1D-1 Calculate the following limits if they exist. If they do not exist, then indicate whether they are $+\infty$, $-\infty$ or undefined.

a) $\lim_{x\to 0} \dfrac{4}{x-1}$ b) $\lim_{x\to 2} \dfrac{4x}{x+1}$ c) $\lim_{x\to -2} \dfrac{4x^2}{x+2}$

d) $\lim_{x\to 2^+} \dfrac{4x^2}{2-x}$ e) $\lim_{x\to 2^-} \dfrac{4x^2}{2-x}$ f) $\lim_{x\to \infty} \dfrac{4x^2}{x-2}$

g) $\lim_{x\to \infty} \dfrac{4x^2}{x-2} - 4x$ i) $\lim_{x\to \infty} \dfrac{x^2+2x+3}{3x^2-2x+4}$ j) $\lim_{x\to 2} \dfrac{x-2}{x^2-4}$

1D-2 For which of the following should one use the one-sided limit? Evaluate it.

a) $\lim_{x\to 0} \sqrt{x}$ b) $\lim_{x\to 1} \dfrac{1}{x-1}$ c) $\lim_{x\to 1} \dfrac{1}{(x-1)^4}$ d) $\lim_{x\to 0} |\sin x|$ e) $\lim_{x\to 0} \dfrac{|x|}{x}$

1D-3 Identify and give the type of the points of discontinuity of each of the following:

a) $\dfrac{x-2}{x^2-4}$ b) $\dfrac{1}{\sin x}$

c) $\dfrac{x^4}{x^3}$ d) $f(x) = \begin{cases} x+a, & x>0 \\ a-x, & x<0 \end{cases}$

e) $f'(x)$, for the $f(x)$ in d) f) $(f(x))^2$, where $f(x) = \dfrac{d}{dx}|x|$

1D-4 Graph the following functions.

a) $\dfrac{4x^2}{x-2}$ (See **1D-1**efg.) b) $\dfrac{1}{x^2+2x+2}$

1D-5 Define $f(x) = \begin{cases} ax+b, & x \geq 1; \\ x^2, & x<1. \end{cases}$

a) Find all values of a, b such that $f(x)$ is continuous.

b) Find all values of a, b such that $f'(x)$ is continuous. (Be careful!)

1D-6 For each of the following functions, find all values of the constants a and b for which the function is differentiable.

a) $f(x) = \begin{cases} x^2+4x+1, & x \geq 0; \\ ax+b, & x<0. \end{cases}$ b) $f(x) = \begin{cases} x^2+4x+1, & x \geq 1; \\ ax+b, & x<1. \end{cases}$

1D - 1 Calculate the following limits if they exists. If they don't indicate whether $+\infty$, $-\infty$, or undefined.

a. $\lim_{x\to 0} \frac{4}{x-1} = \frac{4}{0-1} = -4$

b. $\lim_{x\to 2} \frac{4x}{x+1} = \frac{4(2)}{2+1} = \frac{8}{3}$

c. $\lim_{x\to -2} \frac{4x^2}{x+2} = \lim_{x\to -2^+}, \lim_{x\to -2^-} = -4$ ✗

d. $\lim_{x\to 2^+} \frac{4x^2}{2-x} = \frac{4(2.001)^2}{2-2.001} = -\infty$

e. $\lim_{x\to 2^-} \frac{4x}{2-x} = \frac{4(1.999)^2}{2-1.999} = +\infty$

f. $\lim_{x\to\infty} \frac{4x^2}{x-2} = \frac{\infty}{\infty} =$

g. $\lim_{x\to\infty} \frac{4x^2}{x-2} - 4x =$ i'll re answer this once I understand how infinity arithmetic works

i. $\lim_{x\to\infty} \frac{x^2+2x+3}{3x^2-2x+4} =$

j. $\lim_{x\to 2} \frac{x-2}{x^2-4} = \frac{0}{0} =$

1D - 2 For which of the following should one use the one-sided limit? Evaluate it.

a. $\lim_{x\to 0} \sqrt{x} \Rightarrow \lim_{x\to 0^-} \sqrt{x}$ = undefined

$\Rightarrow \lim_{x\to 0^+} \sqrt{x} = 0$

Answer: right-sided limit

d. $\lim_{x\to 0} |\sin x| \Rightarrow \lim_{x\to 0^-} |\sin x| = 0$

$\Rightarrow \lim_{x\to 0^+} |\sin x| = 0$

Answer: $\lim_{x\to 0} (\sin x) = 0$

b. $\lim_{x\to 1} \frac{1}{x-1} \Rightarrow \lim_{x\to 1^-} \frac{1}{x-1} = -\infty$

$\Rightarrow \lim_{x\to 1^+} \frac{1}{x-1} = +\infty$

Answer: limit does not exist use one-sided limit

e. $\lim_{x\to 0} \frac{|x|}{x} \Rightarrow \lim_{x\to 0^-} \frac{|x|}{x} = -1$

$\Rightarrow \lim_{x\to 0^+} \frac{|x|}{x} = 1$ ✗

Answer: $\lim_{x\to 0} \frac{|x|}{x} = 1$

c. $\lim_{x\to 1} \frac{1}{(x-1)^4} \Rightarrow \lim_{x\to 1^-} \frac{1}{(x-1)^4} = +\infty$

$\Rightarrow \lim_{x\to 1^+} \frac{1}{(x-1)^4} = +\infty$

Answer: $\lim_{x\to 1} f(x) = +\infty$

1D - 3 Identify and give the type of points discontinuity of the following functions.

a. $\frac{x-2}{x^2-4}$ — $x=-2$, $x=2$

Ans. Infinite Discontinuity $[x=2, -2]$

b. $\frac{1}{\sin x}$

Ans. Infinite Discontinuity $[x=0, \pm\pi, \pm 2\pi]$

c. $\frac{x^4}{x^3}$ or $\frac{x^3(x)}{x^2}$

Answer: removable $[x=0]$

d. $f(x) = \begin{cases} x+a, & x>0 \\ a-x, & x<0 \end{cases}$

Ans. removable discontinuity (hole) $[x=0]$

e. $f'(x)$, for the $f(x)$ in d

$f'(x) = \frac{d}{dx}\begin{cases} x+a, & x>0 \\ a-x, & x<0 \end{cases}$

$f'(x) = \begin{cases} 1, & x>0 \\ -1, & x<0 \end{cases}$

Ans. jump discontinuity $[x=0]$

f. $(f(x))^2$, where $f(x) = \frac{d}{dx}|x|$

$f(x) = 1$

Anc. removable $[x=0]$

1D - 4 Graph the following functions.

a. $\frac{4x^2}{x-2}$ ✗

b. $\frac{1}{x^2+2x+2}$

$x^2+2x+2=1$

$x^2+2x+1+2-1=1$

$(x+1)^2=0$

$x=-1$

1D - 5 Define $f(x) = \begin{cases} ax+b, & x\geq 1; \\ x^2, & x<1. \end{cases}$

a. Find all values of a, b such that $f(x)$ is continuous.

$ax+b = x^2$

$a(1)+b = (1)^2$

$a+b=1$

b. Find all values of a, b such that $f'(x)$ is continuous.

$f'(x) = \frac{d}{dx}\begin{cases} ax+b, & x\geq 1; \\ x^2, & x<1 \end{cases}$

$f'(x) = \begin{cases} a, & x\geq 1; \\ 2x, & x<1. \end{cases}$

$a = 2x$

$a = 2(1)$

$a = 2$

Ans.
Condition must be satisfied
1. $a=2$
2. $b=-1$; $a+b=1$

1D - 6 Find all values of the constants a and b for which the function is differentiable.

a. $f(x) = \begin{cases} x^2+ax+1, & x\geq 0; \\ ax+b, & x<0. \end{cases}$

$x^2+ax+1 = ax+b$

$(0)^2+a(0)+1 = a(0)+b$

$b=1$, a is any real no.

b. $f(x) = \begin{cases} x^2+4x+1, & x\geq 1; \\ ax+b, & x<1. \end{cases}$

$x^2+4x+1 = ax+b$

$(1)^2+4(1)+1 = a(1)+b$

$1+4+1 = a+b$

$a+b=6$

Comment: Re answer this part, match the two-sided slopes of $f'(x)$