# Contents

# Contents

# Contents

# Contents

# Contents

# Contents

## Contents

# Contents

## Contents

# Contents

## Contents

# Outline

1. Linear Models
   - Framework and notations
   - Toy example : Housing price prediction
   - Toy example (from the book) : Sales prediction
   - Linear model properties
   - Fitting the regression
   - Solving the regression analytically
   - Solving the regression analytically
   - Gradient descent principles
   - Gradient descent principles
   - Other algorithms : LDA, Polynomial expansion
   - Summary

Starting point :

- Outcome measurement Y (also called dependent variable, response, target, label) ;
- Vector of p predictor measurements X (also called inputs, regressors, covariates, features, independent variables). X is a matrix of dimension (N,p), where N is the number of measurements ;
- In the **regression problem**, Y is quantitative (e.g price,sales, blood pressure) ;
- We have training data (x1,y1),..., (xN,yN). These are observations (examples, instances) of these measurements.

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Linear Regression model with one variable

$Y_i = \beta_0 + \beta_1 X_1$

$Price_{house30} = \beta_0 + \beta_1 Surface_{House30}$

In fact, we could imagine the price depends from severals factors, so we come with Linear Regression with several variables :

$Price_{house30} =$

$K + \beta_1 Surface_{House30} + \beta_2 NbOfRooms_{House30} + \beta_3 Location_{House30}$

In general :

$Y_i = h(X^i) = \beta_0 + \beta_1 X_{i,1} + \beta_2 X_{i,2} + ... + \beta_i X_{i,p}$

Traditionaly p is called the number of features. We will use matrix notation, so there will be double indices.

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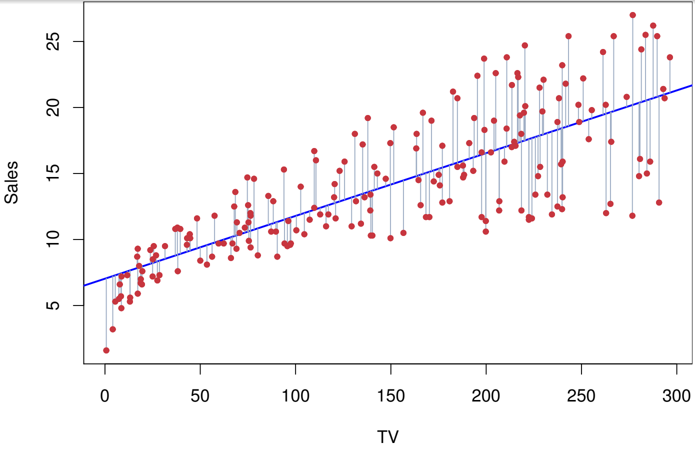


$$Sales \approx \beta_0 + \beta_1 TV$$

# Outline

- The parameters in the linear regression model are very easy to interpret.
- $\beta_j, 1 \leq j \leq p$ is the average increase in Y when $X_j$ is increased by one and all other $X_i$ are held constant.
- Vocabulary : $\beta_0$ is the intercept (i.e. the average value for Y if all the Xs are zero), $\beta_j$ is the slope for the jth variable $X_j$

# Outline

- Historical method : Least Square Regression ;
- Modern method : numerical iterative process : gradient descent and a huge family of similar algorithms (Maths : (Numerical)(Convex or not) Optimization.

Cost function, traditionally noted $J(\beta)$ is given by :
$J(\beta) = \frac{1}{2N} \sum_{i=1}^{N} (h(X^i) - Y_i)^2$
Recall naming and indices are not universal, you will find sometimes n instead of N or p, or m instead of N, etc.

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- We want to minimize the quantity $\sum_{i=1}^{N}(h(X^i) - Y_i)^2$ called MSE Mean Square Error
- Solution in one dimension : write partial derivatives in $\beta_0$ and $\beta_1$ of the cost function. To be done
- Solution in p dimension : Matrix

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- In one dimension you will derive a 2x2 linear system :
$$\begin{cases} \beta_0 \sum_{i=1}^N x_i & +\beta_1 \sum_{i=1}^N x_i^2 = \sum_{i=1}^N x_i y_i, \\ N\beta_0 & +\beta_1 \sum_{i=1}^N x_i = \sum_{i=1}^N y_i. \end{cases}$$
- To recall : you can shift the variables $(x_i, y_i)$ to be centred on the mean, new variables $(\bar{x}_i, \bar{y}_i)$, verifies $\sum_i \bar{x}_i = 0$, $\sum_i \bar{y}_i = 0$ it gives directly the well-known slope coefficient $\beta_1 = \frac{\sum_{i=1}^N x_i \bar{y}_i}{\sum_{i=1}^N \bar{x}_i^2}$
- Solution in p dimensions : Matrix X including a column vector of 1, verify
$(X^T X)\beta = X^T Y$ gives $\beta = (X^T X)^{-1}(X^T Y)$ assuming that the square matrix $X^T X$ is invertible. This is the **Normal Equation**.

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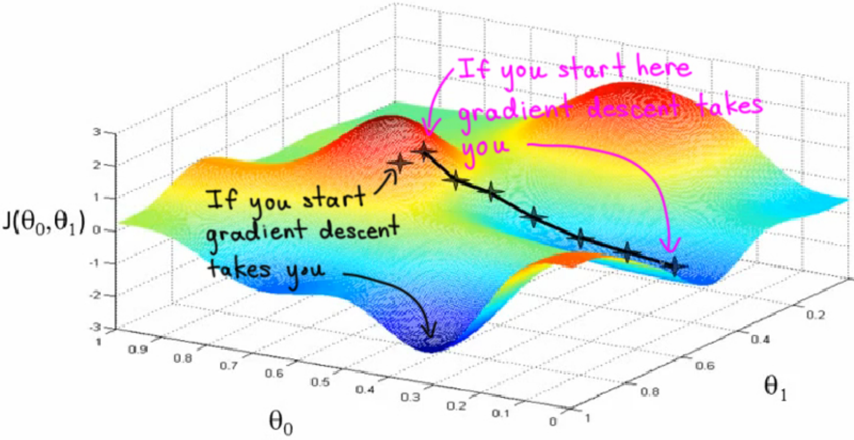
   - Gradient descent principles
   - Other algorithms : LDA, Polynomial expansion
   - Summary


If you start here
gradient descent takes
you
If you start
gradient descent
takes you
$J(\theta_0, \theta_1)$
$\theta_0$
$\theta_1$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.2
0.4
0.6
0.8
1

# Outline

- Levels on a curve or surface, directions of steepest descent
- Stochastic approach steps :
  - Initialize $\beta_0$, $\beta_1$, ... $\beta_p$
  - Compute the new direction : $\beta_j := \beta_j - \alpha_j \frac{\partial J(\beta)}{\partial \beta_j}$ , for $j = 0, ..., p$
  - Evaluate $J(\beta)$ and iterate
- Compare the complexity of the 2 methods

# Outline

- LDA
- Polynomial expansion : beyond the linearity (another chapter)

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- Linearity is a limitation but solving principles are more general ;
- Introducing complexity/flexibility of a model vs interpretability