Activity Score - 4/6

18.01 Exercises *

December 23, 2010

Unit 2. Applications of Differentiation

2A. Approximation

2A-1 Find the linearization of $\sqrt{a+bx}$ at 0, by using (2), and also by using the basic approximation formulas. (Here a and b are constants; assume $a > 0$. Do not confuse this a with the one in (2), which has the value 0.)

2A-2 Repeat Exercise A-1 for the function $\dfrac{1}{a+bx}$, $\quad a \neq 0$.

2A-3 Find the linearization at 0 of $\dfrac{(1+x)^{3/2}}{1+2x}$ by using the basic approximation formulas, and also by using (2).

2A-4 Find the linear approximation for $h \approx 0$ for $w = \dfrac{gW}{(1+h/R)^2}$, the weight of a body at altitude h above the earth's surface, where W is the surface weight and R is the radius of the earth. (Do this without referring to the notes.)

2A-5 Making reasonable assumptions, if a person 5 feet tall weighs on the average 120 lbs., approximately how much does a person $5'1''$ tall weigh?

2A-6 Find a quadratic approximation to $\tan\theta$, for $\theta \approx 0$.

2A-7 Find a quadratic approximation to $\dfrac{\sec x}{\sqrt{1-x^2}}$, for $x \approx 0$.

2A-1 Find the linearization of $\sqrt{a+bx}$ at 0, by using (2), and also by using the basic approximation formulas. (Here a and b are constants; assume $a>0$. Do not confuse this a with the one in (2), which has a value of 0.)

$L(f) \approx f(0) + f'(0)x$

$f(x) = (a+bx)^{1/2}$

$f'(x) = \frac{1}{2}b(a+bx)^{-1/2}$

$L(f) \approx \sqrt{a} + \frac{b}{2\sqrt{a}}x$

$\approx \sqrt{a} + \frac{b\sqrt{a}}{2a}x$

Note: (2) is not provided.

2A-2 Repeat Exercises A-1 for the function $\frac{1}{a+bx}$, $a \neq 0$.

$f(x) = \frac{1}{a+bx}$

$f'(x) = -(a+bx)^{-2} \cdot b$

$= \frac{-b}{(a+bx)^2}$

$L(f) \approx \frac{1}{a} - \frac{b}{a^2}x$

2A-3 Find the linearization at 0 of $\frac{(1+x)^{3/2}}{1+2x}$ by using the basic approximation formulas and also by using (2).

$f(x) = \frac{(1+x)^{3/2}}{1+2x}$

$f'(x) = \frac{\frac{3}{2}(1+x)^{1/2}(1+2x) - (1+x)^{3/2}(2)}{(1+2x)^2}$

$L(f) \approx 1 - \frac{1}{2}x$

2A-4 Find the linear approximation for $h \approx 0$ for $w = \frac{gW}{(1+h/R)^2}$, the weight of a body at altitude h above the earth's surface, where W is the surface weight and R is the radius of the Earth.

$f(h) = \frac{gW}{(1+h/R)^2}$

$f'(h) = \frac{-2hgW(1+h/R)}{r^2(1+h/R)^4}$

$= \frac{-2hgW}{r^2(1+h/R)^3}$

$L(f) \approx gW + 0$

$\approx gW$

$f(h) = \frac{gW}{(1+h/R)^2}$

$f'(h) = \frac{-gW(2)(1+h/R)(1/R)}{(1+h/R)^4}$

$= \frac{-2gW(1/R)}{(1+h/R)^3}$

$f(h) \approx f(0) + f'(0)(h-0)$

$\approx gW - \frac{2gW}{r}(h)$

$\approx gW(1-2h/r)$

Note: the answer keys answer is $g(1-2h/r)$. I don't know what happened to W. Internet sources seems to agree with my red solution

2A-5 Making reasonable assumptions, if a person 5 feet tall weighs on the average 120 lbs., approximately how much does a person 5'1" tall weigh?

Let's assume that weight increases linearly with height. Assumption wrong, ofc answer will also be wrong.

$w = 24h$

$f(h) = 24h$

$f'(h) = 24$

$f(5.083) \approx f(5) + f'(5)(5.083 - 5)$

$\approx 120 + 24(0.083)$

$f(5.083) \approx 121.992$ lbs

$\frac{d}{dx}\left[\frac{\sin\theta}{\cos\theta}\right]$

$\Rightarrow \frac{\cos\theta\cos\theta - \sin\theta(-\sin\theta)}{\cos^2\theta}$

$\Rightarrow \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta} \Rightarrow \frac{1}{\cos^2\theta} \Rightarrow \sec^2\theta$

2A-6 Find a quadratic approximation to $\tan\theta$, for $\theta \approx 0$

$f(\theta) = \tan\theta$

$f'(\theta) = \sec^2\theta$

$f''(\theta) = 2\tan^2\theta\sec\theta$

$Q(f) \approx f(0) + f'(0)x + \frac{1}{2}f''(0)x^2$

$\approx 0 + x + 0$

$Q(f) \approx x$

Why did my variable suddenly changed? Should have been θ

$\frac{d}{dx}\left[\frac{1}{\cos^2\theta}\right]$

$\Rightarrow \frac{2\sin^2\theta}{\cos^4\theta} \Rightarrow \frac{2\sin^2\theta}{\cos^2\theta} \cdot \frac{1}{\cos^2\theta}$

$\Rightarrow 2\tan^2\theta\sec^2\theta$

$\frac{d}{dx}\left[\frac{1}{\cos\theta}\right]$

$\Rightarrow \frac{\sin\theta}{\cos^2\theta} \Rightarrow \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$

$\Rightarrow \tan\theta\sec\theta$

2A-7 Find the quadratic approximation to $\frac{\sec x}{\sqrt{1-x^2}}$, for $x \approx 0$.

$f(x) = \frac{\sec x}{\sqrt{1-x^2}}$

$f'(x) = \tan x\sec x\sqrt{1-x^2} + \sec x(x)(1-x^2)^{-1/2}$

$f''(x) = \left[\sec^3 x + \tan^2 x\sec x\right]\sqrt{1-x^2} + x\tan x\sec x(1-x^2)^{-1/2}$
$+ \frac{\left[x\tan x\sec x + \sec x\right](1-x^2)^{-1/2} - \left[x\sec x(x)(1-x^2)^{-1/2}\right]}{1-x^2}$

$Q(f) \approx f(0) + f'(0)x + \frac{1}{2}f''(0)x^2$

$\approx 1 + 0x + \frac{1}{2}(0)x^2$

$Q(f) \approx 1$

Answer should be $1 + x^2$