Activity Score: 0/4
Recitation: 2/2

Note: Didn't understand the question the first time and tried to find the coefficient for EACH conditions.

To satisfy the concavity requirement,
$f''(x) = -x + 2$ (Eq. 1 & 2)

$\Rightarrow f'(x) = -\frac{1}{2}x^2 + 2x + k$

$f'(1) = -\frac{1}{2}(1)^2 + 2(1) + k$

$0 = -\frac{1}{2} + 2 + k$

$-\frac{3}{2} = k$

$f'(3) = -\frac{1}{2}(9) + 2(3) + k$

$0 = -\frac{9}{2} + \frac{12}{2} + k$

$-\frac{3}{2} = k$

$\Rightarrow f'(x) = -\frac{1}{2}x^2 + 2x - \frac{3}{2}$

$\Rightarrow f(x) = -\frac{1}{6}x^3 + x^2 - \frac{3}{2}x + d$

But since the value of d won't affect the concavity or the graph's rising and falling properties, we can use any value of d.

$\therefore$ $a = -\frac{1}{6}$

$b = 1$

$c = -\frac{3}{2}$

$d = 0$ or any value

Graph Features

The Graph Features mathlet allows you to choose the coefficients of a degree three polynomial and then illustrates where the graph of that polynomial is rising (increasing), falling (decreasing), concave and convex.

Find coefficient values a, b, c and d for a polynomial function:

$$f(x) = ax^3 + bx^2 + cx + d$$

whose graph is:

- convex (smile shaped) for $x < 2$
- concave (frown shaped) for $x > 2$ — $f''(2) = 0$ (1)
- falling when $x < 1$
- rising when $1 < x < 3$ — $f'(1) = 0$ (2) ; $f'(x) < 0$ when $x < 1$ (3), $f'(x) > 0$ when $1 < x < 3$ (4)
- falling when $x > 3$. — $f'(x) < 0$ when $x > 3$ (5) $f'(3) = 0$ (6)

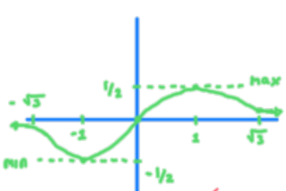
Can you find two different polynomials that satisfy these requirements? Why or why not?

Bonus: Make up a problem similar to this one for a friend to solve.

Recitation:

Sketch the curve

$y = \frac{x}{1+x^2}$



• Plot points

$x = 0$, $y = 0$

$x = 1$, $y = 1/2$

• Ends

$f(1000) = \frac{1000}{1+(1000)^2} << 1$

$\therefore \lim f(\infty) = 0$

$f(-1000) = \frac{-1000}{1+(-1000)^2} >> -1$

$\therefore \lim f(\infty) = 0$

$f(x) = x(1+x^2)^{-1}$

$f'(x) = (1+x^2)^{-1} - (x)(1+x^2)^{-2}(2x)$

$= \frac{(1+x^2) - 2x^2}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$

$f'(1) = 0$; $f'(-1) = 0$ – no other points, means no kinks in the graph.

$f''(x) = \frac{-2x(1+x^2)^2 - (1-x^2)2(1+x^2)2x}{(1+x^2)^4}$

$= \frac{(1+x^2)[-2x(1+x^2) - 4x(1-x^2)]}{(1+x^2)^4}$

$= \frac{(1+x^2)[-2x - 2x^3 - 4x + 4x^3]}{(1+x^2)^4}$

$f''(x) = \frac{2x^3 - 6x}{(1+x^2)^3}$

$2x^3 = 6x$

$x^2 = 3$

$x = \sqrt{3}$