Activity Score: 17/20

Secants and Tangents

We defined the tangent line as a limit of secant lines. We also know that as Δx approaches 0 the secant's slope $\frac{\Delta f}{\Delta x}$ approaches the slope of the tangent line. How close to 0 does Δx have to be for $\frac{\Delta f}{\Delta x}$ to be close to the slope of the tangent line?

We'll use the Secant Approximation mathlet to look at a few examples. Use the dropdown menu in the lower left corner to select the function $f(x) = 0.5x^3 - x$. Use the red and yellow sliders to answer part (a) of each question, then use the Tangent checkbox to answer part (b). Be sure to uncheck Tangent before starting the next problem.

You may find it helps to work with a partner on this exercise.

1. Move the red slider to $x = -0.75$; we'll investigate the slopes of secant lines passing through the point $(-0.75, f(-0.75))$.

   (a) Use the yellow slider to find the value of $\frac{\Delta y}{\Delta x}$ when $x = -0.75$ and Δx has each of the following values:

   $$-0.5, -0.25, 0.25, 0.5.$$

   (b) Use the Tangent checkbox to find the (approximate) slope of the tangent line to the graph of $f(x)$ at $x = -0.75$.

   (c) Find a value of Δx for which the value of $\frac{\Delta y}{\Delta x}$ is within 0.1 units of the slope of the tangent line.

2. Now use the red slider to set $x = 0$.

   (a) Find $\frac{\Delta y}{\Delta x}$ when $x = 0$ and Δx has the values:

   $$-0.5, -0.25, 0.25, 0.5.$$

   (b) Find the slope of the tangent line to the graph of $f(x)$ at $x = 0$.

   (c) Find a value of Δx for which the value of $\frac{\Delta y}{\Delta x}$ is within 0.1 units of the slope of the tangent line.

3. Let $x = 0.75$.

   (a) Find $\frac{\Delta y}{\Delta x}$ when $x = 0.75$ and Δx has the values:

   $$-0.5, -0.25, 0.25, 0.5.$$

   (b) Find the slope of the tangent line to the graph of $f(x)$ at $x = 0.75$.

   (c) Find a value of Δx for which the value of $\frac{\Delta y}{\Delta x}$ is within 0.1 units of the slope of the tangent line.

4. Compare your answers to the previous problems.

   (a) Was your answer to part (c) the same for each problem?

   (b) For some values of x, $\frac{\Delta y}{\Delta x}$ was close to the slope of the tangent line when Δx was 0.5. For others it was not. Can you make any conjectures about when you need a very small value of Δx in order for $\frac{\Delta y}{\Delta x}$ to be close to the slope of the tangent line?

Answers

(1.) a.

Δx	$\Delta y / \Delta x$	
-0.5	0.53	✓
-0.25	0.16	✓
0.25	-0.41	✓
0.5	-0.59	✓

b. $\frac{dy}{dx}\ 0.5x^3 - x = -0.16$ ✓

$x = -0.75$

c. $-0.11 \Delta x \leq 0.08$ ✗

(2.) a.

Δx	$\Delta y / \Delta x$	
-0.5	-0.88	✓
-0.25	-0.97	✓
0.25	-0.97	✓
0.5	-0.88	✓

b. $\frac{dy}{dx}\ 0.5x^3 - x = -1.00$ ✓

$x = 0$

c. $-0.25 \leq \Delta x \leq 0.25$ ✗

(3.) a.

Δx	$\Delta y / \Delta x$	
-0.5	-0.59	✓
-0.25	-0.41	✓
0.25	0.16	✓
0.5	0.53	✓

b. $\frac{dy}{dx}\ 0.5x^3 - x = -0.16$ ✓

$x = 0.75$

c. $-0.10 \leq \Delta x \leq 0.08$ ✓

(4.) a. The range of values do overlap and one can cherry pick the Δx values to have similar answers but the Δx ranges for the three problems were not entirely the same.

✗ wrong explanation because I got the answers wrong

b. You need a small Δx to have $\Delta y / \Delta x$ fairly close to the slope if the slope is "erratic", if the region is fairly regular then it would make sense that having a greater Δx / "error" would still yield a close value to the true slope.

the solution described this as "sharply curved"

18.01SC Single Variable Calculus
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