

Comparing Quadratic Approximations to Calculator Computations

In a previous worked example, we explored linear approximations to the sine function at the point $x = 0$. In this example, we use the quadratic approximation for e^x to calculate values of the exponential function near $x = 0$ and again compare the results to decimal approximations on a scientific calculator.

Find the linear approximation to e^x at the point $x = 0$ and use your answer to approximate the values of $e^{0.01}$, e^1 and e . Check your answer on a calculator.

$$f(x) \approx f(x_0) + f'(x_0)\Delta x + \frac{1}{2}f''(x_0)\Delta x^2$$

$$\text{at } x_0 = 0;$$

$$f(x) \approx f(0) + f'(0)x + \frac{1}{2}f''(0)x^2$$

$$\approx e^0 + e^0x + \frac{1}{2}e^0x^2$$

$$\approx 1 + x + \frac{1}{2}x^2$$

linear approximation	true value
$e^{0.01} \approx 1 + 0.01 + \frac{1}{2}(0.01)^2$	$e^{0.01} = 1.010050167$
≈ 1.01005	

$e^{0.1} \approx 1 + 0.1 + \frac{1}{2}(0.1)^2$	$e^{0.1} = 1.105170918$
≈ 1.105	

$e \approx 1 + 1 + \frac{1}{2}(1)^2$	$e = 2.718281828$
≈ 2.5	

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