

## Solving Equations with $e$ and $\ln x$

We know that the natural log function  $\ln(x)$  is defined so that if  $\ln(a) = b$  then  $e^b = a$ . The *common log* function  $\log(x)$  has the property that if  $\log(c) = d$  then  $10^d = c$ . It's possible to define a logarithmic function  $\log_b(x)$  for any positive base  $b$  so that  $\log_b(e) = f$  implies  $b^f = e$ . In practice, we rarely see bases other than 2, 10 and  $e$ .

Solve for  $y$ :

1.  $\ln(y+1) + \ln(y-1) = 2x + \ln x$
2.  $\log(y+1) = x^2 + \log(y-1)$
3.  $2 \ln y = \ln(y+1) + x$

Solve for  $x$  (hint: put  $u = e^x$ , solve first for  $u$ ):

4.  $\frac{e^x + e^{-x}}{e^x - e^{-x}} = y$
5.  $y = e^x + e^{-x}$

$$(1) \quad \ln[(y+1)(y-1)] = 2x + \ln x$$

$$\begin{aligned} e^{2x} e^{\ln x} &= (y+1)(y-1) \\ x e^{2x} &= y^2 - 1 \\ x e^{2x+1} &= y^2 \end{aligned}$$

$$\begin{aligned} y &= \pm \sqrt{x e^{2x+1}} \\ y &= \sqrt{x e^{2x+1}} \end{aligned}$$

negative roots are not included ;  $\ln(y+1), y-1 > 0$

$$(2) \quad \log(y+1) = x^2 + \log(y-1)$$

$$\begin{aligned} 10^{x^2} 10^{\log(y-1)} &= y+1 \\ (y+1) 10^{x^2} &= y+1 \\ 10^{x^2} y - 10^{x^2} &= y+1 \\ 10^{x^2} y - y &= 10^{x^2} + 1 \\ y(10^{x^2} - 1) &= 10^{x^2} + 1 \\ y &= \frac{10^{x^2} + 1}{10^{x^2} - 1} \end{aligned}$$

$$(3) \quad 2 \ln y = \ln(y+1) + x$$

$$y^2 = e^{\ln(y+1)} e^x$$

$$= (y+1) e^x$$

$$= e^x y + e^x$$

$$y^2 - e^x y - e^x = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{e^x \pm \sqrt{e^{2x} - 4(1)(-e^x)}}{2(1)}$$

$$y = \frac{e^x \pm \sqrt{e^{2x} + 4e^x}}{2}$$

$$y = \frac{e^x + \sqrt{e^{2x} + 4e^x}}{2}$$

$$y > 0 ; \sqrt{e^{2x} + 4e^x} > \sqrt{e^{2x}} = e^x$$

hence  $\sqrt{e^{2x} + 4e^x}$  cannot be negative or  $y$  will be also negative

$$(4) \quad \frac{e^x + e^{-x}}{e^x - e^{-x}} = y$$

$$e^x + e^{-x} = y(e^x - e^{-x})$$

$$e^x + e^{-x} = ye^x - ye^{-x}$$

$$e^x + e^{-x} + ye^{-x} = ye^x$$

$$e^x + e^{-x} + ye^{-x} = ye^$$

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