c) $\dfrac{x+2}{x^2-1}$ d) $\dfrac{x^4+1}{x}$

1F. Chain rule, implicit differentiation

1F-1 Find the derivative of the following functions:

a) $(x^2+2)^2$ (two methods)

b) $(x^2+2)^{100}$. Which of the two methods from part (a) do you prefer?

1F-2 Find the derivative of $x^{10}(x^2+1)^{10}$.

1F-3 Find dy/dx for $y = x^{1/n}$ by implicit differentiation.

1F-4 Calculate dy/dx for $x^{1/3} + y^{1/3} = 1$ by implicit differentiation. Then solve for y and calculate y' using the chain rule. Confirm that your two answers are the same.

1F-5 Find all points of the curve(s) $\sin x + \sin y = 1/2$ with horizontal tangent lines. (This is a collection of curves with a periodic, repeated pattern because the equation is unchanged under the transformations $y \to y + 2\pi$ and $x \to x + 2\pi$.)

1F-6 Show that the derivative of an even function is odd and that the derivative of an odd function is even.

(Write the equation that says f is even, and differentiate both sides, using the chain rule.)

1F-7 Evaluate the derivatives. Assume all letters represent constants, except for the independent and dependent variables occurring in the derivative.

a) $D = \sqrt{(x-a)^2 + {y_0}^2}$, $\dfrac{dD}{dx} = ?$ b) $m = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$, $\dfrac{dm}{dv} = ?$

c) $F = \dfrac{mg}{(1+r^2)^{3/2}}$, $\dfrac{dF}{dr} = ?$ d) $Q = \dfrac{at}{(1+bt^2)^3}$, $\dfrac{dQ}{dt} = ?$

1F-8 Evaluate the derivative by implicit differentiation. (Same assumptions about the letters as in the preceding exercise.)

1E-5 cont...

c. $\frac{d}{dx}\frac{x+2}{x^2-1} = \frac{-x^2-3}{(x^2-1)^2}$

$= \frac{(1)(x^2-1) - (x+2)(2x)}{(x^2-1)^2}$

$= \frac{x^2-1-(2x^2+4x)}{(x^2-1)^2}$ X

$= \frac{-x^2-4x-1}{(x^2-1)^2}$ ← This is my second answer. I got the first one wrong.

d. $\frac{d}{dx}\frac{x^4+1}{x} = \frac{2x^4-1}{x^2}$

$= \frac{4x^3(x) - (x^4+1)(1)}{x^2}$

$= \frac{4x^4 - x^4 - 1}{x^2}$ ✓

$= \frac{3x^4-1}{x^2}$ or $3x^2 - \frac{1}{x^2}$

1F-2 Find the derivative of the following functions.

a. $(x^2+2)^2$

$f'(x^2+2)^2 = f'((x^2+2)^2) \cdot f'(x^2+2)$

$= 2(x^2+2)(2x)$

$= 4x(x^2+2)$

$\frac{d}{dx}(x^2+2)^2 = \frac{du}{dy}\frac{dy}{dx}$

$= 2(x^2+2)\,2x$

$= 4x(x^2+2)$ ✓

b. $(x^2+2)^{100}$

$\frac{d}{dx}(x^2+2)^{100} = 100(x^2+2)^{99}(2x)$

$= 200x^{99(2)+1} + 200x(2^{99})$

$= 200x(x^{198} + 2^{99})$ or $200x(x^2+2)^9$

1F-1 Find the derivative of $x^{10}(x^2+1)^{10}$

$f'(x^{10}(x^2+1)^{10}) = f'(x^{30} + x^{10})$ ← it wrong!: Use binomial theorem here

$= 30x^{29} + 10x^9$

$= 10x^9(3x^{20} + 1)$ ← hence this is shorter, easier

$f'(x^{10}(x^2+1)^{10}) = f'(x^{10})(x^2+1)^{10} + x^{10}f'((x^2+1)^{10})$

$= 10x^9(x^2+1)^{10} + x^{10}((10(x^2+1)^9)(2x))$

$= 10(3x^2+1)x^9(x^2+1)^9$

This is from the answer key. I don't understand how it got simplified to that. Will come back to this at some point.

1F-3 Find dy/dx for $y = x^{1/n}$ by implicit differentiation

1F-4 Calculate dy/dx for $x^{1/3} + y^{1/3} = 1$ by implicit differentiation

1F-5 Find all points of the curves $\sin x + \sin y = 1/2$ with horizontal tangent lines.

NOT COVERED YET

1F-6 Show that the derivative of an even function is odd and that the derivative of an odd function is even

Even function

$f(x) = f(-x)$ $\qquad f'(x) = f'(-x)$

Odd function

$f(x) = -f(-x)$