b) Show using your expression for λ that if at time t_1 the amount is y_1, then at time $t_1 + \lambda$ it will be $y_1/2$, no matter what t_1 is.

1H-2 If a solution containing a heavy concentration of hydrogen ions (i.e., a strong acid) is diluted with an equal volume of water, by approximately how much is its pH changed? (Express $(\mathrm{pH})_{\mathrm{diluted}}$ in terms of $(\mathrm{pH})_{\mathrm{original}}$.)

1H-3 Solve the following for y:

a) $\ln(y+1) + \ln(y-1) = 2x + \ln x$ b) $\log(y+1) = x^2 + \log(y-1)$

c) $2 \ln y = \ln(y+1) + x$

1H-4 Solve $\dfrac{\ln a}{\ln b} = c$ for a in terms of b and c; then repeat, replacing ln by log.

1H-5 Solve for x (hint: put $u = e^x$, solve first for u):

a) $\dfrac{e^x + e^{-x}}{e^x - e^{-x}} = y$ b) $y = e^x + e^{-x}$

1H-6 Evaluate from scratch the number $A = \log e \cdot \ln 10$. Then generalize the problem, and repeat the evaluation.

1H-7 The decibel scale of loudness is

$$L = 10 \log_{10}(I/I_0)$$

where I, measured in watts per square meter, is the intensity of the sound and $I_0 = 10^{-12}$ watt/m^2 is the softest audible sound at 1000 hertz. Classical music typically ranges from 30 to 100 decibels. The human ear's pain threshold is about 120 decibels.

a) Suppose that a jet engine at 50 meters has a decibel level of 130, and a normal conversation at 1 meter has a decibel level of 60. What is the ratio of the intensities of the two sounds?

b) Suppose that the intensity of sound is proportional to the inverse square of the distance from the sound. Based on this rule, calculate the decibel level of the sound from the jet at a distance of 100 meters, at distance of 1 km.[1]

[1]The inverse square law is justified by the fact that the intensity is measured in energy per unit time per unit area. When the sound has travelled a distance r, the energy of a sound spread over a sphere of radius r centered at the source. The area of that sphere is proportional to r^2, so the average intensity is proportional to $1/r^2$. Fortunately for people who live near airports, sound

1H-2 If a solution containing a heavy concentration of hydrogen ions (i.e., a strong acid) is diluted with an equal volume of water, by approximately how much is its pH changed? (Express $(pH)_{diluted}$ in terms of $(pH)_{original}$.)

$pH = -\log_{10}[H^+]$; $[H^+]_{dil} = \frac{1}{2}[H^+]_{orig}$

$-\log[H^+]_{dil} = -\log(\frac{1}{2}[H^+]_{orig})$

$-\log[H^+]_{dil} = -\log[H^+]_{orig} + \log 2$

$pH_{dil} = pH_{orig} + \log 2$

my god, it is lovely

this is really what math should be, not some procedures that we just have to memorize and swear to be true.

1H-3 Solve the following for y:

a. $\ln(y+1) + \ln(y-1) = 2x + \ln x$

$\ln[(y+1)(y-1)] = 2x + \ln x$

$e^{2x + \ln x} = (y+1)(y-1)$

$xe^{2x} = (y+1)(y-1)$

$xe^{2x} = y^2 - 1$

$\sqrt{xe^{2x} - 1} = y$

b. $\log(y+1) = x^2 + \log(y-1)$

$x^2 = \log(y+1) - \log(y-1)$

$x^2 = \log\left[\frac{(y+1)}{(y-1)}\right]$

$10^{x^2} = \frac{(y+1)}{(y-1)}$

$y + 1 = 10^{x^2}y - 10^{x^2}$

$y - 10^{x^2}y = -10^{x^2} - 1$

$y(-10^{x^2} + 1) = -10^{x^2} - 1$

$y = \frac{-10^{x^2} - 1}{-10^{x^2} + 1}$ or $\frac{10^{x^2} + 1}{10^{x^2} - 1}$

idk why this is wrong, i know the reason. Apparently in the answer key the original expression translates to

$\log(y+1) - \log(y-1) = -x^2$

i think this should be positive !!

c. $2\ln y = \ln(y+1) + x$

$\ln y^2 - \ln(y+1) = x$

$\ln\left(\frac{y^2}{y+1}\right) = x$

$\frac{y^2}{y+1} = e^x$

$e^x y + e^x = y^2$

$e^x = y^2 - e^x y$

$0 = y^2 - e^x y - e^x$

$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{e^x \pm \sqrt{e^{2x} - 4(e^x)}}{2(1)}$

$y = \frac{e^x \pm \sqrt{e^{2x} - 4e^x}}{2}$

quadratic equation "special case"

this is a gap in my knowledge, i need to look at this first

1H-4 Solve $\frac{\ln a}{\ln b} = c$ for a in terms of b and c; then repeat, replacing ln by log.

$\frac{\ln a}{\ln b} = c$

$\log_b a = c$

$b^c = a$

$\frac{\log a}{\log b} = c$

$\log_b a = c$

$b^c = a$

1H-5 Solve for x (hint: put $u = e^x$, solve first for u):

a. $\frac{e^x + e^{-x}}{e^x - e^{-x}} = y \Big|_{u = e^x} = \frac{u + u^{-1}}{u - u^{-1}} \cdot \frac{u}{u}$

$y = \frac{u^2 + 1}{u^2 - 1}$

$u^2 y - y = u^2 + 1$

$u^2 y - u^2 = y + 1$

$u^2(y-1) = y + 1$

$u^2 = \frac{y+1}{y-1}$

$e^x = \sqrt{\frac{y+1}{y-1}}$

$x = \ln\left|\sqrt{\frac{y+1}{y-1}}\right|$ or $\frac{1}{2}\ln\left[\frac{(y+1)}{(y-1)}\right]$

b. $y = e^x + e^{-x}$

$y = u + u^{-1} \cdot \frac{u}{u}$

$y = \frac{u^2 + 1}{u}$

$yu = u^2 + 1$

$0 = u^2 - yu + 1$

$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$e^x = \frac{y \pm \sqrt{y^2 - 4}}{2}$

$x = \ln\left[\frac{y \pm \sqrt{y^2 - 4}}{2}\right]$

1H-6 Evaluate from scratch the number $A = \log e \cdot \ln 10$. Then generalize the problem, and repeat the evaluation.

$A = \log e \cdot \ln 10$

$= \frac{\ln e}{\ln 10} \cdot \ln 10$

$A = \ln e$

$A = 1$

Generalization

$A = \log_b e \cdot \ln b$

$= \log_b e \cdot \frac{\log_b b}{\log_b e}$

$= \log_b b$

$A = 1$

PSET SCORE (Page)

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1H-7 The decibel scale of loudness is

$$L = 10\log_{10}(I/I_0)$$

where I, measured in watts per square meter, is the intensity of sound and $I_0 = 10^{-12}$ watt/m^2 is the softest audible sound at 1000 hertz. Classical music typically ranges from 30 to 100 decibels. The human ear's pain threshold is about 120 decibels.

a. Suppose that a jet engine at 50 meters has a decibel level of 130, and a normal conversation at 1 meter has a decibel level of 60. What is the ratio of the intensities of two sounds?

$L = 10\log(I/I_0)$

$L/10 = \log(I/I_0)$

$10^{L/10} = I/I_0$

$I = 10^{L/10} I_0$

$I = 10^{L/10} I_0 r^2$

$I_{130} = 10^{130/10} I_0$

$I_{130} = 10^{13} I_0$

$I_{60} = 10^{60/10} I_0$

$I_{60} = 10^6 I_0$

$R = \frac{I_{130}}{I_{60}} = \frac{10^{13} I_0}{10^6 I_0}$

$R = 10^7$

b. Suppose that the intensity of sound is proportional to the inverse square of the distance from the sound. Based on this rule, calculate the decibel level of the sound from the jet at a distance of 100 meters, at a distance of 1 km.

$I = \frac{c}{r^2}$ and $I = I_1$ when $r = 50$;

$C = I_1 50^2 \Rightarrow I = I_1 50^2 / r^2$

$I = \frac{I_1 50^2}{100^2} = I_1/4$

$I = \frac{I_1 50^2}{1000^2} = I_1/400$

$L = 10\log(I/I_0)$

$= 10\log(I/I_0)$

$= 10\log(I_1/4I_0)$

$= 10\log(I_1/I_0) - 10\log 4$

$= 130 - 6$

$L_{100} = 124$

$L = 10\log(I/I_0)$

$= 10\log(I_1/400I_0)$

$= 10\log(I_1/I_0) - 10\log 400$

$= 130 - 26$

$L_{1000} = 104$

My first "answer" was wrong. I looked at the answer key for this one.