Circuit Theory and Electronics Fundamentals

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First Laboratory Report

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1 Introduction

In this laboratory assignment a circuit will be analysed theoretically and simulated using the circuit simulator tool NGSPICE. Its main objective is to compare the values of an operating point analysis (voltage and current) from a theoretical analysis using the nodal and mesh methods with the results obtained in NGSPICE circuit simulation. The circuit is composed of one independent voltage source, one independent current source, one dependent voltage source, one dependent current source, and seven resistors. In Figure 1 the circuit is represented, as well as the designations for the nodes and meshes adopted on the analysis.

In Section 2 of this report, the mesh and nodal methods from theoretical analysis of the circuit are presented. In Section 3, the circuit is analysed by simulation, and the results are commented and compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

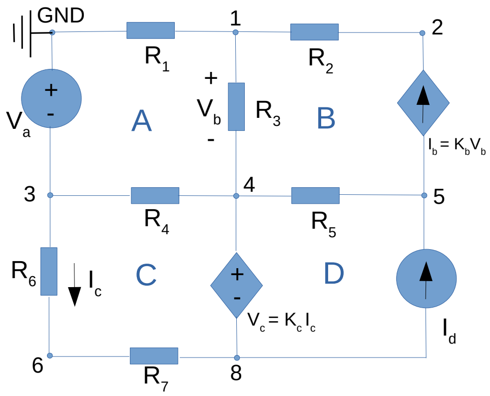

Figure 1: Circuit T1.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically using the mesh and nodal methods.

2.1 Mesh Analysys

The application of the mesh method in this circuit requires 4 currents to be determined. Each one of them will loop in one of the elementary meshes. In order to facilitate the aplication of the method, each mesh was labeled with a letter, as shown in Figure 1, and the designations I_A, I_B, I_C and I_D were assigned to the currents in meshes A, B, C and D, respectively.

Applying the Kirchhoff's Voltage Law (KVL) to each of these meshes, the following equations are obtained:

Mesh A:

$$(R_1 + R_3 + R_4)I_A - R_3 I_B - R_4 I_C = -V_a \tag{1}$$

Mesh B:

$$-K_b R_3 I_A + (K_b R_3 - 1)I_B = 0 \tag{2}$$

Mesh C:

$$R_4 I_A + (-R_6 - R_7 + K_c - R_4)I_C = 0 \tag{3}$$

Mesh D:

$$I_D = I_d \tag{4}$$

Solving this equations as a system of equations matrix in OCTAVE, the following values to the currents are obtained:

Name	Value [A]
I_A	-0.000195
I_B	-0.000204
I_C	0.001010
I_D	0.001005

Table 1: Mesh currents from mesh analysis.

Now, applying the Ohm's Law, it's possible to obtain the node voltages. The following equations result of this application to each node:

Node 1:

$$V_1 = R_1 I_A \tag{5}$$

Node 2:

$$V_2 = V_1 + R_2 I_B \tag{6}$$

Node 3:

$$V_3 = -V_a \tag{7}$$

Node 4:

$$V_4 = V_1 - R_3(-I_A + I_B) \tag{8}$$

Node 5:

$$V_5 = V_4 - R_5(I_B - I_D) \tag{9}$$

Node 6:

$$V_6 = V_3 - R_6 I_C \tag{10}$$

Node 8:

$$V_8 = V_6 - R_7 I_C \tag{11}$$

The Table 2 shows the results from these equations:

Name	Value [V]
V_1	-0.202617
V_2	-0.619394
V_3	-5.201027
V_4	-0.174256
V_5	3.485906
V_6	-7.293091
V_8	-8.322033

Table 2: Node voltages from mesh analysis.

The previous equations already included the relations necessary to determine the current of each component of the circuit using the currents of the meshes. As an exemple, the current of R_3 can be determined using the relation:

$$I(R_3) = I_B - I_A \tag{12}$$

2.2 Node Analysys

In order to theoretically analyse the circuit using the nodal method, 8 nodes have been identified, as shown in Figure 1. For 5 of these, the respective equations have been written. The node on the upper left corner was considered to have 0V. This way, its equation doesn't need to be written. The nodes 4 and 8, located on the terminals of the dependent voltage source, were grouped in a supernode in order to facilitate writing the equations. Also, the equation that describes the dependent voltage source was considered and an additional equation was written in node 3.

This way, a 7 equation and 7 unknowns system was determined and put in matrix form in order to solve it in Octave:

$$\begin{bmatrix} -\frac{1}{R_2}-\frac{1}{R_3}-\frac{1}{R_1} & \frac{1}{R_2} & 0 & \frac{1}{R_3} & 0 & 0 & 0 \\ K_b+\frac{1}{R_2} & -\frac{1}{R_2} & 0 & -K_b & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{R_3} & 0 & \frac{1}{R_4} & -\frac{1}{R_4}-\frac{1}{R_3}-\frac{1}{R_5} & \frac{1}{R_5} & \frac{1}{R_7} & -\frac{1}{R_7} \\ K_b & 0 & 0 & -\frac{1}{R_5}-K_b & \frac{1}{R_5} & 0 & 0 \\ 0 & 0 & \frac{1}{R_6} & 0 & 0 & -\frac{1}{R_6}-\frac{1}{R_7} & \frac{1}{R_7} \\ 0 & 0 & \frac{K_c}{R_6} & -1 & 0 & -\frac{K_c}{R_6} & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -V_a \\ I_d \\ I_d \\ 0 \\ 0 \end{bmatrix}$$

Node equations determined using the Kirchhoff Current Law (KCL):

Node 1:

$$(-\frac{1}{R_2}-\frac{1}{R_3}-\frac{1}{R_1})V_1+\frac{1}{R_2}V_2+\frac{1}{R_3}V_4=0 \tag{13}$$

Node 2:

$$(K_b+\frac{1}{R_2})V_1-\frac{1}{R_2}V_2-K_bV_4=0 \tag{14}$$

Supernode:

$$\frac{1}{R_3}V_1+\frac{1}{R_4}V_3+(-\frac{1}{R_4}-\frac{1}{R_3}-\frac{1}{R_5})V_4+\frac{1}{R_5}V_5+\frac{1}{R_7}V_6-\frac{1}{R_7}V_8=I_d \tag{15}$$

Node 5:

$$K_bV_1+(-\frac{1}{R_5}-K_b)V_4+\frac{1}{R_5}V_6=I_d \tag{16}$$

Node 6:

$$\frac{1}{R_6}V_3+(-\frac{1}{R_6}-\frac{1}{R_7})V_6+\frac{1}{R_7}V_8=0 \tag{17}$$

Additional equations:

$$\frac{K_c}{R_6}V_3-V_4-\frac{K_c}{R_6}V_6+V_8=0 \tag{18}$$

$$V_3=-V_a \tag{19}$$

The results of the previous system of equations are shown in Table 3:

Name	**Value [V]**
V_1	-0.202617
V_2	-0.619394
V_3	-5.201027
V_4	-0.174256
V_5	3.485906
V_6	-7.293091
V_8	-8.322033

Table 3: Node voltages from node analysis.

After finding the voltages of every node, Kirchoff's Voltage Law was applied in order to check the results. On the meshes A and D, the sum of all potential differences was, in fact, 0. On the meshes B and C, the result was, respectively, $-5.551115123125783e-17$ and $-8.881784197001252e-16$, which are both very close to 0.

The information of the node voltages is enough to determine the current of each component of the circuit, using the Ohm's Law. As as example, the following relations can be used to determine the current of R_3:

$$V(R_3) = V_1 - V_4 \tag{20}$$

$$I(R_3) = \frac{V(R_3)}{R_3} \tag{21}$$

3 Simulation Analysis

3.1 Operating Point Analysis

Table 4 shows the simulated operating point results for the circuit under analysis.

Comparing the simulation results of the node voltages to the ones obtained in the theoretical analysis (Table 2 and Table 3) we can realize that they are the same or virtually the same. Using the relations refered in Section 2 we can also conclude that since the node voltages are the same, then the currents of each component of the circuit are also the same as expected using the theoretical methods.

Name	**Value [A or V]**
@gib[i]	-2.03930e-04
@id[current]	1.004605e-03
@r1[i]	1.948215e-04
@r2[i]	-2.03930e-04
@r3[i]	-9.10887e-06
@r4[i]	1.205214e-03
@r5[i]	-1.20854e-03
@r6[i]	1.010392e-03
@r7[i]	1.010392e-03
v(1)	-2.02617e-01
v(2)	-6.19394e-01
v(3)	-5.20103e+00
v(4)	-1.74256e-01
v(5)	3.485906e+00
v(6)	-7.29309e+00
v(7)	-7.29309e+00
v(8)	-8.32203e+00

Table 4: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

The circuit was built through a Ngspice script with a ”netlist” where the various components were introduced through their element type label, which is predefined by Ngspice, followed by a name for the component to differentiate it from the others. Besides the type and name, each component was identified with its two nodes on the circuit and the values of its characteristic parameters. For some components, NGSPICE need some more information to be provided. An example of this are the dependent sources, which are present in the circuit in study. For the voltage-controlled current source (VCCS) it was also necessary to provide information of the two nodes where the control-voltage was. For the current-controlled voltage source (CCVS) it was necessary the addition of a voltage source with 0V between the node 6 and the resistor 7, and subsequently the addition of a new node, named node 7, placed before the resistor 7. This addition is crucial for the CCVS input because this new voltage source will sense its control current.

The analysis done is an operating point analysis, which gives as outputs the voltage at each node and the current of the various components of the circuit. This is a type of DC analysis, a kind of analysis that does not consider any time dependence on any sources within the circuit.

4 Conclusion

In this laboratory assignment the objective of analysing a circuit using the Ngspice, and both the mesh and node theoretic methods has been achieved.

Static analysis has been performed both theoretically using the Octave maths tool and by circuit simulation using the Ngspice tool. The simulation results and the theoretical results were identical. The reason for this similarity is the fact that this is a straightforward and simple circuit containing only linear components, so the theoretical and simulation models don't differ. The static analysis done prevents also a lot of variables that can affect a circuit and it's components like time progression and other external and internal factors.

Also given the similarity between the two type of analysis is safe to assume that the Ngspice engine uses the same methods used theoretically, mainly the node analysis because it's easier the engine to identify the different nodes than to identify the meshes. So because of this, the values are, naturally, identical given that they are achieved using the same equations and methods. All the values obtained for the various components and nodes follow the various laws that characterize a circuit, KVL and KCL with a error of 0% or very close to it, and are what was expected given the values given to study the circuit in terms of grandness and direction of currents, which leads to believe that the values achieved are correct and the objective of this laboratory assignment was achieved.