Implicit Differentiation and the Second Derivative

Calculate y'' using implicit differentiation; simplify as much as possible.

$$x^2 + 4y^2 = 1$$

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$y' = \frac{d}{dx}\left(x^2 + 4y^2 = 1\right) =$

$\frac{d}{dx}(2x + 8yy' = 0)$

$2 + 8y'y' + 8yy'' = 0$

$2 + 8(y')^2 + 8yy'' = 0$

$y'' = \frac{-8(y')^2 - 2}{8y}$

$= \frac{-8\left(\frac{-x}{4y}\right)^2 - 2}{8y}$

$= \frac{\frac{-8x^2}{(16y^2)} - 2}{8y}$

$= \frac{\frac{-x^2}{2y^2} - 2}{8y}$

$= \frac{-x^2 - 4y^2}{16y^3}$

$= \frac{-x^2 - 4\left(\frac{\sqrt{1-x^2}}{2}\right)^2}{16y^3}$

$y'' = \frac{-x^2 - 1 + x^2}{16y^3} = \frac{-1}{16y^3}$

$y'' = \frac{-1}{16\left(\frac{\sqrt{1-x^2}}{2}\right)^3}$

$y'' = \frac{-1}{2\sqrt{1-x^2} - 2x^2\sqrt{1-x^2}}$

$x^2 + 4y^2 = 1$

$y^2 = \frac{1-x^2}{4}$

$y = \frac{\sqrt{1-x^2}}{2}$

$\frac{d}{dx}(x^2 + 4y^2 = 1)$

$2x + 8yy' = 0$

$y' = \frac{-2x}{8y}$

$y' = \frac{-x}{4y}$

$y'' = \frac{(-1)(4y) - (-x)(4y')}{(4y)^2}$

$= \frac{-4y + 4xy'}{(4y)^2} = \frac{-y + xy'}{4y^2}$

$= \frac{-y + x\left(\frac{-x}{4y}\right)}{4y^2}$

$= \frac{\frac{-4y^2 - x^2}{4y}}{4y^2} = \frac{-x^2 - 4y^2}{16y^3}$

$= \frac{-x^2 - 4\left(\frac{\sqrt{1-x^2}}{2}\right)^2}{16y^3} = \frac{-x^2 - 1 + x^2}{16y^3}$

$y'' = \frac{-1}{16y^3}$

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