Comparing Linear Approximations to Calculator Computations

In lecture, we explored linear approximations to common functions at the point $x = 0$. In this worked example, we use the approximations to calculate values of the sine function near $x = 0$ and compare the answers to those on a scientific calculator.

Find the linear approximation to $\sin(x)$ at the point $x = 0$ and use your answer to approximate the values of $\sin(.01), \sin(.1)$ and $\sin(1)$. Check your answer on a calculator.

$f(x) \approx f(0) + f'(0)x$

$f(x) \approx \sin(0) + \cos(0)x$

$f(x) \approx x$

$\sin(.01) \approx 0.01$

$\sin(0.1) \approx 0.1$

$\sin(1) \approx 1$

} linear approximation at points close to zero

$\sin(0.01) = 0.00999$

$\sin(0.1) = 0.0998$

$\sin(1) = 0.8417$

} true value of $f(x) = \sin x$

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