a) $V = \frac{1}{3}\pi r^2 h, \qquad \frac{dr}{dh} = ?$ b) $PV^c = nRT, \qquad \frac{dP}{dV} = ?$

c) $c^2 = a^2 + b^2 - 2ab\cos\theta, \qquad \frac{da}{db} = ?$

1G. Higher derivatives

1G-1 Calculate y'' for the following functions.

a) $3x^2 + 2x + 4\sqrt{x}$ b) $\frac{x}{x+5}$

c) $\frac{-5}{x+5}$ d) $\frac{x^2+5x}{x+5}$

1G-2 Find all functions $f(x)$ whose third derivative $f'''(x)$ is identically zero. ("Identically" is math jargon for "always" or "for every value of x".)

1G-3 Calculate y'' using implicit differentiation and simplify as much as possible.

$$x^2a^2 + y^2b^2 = 1$$

1G-4 Find the formula for the nth derivative $y^{(n)}$ of $y = 1/(x+1)$.

1G-5 Let $y = u(x)v(x)$.

a) Find y', y'', and y'''.

b) The general formula for $y^{(n)}$, the n-th derivative, is called *Leibniz' formula*: it uses the same coefficients as the binomial theorem , and looks like

$$y^{(n)} = u^{(n)}v + \binom{n}{1}u^{(n-1)}v^{(1)} + \binom{n}{2}u^{(n-2)}v^{(2)} + ... + uv^{(n)}$$

Use this to check your answers in part (a), and use it to calculate $y^{(p+q)}$, if $y = x^p(1+x)^q$.

1H. Exponentials and Logarithms: Algebra

1H-1 The *half-life* λ of a radioactive substance decaying according to the law $y = y_0e^{-kt}$ is defined to be the time it takes the amount to decrease to 1/2 of the initial amount y_0.

a) Express the half-life λ in terms of k. (Do this from scratch — don't just plug into formulas given here or elsewhere.)

PSET Score (page 7) = 7/12

IG-1 Calculate y'' of the following functions

a. $3x^2+2x+4\sqrt{x}$

$y''=\frac{d}{dx}\left[\frac{d}{dx}(3x^2+2x+4x^{1/2})\right]$

$=\frac{d}{dx}(6x+2+2x^{-1/2})$

$y''=6-x^{-3/2}$

b. $\frac{x}{x+5}$

$y''=\frac{d}{dx}\left[\frac{d}{dx}(x(x+5)^{-1})\right]$

$=\frac{d}{dx}\left[(1)(x+5)^{-1}+(x)(-1)(x+5)^{-2}(1)\right]$

$=\frac{d}{dx}\left[(x+5)-x\right](x+5)^{-2}=\frac{d}{dx}(5)(x+5)^{-2}$

$=5(-2)(x+5)^{-3}(1)$

$y''=-10(x+5)^{-3}$

c. $\frac{-5}{x+5}$

$y''=\frac{d}{dx}\left[\frac{d}{dx}(-5)(x+5)^{-1}\right]$

$=\frac{d}{dx}\left[(-5)(-1)(x+5)^{-2}(1)\right]=\frac{d}{dx}5(x+5)^{-2}$

$=5(-2)(x+5)^{-3}$

$y''=-10(x+5)^{-3}$

d. $\frac{x^2+5x}{x+5}$

$y''=\frac{d}{dx}\left[\frac{d}{dx}(x^2+5x)(x+5)^{-1}\right]$

$=\frac{d}{dx}(2x+5)(x+5)^{-1}+(x^2+5x)(-1)(x+5)^{-2}$

$=(2)(x+5)^{-1}+(2x+5)(-1)(x+5)^{-2}-\left[(2x+5)(x+5)^{-2}+(x^2+5x)(-2)(x+5)^{-3}\right]$

$=2(x+5)^{-1}-(2x+5)(x+5)^{-2}-(2x+5)(x+5)^{-2}+2(x^2+5x)(x+5)^{-3}$

$=\left[2(x+5)^2-(2x+5)(x+5)-(2x+5)(x+5)+2x(x+5)\right](x+5)^{-3}$

$=\left[(x+5)(2x+10-2x-5-2x-5+2x)\right](x+5)^{-3}$

$=\left[(x+5)(0)\right](x+5)^{-3}$

$y''=0$

IG-2 Find all functions $f(x)$ whose third derivative is identically zero.

will consult answer key !

IG-3 Calculate y'' using implicit differentiation and simplify as much as possible

$x^2a^2+y^2b^2=1$ Wrong given, maybe ...

$\frac{d}{dx}(x^2a^2+y^2b^2=1)$

$2a^2x+2b^2yy'=0$

$y'=\frac{-2a^2x}{2b^2y}$

$y'=\frac{-a^2x}{b^2y}$

$\frac{d}{dx}(-a^2x)(b^2y)^{-1}$

$-a^2(-1)(b^2y)^{-2}(b^2)(y')$

$y''=\frac{a^2b^2y'}{(b^2y)^2}$

$=\frac{a^2b^2(-a^2x)(b^2y)^{-1}}{(b^2y)^2}$

$y''=\frac{-a^4b^2x}{(b^2y)^3}$

IG-4 Find the formula for the nth derivative $y^{(n)}$ of $y=1/(x+1)$.

$y^{(1)}=-(x+1)^{-2}$

$y^{(2)}=2(x+1)^{-3}$

$y^{(3)}=-6(x+1)^{-4}$

$y^{(4)}=24(x+1)^{-5}$

$\boxed{y=(-1)^n(n!)(x+1)^{-n-1}}$

IG-5 Let $y=u(x)v(x)$

a. Find y', y'' and y'''

got the pattern from oies.com; just didn't thought it to be right XD

$y'=\frac{d}{dx}(u(x)v(x))$

$y'=u'(x)v(x)+u(x)v'(x)$

$y''=\frac{d}{dx}u'(x)v(x)+u(x)v'(x)$

$y''=u''(x)v(x)+u'(x)v'(x)+u'(x)v'(x)+u(x)v''(x)$

or $u''v+2u'v'+uv''$

got this wrong the first time

$y'''=\frac{d}{dx}y''$

$=u^{(3)}v+u^{(2)}v^{(1)}+2(u^{(2)}v^{(1)}+u^{(1)}v^{(2)})+u^{(1)}v^{(2)}+uv^{(3)}$

$=u^{(3)}v+3u^{(2)}v^{(1)}+3u^{(1)}v^{(2)}+uv^{(3)}$

b. The general formula for $y^{(n)}$, the nth derivative, is called the Leibniz' formula. It uses the same coefficients as the binomial theorem.

$y^{(n)}=u^{(n)}v+\binom{n}{1}u^{(n-1)}v^{(1)}+\binom{n}{2}u^{(n-2)}v^{(2)}\dots uv^{(n)}$

Use this to check your answer in part (a), and calculate $y^{(p+q)}$, if $y=x^p(1+x)^q$

$y^{(p+q)}\Big|_{p+q=n}=x^{(p+q)}(1+x)+\binom{n}{1}x^{(p+q-1)}(1+x)^{(1)}$
$+\binom{n}{2}x^{(p+q-2)}(1+x)^{(2)}\dots x(1+x)^{(p+q)}$

, because $(1+x)^{(n)}=0 \Leftrightarrow n>1$

$=x^{(p+q)}(1+x)+\binom{n}{1}x^{(p+q-1)}$

IH-1 The half-life λ of a radioactive substance decaying according to the law $y=y_0e^{-kt}$ is defined to be the time it takes the amount to decrease to 1/2 of the initial amount y_0.

a. Express the half-life λ in terms of k. (Do this from scratch - don't just plug into formulas given here or anywhere.

$\frac{y}{y_0}=e^{-kt}$

$-kt=\ln\left(\frac{y}{y_0}\right)$

$-k\lambda=\ln\left(\frac{1}{2}\right)$

$-k=\frac{\ln 1-\ln 2}{\lambda}$

$k=\frac{\ln 2-\ln 1}{\lambda}=\frac{\ln 2}{\lambda}\Rightarrow\lambda=\frac{\ln 2}{k}$

b. Show using your expression for λ that if at time t_1 the amount is y_1 then at time $t_1+\lambda$ it will be $y_1/2$ no matter what t_1 is.

$\lambda=\frac{\ln y_0-\ln y}{k}$

$t_1=\frac{\ln y_0-\ln y_1}{k}$

$t_1+\lambda=\frac{\ln y_0-\ln y_1}{k}+\frac{\ln 2}{k}$

$k(t_1+\lambda)=\ln y_0-\ln y_1+\ln 2$

$-k(t_1+\lambda)=\ln y_1-\ln y_0-\ln 2$

$-k(t_1+\lambda)=\ln\left(\frac{y_1}{2y_0}\right)$

$e^{-k(t_1+\lambda)}=y_1/2y_0$

$y_0e^{-k(t_1+\lambda)}=\frac{y_1}{2}$