Papers I'm reading and ideas about them

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Abstract

List of papers I'm reading and important ideas about them.

1 The Burnside problem for $\mathbf{Diff}_\omega^\infty(\mathbb{S}^2)$

1.1 Introduction to the Burnside problem

Let G be a finitely generated group such that every element has finit order, is G necessarly finite? The answer in general is no.

The paper focuses on three different cases of the following question: if the order of the above group is bounded, is the group finite.

The cases are:

1.2 $G < \mathbf{Diff}_\omega^\infty(\mathbb{S}^2)$

G is a subgroup of area preserving diffeomorphisms. In this case the answer to the last question is "YeS"". In order to prove it one considers $S = \{s_1, \ldots, s_n\}$ a symmetric set of generators of the group, the shift space $\Sigma = S^{\mathbb{Z}}$: $F : \Sigma \times M \to \Sigma \times M$ given by $F(w, x) = (\sigma w, w_0 x)$. Since every element of G has finite order, the cocycle cannot admit hyperbolic fixed points.

A group without hyperbolic fixed points is called elliptic. For G elliptic, one shows that the growth of derivatives of G is subexponential. In order to do this on asssumes the contrary and constructs a hyperbolic invariant ergodic measure and applies the following theorem

Theorem 1.1 (Katok) *IF f is a $C^{1+\alpha}$ diffeomorphism on a compact smooth manifold and μ is a hyperbolic ergodic invariant measure, then f admits hyperbolic periodic points and we can take them close to $supp(\mu)$.*