1D-7 Find the values of the constants a, b and c for which the following function is differentiable. (Give a and b in terms of c.)

$$f(x) = \begin{cases} cx^2 + 4x + 1, \ x \geq 1; \\ ax + b, \ x < 1. \end{cases}$$

1D-8 For each of the following functions, find the values of the constants a and b for which the function is continuous, but *not* differentiable.

a) $f(x) = \begin{cases} ax + b, \ x > 0; \\ \sin 2x, \ x \leq 0. \end{cases}$ b) $f(x) = \begin{cases} ax + b, \ x > 0; \\ \cos 2x, \ x \leq 0. \end{cases}$

1D-9 Find the values of the constants a and b for which the following function is differentiable, but *not* continuous.

$$f(x) = \begin{cases} ax + b, \ x > 0; \\ \cos 2x, \ x \leq 0. \end{cases}$$

1D-10* Show that

$g(h) = \dfrac{f(a+h) - f(a)}{h}$ has a removable discontinuity at $h = 0$ $\iff$ $f'(a)$ exists.

1E. Differentiation formulas: polynomials, products, quotients

1E-1 Find the derivative of the following polynomials.

a) $x^{10} + 3x^5 + 2x^3 + 4$ b) $e^2 + 1$ (e = base of natural logs)

c) $x/2 + \pi^3$ d) $(x^3 + x)(x^5 + x^2)$

1E-2 Find the antiderivative of the following polynomials.

a) $ax + b$ (a and b are constants.) b) $x^6 + 5x^5 + 4x^3$

c) $(x^3 + 1)^2$

1E-3 Find the points (x, y) of the graph $y = x^3 + x^2 - x + 2$ at which the slope of the tangent line is horizontal.

1E-4 For each of the following, find all values of a and b for which $f(x)$ is differentiable.

a) $f(x) = \begin{cases} ax^2 + bx + 4, \ x \leq 0; \\ 5x^5 + 3x^4 + 7x^2 + 8x + 4, \ x > 0. \end{cases}$ b) $f(x) = \begin{cases} ax^2 + bx + 4, \ x \leq 1; \\ 5x^5 + 3x^4 + 7x^2 + 8x + 4, \ x > 1. \end{cases}$

1E-5 Find the derivatives of the following rational functions.

a) $\dfrac{x}{1+x}$ b) $\dfrac{x+a}{x^2+1}$ (a is constant)

1D-7 Find the values of the constants a, b and c for which the following function is differentiable. Give a and b in terms of c.

$$f(x) = \begin{cases} cx^2+4x+1 & , x \geq 1; \\ ax+b & , x < 1. \end{cases}$$

* a, b, c value for continuity

$cx^2+4x+1 = ax+b$

$c(1)^2+4(1)+1 = a(1)+b$

$c+4+1 = a+b$

$c+5 = a+b$

* f'(x) two-sided limits should be equal

$\lim_{x\to1^+} \frac{d}{dx} cx^2+4x+1 = \lim_{x\to1^-} ax+b$

$\lim_{x\to1^+} 2cx+4 = \lim_{x\to1^-} a$

$2c+4 = a$

$c+5 = a+b$

$c+5 = 2c+4+b$

$c+5-2c-4 = b$

$b = -c+1$

1D-8 Find the values of the constants a and b for which the function is continuous but not differentiable.

a. $f(x) = \begin{cases} ax+b & , x>0; \\ \sin 2x & , x \leq 0. \end{cases}$

* a and b for continuity

$ax+b = \sin 2x$

$a(0)+b = \sin 2(0)$

$b = 0$

* f'(x) two-sided limit

$\lim_{x\to0^-} \frac{d}{dx} \sin 2x \neq \lim_{x\to0^+} \frac{d}{dx} ax+b$

$\lim_{x\to0^-} 2\cos 2x \neq \lim_{x\to0^+} a$

$a \neq 0$ 2

$\frac{\Delta f}{h} = \frac{\sin 2(x+h) - \sin 2x}{h}$

$= \frac{\sin 2x \cos 2h + \sin 2h \cos 2x - \sin 2x}{h}$

$= \frac{\sin 2x(\cos 2h - 1)}{h} + \frac{\sin 2h \cos 2x}{h}$

$= 2\cos 2x$, as h approaches 0

b. $f(x) = \begin{cases} ax+b, & x>0; \\ \cos 2x, & x \leq 0; \end{cases}$

* a and b for continuity

$ax+b = \cos 2x$

$a(0)+b = \cos 2(0)$

$b = 1$

* f'(x) two-sided limit

$\lim_{x\to0^+} \frac{d}{dx} ax+b = \lim_{x\to0^-} \cos 2x$

$\lim_{x\to0^+} a \neq \lim_{x\to0^-} 2\sin 2x$

$a \neq 0$

1D-9 Find the constants a and b for which the function is differentiable, but not continuous.

$f(x) = \begin{cases} ax+b, & x>0; \\ \cos 2x, & x \leq 0. \end{cases}$

Ans. A function that is not continuous means it is not differentiable.

1D-10 Show that

$g(h) = \frac{f(a+h) - f(a)}{h}$ has a removable discontinuity at $h=0 \Leftrightarrow f'(a)$ exists

$f'(a) = \lim_{a\to h} g(h)$

Ans. Differentiability, the fact that f'(a) exist, implies continuity.

1E-1 Find the derivative of the following polynomials.

a. $\frac{d}{dx} x^{10}+3x^5+2x^3+4 = 10x^9+15x^4+6x^2$

b. $\frac{d}{dx} e^2+1 = 2e$ $e^2+1 \approx 8.4$ is a CONSTANT, hence f'(x) is zero

c. $\frac{d}{dx} \frac{x}{2} + \pi^3 = \frac{1}{2} + 3\pi^2$ π^3 is a constant as well

$\frac{d}{dx}(u/v) = \frac{u'v - uv'}{v^2}$

$= \frac{(1)2 - x(0)}{(2)^2}$

$= \frac{2}{4} = \frac{1}{2}$

d. $(x^3+x)(x^5+x^2) = 20x^7+6x^5+5x^4+3x^2$

$\frac{d}{dx}(u \cdot v) = u'v + uv'$

$= 3x^2+1(x^5+x^2) + x^3+x(5x^4+2x)$

$= 15x^7+3x^4+x^5+x^2+5x^7+2x^4+5x^5+2x^2$

$= 20x^7+5x^4+6x^5+3x^2$

$= 20x^7+6x^5+5x^4+3x^2$

like that

do not use the PRODUCT RULE, multiply first instead

$f(x) = (x^3+x)(x^5+x^2)$

$= x^8+x^5+x^6+x^3$

$f'(x) = 8x^7+5x^4+6x^5+3x^2$

if you really want to use the product rule

$f'(x) = \frac{d}{dx}(x^3+x) \cdot \frac{d}{dx}(x^5+x^2)$

$= 3x^2+1(x^5+x^2) + x^3+x(5x^4+2x)$

$= 3x^7+3x^4+x^5+x^2+5x^7+2x^4+5x^5+2x^2$

$f'(x) = 8x^7+6x^5+5x^4+3x^2$

1E-2 Find the antiderivative of the following polynomials

a. $ax+b$ (a and b are constants)

b. $x^6+5x^5+4x^3$ not covered yet. will skip it.

c. $(x^3+1)^2$

1E-3 Find the points (x, y) of the graph $y = x^3+x^2-x+2$ at which the slope of the tangent line is horizontal.

$f'(x) = \frac{d}{dx} x^3+x^2-x+2$

$f'(x) = 3x^2+2x-1$

$0 = 3x^2+2x-1$

$\frac{1}{3}(0) = \frac{1}{3}(3x^2+2x-1)$

$0 = x^2+\frac{2}{3}x-\frac{1}{3}$

$= x^2+\frac{2}{3}x+\left(\frac{2}{6}\right)^2 - \frac{4}{36} - \frac{1}{3}$

$= \left(x+\frac{2}{6}\right)^2 - \frac{4}{36} - \frac{12}{36}$

$= \left(x+\frac{2}{6}\right)^2 - \frac{16}{36}$

$0 = \left(x+\frac{2}{6}\right)^2 - \frac{4}{9}$

$\sqrt{\frac{4}{9}} = \sqrt{\left(x+\frac{2}{6}\right)^2}$

$\pm\frac{2}{3} = x+\frac{2}{6}$

$x = \frac{2}{3} - \frac{2}{6}$

$x = \frac{4-2}{6}$

$x = \frac{2}{6} = \frac{1}{3}$

$-\frac{2}{3} = x+\frac{2}{6}$

$-\frac{2}{6} - \frac{2}{3} = x$

$\frac{-6}{6} = x$

$-1 = x$

$y = x^3+x^2-x+2$

$= \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^2 - \frac{1}{3} + 2$

$= \frac{1}{27} + \frac{1}{9} - \frac{1}{3} + 2$

$= \frac{1+3-9+54}{27}$

$y = \frac{49}{27}$

Ans. m at $\left(\frac{1}{3}, \frac{49}{27}\right)$ is zero.

m at $(-1, 3)$ is zero

DO NOT forget to include negative roots

$y = x^3+x^2-x+2$

$= (-1)^3+(-1)^2-(-1)+2$

$= -1+1+1+2$

$= 3$

Total score (page

1E-4 Find all values of a and b for which f(x) is differentiable.

a. $f(x) = \begin{cases} ax^2+bx+4 & , x \leq 0; \\ 5x^5+3x^4+7x^2+8x+4 & , x>0. \end{cases}$

a and b for continuity

$ax^2+bx+4 = 5x^5+3x^4+7x^2+8x+4$

$a(0)+b(0)+4 = 5(0)^5+3(0)^4+7(0)^2+8(0)+4$

$4 = 4$; a and b could be any real number

a and b values such that $\lim_{x\to0^+} f'(x) = \lim_{x\to0^-} f'(x)$

$\lim_{x\to0^-} \frac{d}{dx} ax^2+bx+4 = \lim_{x\to0^+} \frac{d}{dx} 5x^5+3x^4+7x^2+8x+4$

$\lim_{x\to0^-} 2ax+b = \lim_{x\to0^+} 25x^4+12x^3+14x+8$

$2a(0)+b = 25(0)^4+12(0)^3+14(0)+8$

$b = 8$; a could be any real number

b. $f(x) = \begin{cases} ax^2+bx+4 & , x \leq 1; \\ 5x^5+3x^4+7x^2+8x+4 & , x>1. \end{cases}$

a and b for continuity

$ax^2+bx+4 = 5x^5+3x^4+7x^2+8x+4$

$a(1)^2+b(1)+4 = 5(1)^5+3(1)^4+7(1)^2+8(1)+4$

$a+b = -4+5+3+7+8+4$

$a+b = 23$

a and b for equal left, right-sided f'(x) limits

$\lim_{x\to1^-} \frac{d}{dx} ax^2+bx+4 = \lim_{x\to1^+} \frac{d}{dx} 5x^5+3x^4+7x^2+8x+4$

$\lim_{x\to1^-} 2ax+b = \lim_{x\to1^+} 25x^4+12x^3+14x+8$

$2a(1)+b = 25(1)^4+12(1)^3+14(1)+8$

$2a+b = 25+12+14+8$

$2a+b = 59$

Subtract the first equation from the second to get a.

$2a+b = 59$

$-\ a+b = 23$

$a = 36$

$a+b = 23$

$36+b = 23$

$b = 23-36$

$b = -13$

I forgot how to do this. I know I am lacking something, I just can't figure out what it is.

1E-5 Find the derivatives of the following rational function

a. $f'\left(\frac{x}{1+x}\right) = \frac{1}{(x+1)^2}$

$f'(u/v) = \frac{u'v - uv'}{v^2}$

$= \frac{(1)(x+1) - (x)(1)}{(x+1)^2}$

$= \frac{1}{(x+1)^2}$

b. $f'\left(\frac{x+a}{x^2+1}\right) =$

$f'(u/v) = \frac{u'v - uv'}{v^2}$

$= \frac{(1)(x^2+1) - (x+a)(2x)}{(x^2+1)^2}$

$= \frac{x^2+1 - (2x^2+2ax)}{(x^2+1)^2}$

$= \frac{x^2+1-2x^2-2ax}{(x^2+1)^2}$

$= \frac{-x^2-2ax+1}{(x^2+1)^2}$