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Universidade Federal do Ceará - UFC
Departamento de Matemática

CB0536 - Cálculo Diferencial e Integral III - Turma 08 - 2023.1
Avaliação Parcial 3 - 03.07.2023

Use caneta para redigir essa prova

Aluno(a): ______________________ **Matrícula.** _

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1. Considere o campo de vetores

$$\mathbf{F}(x,y) = \left(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2}\right)$$

definido para todo $(x,y) \neq (0,0)$. Encontre uma função $\varphi(x,y)$ tal que $\nabla\varphi = \mathbf{F}$ e calcule a **integral de linha** de $\mathbf{F}$ ao longo da curva $y = x^2 - 1$ do ponto $(-1,0)$ ao ponto $(2,3)$.

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2. Use o **Teorema da Divergência** para calcular a **integral de superfície**

$$\int\int_S \mathbf{F}\cdot d\mathbf{S}$$

onde $\mathbf{F}(x,y,z) = (4x^3z, 4y^3z, 3z^4)$ e $\mathbf{S}$ é a esfera com centro na origem e raio $R > 0$, orientada para fora.

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3. Use o **Teorema da Divergência** para calcular a **integral de superfície**

$$\int\int_S \mathbf{F}\cdot d\mathbf{S}$$

onde $\mathbf{F}(x,y,z) = (x+y, z^2, x^2)$ e $\mathbf{S}$ é a superfície $x^2+y^2+z^2 = 1$, $z > 0$. Observe que $\mathbf{S}$ não é uma superfície fechada.

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4. Use o **Teorema de Stokes** para calcular a **integral de superfície**

$$\iint_S (\text{rot}\mathbf{F})\cdot d\mathbf{S}$$

onde $\mathbf{S}$ é o hemisfério superior $x^2+y^2+z^2 = 1,\ z \geq 0$, e $\mathbf{F}$ é o campo $\mathbf{F}(x,y,z) = (y^3, -x^3, z^3)$.

♠

~~4a) $z=0$: $x^2+y^2=1$ $x=r\cos\theta$~~
$y=r\,\text{sen}\,\theta$

4a) $r=(\cos\theta, \text{sen}\,\theta, 0)$ $z=0\ [x^2+y^2=1]$

$r'=(-\text{sen}\,\theta, \cos\theta, 0)$

$F(r)=(\text{sen}^3\theta, -\cos^3\theta, 0)$

$$\int F\cdot dr = \int_0^{2\pi} -\text{sen}^4\theta - \cos^4\theta\,d\theta = -\int_0^{2\pi}\text{sen}^2\cdot\text{sen}^2\,d\theta\,(-)\int_0^{2\pi}\cos^2\cos^2\,d\theta$$

$$= -\int_0^{2\pi}\left(1-\frac{\cos(2\theta)}{2}\right)^2 d\theta - \int_0^{2\pi}\left(1-\frac{\text{sen}(2\theta)}{2}\right)^2$$

$$= -\int_0^{2\pi} 1-\cos(2\theta)+\frac{\cos^2(2\theta)}{4}\,d\theta - \int_0^{2\pi} 1-\text{sen}(2\theta)+\frac{\text{sen}^2(2\theta)}{4}\,d\theta$$

$$= -2\pi - \frac{1}{2}\overset{0}{\cancel{\text{sen}(2\theta)}}\Big|_0^{2\pi} + \frac{1}{4}\int_0^{2\pi}\underset{\int=0}{\cos^2(2\theta)} - 2\pi + \frac{\overset{0}{\cancel{\cos(2\theta)}}}{2}\Big|_0^{2\pi} + \frac{1}{4}\int_0^{2\pi}\underset{\int=0}{\text{sen}^2(2\theta)}\,d\theta$$

$$= -4\pi + \frac{1}{4}\left[-\int_0^{2\pi} 1-\frac{\text{sen}^2\cancel{(4\theta)}}{\cancel{2}}\,d\theta - \int_0^{2\pi} 1-\frac{\cos^2\cancel{(4\theta)}}{\cancel{2}}\,d\theta\right]$$

$$= -4\pi + \frac{1}{4}\left[-2\pi - 2\pi\right] = -5\pi$$

?

1ª) $F = \nabla f$??

$u = x^2 + y^2$

$\int F_x = \int \frac{x}{x^2+y^2}\, dx = \frac{1}{2}\int \frac{1}{u}\, du = \frac{1}{2} \ln|x^2+y^2|$ $du = 2x\,dx$

$\int F_y = \int \frac{y}{x^2+y^2}\, dy = \frac{1}{2} \ln|x^2+y^2|$

Logo, temos que $f = \frac{1}{2} \ln|x^2+y^2|$

$P_a = (-1, 0)$

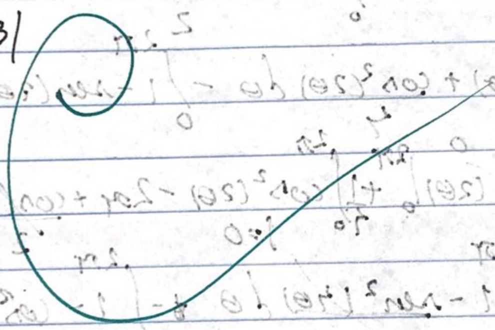
Portanto:

$P_b = (2, 3)$

$\int \nabla f \cdot dr = f(r(b)) - f(r(a))$

$\int \nabla f \cdot dr = \frac{1}{2} \ln|2^2+3^2| - \frac{1}{2} \ln|(-1)^2+0^2|$

$\int \nabla f \cdot dr = \frac{1}{2} \ln|13|$



2ª) $\text{div } F = 12x^2 z + 12y^2 z + 12z^3$

$x = \rho \,\text{sen}\,\phi \cos\theta$

$y = \rho \,\text{sen}\,\phi \,\text{sen}\,\theta$

$z = \rho \cos\phi$

$0 \leq \rho \leq R \quad 0 \leq \phi \leq \pi \quad 0 \leq \theta \leq 2\pi$ ✓

$$\iiint \text{div } F \cdot dv = 12 \iiint [(\rho^2 \text{sen}^2\phi \cos^2\theta)\rho\cos\phi + (\rho^2 \text{sen}^2\phi \,\text{sen}^2\theta)\rho\cos\phi + \rho^3\cos^3\phi]\,\rho^2 \text{sen}\,\phi \, d\rho\, d\phi\, d\theta$$

$$\iiint \text{div } F\, dV = 12 \iiint [\rho^3 \text{sen}^2\phi\cos\phi + \rho^3\cos^3\phi]\,\rho^2\text{sen}\,\phi\, d\rho\, d\phi\, d\phi$$

$$= 24\pi \iint \rho^5 [\text{sen}^3\phi\cos\phi + \cos^3\phi\,\text{sen}\,\phi]\, d\rho\, d\phi$$

$$= 24\pi \frac{R^6}{6}\left[\int_0^{\pi} [\text{sen}^3\phi\cos\phi + \cos^3\phi\,\text{sen}\,\phi]\, d\phi\right.$$

$u = \cos\phi$

$du = -\text{sen}\,\phi\, d\phi$

$v = \text{sen}\,\phi$

$dv = \cos\phi\, d\phi$

$$= 4\pi R^6 \left[\underset{0}{\int_0^{\pi} v^3\, dv} + \underset{0}{\int_0^{\pi} -u^3\, du}\right]$$

$$= 4\pi R^6 \left[\cancel{\frac{\text{sen}^4\phi}{4}}\Big|_0^{\pi} - \cancel{\frac{\cos^4\phi}{4}}\Big|_0^{\pi}\right]$$

~~$= 4\pi R^6 \left[\frac{1}{4} - \left[0 - \frac{1}{4}\right]\right]$~~

~~$= 4\pi R^6 \cdot \frac{1}{2} = 2\pi R^6$~~

$\iint \text{div } F V = 0$

3a) ~~divF~~ $\operatorname{div}F = 1$

$$\iiint \operatorname{div}F = \int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^1 \rho^2 \operatorname{sen}\phi \, d\rho \, d\phi \, d\theta + \int_0^{2\pi}\int_0^1 r \, dr \, d\theta$$

$$= 2\pi \frac{1}{3}\int_0^{\frac{\pi}{2}} \operatorname{sen}\phi \, d\theta + 2\pi \frac{1}{2} = \frac{2\pi}{3}\left[-\cos\phi\Big|_0^{\frac{\pi}{2}}\right] + \pi = \frac{5\pi}{3}$$

ATK SPD HP — RAY

ATK ~~DEF~~ ~~SP.DEF~~ SP.ATK — DEOXYS

SP.ATK — GARDE

GYAR — HP SP.DEF