Solving Equations with e and $\ln x$

We know that the natural log function $\ln(x)$ is defined so that if $\ln(a) = b$ then $e^b = a$. The *common log* function $\log(x)$ has the property that if $\log(c) = d$ then $10^d = c$. It's possible to define a logarithmic function $\log_b(x)$ for any positive base b so that $\log_b(e) = f$ implies $b^f = e$. In practice, we rarely see bases other than 2, 10 and e.

Solve for y:

1. $\ln(y + 1) + \ln(y - 1) = 2x + \ln x$
2. $\log(y + 1) = x^2 + \log(y - 1)$
3. $2 \ln y = \ln(y + 1) + x$

Solve for x (hint: put $u = e^x$, solve first for u):

4. $\dfrac{e^x + e^{-x}}{e^x - e^{-x}} = y$
5. $y = e^x + e^{-x}$

1. $\ln[(y-1)(y+1)] = 2x + \ln x$

$$e^{2x} e^{\ln x} = (y-1)(y+1)$$
$$xe^{2x} = y^2 - 1$$
$$xe^{2x} + 1 = y^2$$
$$y = \pm\sqrt{xe^{2x}+1}$$
$$y = \sqrt{xe^{2x}+1}$$

negative roots are not included ; $\ln(y-1)$, $y-1>0$

2. $\log(y+1) = x^2 + \log(y-1)$

$$10^{x^2} 10^{\log(y-1)} = y+1$$
$$(y-1)10^{x^2} = y+1$$
$$10^{x^2} y - 10^{x^2} = y+1$$
$$10^{x^2} y - y = 10^{x^2} + 1$$
$$y(10^{x^2} - 1) = 10^{x^2} + 1$$
$$y = \frac{10^{x^2}+1}{10^{x^2}-1}$$

3. $2\ln y = \ln(y+1) + x$

$$y^2 = e^{\ln(y+1)} e^x$$
$$= (y+1)e^x$$
$$= e^x y + e^x$$
$$y^2 - e^x y - e^x = 0$$
$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{e^x \pm \sqrt{e^{2x} - 4(1)(-e^x)}}{2(1)}$$
$$y = \frac{e^x \pm \sqrt{e^{2x} + 4e^x}}{2}$$
$$y = \frac{e^x + \sqrt{e^{2x} + 4e^x}}{2}$$

$y > 0$; $\sqrt{e^{2x} + 4e^x} > \sqrt{e^{2x}} = e^x$

hence $\sqrt{e^{2x} + 4e^x}$ cannot be negative or y will be also negative

4. $$\frac{e^x + e^{-x}}{e^x - e^{-x}} = y$$