c) $\dfrac{x+2}{x^2-1}$ d) $\dfrac{x^4+1}{x}$

1F. Chain rule, implicit differentiation

1F-1 Find the derivative of the following functions:

a) $(x^2+2)^2$ (two methods)

b) $(x^2+2)^{100}$. Which of the two methods from part (a) do you prefer?

1F-2 Find the derivative of $x^{10}(x^2+1)^{10}$.

1F-3 Find dy/dx for $y = x^{1/n}$ by implicit differentiation.

1F-4 Calculate dy/dx for $x^{1/3} + y^{1/3} = 1$ by implicit differentiation. Then solve for y and calculate y' using the chain rule. Confirm that your two answers are the same.

1F-5 Find all points of the curve(s) $\sin x + \sin y = 1/2$ with horizontal tangent lines. (This is a collection of curves with a periodic, repeated pattern because the equation is unchanged under the transformations $y \to y + 2\pi$ and $x \to x + 2\pi$.)

1F-6 Show that the derivative of an even function is odd and that the derivative of an odd function is even.

(Write the equation that says f is even, and differentiate both sides, using the chain rule.)

1F-7 Evaluate the derivatives. Assume all letters represent constants, except for the independent and dependent variables occurring in the derivative.

a) $D = \sqrt{(x-a)^2 + {y_0}^2}$, $\dfrac{dD}{dx} = ?$ b) $m = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$, $\dfrac{dm}{dv} = ?$

c) $F = \dfrac{mg}{(1+r^2)^{3/2}}$, $\dfrac{dF}{dr} = ?$ d) $Q = \dfrac{at}{(1+bt^2)^3}$, $\dfrac{dQ}{dt} = ?$

1F-8 Evaluate the derivative by implicit differentiation. (Same assumptions about the letters as in the preceding exercise.)

Activity Score - 13/15

1E-5 cont...

c. $\frac{d}{dx}\frac{x+2}{x^2-1} = \frac{-x^2-3}{(x^2-1)^2}$

$= \frac{(1)(x^2-1) - (x+2)(2x)}{(x^2-1)^2}$

$= \frac{x^2-1-(2x^2+4x)}{(x^2-1)^2}$

$= \frac{-x^2-4x-1}{(x^2-1)^2}$ This is my second answer. I got the first one wrong.

d. $\frac{d}{dx}\frac{x^4+1}{x} = \frac{3x^4-1}{x^2}$

$= \frac{4x^3(x) - (x^4+1)(1)}{x^2}$

$= \frac{4x^4-x^4-1}{x^2}$

$= \frac{3x^4-1}{x^2}$ or $3x^2-\frac{1}{x^2}$

1F-1 Find the derivative of the following functions.

a. $(x^2+2)^2$

$f'(x^2+2)^2 = f'((x^2+2)^2)\, f'(x^2+2)$

$= 2(x^2+2)(2x)$

$= 4x(x^2+2)$

$\frac{d}{dx}(x^2+2)^2 = \frac{du}{dy}\frac{dy}{dx}$

$= 2(x^2+2)\,2x$

$= 4x(x^2+2)$

b. $(x^2+2)^{100}$

$\frac{d}{dx}(x^2+2)^{100} = 100(x^2+2)^{99}(2x)$

$= 200x^{99(2)+1} + 200x(2^{99})$

$= 200x(x^{198}+2^{99})$ or $200x(x^2+2)^9$

1F-1 Find the derivative of $x^{10}(x^2+1)^{10}$

$f'(x^{10}(x^2+1)^{10}) = f'(x^{30}+x^{10})$ wrong!! use binomial theorem here

$= 30x^{29}+10x^9$

$= 10x^9(3x^{20}+1)$

hence this is shorter; easier

$f'(x^{10}(x^2+1)^{10}) = f'(x^{10})(x^2+1)^{10} + x^{10}f'((x^2+1)^{10})$

$= 10x^9(x^2+1)^{10} + x^{10}((10(x^2+1)^9)(2x))$

$= 10(3x^2+1)x^9(x^2+1)^9$

This is from the answer key. I don't understand how it got simplified to that. Will come back to this at some point.

1F-3 Find dy/dx for $y=x^{1/n}$ by implicit differentiation

$y=x^{1/n}$

$\frac{d}{dx}(y=x^{1/n})$

$y' = \frac{1}{n}x^{1/n-1}$

1F-4 Calculate dy/dx for $x^{1/3}+y^{1/3}=1$ by implicit differentiation

$\frac{d}{dx}(x^{1/3}+y^{1/3}=1)$

$\frac{1}{3}x^{-2/3}+\frac{1}{3}y^{-2/3}y' = 0$

$y' = -\frac{1}{3}x^{-2/3}(3y^{2/3})$

$y' = -x^{-2/3}y^{2/3}$

$y^{1/3} = 1-x^{1/3}$

$y = (1-x^{1/3})^3$

$\frac{d}{dx}(1-x^{1/3})^3 = 3(1-x^{1/3})^2(-\frac{1}{3}x^{-2/3})$

$= -x^{-2/3}(1-x^{1/3})^2$

1F-5 Find all points of the curves $\sin x + \sin y = 1/2$ with horizontal tangent lines.

$\frac{d}{dx}(\sin x + \sin y = \frac{1}{2})$

$\cos x + \cos y\, y' = 0$

$y' = \frac{-\cos x}{\cos y}$

$y' = 0 \Leftrightarrow -\cos x = 0$

we're looking for points not x-values.

$x = \pi/2 + k\pi$, for every integer k

$\sin(\pi/2+k\pi) = (-1)^k$, 1 if k is even, -1 if k is odd.

Thus at $x=\pi/2+k\pi$, $\pm 1+\sin y = \frac{1}{2}$ or $\sin y = \mp 1+\frac{1}{2}$

$x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$

$\sin y = -1+\frac{1}{2}$

$y = \sin^{-1}[-1+\frac{1}{2}]$

How do you convert this to $y = a+b\pi$??

1F-6 Show that the derivative of an even function is odd and that the derivative of an odd function is even

Even function

$f(x) = f(-x)$

$f(x)' = \frac{d}{dx}(-x)\cdot\frac{d}{dx}(f(-x))$

$f(x)' = (-1)f(-x)'$

$f(x)' = -f(-x)'$

Odd Function

$f(x) = -f(-x)$

$f(x)' = (0)(-f(-x)) + (-1)f(-x)'$

$f(x)' = (-1)\cdot\frac{d}{dx}(-x)\cdot\frac{d}{dx}(f(-x))$

$f(x)' = (-1)(-1)f(-x)'$

$f(x)' = f(-x)'$

I need to change my notation from $f(x)'$ to $f'(x)$.

1F-7 Evaluate the derivatives. Assumes all letters represent constants, except for the independent and dependent variables occurring in the derivative.

a. $D = \sqrt{(x-a)^2+y_0^2}$, $\frac{dD}{dx} = ?$

$\frac{d}{dx}[(x-a)^2+y_0^2]^{\frac{1}{2}} = \frac{1}{2}[(x-a)^2+y_0^2]^{-\frac{1}{2}}\cdot\frac{d}{dx}(x-a)^2$

$= \frac{2(x-a)}{2\sqrt{(x-a)^2+y_0^2}}$

$= (x-a)[(x-a)^2+y_0^2]^{-\frac{1}{2}}$

b. $m = \frac{m_0}{\sqrt{1-v^2/c^2}}$, $\frac{dm}{dv} = ?$

$\frac{d}{dx}(m_0[(1-v^2/c^2)]^{-\frac{1}{2}}) = m_0\cdot\frac{d}{dv}((1-v^2/c^2)^{-\frac{1}{2}})$

v^2c^{-2}

$2vc^{-2}+v^2 0$

$= m_0[-\frac{1}{2}(1-v^2[c^{-2}])^{-\frac{3}{2}}\cdot(0-\boxed{2v[c^{-2}]})]$

c^{-2} suddenly missing

$= \frac{-2m_0v}{-2(1-v^2/c^2)^{3/2}}$

$= \frac{m_0v}{(1-v^2/c^2)^{3/2}}$

Answer: $\frac{m_0v}{c^2(1-v^2/c^2)^{3/2}}$

c. $F = \frac{mg}{(1+r^2)^{3/2}}$, $\frac{dF}{dr} = ?$

$\frac{d}{dr}[mg(1+r^2)^{-3/2}] = mg\cdot\frac{d}{dr}(1+r^2)^{-3/2}$

$= mg[-\frac{3}{2}(1+r^2)^{-5/2}(2r)]$

$= -\frac{6rmg}{2(1+r^2)^{5/2}}$

$= \frac{-3rmg}{(1+r^2)^{5/2}}$

d. $Q = \frac{at}{(1+bt^2)^3}$, $\frac{dQ}{dt} = ?$

$\frac{d}{dt}(at(1+bt^2)^{-3}) = at\cdot\frac{d}{dt}(1+bt^2)^{-3}$

$= at[-3(1+bt^2)^{-4}(2bt)]$

$= \frac{-6abt^2}{(1+bt^2)^{-4}}$

$= at[(-3)(1+bt^2)^{-4}(2bt)] + a[(1+bt^2)^{-3}]$

$= -6abt^2(1+bt^2)^{-4} + a(1+bt^2)^{-3}$

$= [-6abt^2 + a(1+bt^2)](1+bt^2)^{-4}$

$= [a(-6bt^2+1+bt^2)](1+bt^2)^{-4}$

$= [a(1-5bt^2)](1+bt^2)^{-4}$

1F-8 Evaluate the derivative by implicit differentiation. Same assumptions about the letters in the previous section.

a. $V = \frac{1}{3}\pi r^2h$, $\frac{dr}{dh} = ?$

$\frac{dr}{dh}(V = \frac{1}{3}\pi r^2h)$

$0 = \frac{1}{3}\pi[2rr'h + r^2(1)]$

$0 = 2rhr' + r^2$

$2rhr' = -r^2$

$r' = \frac{-r^2}{2rh}$

$r' = \frac{-r}{2h}$

$r' = \sqrt{(3V)(\pi h)^{-1}}\,(2h)^{-1}$

$r^2 = \frac{V}{\frac{1}{3}\pi h}$

$r = \sqrt{\frac{3V}{\pi h}}$

b. $PV^c = nRT$, $\frac{dP}{dV} = ?$

$\frac{dP}{dV}(PV^c = nRT)$

$(1)(P')(V^c) + P(cV^{c-1}) = 0$

$P' = -\frac{cV^{c-1}P}{V^c}$

$P' = -\frac{cP}{V}$

c. $c^2 = a^2+b^2-2ab\cos\theta$, $\frac{da}{db} = ?$

$\frac{da}{db}(c^2 = a^2+b^2-2ab\cos\theta)$

$0 = 2aa' + 2b - (2\cos\theta)(a'b + a(1))$

$= 2aa' + 2b - 2\cos\theta\, a'b - 2\cos\theta\, a$

$= 2aa' - 2\cos\theta\, a'b + 2b - 2\cos\theta\, a$

$= a'(2a - 2\cos\theta\, b) + 2(b - \cos\theta\, a)$

$a' = \frac{-2(b-\cos\theta\, a)}{2(a-\cos\theta\, b)}$

$a' = \frac{\cos\theta\, a - b}{a - \cos\theta\, b}$

$\frac{a\cos\theta - b}{a - b\cos\theta}$ - to avoid confusion i.e $\cos(\theta a)$