Notes on rotation theory

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Abstract

Some important results in rotation theory in low dimensions.

1 Plane homeomorphisms

Plane homeomorphisms will be important to the study of anulli homeomorphisms, a fundamental result used in other, more complicated proofs is this result by Brower:

Theorem 1.1 *For $f : \mathbb{R}^2 \to \mathbb{R}^2$ an orientation presenving homeomorphism, if f has no periodic point then it has no fixed point.*

Fundamental to the proof of this theorem is a result stating that if f has a periodic disk chain, then f has a fixed point.

Definition 1.1 (Periodic disk chain) *A periodic disk chain for f is a family $U_1, \ldots, U_n$ os disks with*

1. $f(U_i) \cap U_i = \emptyset$
2. $U_i \cap U_j = \emptyset$ *for* $i \neq j$
3. *For all i, there exists* m_{i-1} *such that* $f^{m_{i-1}} U_{i-1} \cap U_i \neq \emptyset$

One example of homeomorphism without fixed points is given by translations, we have a strong result due to Brower that says that in a sense every homeomorphism without fixed points acts like a translation.

For this we need:

Definition 1.2 (Translation domain) *For a properly embedded line L, we say the open connected region whose boundary is $L \cup f(L)$ is a translation domain for f if L separates $f(L), f^{-1}(L)$.*

Theorem 1.2 (Brower translation theorem) *If f is a plane homeomorphisms, then every point is contained in a translation domain.*