Self-programmed Implementation of PCA Algorithm

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Abstract—The abstract goes here.

Index Terms—Computer Society, IEEEtran, journal, LaTeX, paper, template.

1 Introduction

Principal Component Analysis (PCA) have long been used in pattern recognition. An $n \times n$ image has n^2 dimensions, assume each pixel has a intensity domain ranges from 0 to 255, the whole image space will contain $256^{n \times n}$ images in total, which is a great waste considering a face image only occupies a small part of this space. Thus, the PCA algorithm is proposed, using the principal component set of face set, where the principals are orthogonal to each other, to get a coordinate system. Each axis is an image, also called eigenpicture. The corresponding coordinate system is called eigenspace and the projection coefficient set obtained by project actual faces to this coordinate system is called principal component representation of face image. Obviously, this method reduces great amount of redundancy. [1]

2 The Principal of PCA Algorithm

The thinking in PCA algorithm is to map the n-dimensional characteristics to a k-dimensional space, briefly speaking, PCA is a dimensionality reduction process. It can be defined as an orthogonal projection of data onto a low dimensional linear space, which is called principal subspace.

2.1 Maximum Variance Theory

Assume a unit vector u, whose dimension is n, defines the direction of a 1-dimensional space. Original sample point $x^{(i)}$ is also a n-dimensional vector, it is easy to know that the distance of projection of x onto u from origin is $x^{(i)T}u$. Original data set is $X_{m\times n}$, we aim to find the best projection space $W_{n\times k} = (w_1, w_2, \dots, w_k)$, in which w_i is a vector of unit length and w_i is orthogonal to $w_j(i \neq j)$[2]. The best projection space means maximizing the variance of sample points after the projection.

The variance after the projection in total is:

$$\frac{1}{m}\sum_{i=1}^{m}(x^{(i)T}w)^2 = \frac{1}{m}\sum_{i=1}^{m} w^T x^{(i)} x^{(i)T} w = \sum_{i=1}^{m} w^T(\frac{1}{m}x^{(i)}x^{(i)T})w \tag{1}$$

Meanwhile, $\frac{1}{m}x^{(i)}x^{(i)T}$ is exactly the covariance matrix of original data set X. Let $\lambda = \frac{1}{m}\sum_{i=1}^{m}(x^{(i)T}w)^2, \sum = \frac{1}{m}x^{(i)}x^{(i)T}$, we have

$$\lambda = w^T \sum w \tag{2}$$

Multiply both left sides of this equation by w, and $ww^T = 1(unitvector)$, we have

$$\lambda w = \sum w \tag{3}$$

Thus, w is the feature vector to which the feature value of matrix $\sum$ corresponds to. So the best projection vector w is the corresponding one then feature value λ reaches its peak. Consequently, when we set vector w equal to the feature vector with biggest feature value λ.

3 Procedure of PCA Algorithm

• Input data set$X_{m\times n}$

• Calculate the mean value$X_{mean}$of$X_{m\times n}$, then let $X_{new} = X - X_{mean}$

• Calculate the covariance matrix of X_{new} , denote by Cov

• Calculate the feature value and feature vector of covariance matrix Cov

• Sort the feature values in order from largest to smallest and select k largest ones, then make up a feature vector matrix $W_{n\times k}$with their corresponding feature vectors as column vector.

• Calculate $X_{new}W$, which is to project data set X_{new} to the selected feature vector.

4 Implementation with MATLAB

Matlab is a powerful tool for arithetic and image processing, as indecated by its name, matlab is good at matrix operations. The source code will be appended in another doc.

5 Result

0.2552	-0.1548
0.2571	-0.156
0.2686	-0.1480
0.2686	-0.1154
0.2505	-0.0988
0.2092	-0.0798

This is just a little part of output data, more will be in the appendix. And the reconstruction is showed below:

References

[1] Shen Li, Liu Yiguang, Xiong Zhiyong.FUNDAMENTALS AND ALGORITHMS OF FACE RECOGNITION, published:2014 P63~65 FUNDAMENTALS AND ALGORITHMS OF FACE RECOGNITION

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