

Comparing Quadratic Approximations to Calculator Computations

In a previous worked example, we explored linear approximations to the sine function at the point $x = 0$. In this example, we use the quadratic approximation for e^x to calculate values of the exponential function near $x = 0$ and again compare the results to decimal approximations on a scientific calculator.

Find the linear approximation to e^x at the point $x = 0$ and use your answer to approximate the values of $e^{0.1}$, e^{-1} and e . Check your answer on a calculator.

$$f(x) \approx f(x_0) + f'(x_0)\Delta x + \frac{1}{2}f''(x_0)\Delta x^2$$

at $x_0 = 0$;

$$f(x) \approx f(0) + f'(0)x + \frac{1}{2}f''(0)x^2$$

$$\approx e^0 + e^0x + \frac{1}{2}e^0x^2$$

$$\approx 1 + x + \frac{1}{2}x^2 \text{ when } x \approx 0$$

linear approximation	true value
$e^{0.01} \approx 1 + 0.01 + \frac{1}{2}(0.01)^2$	$e^{0.01} = 1.010050167$
≈ 1.01005	

$e^{-0.1} \approx 1 + (-0.1) + \frac{1}{2}(-0.1)^2$	$e^{-0.1} = 1.105170918$
≈ 1.105	

$e \approx 1 + 1 + \frac{1}{2}(1)^2$	$e = 2.718281828$
≈ 2.5	

Recitation:

What is the quadratic approximation of $f(x) = e^{x+x^2}$ near $x = 0$?

$$\begin{aligned} f(x) &\approx f(x_0) + f'(x_0)(x_1 - x_0) + \frac{1}{2}f''(x_0)(x_1 - x_0)^2 \\ &\approx e^{x+x^2} + e^{x+x^2}(1+2x)(x_1 - x) + \frac{1}{2}[e^{x+x^2}(1+2x)^2 + 2e^{x+x^2}](x_1 - x)^2 \\ &\approx 1 + [1 + 2(0)](x_1 - 0) + \left[\frac{1}{2}(1)[1 + 2(0)]^2 + (1)(1)\right]x_1^2 \\ &\approx 1 + x_1 + \frac{1}{2}x_1^2 + x_1^2 \end{aligned}$$

$$\approx 1 + x_1 + \frac{3}{2}x_1^2$$

$$f(x) \approx 1 + x + \frac{3}{2}x^2$$

Notes: Had a minor calculation error and fixed it after I had a peek of the answer so 1/2 pts for this.

Additional Exercise:

Solve for quadratic approximation of $f(x) = e^{x^2}$ near $x = 0$.

$$\begin{aligned} f(x) &\approx f(0) + f'(0)x + \frac{1}{2}f''(0)x^2 \\ &\approx 1 + 0x + \frac{1}{2}(2)x^2 \end{aligned}$$

$$f(x) \approx 1 + x^2$$

$$\begin{aligned} f(x) &= e^{x^2} \\ f'(x) &= 2xe^{x^2} \\ f''(x) &= 2e^{x^2} + 4x^2e^{x^2} \end{aligned}$$

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