The Unite and Conquer GMRES-LS/ERAM method to solve sequences of Linear Systems

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Resolving large scale non-Hermitian linear algebra problems often requires to solve in sequence several linear systems, with different right-hand sides but a unique matrix. Efficient resolution of such problems on the faster supercomputers often requires to optimize communications and mix distributed and parallel algorithms.

In this talk we first present a recent development of a unite and conquer method [1], asynchronously computing in parallel the eigenvalues of the matrix, using the ERAM method, and minimizing the residuals using the GMRES method [2] . The approximated eigenvalues allow to accelerate the convergence of the method using a LS polynomial acceleration [3]. These methods run asynchronously on subsets of processors and the generated communications are then non-blocking ones. We introduced this distributed and parallel method as the Unite and Conquer GMRES-ERAM/LS method (UCGLE). We then focus in this talk on the resolution of sequences of linear systems using the UCGLE method. The eigenvalues computed during the resolution associated with one given right-hand vector are reused, and sometime improved, to resolve following sequences of different linear systems. The convergence may increase as the number of different right hand sides because the number of computed eigenvalue and their accuracies increase with this number of linear systems we solve in sequence. Then, it provides a continuous amelioration to solve large sparse non-Hermitian linear systems with different right-hand sides.

We present some experiments on supercomputers, using a software engine we developed which distributes parallel implementations of Krylov methods, using PETSc, into hundreds of computing nodes. We experimented our method with news generated large scale sparse matrices, evaluating the importance of some parameters, and showing that some would have to be auto-tuned. We obtained better convergence and performance than using classical preconditioners.

References

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