d) $x \ln x - x$ e) $\ln(x^2)$ f) $(\ln x)^2$
g) $(e^{x^2})^2$ h) x^x i) $(e^x + e^{-x})/2$
j) $(e^x - e^{-x})/2$ k) $\ln(1/x)$ l) $1/\ln x$
m) $(1 - e^x)/(1 + e^x)$

1I-2 Graph the function $y = (e^x + e^{-x})/2$.

1I-3 a) Evaluate $\lim_{n\to\infty} n \ln(1 + \frac{1}{n})$.

Hint: Let $h = 1/n$, and use $(d/dx) \ln(1 + x)|_{x=0} = 1$.

b) Deduce that $\lim_{n\to\infty} (1 + \frac{1}{n})^n = e$.

1I-4 Using $\lim_{n\to\infty} (1 + \frac{1}{n})^n = e$, calculate

a) $\lim_{n\to\infty} (1 + \frac{1}{n})^{3n}$ b) $\lim_{n\to\infty} (1 + \frac{2}{n})^{5n}$ c) $\lim_{n\to\infty} (1 + \frac{1}{2n})^{5n}$

1I-5* If you invest P dollars at the annual interest rate r, then after one year the interest is $I = rP$ dollars, and the total amount is $A = P + I = P(1 + r)$. This is *simple interest.*

For *compound interest*, the year is divided into k equal time periods and the interest is calculated and added to the account at the end of each period. So at the end of the first period, $A = P(1 + r(\frac{1}{k}))$; this is the new amount for the second period, at the end of which $A = P(1 + r(\frac{1}{k}))(1 + r(\frac{1}{k}))$, and continuing this way, at the end of the year the amount is

$$A = P\left(1 + \frac{r}{k}\right)^k .$$

The compound interest rate r thus earns the same in a year as the simple interest rate of

$$\left(1 + \frac{r}{k}\right)^k - 1 ;$$

this equivalent simple interest rate is in bank jargon the "annual percentage rate" or APR.[3]

a) Compute the APR of 5% compounded monthly, daily,[4] and continuously. Continuous compounding means the limit as k tends to infinity.

b) As in part (a), compute the APR of 10% compounded monthly, biweekly (k=26), daily, and continuously. (We have thrown in the biweekly rate because loans can be paid off biweekly.)

[3] Banks are required to reveal this so-called APR when they offer loans. The APR also takes into account certain bank fees known as points. Unfortunately, not all fees are included in it, and the true costs are higher if the loan is paid off early.

[4] For daily compounding assume that the year has 365 days, not 365.25. Banks are quite careful about these subtle differences. If you look at official tables of rates from precalculator days you will find that they are off by small amounts because U.S. regulations permitted banks to pretend that a year has 360 days.

II-1 Calculate the derivatives

a. xe^x

$\frac{d}{dx}(xe^x) = e^x + xe^x$

$= e^x(x+1)$

b. $(2x-1)e^{2x}$

$\frac{d}{dx}(2x-1)e^{2x} = 2e^{2x} + (2x-1)(2e^x)e^x$

$= 2e^{2x} + 4xe^{2x} - 2e^{2x}$

$= 4xe^{2x}$

c. e^{-x^2} | $u = e^{-x^2}$

$(\ln u)' = \frac{u'}{u}$

$(\ln e^{-x^2})' u = u'$

$-2x(e^{-x^2}) = u'$

d. $x\ln x - x$

$\frac{d}{dx}(x\ln x - x) = \ln x + \frac{1}{x} - 1$

$= \frac{x\ln x + 1 - x}{x}$

e. $\ln(x^2)$

$\frac{d}{dx}\ln(x^2) = \frac{2x}{x^2}$

$= \frac{2}{x}$

f. $(\ln x)^2$

$\frac{d}{dx}(\ln x)^2 = 2\ln x \cdot \frac{1}{x}$

$= \frac{2\ln x}{x}$

$= \frac{\ln(x^2)}{x}$

g. $(e^{x^2})^2$

$2\ln e^{x^2}$

$2x^2$

$(\ln u)' u = u'$

$(2x^2)'(e^{x^2})^2 = u'$

$4x(e^{x^2})^2 = u'$

h. x^x (Log Differentiation Applied)

$v = x^x$

$\ln v = \ln x^x = x\ln x$ This is godly. Got some help with my notes XD

$(\ln v)' = (x\ln x)' = \frac{v'}{v}$

$x^x(\ln x + 1) = v'$

$v' = x^x(1+\ln x)$

i. $(e^x + e^{-x})/2$

$\frac{d}{dx}(e^x + e^{-x})(2)^{-1} = (e^x - e^{-x})(2)^{-1}$

$\frac{e^x - e^{-x}}{2}$

j. $(e^x - e^{-x})/2$

$\frac{d}{dx}(e^x - e^{-x})(2)^{-1} = (e^x + e^{-x})(2)^{-1}$

$\frac{e^x + e^{-x}}{2}$

k. $\ln(1/x)$

$\frac{d}{dx}\ln(1/x) = \frac{1}{\frac{1}{x}} = x$

l. $1/\ln x$

$\frac{d}{dx}(1)(\ln x)^{-1} = (1)(-\ln x)\frac{1}{x}$

$= -\frac{\ln x}{x}$

m. $(1-e^x)/(1+e^x)$

$\frac{d}{dx}(1-e^x)(1+e^x)^{-1}$

$-e^x(1+e^x)^{-1} + (1-e^x)(-1)(1+e^x)^{-2}(e^x)$

$-e^x(1+e^x)^{-1} - e^x(1+e^x)^{-2} \cdot \frac{(1+e^x)^2}{(1+e^x)^2}$

$\frac{-e^x(1+e^x) - e^x}{(1+e^x)^2} = \frac{-e^x[(1+e^x)+1]}{(1+e^x)^2}$

II-2 Graph the function $y = (e^x + e^{-x})/2$

II-3 a.) Evaluate $\lim_{n\to\infty} n\ln(1+\frac{1}{n})$.

Hint: Let $h = 1/n$, and use $(d/dx)\ln(1+x)|_{x=0} = 1$

$\lim_{n\to\infty} n\ln(1+\frac{1}{n})\Big|_{h=\frac{1}{n}} = \lim_{h\to 0}\frac{\ln(1+h) - \ln 1}{h}$

$= \frac{d}{dx}\ln x\Big|_{x=1} = \frac{1}{1} = 1$

b.) Deduce that $\lim_{n\to\infty}(1+\frac{1}{n})^n = e$.

$\ln\left[\lim_{n\to\infty}(1+\frac{1}{n})^n\right] = \ln q$

$\lim_{n\to\infty} n\ln(1+\frac{1}{n}) = \ln q$

$1 = \ln q$

$e^1 = q = e$

II-4 Using $\lim_{n\to\infty}(1+\frac{1}{n})^n = e$, calculate

a. $\lim_{n\to\infty}(1+\frac{1}{n})^{3n} = \left[\lim_{n\to\infty}(1+\frac{1}{n})^n\right]^3$

$= e^3$

b. $\lim_{n\to\infty}(1+\frac{2}{n})^{5n} = \lim_{k\to\infty}(1+\frac{1}{k})^{10k}$

$\frac{2}{n} = \frac{1}{k}$

$2k = n$

$= \left[\lim_{k\to\infty}(1+\frac{1}{k})^k\right]^{10}$

$= e^{10}$

c. $\lim_{n\to\infty}(1+\frac{1}{2n})^{5n} = \lim_{k\to\infty}(1+\frac{1}{k})^{\frac{5}{2}k}$

$\frac{1}{2n} = \frac{1}{k}$

$2n = k$

$n = \frac{k}{2}$

$= \left[\lim_{k\to\infty}(1+\frac{1}{k})^k\right]^{5/2}$

$= e^{5/2}$

II- a. Compute the APR of 5% compounded monthly, daily and continuously.

$APR_{30} = (1+\frac{r}{k})^k - 1$

$= (1+\frac{0.05}{\ }) - 1$

$= 5.116\%$

$APR_{365} = (1+\frac{r}{k})^k - 1$

$= (1+\frac{0.05}{365})^{365} - 1$

$= 5.127\%$

$APR = \lim_{k\to\infty}(1+\frac{r}{k})^k - 1$

$= \lim_{n\to\infty}(1+\frac{1}{n})^{0.05n} - 1$

$= e^{0.05} - 1$

$= 5.127\%$

b. As in part(a), compute the APR of 10% compounded monthly, biweekly ($k = 26$), daily, and continuously.

$APR_{30} = (1+\frac{r}{k})^k - 1$

$= (1+\frac{0.1}{12})^{12} - 1$

$= 10.471\%$

$APR_{26} = (1+\frac{r}{k})^k - 1$

$= (1+\frac{0.1}{26})^{26} - 1$

$= 10.496\%$

$APR_{26} = (1+\frac{r}{k})^k - 1$

$= (1+\frac{0.1}{365})^{365} - 1$

$= 10.516\%$

$APR_{k\to\infty} = e^{0.1} - 1$

$= 10.517\%$