

Consider a parity game G on state space $X = \{1, \dots, n\}$ with priority function $\Omega : X \rightarrow \{1, \dots, d\}$.

Algo 1 Consider the following algorithm that takes a game as input and update W

1. Let p be the maximal priority
2. Let $i = p \bmod 2$ be the corresponding player.
3. Let

$$g(x) = (-1)^{\Omega(x)} \mathbb{1}_{\Omega(x)=p}$$

and

$$Tv = g + \max_{\mu} \min_{\nu} P_{\mu, \nu} v.$$

4. Compute $v = T^n 0$
5. Let $A = \{x; T^n 0(x) = 0\}$
6. Let $B = (1 - i) - Attr(A)$
7. Let $W(p) = G \setminus B$
8. Recursive call on game G restricted to A
9. Propagate the values on B .

Algo 2 Consider the Bellman operator T of Puri's reduction.

1. Compute $v = T^n 0$.
2. Compute $A = \{x; v(x) = -n\}$. If A is not empty, then we know that the game is won with priority 1 from x . So also from $1 - Attr(A)$, which we can remove from the game.
3. Compute $A = \{x; n^2 - (n - 1) \leq v(x) \leq n^3\}$. If A is not empty, then we know that the game is won with priority 2 from y . So also from $0 - Attr(B)$, which we can remove from the game.
4. Compute $A = \{x; -n^4 \leq v(x) \leq -n^3 + (n - 1)n^2 = n^2\} \dots$
5. Compute $A = \{x; n^4 - (n - 1)n^3 = -n^3 \leq v(x) \leq n^5\} \dots$