Activity Score = 0/2

Note: The answers were correct and I solved them myself. But I didn't give credit because I needed to looked at the solution pdf to answer. (Didn't realized it was just laws of exponents and substitution applied)

Evaluating an Interest Using the Limit

Recall that the formula for *compound interest* is:

$$A = P\left(1 + \frac{r}{k}\right)^k$$

and the anual percentage rate is:

$$\text{APR} = \left(1 + \frac{r}{k}\right)^k - 1.$$

Here P is the principal invested, r is the annual "simple" interest rate, A is the amount in the account at a given time, and k determines the frequency with which interest is added to the account.

As k approaches infinity interest is added more and more often; in the limit we say that the interest is *compounded continuously*.

1. Use the fact that $\lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n = e$ to compute the APR of 5% compounded continuously.
2. Compute the APR of 10% compounded continuously.

(1.)

$$\text{APR of 5\%} = \lim_{k\to\infty}\left(1 + \frac{r}{k}\right)^k - 1 \qquad \frac{0.05}{k} = \frac{1}{n}, \quad k = 0.05n$$

$$= \lim_{k\to\infty}\left(1 + \frac{0.05}{k}\right)^k - 1$$

$$= \lim_{k\to\infty}\left(1 + \frac{0.05}{k}\right)^k - 1$$

$$= \lim_{k\to\infty}\left(1 + \frac{0.05}{0.05n}\right)^{0.05n} - 1$$

$$= \lim_{k\to\infty}\left(1 + \frac{1}{n}\right)^{0.05n} - 1$$

$$= \lim_{k\to\infty}\left[\left(1 + \frac{1}{n}\right)^n\right]^{0.05} - 1$$

$$= e^{0.05} - 1$$

$$= 5.127\%$$

(2.)

$$\text{APR of 5\%} = \lim_{k\to\infty}\left(1 + \frac{0.1}{k}\right)^k - 1 \qquad \frac{1}{n} = \frac{0.1}{k}, \quad k = 0.1(n)$$

$$= \lim_{k\to\infty}\left(1 + \frac{0.1}{0.1n}\right)^{0.1n} - 1$$

$$= \lim_{k\to\infty}\left(1 + \frac{1}{n}\right)^{0.1n} - 1$$

$$= \lim_{k\to\infty}\left[\left(1 + \frac{1}{n}\right)^n\right]^{0.1} - 1$$

$$= e^{0.1} - 1$$

$$= 10.517\%$$

Generalization

$$\text{APR} = \lim_{n\to\infty}\left(1 + \frac{r}{k}\right)^K - 1 \qquad \frac{r}{n} = \frac{1}{h}$$

$$= \lim_{n\to\infty}\left(1 + \frac{r}{rn}\right)^{rn} - 1$$

$$= \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^{rn} - 1$$

$$= \lim_{n\to\infty}\left[\left(1 + \frac{1}{n}\right)^n\right]^r - 1$$

$$\text{APR} = e^r - 1 \quad \text{[compounded continuously, } k\to\infty\text{]}$$

18.01SC Single Variable Calculus
Fall 2010