

NOTES ON MAGNETIC FIELD DESIGN ON PROCURE

BRUNO XIMENEZ

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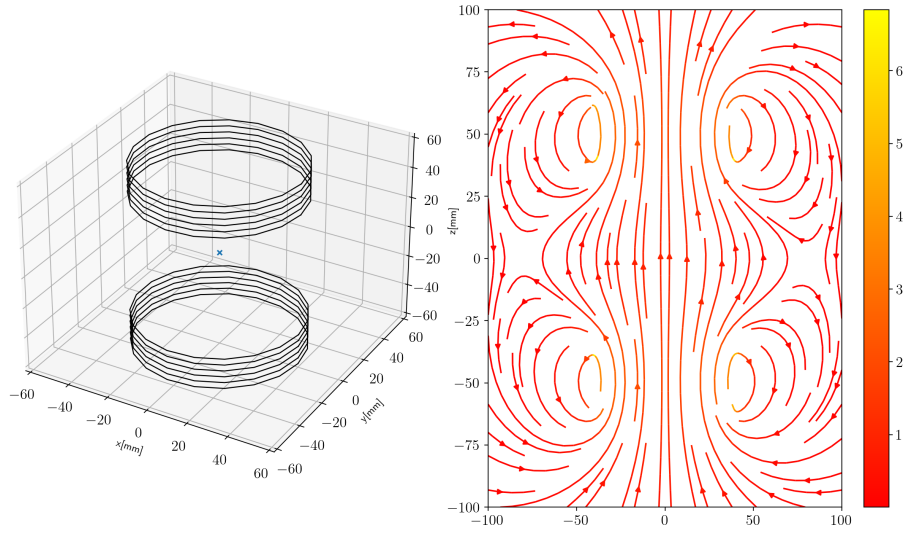


Figure 1: The Helmholtz coils setup is represented on the left and on the right the line fields of the magnetic field on the XZ plane.

1 \vec{B} FIELD CONFIGURATION STUDY

Here is an initial investigation on the magnetic field produced by the under-vacuum coils designed for the PROCURE machine. The setup is a pair of solenoids with about 5 turns. I consider here as an example, solenoids with radius of 40 mm and a distance between the two solenoids (in a Helmholtz configuration) of about 80 mm or the diameter of the coils. The current chosen to plot the graphs is 20 A. On figure 1. This setup generates a highly homogeneous magnetic field at the trapping site, which has a cubic volume with a length of $l = 30 \times 5 \mu\text{m} \approx 0.15 \text{ mm}$.

For those parameters the magnetic field produced at the trap is on the order of 1 mT. Varying the distance between the coils by 2 mm leads to a change on the magnitude of the field on the order of 10^{-2} mT but the homogeneity is kept nearly the same, which is on the order of μT . Results are shown on figure 2.

Next I tilt the one of the coils by about 10 degrees. On figure 3 is shown the geometric deformation and on figure 4 is the magnitude of the magnetic field on the z-axis. There is a shift on the minimum of the magnetic field and variations on the magnetic field on the level of 10^{-3} mT.

We can conclude that we don't need to worry too much about the exact geometry of the solenoid for homogeneous magnetic field at the level of μT (or better than 0.1 Gauss), in this approximation and in the trapping region of about 0.15 mm.

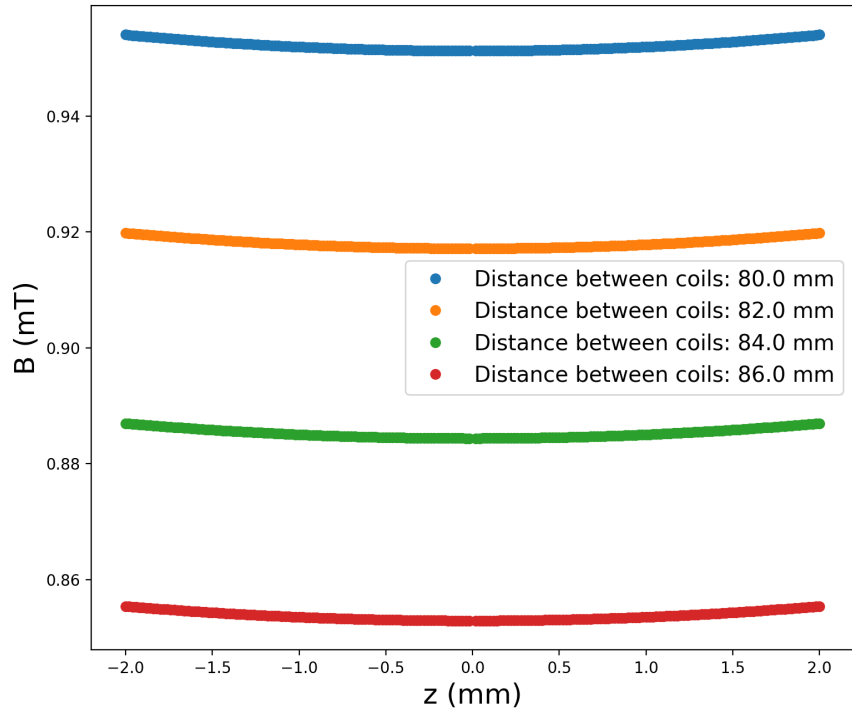


Figure 2: Magnitude of the magnetic field produced on the z-axis when varying the distance between the coils.

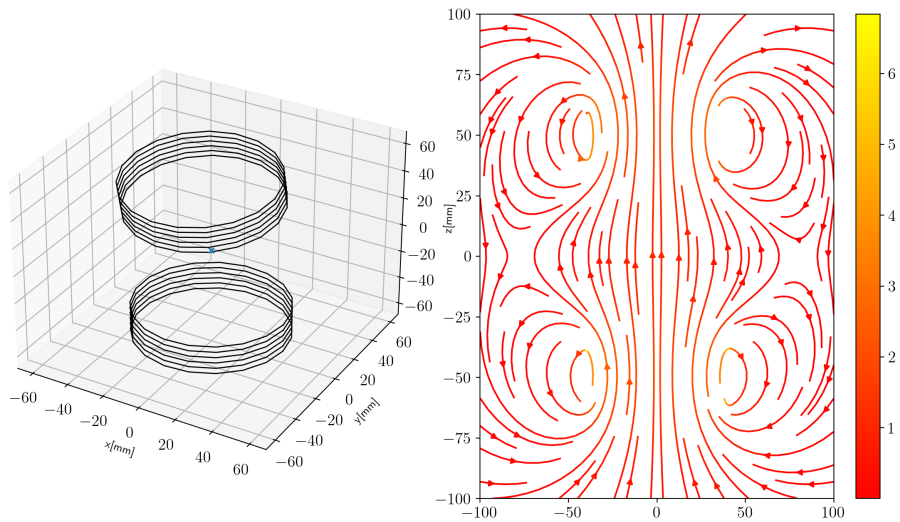


Figure 3: Tilted coil of 10 degrees.

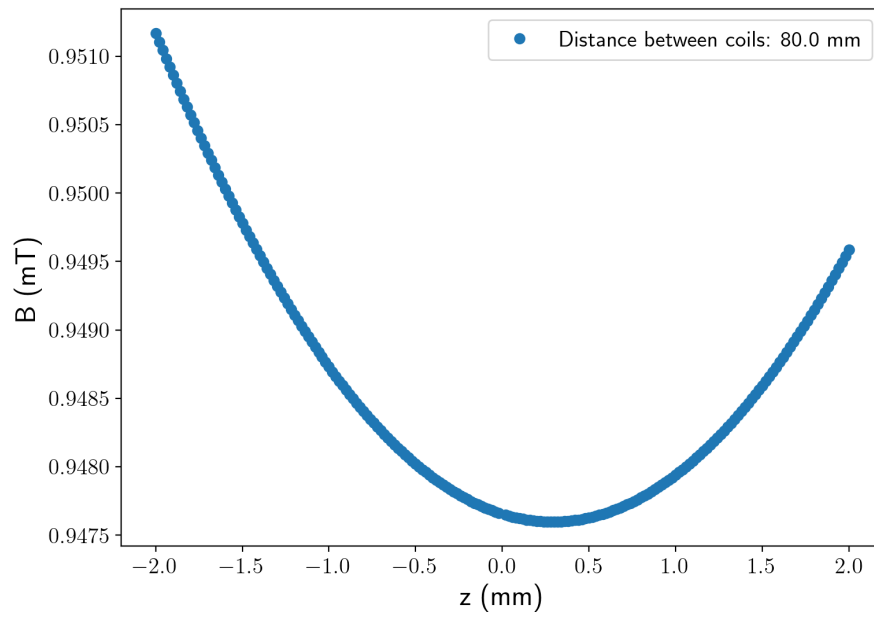


Figure 4: Tilted coil of 10 degrees.

2 HEAT DISSIPATED ON THE COILS

Considering ideal coils with N turns copper wire with a given diameter, the resistance of the solenoid is given by:

$$R_{\text{solenoid}} = \frac{L\rho}{\pi r_{\text{wire}}^2}, \quad (1)$$

where ρ is the resistance of the copper, L is the length of the wire making the solenoid and r_{wire} is the radius of the wire. The length of the wire can be approximated as $L = 2\pi r_{\text{solenoid}} N$, being r_{solenoid} the radius of each turn.

The total resistance of the solenoid is:

$$R_{\text{solenoid}} = \frac{2\rho_{\text{solenoid}} N \rho_{\text{copper}} I^2}{r_{\text{wire}}^2} \quad (2)$$

The power dissipated is just RI^2 and as a scaling law:

$$P \approx N(I/r_{\text{wire}})^2 \quad (3)$$

If we want to dissipate as minimum heat as we can, thicker wire and maximize the number of turns in the solenoid is desired. If the number of turn in the solenoid is increased by a factor of f then the dissipated power is reduced by a fact of f . Considering a 1.5 mm radius wire and $\rho_{\text{copper}} = 0.0171 \, \Omega \text{ mm}^2/\text{m}$, and a solenoid with 4 turns, that gives a total lentgh of about 1 m, a total resistance of about 2 m Ω and $I = 20 \text{ A}$, then 1 W will be dissipated as heat on each solenoid.

3 ZEEMAN EFFECT ON ^{87}Rb

We turn now our attention to the calculation of the Zeeman shift on the energy levels of ^{87}Rb . Two Hamiltonians will be of relevance: the hyperfine structure and the Zeeman effect. The hyperfine structure is simply given by:

$$\mathcal{H}_{\text{hfs}} = A_{\text{hfs}} \vec{I} \cdot \vec{J} + B_{\text{hfs}} \frac{3(\vec{I} \cdot \vec{J})^2 + 3\vec{I} \cdot \vec{J} - I(I+1)J(J+1)}{2I(2I-1)J(2J-1)} \quad (4)$$

The quantum numbers are the total angular momentum of the atom which, for a hydrogen-like system, would be just the several angular momentum of the outer electron. The above expression can be written in a simpler manner from the calculation point of view:

$$\mathcal{H}_{\text{hfs}} = \frac{1}{2} A_{\text{hfs}} K + B_{\text{hfs}} \frac{3/2 K(K+1) - 2I(I+1)J(J+1)}{4I(2I-1)J(2J-1)} \quad (5)$$

with $K = F(F+1) - I(I+1) - J(J+1)$. For the ground state $5S_{1/2}$, only the first term is relevant and for this specific isotope, $I = 3/2$. The usual quantum mechanics relation remains: $F = J + I$, $J = L + S$. For the ground state, there are two hyperfine levels, $F = 1, 2$ with the respective Zeeman sublevels that are degenerated for zero magnetic field. The hyperfine splitting can be calculated by extracting the eigenvalues of the Hamiltonian with the values:

Magnetic dipole constant	$A_{5S_{1/2}}$	3.417 341 305 452 15(5) GHz
Magnetic dipole constant	$A_{5P_{1/2}}$	408.328(15) MHz
Magnetic dipole constant	$A_{5P_{3/2}}$	84.7185(20) MHz
Electric Quadrupole constant	$B_{5P_{3/2}}$	12.4965(37) MHz

The ground state splitting is about 6.8 GHz. We now include in the Hamiltonian the contribution due to a non-zero magnetic field and for energy shift that are smaller than the fine structure splitting:

$$\mathcal{H}_z = \mu_B B_0 (g_J J_z + g_I I_z) \quad (6)$$

with,

$$g_J \sim 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \quad (7)$$

For the condition where the Zeeman shift of the Zeeman sublevels is small compared to the hyperfine splitting between $F = 2$ and $F = 1$, we can consider these two level as being independent (or not mixed) and the calculation can be approximated to:

$$\mathcal{H}_z = \mu_B B_0 g_F m_F \quad (8)$$

with,

$$g_F \sim g_J \frac{F(F+1) - I(I+1) + J(J+1)}{2F(F+1)} \quad (9)$$

In this approximation,

$$\Delta \mathcal{E}_z = \begin{cases} 7m_F \text{ MHz / mT, } F = 2 \\ -7m_F \text{ MHz / mT, } F = 1 \end{cases}$$

If we consider the qubit transition, for example, $|5S_{1/2}, F = 1, m_F = 1\rangle \rightarrow |5S_{1/2}, F = 2, m_F = 2\rangle$, this transition will be shifted by 21 MHz/mT. On the μT regime, this is about 21 kHz.

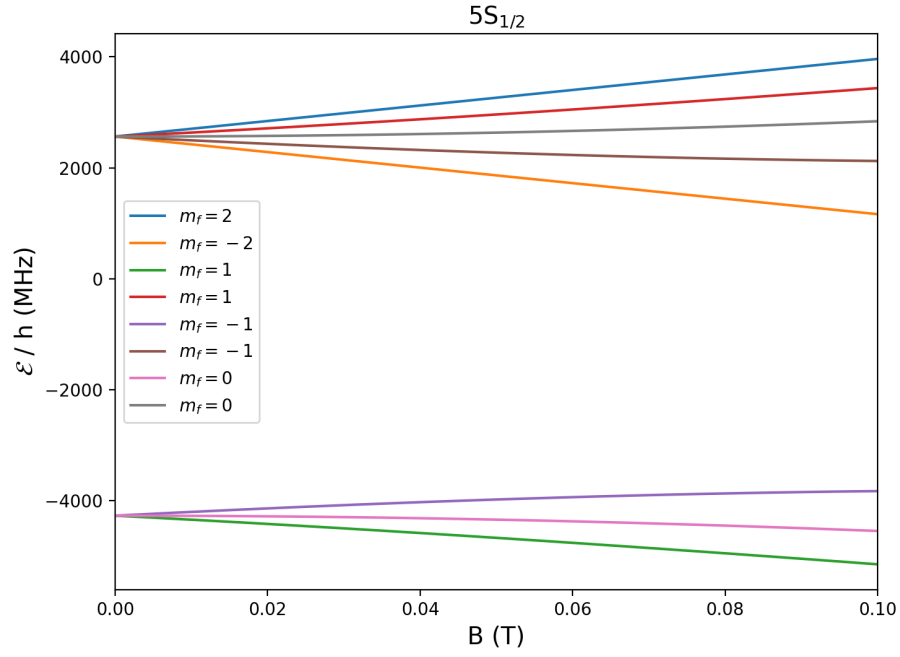


Figure 5: Energy levels of the $5S_{1/2}$ level in a magnetic field.

Considering higher fields, then the Zeeman Hamiltonian will mix the states of $F = 2$ and $F = 1$ with the same m_f . In this case, we use the expression 6 and diagonalize $\mathcal{H}_{\text{hfs}} + \mathcal{H}_Z$. On figure 5 the levels are displayed as a function of the magnetic field. We can see that, by considering the mix between the quantum states with different F 's, the states with $m_f = 0$ starts to being shifted by the magnetic field at the level of 100 mT.

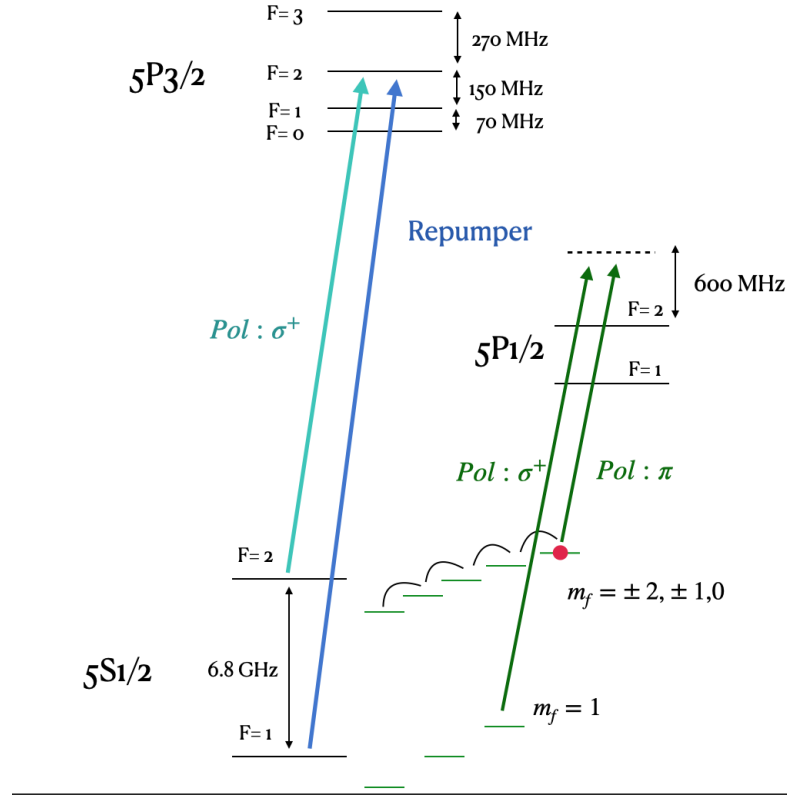


Figure 6: Optical pumping scheme to initialize the qubits in the $|5S_{1/2}, F=2, m_f=2\rangle$ state.

4 QUANTUM STATE PREPARATION – OPTICAL PUMPING

The goal is to initialize the qubits in a given state. Considering optical pumping techniques, it is chosen to prepare the atoms in the state $|5S_{1/2}, F, m_f=2, 2\rangle$ and the pumping scheme is shown on figure 6. The experiment starts with the atoms populating all the Zeeman levels in the ground hyperfine states. The optical pumping laser at 780 nm is prepared such that it addresses σ^+ transitions from $|5S_{1/2}, F=2\rangle \rightarrow |5P_{3/2}, F=2\rangle$ with $\Delta m_f = +1$. In this way, when the an atom populate the level $|5S_{1/2}, F=2, m_f=2\rangle$, the laser-atom interaction is stopped, avoiding unnecessary heating.

The atoms initially populated in the $|5S_{1/2}, F=1\rangle$ are not excited by the optical pumping laser, being off resonance. In order to pump those atoms into the deired state, a repumper laser is used to pump those atoms in the $|5P_{3/2}, F=2\rangle$ and then they eventually decay to $|5S_{1/2}, F=2\rangle$ where they enter in the optical pumping system.

The diagnostics is performed with