



Homework 1

Notes:

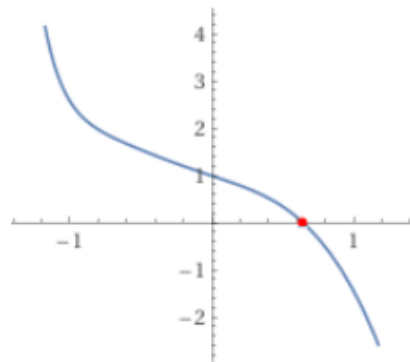
1. Do all assigned problems.
 2. The set is worth 100 points.
 3. No late HW is accepted.
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Note: I'm posting something for a sneak preview to allow people to determine if ams326 is his/her cup of tea. Please do not act on it until Feb 7 when I'll finalize the set.

Problems 1.1 (60 points, 15 for each method) Given the following function,

$$f(x) = e^{-x^3} - x^4 - \sin x$$

we know it has one root at $r \approx 0.641583$ in the interval $x \in (0, 1)$, i.e., $f(r) \approx 0$, as shown,



Please write program(s) for each of the following methods to find the root correct within 4 decimal places, i.e., your root x_c should satisfy $|x_c - r| < 0.5 \times 10^{-4}$. Also, please indicate how many iteration steps and estimate (either by hand count or by computer programming) how many floating-point operations needed to achieve your results from the initial (given) conditions:

Method 1 Use the bisection method with a given interval $a_0 = -1$ and $b_0 = 1$.

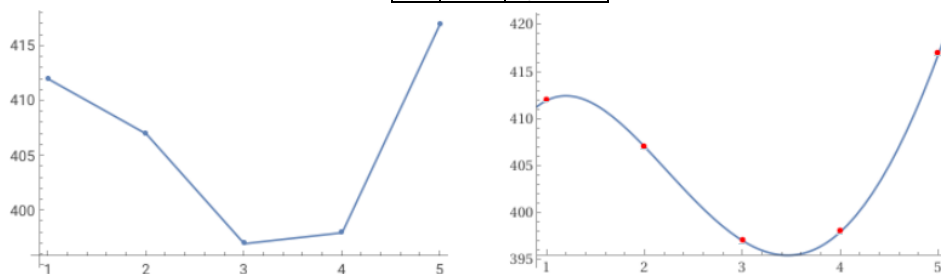
Method 2 Use Newton's method with the given initial root $x_0 = 0$.

Method 3 Use the Secant method with two given initial roots $x_{0,1} = -1, 1$.

Method 4 Use Monte Carlo method for a given range $[0.50, 0.75]$. A narrowed range is given to not blow up your laptop.

Problem 1.2 (20 points, 10 for each sub-problem): The following table is the Tesla stock closings during five consecutive trading sessions. Please complete the following

t	y	Date
05	417	Jan 29
04	398	Jan 28
03	397	Jan 27
02	407	Jan 24
01	412	Jan 23

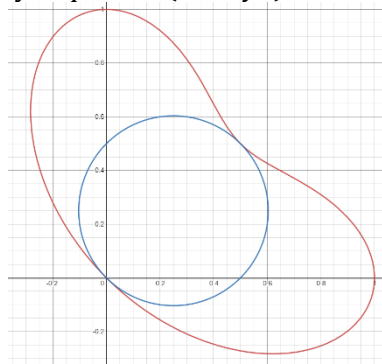


(Note: I made up the last day's stock value to make the problem more interesting!)

Please complete the following:

- (1) Interpolate the data in a polynomial $P_4(t)$ and compute $P_4(t = 6)$ using your $P_4(t)$
- (2) Make a quadratic fit of the data $Q_2(t) = a_0 + a_1t + a_2t^2$ and compute $Q_2(t = 6)$ using $Q_2(t)$.

Problem 1.3 (20 Points) The “kidney” equation $(x^2 + y^2)^2 = x^3 + y^3$ (red curve) can be graphed as



Dig a disc from the kidney. The disc equation is $(x - 0.25)^2 + (y - 0.25)^2 = 0.125$ (blue).

- (1) Write a program to use the rectangle method to compute the area of the remaining kidney (4 significant digits).
- (2) Write a computer program to use the trapezoidal method to compute the area of the remaining kidney (4 significant digits).