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Outline

In fact, this is the same notation as in the linear models chapter, except that now Y_i is discrete and represent generally a class number, a qualitative variable $Y_i \in \{0, 1, ...K\}$, $Y_i \in \{$ spam, ham$\}$
$Y_i = h(X^i) = \beta_0 + \beta_1 X_{i,1} + \beta_2 X_{i,2} + ... + \beta_i X_{i,p}$
Moreover the prediction is a probability to be in the class k. In a two-class problem, you can decide Y_i is true if probability is > 0.5

Outline

1. Classification
   - Framework and notations
   - Confusion matrix
   - ROC Curve
   - Logistic Regression motivation
   - Logistic Regression algorithm
   - Other approach K-Nearest Neighbours (KNN)
   - Summary

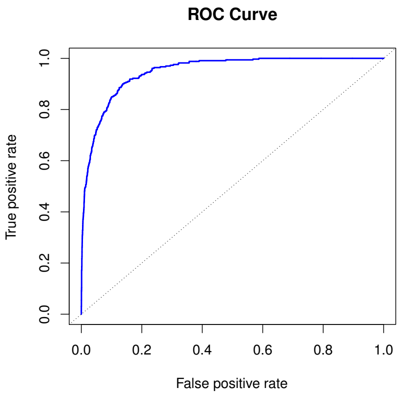
Vocabulary :

		Predicted condition		
	Total population	Predicted Condition positive	Predicted Condition negative	Prevalence $= \frac{\Sigma \text{ Condition positive}}{\Sigma \text{ Total population}}$
True condition	condition positive	**True positive**	**False Negative** (Type II error)	True positive rate (TPR), Sensitivity, Recall $= \frac{\Sigma \text{ True positive}}{\Sigma \text{ Condition positive}}$
	condition negative	**False Positive** (Type I error)	**True negative**	False positive rate (FPR), Fall-out $= \frac{\Sigma \text{ False positive}}{\Sigma \text{ Condition negative}}$
	Accuracy (ACC) = $\frac{\Sigma \text{ True positive} + \Sigma \text{ True negative}}{\Sigma \text{ Total population}}$	Positive predictive value (PPV), Precision $= \frac{\Sigma \text{ True positive}}{\Sigma \text{ Test outcome positive}}$	False omission rate (FOR) $= \frac{\Sigma \text{ False negative}}{\Sigma \text{ Test outcome negative}}$	Positive likelihood ratio (LR+) $= \frac{\text{TPR}}{\text{FPR}}$
		False discovery rate (FDR) $= \frac{\Sigma \text{ False positive}}{\Sigma \text{ Test outcome positive}}$	Negative predictive value (NPV) $= \frac{\Sigma \text{ True negative}}{\Sigma \text{ Test outcome negative}}$	Negative likelihood ratio (LR−) $= \frac{\text{FNR}}{\text{TNR}}$

Beware : In the book Predicted is given horizontally
From wikipedia-en on ROC

Outline

From communication theory Receiver Operator Curve :



The associated measure is the Area Under the Curve (**AUC**), need to be near 1.

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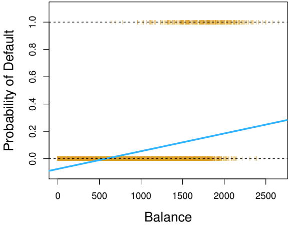

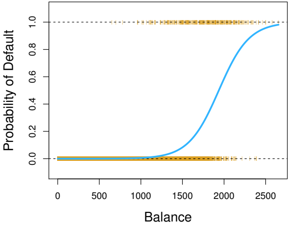

- To avoid values outside $[0, 1]$, we use a specific function called **sigmoid** or **logistic function**
- $\text{sigmoid}(x) = \frac{1}{1+e^{-x}} = \frac{e^x}{e^x+1} \in [0, 1]$
- The Linear Regression is "filtered" by this function.

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- After some manipulation the model is : $\ln(\frac{p(X)}{1-p(X)}) = \beta_0 + \beta_1 X_1 + \epsilon$ where $p(X)$ is the probability that the response is 1
- Instead of using Least Square algorithm to adjust the β_i parameters, here we use a more statistical way, the Maximum Likelihood algorithm
- Obviously we can have multiple predictors : $\ln(\frac{p(X)}{1-p(X)}) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_i X_p + \epsilon$

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- In order to make a prediction for an observation $X = x$, the K training observations that are closest to x are identified.
- Then X is assigned to the class to which the plurality of these observations belong. Hence KNN is a completely **non-parametric** approach : no assumptions are made about the shape of the decision boundary.
- There is also a KNN Regression algorithm (book)

Outline

- Prediction of probabilities ;
- Same minimization frameworks ;
- In general, all the algorithms have a Regression and a Classification behaviour.