Activity Score: $\frac{2}{2}$

Smoothing a Piecewise Polynomial

For each of the following, find all values of a and b for which $f(x)$ is differentiable.

a) $f(x) = \begin{cases} ax^2 + bx + 6, & x \leq 0; \\ 2x^5 + 3x^4 + 4x^2 + 5x + 6, & x > 0. \end{cases}$

b) $f(x) = \begin{cases} ax^2 + bx + 6, & x \leq 1; \\ 2x^5 + 3x^4 + 4x^2 + 5x + 6, & x > 1. \end{cases}$

Note: Had to search and understand limits first before I started this problem. I was also a bit confused on how to evaluate the slope at x_0 but turned out it was trivial.

A f'n is differentiable at x_0 if:

- continuous at x_0
- if $\displaystyle f'(x)_{\Delta x \to x_0^-} = f'(x)_{\Delta x \to x_0^+}$ at x_0

a. $g(x) = ax^2 + bx + 6 \qquad h(x) = 2x^5 + 3x^4 + 4x^2 + 5x + 6$

$$\lim_{x \to 0^-} ax^2 + bx + 6 = \lim_{x \to 0^+} 2x^5 + 3x^4 + 4x^2 + 5x + 6$$

$$\cancel{a(0)^2} + \cancel{b(0)^2} + 6 = \cancel{2(0)^5} + \cancel{3(0)^4} + \cancel{4(0)^2} + \cancel{5(0)} + 6$$

$$6 = 6$$

$$f'(x)_{x \to 0^-} = f'(x)_{x \to 0^+}$$

$$2ax + b = 10x^4 + 12x^3 + 8x + 5$$

$$2a(0) + b = 10(0)^4 + 12(0)^3 + 8(0) + 5$$

$$b = 5$$

$\boxed{\text{a. } a \text{ is any real number and } b = 5,\ f(x) \text{ is differentiable}}$

b. $f(x) = \begin{cases} ax^2 + bx + 6 & x \le 1; \\ 2x^5 + 3x^4 + 4x^2 + 5x + 6 & , x > 1. \end{cases}$

$$\lim_{x \to 1^-} ax^2 + bx + 6 = \lim_{x \to 1^+} 2x^5 + 3x^4 + 4x^2 + 5x + 6$$

$$a(1)^2 + b(1) + 6 = 2(1)^5 + 3(1)^4 + 4(1)^2 + 5(1) + 6$$

$$a + b + 6 = 20$$

f(x) is continuous if $a + b = 14$

$$\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^+} f'(x)$$

$$2ax + b = 10x^4 + 12x^3 + 8x + 5$$

$$2a(1) + b = 10(1)^4 + 12(1)^3 + 8(1) + 5$$

$$2a + b = 35$$

f(x) is differentiable if the two conditions were satisfied;

$$a + b - 14 = 2a + b - 35$$

$$35 - 14 = \cancel{2}a + \cancel{b} - \cancel{a} - \cancel{b}$$

$$21 = a$$

and $-7 = b$

$a + b = 14$

$21 + b = 14$

$b = 14 - 2$

$b = -7$

$\boxed{\text{for } f(x) \text{ to be differentiable, } a = 21 \text{ and } b = -7}$