

Implicit Differentiation and the Second Derivative

Calculate y'' using implicit differentiation; simplify as much as possible.

$$x^2 + 4y^2 = 1$$

$$\begin{aligned} x^2 + 4y^2 &= 1 \\ y' &= \frac{d}{dx}(x^2 + 4y^2 = 1) = \\ &\frac{d}{dx}(2x + 8y y') = 0 \\ 2 + 8y'y' + 8y y'' &= 0 \\ 2 + 8(y'^2 + 8y y'') &= 0 \\ y'' &= \frac{-8(y'^2 - 2)}{8y} \\ &= -\frac{8\left(\frac{-x}{4y}\right)^2 - 2}{8y} \\ &= \frac{-8\frac{x^2}{16y^2} - 2}{8y} \\ &= \frac{-\frac{x^2}{2y^2} - 2}{8y} \\ &= \frac{-x^2 - 4y^2}{16y^3} \\ &= \frac{-x^2 - 4\left(\frac{1-x^2}{2}\right)^2}{16y^3} \\ y'' &= \frac{-x^2 - 1 + x^2}{16y^3} = \frac{-1}{16y^3} \end{aligned}$$

$$\begin{aligned} x^2 + 4y^2 &= 1 \\ y^2 &= \frac{1-x^2}{4} \\ y &= \frac{\sqrt{1-x^2}}{2} \\ \frac{d}{dx}(x^2 + 4y^2 = 1) &= 0 \\ 2x + 8y y' &= 0 \\ y' &= \frac{-2x}{8y} \\ y' &= \frac{-x}{4y} \\ y'' &= \frac{(-1)(4y) - (-x)(4y')}{(4y)^2} \\ &= \frac{-4y + 4xy'}{(4y)^2} = \frac{-y + xy'}{4y^2} \\ &= \frac{-y + x\left(\frac{-x}{4y}\right)}{4y^2} \\ &= \frac{-4y^3 - x^2}{16y^3} = \frac{-x^2 - 4y^2}{16y^3} \\ &= \frac{-x^2 - 4\left(\frac{\sqrt{1-x^2}}{2}\right)^2}{16y^3} = \frac{-x^2 - 1 + x^2}{16y^3} \\ y'' &= \frac{-1}{16y^3} \end{aligned}$$

MIT OpenCourseWare
<http://ocw.mit.edu>

18.01SC Single Variable Calculus
Fall 2010

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.