SM 2023-09-25

Série de Fourier

$j = \sqrt{-1}$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$(jx)^2 = -x^2$$
$$(jx)^3 - j x^3$$

$$e^{jx} = 1 + jx - \frac{x^2}{2!} - j\frac{x^3}{3!} + \frac{x^4}{4!} \dots$$

$$\boxed{e^{jx} = \cos x + j \sin x}$$

$$e^{-jx} = \cos x - j \sin x$$

$$\boxed{\begin{aligned} \cos x &= \frac{e^{jx} + e^{-jx}}{2} \\ \sin x &= \frac{e^{jx} - e^{-jx}}{2j} \end{aligned}}$$

Série clássica de Fourier

função periódica de período $T \rightarrow$

$$x(t+T) = x(t), \quad \forall t$$

período fundamental é o período mais pequeno possível.

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{+\infty} a_n \cos\left(2\pi n \frac{t}{T}\right) + \sum_{n=1}^{+\infty} b_n \sin\left(2\pi n \frac{t}{T}\right)$$

$$x(t) = \sum_{n=-\infty}^{+\infty} c_n \, e^{j2\pi n \frac{t}{T}} \qquad c_n = \frac{1}{2\pi}\int_0^T x(t)\, e^{-j2\pi n \frac{t}{T}}\, dt$$

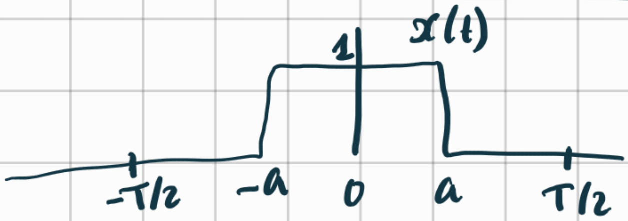
Errado

$$\langle x(t), y(t) \rangle = \cancel{\frac{1}{2\pi}}\,\frac{1}{T} \int_0^T x(t)\, y^*(t)\, dt$$

$$\|x(t)\| = \sqrt{\langle x, x\rangle}$$

$$x_n(t) = e^{j2\pi n \frac{t}{T}}$$

$$\langle x_n(t), x_m(t) \rangle = \begin{cases} 1, & n = m \\ 0, & n \neq m \end{cases}$$



$0 < a < \frac{T}{2}$

função par

$f(x) = f(-x)$

ímpar

$f(x) = -f(-x)$

$$c_n = \cancel{\frac{1}{2\pi}}\,\frac{1}{T} \int_{-a}^{+a} e^{-j2\pi n \frac{t}{T}}\, dt$$

$$= \cancel{\frac{1}{2\pi}}\,\frac{1}{T} \left(\frac{e^{-j2\pi n \frac{t}{T}}}{-j2\pi \frac{n}{T}} \right) \Bigg|_{-a}^{+a}$$

$$= \frac{T}{n(-j)} \left(e^{-j2\pi n \frac{a}{T}} - e^{+j2\pi n \frac{a}{T}} \right) = \frac{2T}{n} \underbrace{\frac{e^{j2\pi n \frac{a}{T}} - e^{-j\cdots}}{2j}}_{\sin\left(2\pi n \frac{a}{T}\right)}$$

$$c_n = \frac{2T}{n} \sin\left(2\pi n \frac{a}{T}\right) \frac{1}{(2\pi)^2}$$

(A confusão deve-se a que eu estou habituado a que o período seja 2π ....)

$$c_n = \frac{1}{T}\int_{-a}^{+a} e^{-j2\pi n\frac{t}{T}}\,dt = \frac{1}{T}\left.\frac{e^{-j2\pi\frac{n}{T}t}}{-j2\pi\frac{n}{T}}\right|_{-a}^{+a}$$

$$= \frac{1}{n\pi}\,\frac{e^{-j2\pi n\frac{a}{T}} - e^{+j2\pi n\,a/T}}{-j2}$$

$$= \frac{\sin\left(2\pi n\frac{a}{T}\right)}{\pi n}$$

Para $n=0$ $\quad c_0 = \frac{2a}{T} = \lim_{n\to 0}\frac{\sin(2\pi n\,a/T)}{\pi n} = \frac{2\pi\frac{a}{T}\cos(0)}{\pi}$ ✓

[Peço desculpa pelo erro na aula teórica...]