1A-8 Suppose $f(x)$ is odd and periodic. Show that the graph of $f(x)$ crosses the x-axis infinitely often.

1A-9 a) Graph the function f that consist of straight line segments joining the points $(-1,-1)$, $(1,2)$, $(3,-1)$, and $(5,2)$. Such a function is called piecewise linear.

b) Extend the graph of f periodically. What is its period?

c) Graph the function $g(x) = 3f((x/2)-1)-3$.

1B. Velocity and rates of change

1B-1 A test tube is knocked off a tower at the top of the Green building. (For the purposes of this experiment the tower is 400 feet above the ground, and all the air in the vicinity of the Green building was evacuated, so as to eliminate wind resistance.) The test tube drops $16t^2$ feet in t seconds. Calculate

a) the average speed in the first two seconds of the fall

b) the average speed in the last two seconds of the fall

c) the instantaneous speed at landing

1B-2 A tennis ball bounces so that its initial speed straight upwards is b feet per second. Its height s in feet at time t seconds is given by $s = bt - 16t^2$

a) Find the velocity $v = ds/dt$ at time t.

b) Find the time at which the height of the ball is at its maximum height.

c) Find the maximum height.

d) Make a graph of v and directly below it a graph of s as a function of time. Be sure to mark the maximum of s and the beginning and end of the bounce.

e) Suppose that when the ball bounces a second time it rises to half the height of the first bounce. Make a graph of s and of v of both bounces, labelling the important points. (You will have to decide how long the second bounce lasts and the initial velocity at the start of the bounce.)

f) If the ball continues to bounce, how long does it take before it stops?

1 A - 8

Suppose $f(x)$ is odd and periodic. Show that the graph of $f(x)$ crosses x-axis infinitely often.

Periodic Function

$f(x) = f(x+T)$; T is positive real number

Odd Function

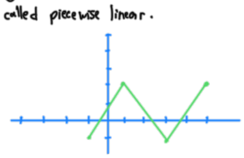
$f(x) = -f(-x)$

if $f(x)$ is odd and periodic
$f(0) \Rightarrow f(0) = -f(0) \Rightarrow f(0) = 0.$

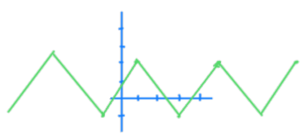
$f(0+T) = f(2T) = \cdots = 0$, by periodicity where T is the period

1 A - 9

a. Graph the function f that consist of straight line segments joining the points $(-1,-1)$, $(1,2)$, $(3,-1)$, and $(5,2)$. Such a function is called piecewise linear.

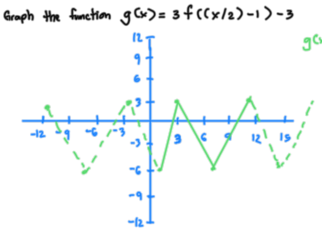


b. Extend the graph of f periodically. What is its period?



Period = 6

c. Graph the function $g(x) = 3f((x/2)-1)-3$



$g(x) = 3f(\frac{1}{2}x - 1) - 3$

$(-1,-1) \rightarrow (-1,-6)$
$(1,2) \rightarrow (3,3)$
$(3,-1) \rightarrow (7,-6)$
$(5,2) \rightarrow (11,3)$

1 B - 1 Velocity and rate of change

a. the average speed in the first two seconds of the fall

$h = 400$ ft
$y = 16t^2$

$$S_2 = \frac{\Delta y}{\Delta t} = \frac{16(2)^2 - 16(0)^2}{2-0}$$

$S_2 = 32$ ft/sec

b. the average speed in the last two seconds of the fall.

$400 = 16t^2$
$t^2 = 25$
$t = 5$

$$S_5 = \frac{\Delta y}{\Delta t} = \frac{16(5)^2 - 16(3)^2}{5-3}$$

$S_5 = 128$ ft/sec

c. the instantaneous speed at landing

$\frac{dy}{dx} 16t^2 = 32t$

$\lim_{t \to 5} = 32(5)$

$S_i = 160$ ft/sec

1 B - 2

(a) Find the velocity $v = ds/dt$ at time t.

$s = bt - 16t^2$

$v = \frac{d}{dy} bt - 16t^2$

$v = b - 32t$

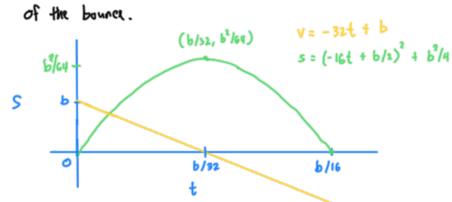
(b) Find the time at which the height of the ball is at its maximum height.

$0 = b - 32t$
$b = 32t$
$t_{max} = \frac{b}{32}$

(c) Find the maximum height

$s = bt - 16t^2$
$= b\left(\frac{b}{32}\right) - 16\left(\frac{b}{32}\right)^2$
$= \frac{b^2}{32} - \frac{b^2}{64}$
$= \frac{2b^2}{64} - \frac{b^2}{64}$
$s = \frac{b^2}{64}$

(d) Make a graph of v and directly below it a graph of s as a function of time. Be sure to mark the maximum of s and the beginning and end of the bounce.



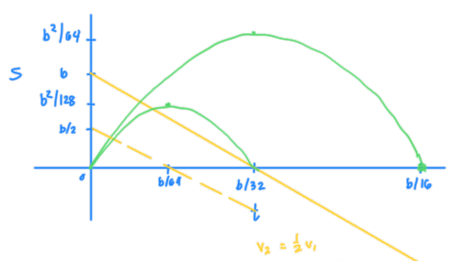
$s = -16t^2 + bt$
$= -16\left(t^2 - \frac{bt}{16}\right)$
$= -16\left(t^2 - \frac{bt}{16} + \left(\frac{b}{32}\right)^2 - \left(\frac{b}{32}\right)^2\right)$
$= -16\left((t - b/32)^2 - (b/32)^2\right)$
$= (-16t + b/2)^2 + b^2/4$

$0 = (-16t + b/2)^2 + (b/2)^2$
$= -16t + \frac{b}{2} \pm \frac{b}{2}$

$16t = b$
$t = \frac{b}{16}$

$16t = \frac{b}{2} - \frac{b}{2}$
$t = 0$

(e) Suppose that when the ball bounces a second time it rises to half the height of the first bounce. Make a graph of s and v of both bounces, labelling the important points. (You will have to decide how long the second bounce lasts and the initial velocity at the start of the bounce.



$v_2 = \frac{1}{2} v_1$
$v_1 = -32t + b$
$v_2 = -16t + b/2$
$s_1 = (-16t + b/2)^2 + b^2/4$
$s_2 = (-8t + b/4)^2 + b^2/8$

(f) If the ball continues to bounce, how long does it take before it stops.