}$$

With these equations and the previously obtained in subsection 2.1 we can build the following system of matix:

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{R_1}-\frac{1}{R_2}-\frac{1}{R_3} & \frac{1}{R_2} & \frac{1}{R_3} & 0 & 0 & 0 \\
0 & K_b+\frac{1}{R_2} & -\frac{1}{R_2} & -K_b & 0 & 0 & 0 \\
0 & -K_b & 0 & \frac{1}{R_5}+K_b & -\frac{1}{R_5}-\frac{1}{Z_C} & 0 & \frac{1}{Z_C} \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{R_6}-\frac{1}{R_7} & \frac{1}{R_7} \\
0 & -\frac{1}{R_3} & 0 & -\frac{1}{R_3}-\frac{1}{R_4}-\frac{1}{R_5} & \frac{1}{R_5}+\frac{1}{Z_C} & \frac{1}{R_7} & -\frac{1}{R_7}-\frac{1}{Z_C} \\
0 & 0 & 0 & 1 & 0 & \frac{K_d}{R_6} & -1
\end{bmatrix}
\begin{bmatrix} \tilde{V}_1 \\ \tilde{V}_2 \\ \tilde{V}_3 \\ \tilde{V}_5 \\ \tilde{V}_6 \\ \tilde{V}_7 \\ \tilde{V}_8 \end{bmatrix}
=
\begin{bmatrix} \tilde{V}_s \\ -\frac{\tilde{V}_s}{R_1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\tag{44}
$$

Using Octave we obtain the phasor voltages in every node. The Table 4 shows the magnitude of the node phasors and the Table 5 shows the phase of the node phasors.

Name	**Value [V]**
V_1	1.000000
V_2	0.961043
V_3	0.880909
V_5	0.966496
V_6	0.600075
V_7	0.402240
V_8	0.600075

Table 4: Magnitude of the node phasors.

Name	**Value [Radians]**
ϕ_{V_1}	1.570796
ϕ_{V_2}	1.570796
ϕ_{V_3}	1.570796
ϕ_{V_5}	1.570796
ϕ_{V_6}	-1.570649
ϕ_{V_7}	-1.570796
ϕ_{V_8}	-1.570796

Table 5: Phase of the node phasors.

And finnaly we can compute the forced solution v_{6f} on the time interval [0, 20] ms using:

$$
V_{6f}(t) = V_6 cos(wt - \phi_{V_6}) \tag{45}
$$

and plotting this in octave we obtain the graphic of the Figure 3.

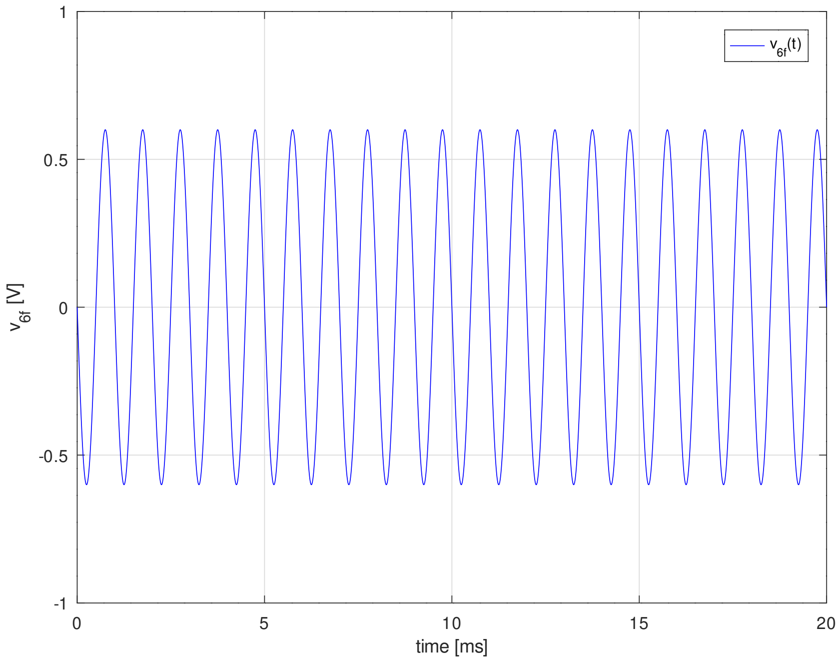

Figure 3: Force response for V_6 as a function of time in [0,20] ms

2.5 Total solution $V_6(t)$

To acquire the total solution of $V_6(t)$ on [-5,20] ms we need to convert the phasors to real time functions for $f = 1000Hz$, and superimpose the natural and forced solutions already determined.

The equation used to obtain the total solution is:

$$V_6(t) = V_{6f}(t) + V_{6n}(t). \tag{46}$$

The graphic of the Figure 4 shows the plot of total solution for $V_6(t)$ along with the plot of $V_s(t)$ on the time interval[-5,20] ms.

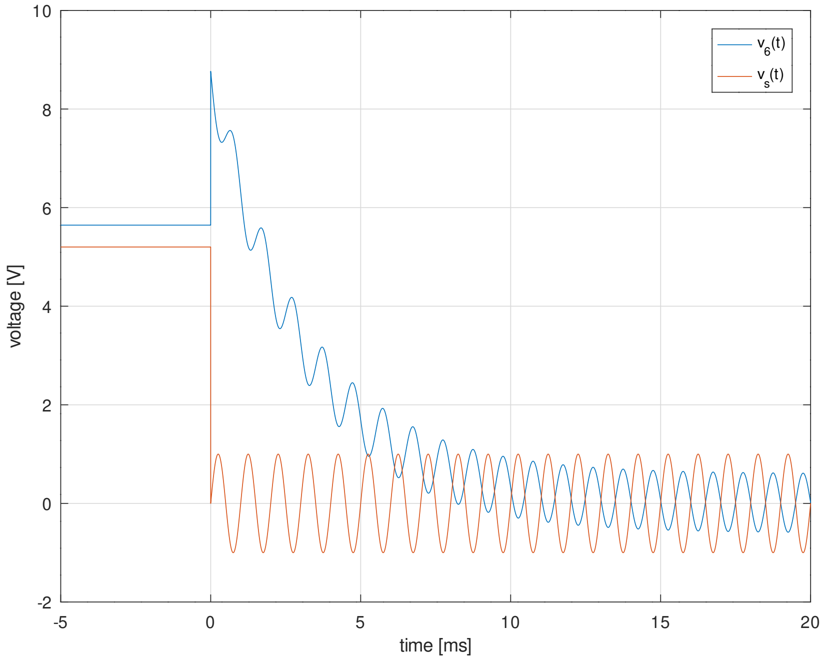

Figure 4: Total response for V_6 and V_s as a function of time in [-5,20] ms

2.6 Frequency response $v_c(f)$, $v_s(f)$ and $v_6(f)$

In this section we study how the phasor voltages v_c, v_s and v_6 behave with the variation of the frequency.

The variation of the amplitude and the variation of the phase of $v_c(f)$, $v_s(f)$ and $v_6(f)$ with the frequency in a range from 0.1Hz (very low frequency) to 1MHz (very high frequency) can be seen in the graphics of the Figures 5 and 6,respectively.

Analysing this graphics we realize that the amplitude and phase of $v_s(f)$ keeps constant, this can be explained with the fact that they don't depend on the frequency as we can see through its equation:

$$V_s(t) = V_s sin(2\pi ft) \tag{47}$$

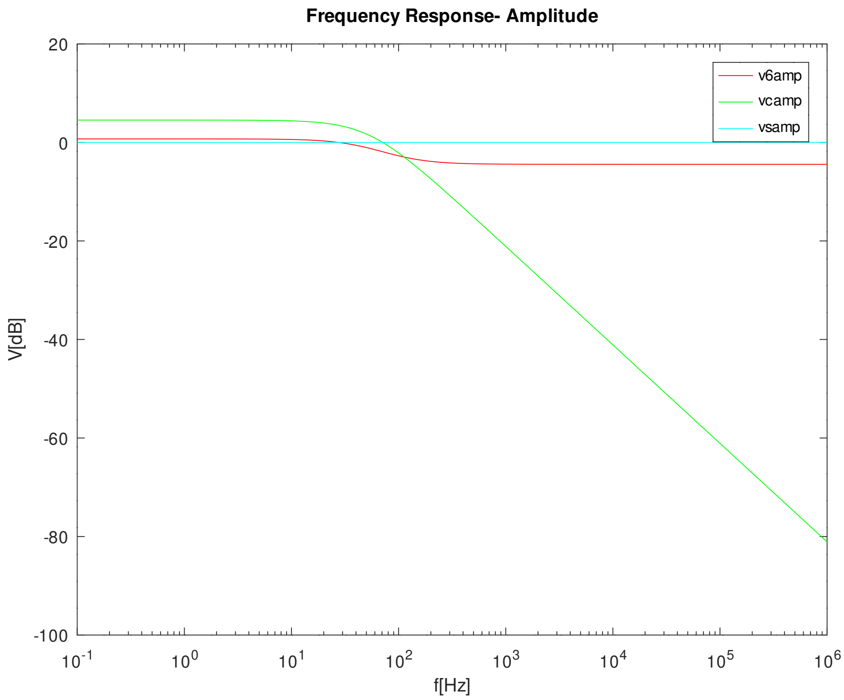

Figure 5: Amplitude response of v_c, v_s and v_6 for frequencies from 0.1Hz to 1MHz (logarithmic scale)

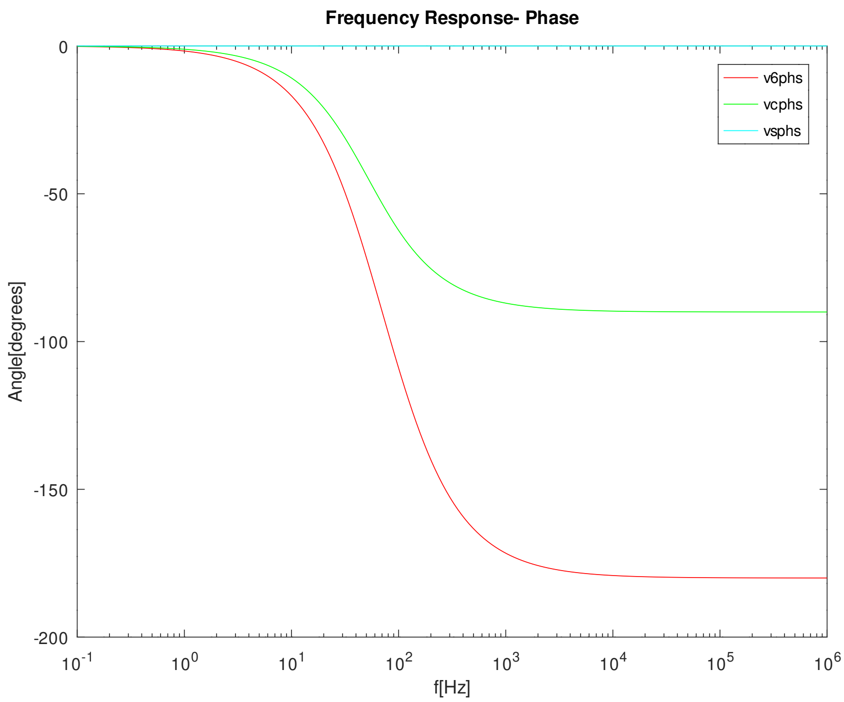

Figure 6: Phase response of v_c, v_s and v_6 for frequencies from 0.1Hz to 1MHz (logarithmic scale)

3 Simulation Analysis

3.1 Operating Point Analysis for t $<$ 0

The circuit was simulated and analysed using the Ngspice software.

The results for step (1), which asked to simulate the operating point for t<0, are shown in the Table 6.

Name	Value [A or V]
@c[i]	0.000000e+00
@gb[i]	-2.03931e-04
@r1[i]	1.948216e-04
@r2[i]	-2.03931e-04
@r3[i]	-9.10945e-06
@r4[i]	-1.20521e-03
@r5[i]	-2.03931e-04
@r6[i]	1.010393e-03
@r7[i]	1.010393e-03
v(1)	5.201027e+00
v(2)	4.998410e+00
v(3)	4.581631e+00
v(4)	-2.09206e+00
v(5)	5.026773e+00
v(6)	5.644397e+00
v(7)	-2.09206e+00
v(8)	-3.12101e+00

Table 6: Operating point for t<0. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

By comparing this table with the one from the theoretical analysis, we can see slightly different values, which are probably caused by different approximations in Octave and Ngspice.

3.2 Operating Point Analysis for t $=$ 0

In step (2) the capacitor was replaced by a voltage source with the same voltages obtained in step (1) ($V_x = V_6 - V_8$) and V_s was set to 0. The results are shown in the table.

Name	Value [A or V]
@gb[i]	0.000000e+00
@r1[i]	0.000000e+00
@r2[i]	0.000000e+00
@r3[i]	0.000000e+00
@r4[i]	0.000000e+00
@r5[i]	-2.89422e-03
@r6[i]	0.000000e+00
@r7[i]	0.000000e+00
v(1)	0.000000e+00
v(2)	0.000000e+00
v(3)	0.000000e+00
v(4)	0.000000e+00
v(5)	0.000000e+00
v(6)	8.765400e+00
v(7)	0.000000e+00
v(8)	0.000000e+00

Table 7: Operating point for t=0. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

This step is needed to calculate the time constant.

3.3 Natural solution for V_6 using transient analysis

In step (3), the natural solution was simulated. The result of the transient analysis in the time interval [0s,20s] is shown in the graphic of the Figure 7.

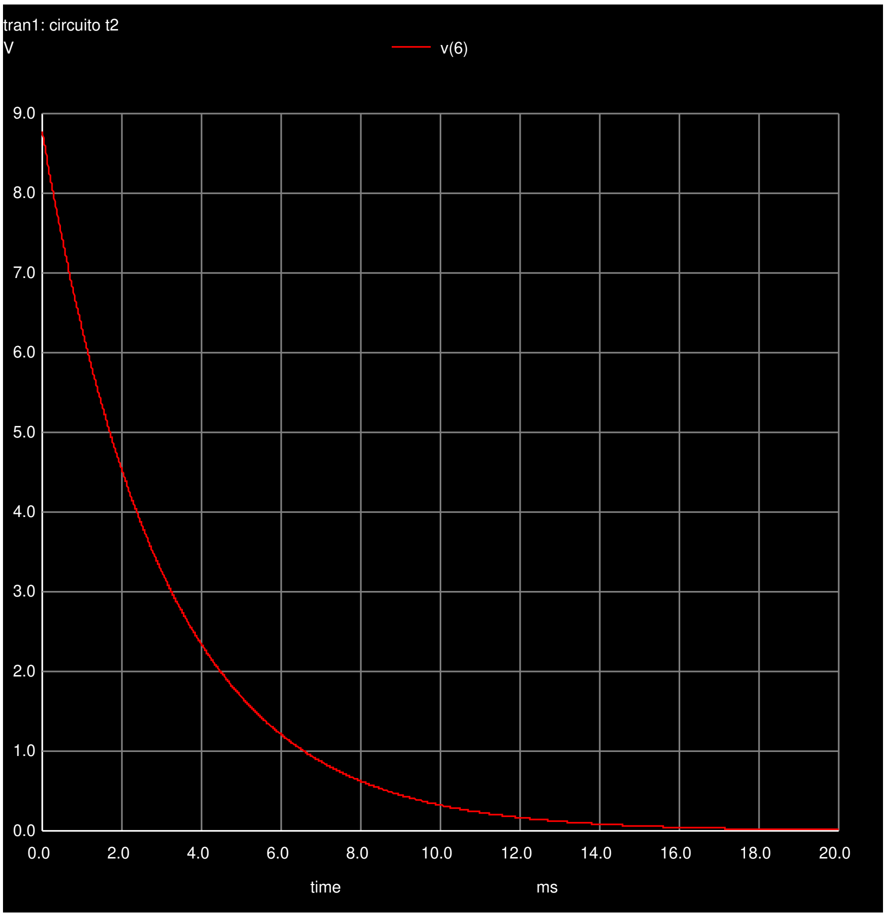

Figure 7: Simulated natural response of $V_6(t)$ in the interval [0,20] ms. The *x axis* represents the time in miliseconds and the *y axis* the Potencial in node 6 in Volts.

Comparing to the graphic obtained with Octave, we can see there is no difference.

3.4 Total solution for V_6 using transient analysis

Step (4) asked us to simulate the total (natural+forced) solution. For that, the frequency was given. We can see, in the graphic of the Figure 8, the stimulus and the response.

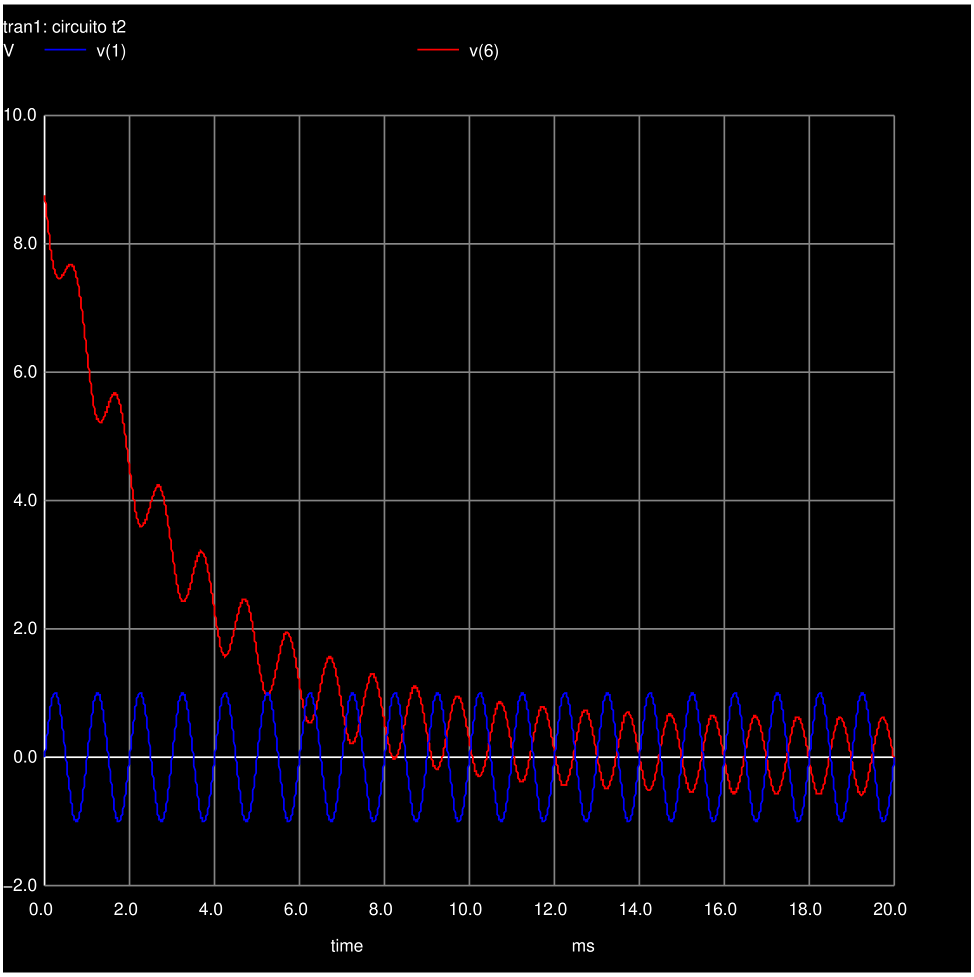

Figure 8: Simulated total response of $V_6(t)$ in the interval [0,20] ms. The *x axis* represents the time in miliseconds and the *y axis* the Potencial in node 6 in Volts.

Once again, the graphic obtained in Octave is the same.

3.5 Frequency response in node 6

Finally, in step (5), it was asked to simulate the frequency response between 0.1 Hz and 1 MHz.In the following figures, the amplitude frequency response and the phase response were plotted. It is important to notice that the frequency logscale has its magnitude in dB units and the phase is presented in degrees.

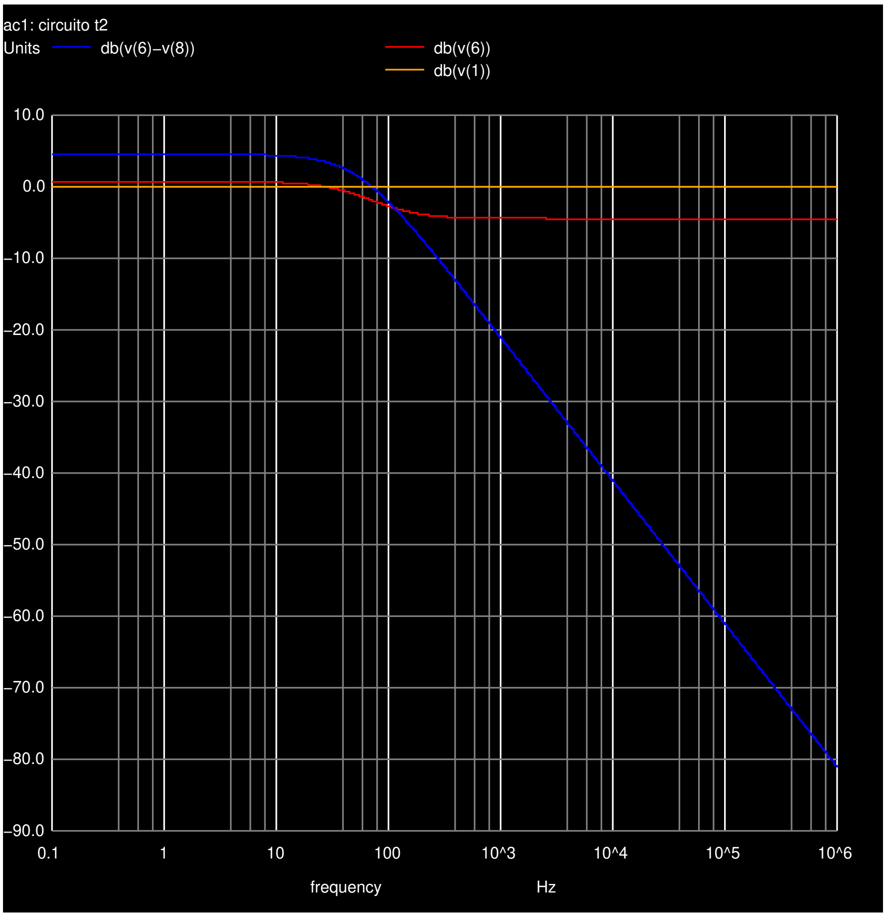

Figure 9: Amplitude frequency response for $v_s(f)$, $v_6(f)$ and $v_c(f)$ in the interval [0.1, 1M]Hz. The x axis has the frequency in Hz (logarithmic scale) and the y axis has the amplitude in dB.

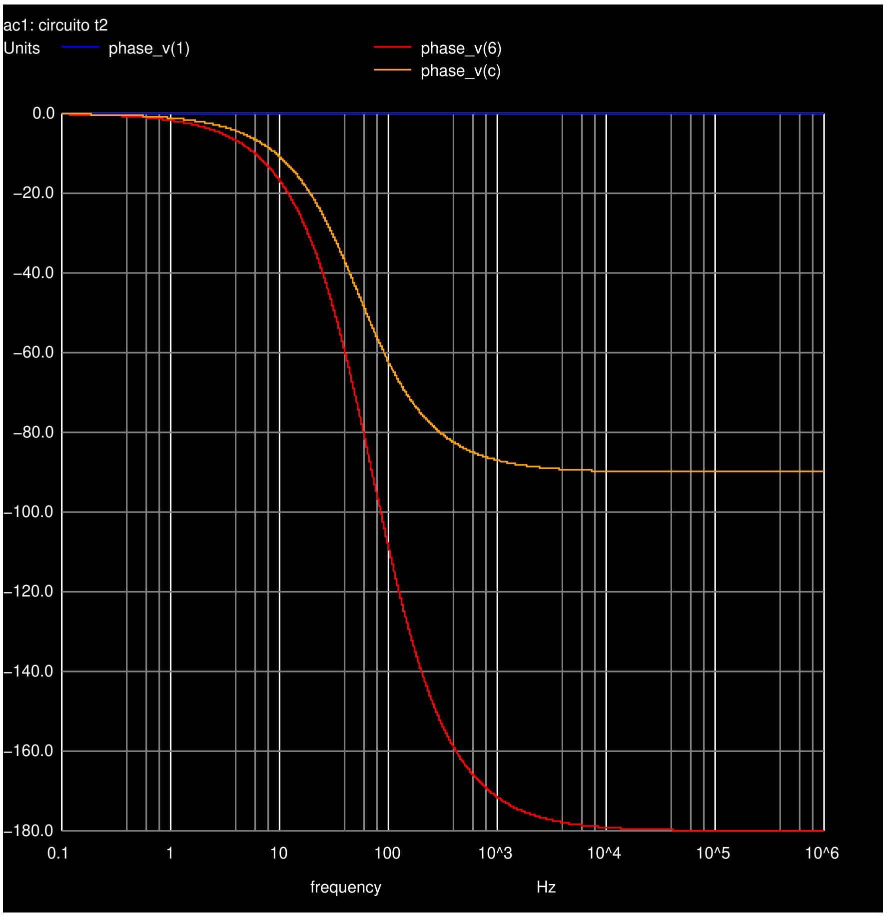

Figure 10: Phase response for $v_s(f)$, $v_6(f)$ and $v_c(f)$ in the interval [0.1, 1M]Hz. The x axis has the frequency in Hz (logarithmic scale) and the y axis has the amplitude in dB.

4 Conclusion

In this laboratory assignment the objective of analysing an RC circuit has been achieved. Static, time and frequency analyses have been performed both theoretically using the Octave maths tool and by circuit simulation using the Ngspice tool. The simulation results matched the theoretical results precisely. The reason for this perfect match is the fact that this is a straightforward circuit containing only linear components, so the theoretical and simulation models cannot differ. For more complex components, the theoretical and simulation models could differ but this is not the case in this work.