NUMBERS IN NATURE: FIBONACCI SEQUENCE

LEONARDO PISANO BOGOLLO

- -lived between 1170 and 1250 in Italy.
- -his nickname, “Fibonacci” roughly means “Son of Bonacci”(Fibonacci Sequence, 2016)
- -helped spread Hindu Arabic numerals through Europe in place of Roman numerals
- -he developed the famous Fibinacci Sequence (sum of the two numbers which precede it)
- 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, …

The Fibonacci Sequence

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots ,$$

$$a_0, a_1, a_2, a_3, a_4, \ldots$$

Starting with 0 and 1, each term is the sum of the two previous terms.

$$a_0 = 0$$

$$a_0 = 1$$

$$a_N = a_{N-2} + a_{N-1}$$

$$a_3 = a_{3-2} + a_{3-1}$$

$$a_3 = a_1 + a_2 = 1 + 1 = 2$$

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The Fibonacci Sequence

The Fibonacci sequence is probably the most famous number sequence. It is named after Italian Mathematician Leonardo Pisano of Pisa, known as Fibonacci. His 1202 book *Liber Abaci* introduced the sequence to Western European mathematics, although the sequence has been described earlier in Indian mathematics.

Sunflower Seed Pattern

Some sunflowers have 21 and 34 spirals; some have 55 and 89 or 89 and 144 depending on the species. These pair of number of spirals actually forms two consecutive numbers of the Fibonacci sequence.

However, this pattern is not true for all sunflowers. Using 657 sunflowers, Swinton et al. (2016) found out that one in five flowers did not conform to the Fibonacci sequence.

Fibonacci numbers in Flowers

Figure 4. Close-up photo of sagebrush mariposa lily in Southeastern Washington in the valley of the Grande Ronde River (06/28/06)

(Source: http://science.halleyhosting.com/nature/basin/3petal/lily/calo/maculosus.htm)

Fibonacci numbers in Flowers

Figure 5. Pink Gumamela
(Source: https://www.publicdomainpictures.net/en/view-image.php?image=205827&picture=pink-gumamela)

Fibonacci numbers in Flowers

Figure 6. Kalachuchi
(Source:
https://www.publicdomainpictures.net/en/view-image.php?image=205827&picture=pink-gumamela)

Fibonacci numbers in Flowers

Figure 7. Corn marigold with 13 petals
(Source: https://pixabay.com/en/corn-marigold-coleostephus-myconis-759453/)

Fibonacci numbers in Fruits

Figure 8. Banana

(Source: https://liaisonwithalison.files.wordpress.com/2015/03/banana-e1427176951523.jpg)

Fibonacci numbers in Fruits

Figure 9. The pericarp of an apple in the form of five-pointed star
(Source: http://www.eniscuola.net/en/2015/12/17/the-perfection-of-the-snail/)

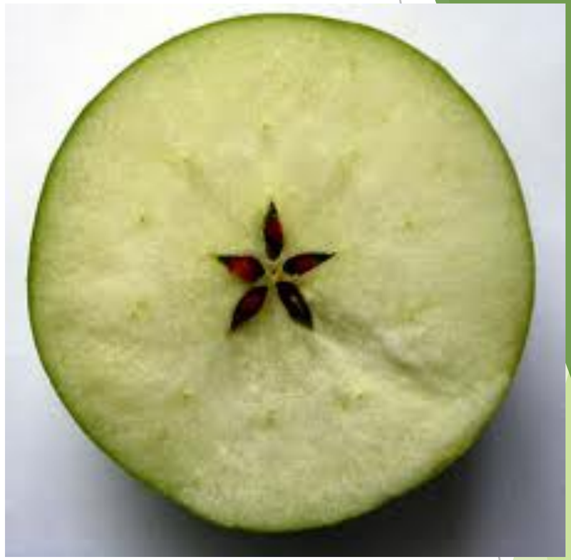


Figure 10. The pericarp of an apple
(Source: https://momanddadacademy.files.wordpress.com/2012/03/fibonacci-apple.jpg)

The Fibonacci Sequence

0 1 1 2 3 5 8 13 21 34 55 . . .

0 1 1 4 9 25 64 169 441 1156 3025 . . .

$$1 + 1 = 2$$

$$1 + 4 = 5$$

$$4 + 9 = 13$$

$$9 + 25 = 34$$

$so\ on \ldots.$

The Fibonacci Sequence

0 1 1 2 3 5 8 13 21 34 55 . . .

0 1 1 4 9 25 64 169 441 1156 3025 . . .

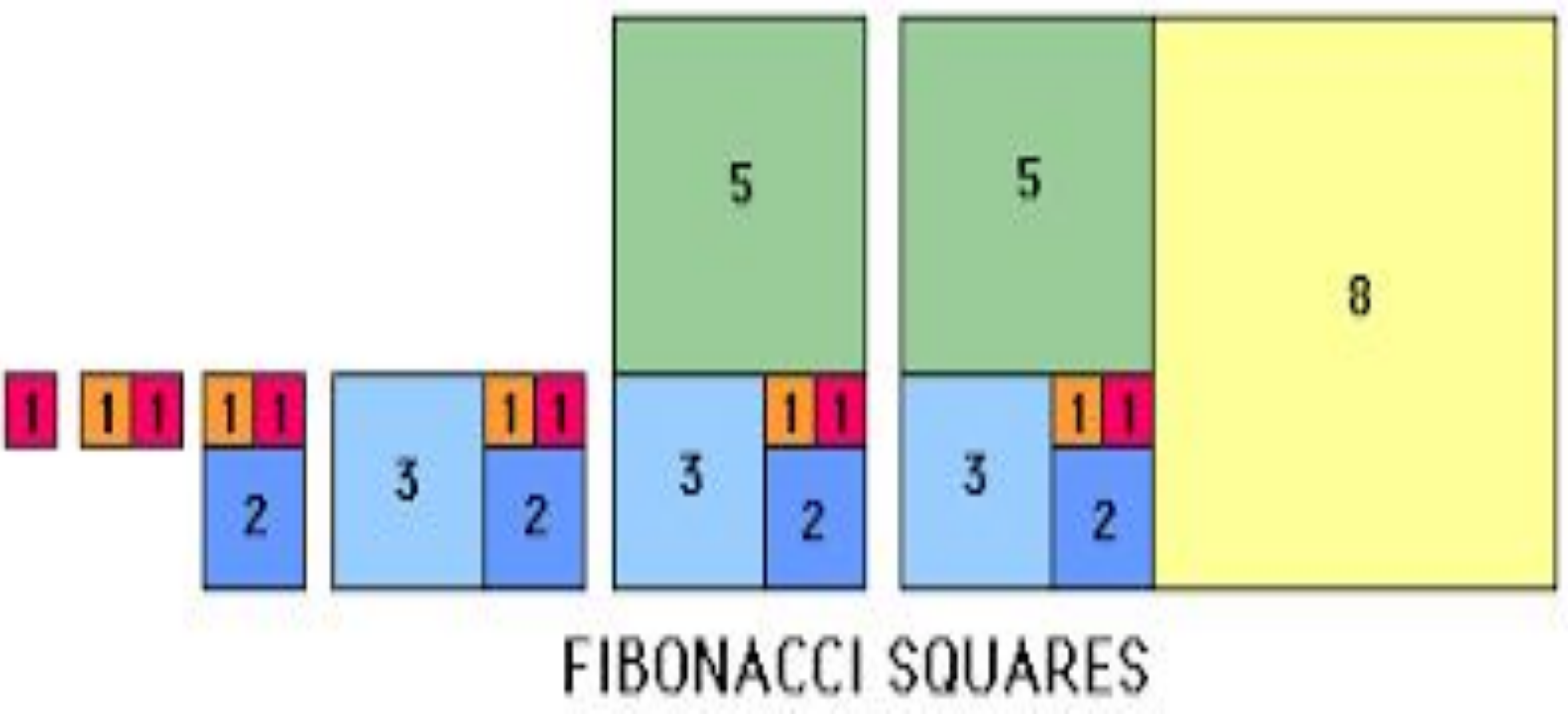
$$1 + 1 + 4 = 6 \quad = 2\ x\ 3$$

$$1 + 1 + 4 + 9 = 15 \quad = 3\ x\ 5$$

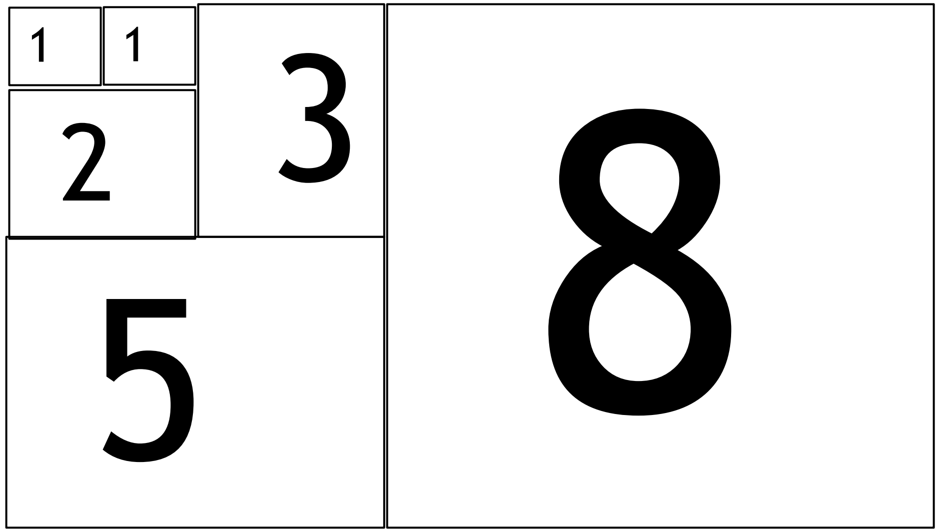
$$1 + 1 + 4 + 9 + 25 = 40 \quad = 5\ x\ 8$$

$$1 + 1 + 4 + 9 + 25 + 64 = 104 = 8\ x\ 13$$

$$1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 = 8\ x\ 13$$


1
1 1
1 1 2
3 1 1 2
5 3 1 1 2
5 3 1 1 2 8
FIBONACCI SQUARES

The Fibonacci Sequence



What is the area of the rectangle?

$$Area = 1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 =$$

$$Area = 1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 = \mathbf{H \ x \ B}$$

$$= 8 \ \mathrm{x} \ (5 + 8) = 8 \ x \ 13$$

Fibonacci in birth rate of rabbits

Fibonacci created a problem that concerns the birth rate of rabbits.

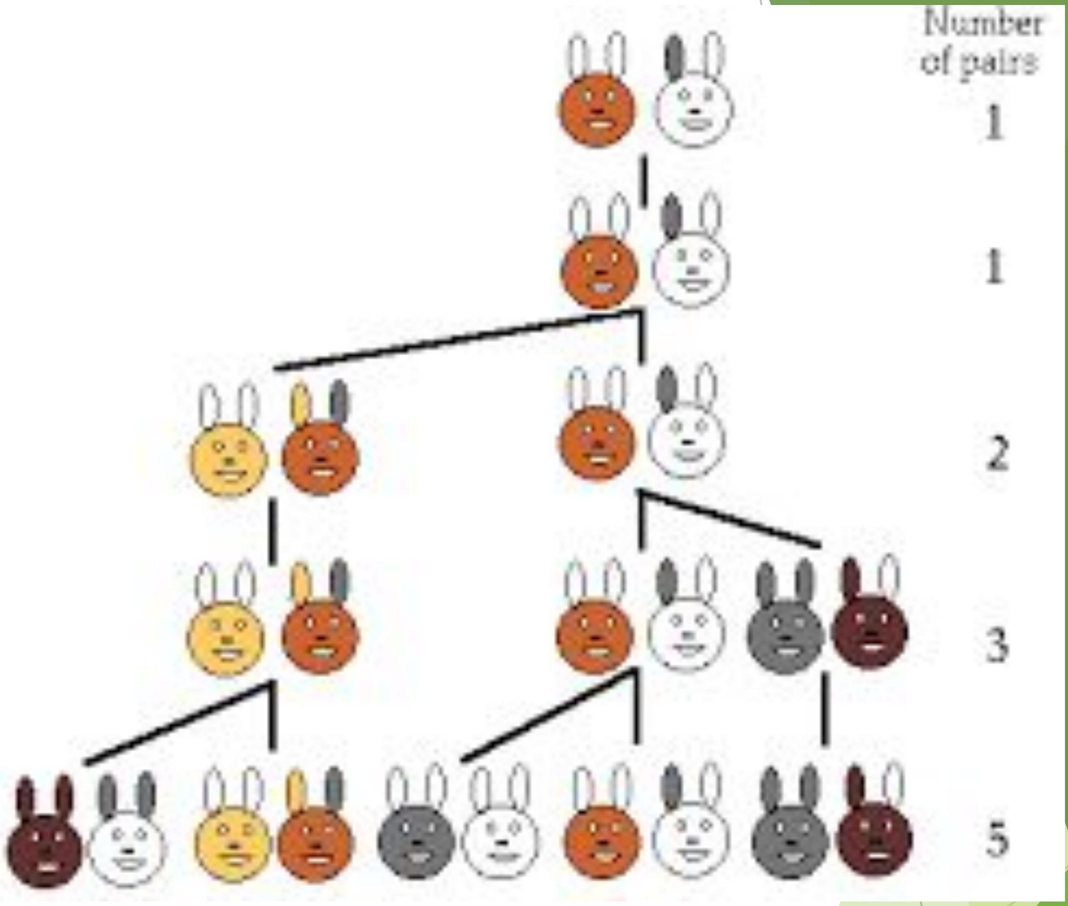
At the beginning of a month, you are given a pair of newborn rabbits. After a month the rabbits have produced no offspring; however every month thereafter, the pair of rabbits produces another pair of rabbits. The offspring reproduce in exactly the same manner. If none of the rabbit dies, how many pairs of rabbits will there be at the start of each succeeding month?

Fibonacci in birth rate of rabbits

Fibonacci then discovered that the number of pairs of rabbits for any month after the first two months can be determined by adding the numbers of pairs of rabbits in each of the *two previous* months.

The Original Rabbit Problem

A newly-born pair of rabbit (1 male, 1 female) are put into a field. Rabbits are able to mate at the age of 1 month, which means at the end of the 2nd month a female can produce another pair of rabbit. Suppose the rabbits never die and the female always produce 1 male and 1 female.



The number of pairs of rabbits at the start of each month is 1, 1, 2, 3, 5,8, 13, 21, 34, …

Fibonacci numbers and the Golden Ratio

Golden ratio (also known as Divine Proportion) exists when a line is divided into two parts and the ratio of the longer part "a" to shorter part "b" is equal to the ration of the sum "a+b" to "b".

Fibonacci numbers and the Golden Ratio

$$\phi = \frac{a}{b} = \frac{b}{a + b}$$

- The value of the Golden Ratio is given by the irrational number $\phi = 1.6180339887\ldots$ or $\phi \approx 1.618034$

THE GOLDEN RATIO

- The ratio of any two successive Fibonacci Numbers is very close to the Golden Ratio, referred to and represented as phi (φ) which approximately equal to 1.618034…

A	B	B/A = φ
2	3	1.5
3	5	1.666667
5	8	1.6
8	13	1.625
…	…	…
233	377	1.6180257511
196418	317811	1.6180339887

Fibonacci numbers and the Golden Ratio

Any two successive numbers in the Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89 ...) have a ratio very close to the golden ratio.

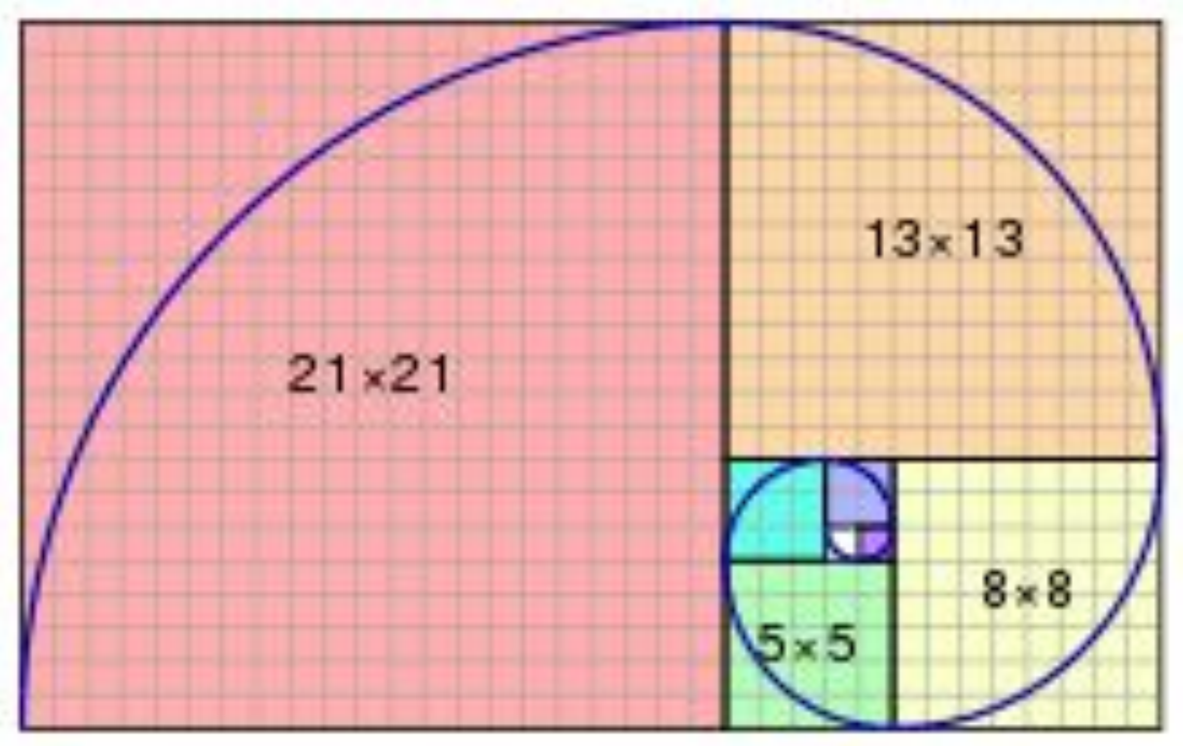
$$\frac{5}{3} = 1.6667$$

$$\frac{8}{5} = 1.6000$$

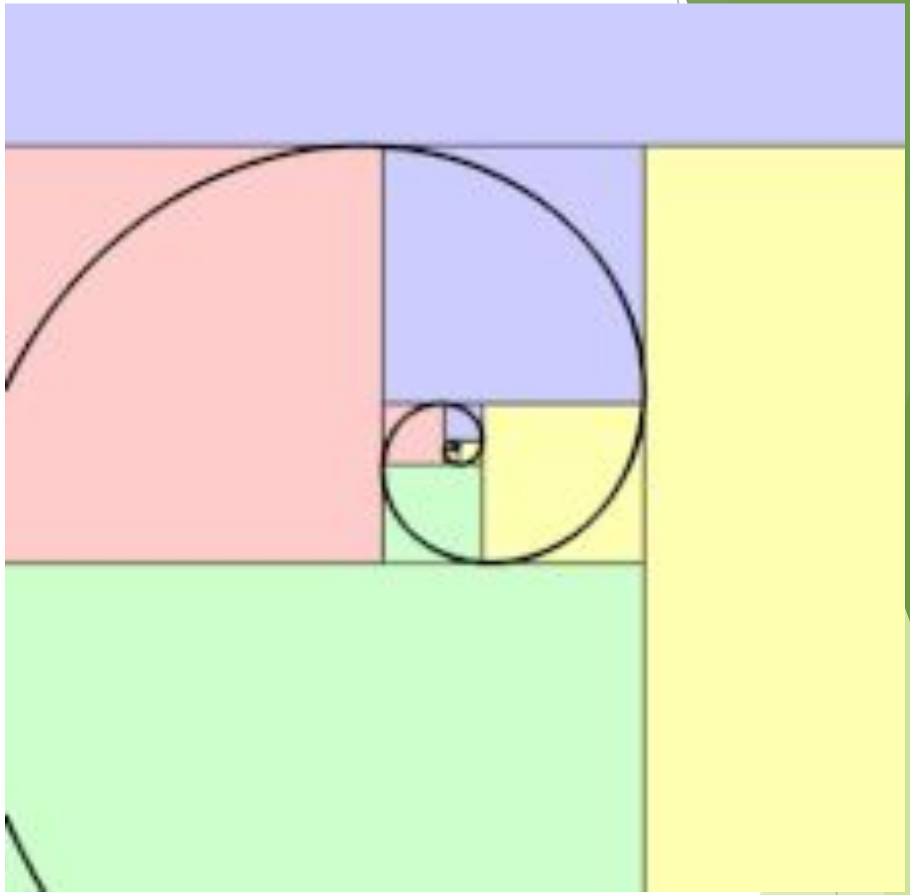
$$\frac{13}{8} = 1.6250$$

$$\frac{21}{13} = 1.6154$$

A Fibonacci spiral approximates the golden spiral using quarter circle arcs inscribed in squares of integer Fibonacci number side, shown for square sizes 1, 1, 2, 3, 5, 8, 13, and 21.



A golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio. The spiral gets wider by a factor of φ every quarter turn.



$$\varphi = \frac{(1+\sqrt{5})}{2} = 1.618034$$

Fibonacci numbers in Nature

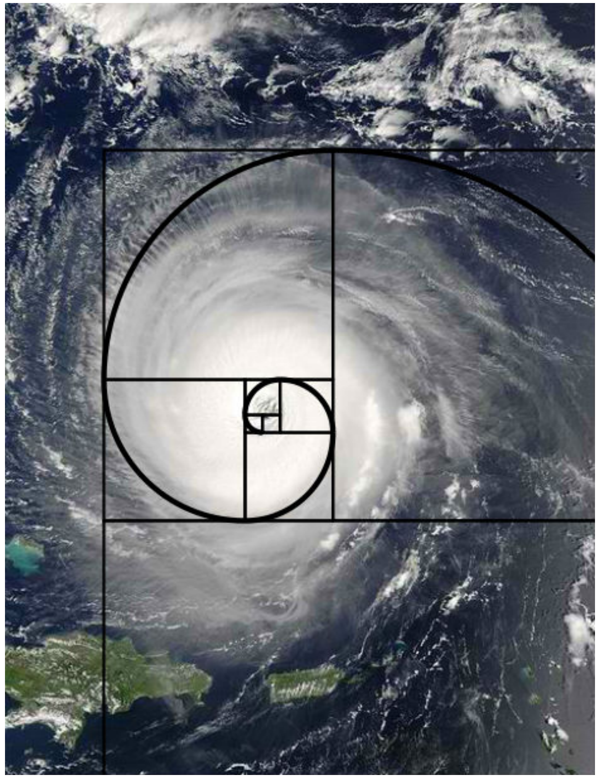


Figure 12. Satellite image of Hurricane Isabel
(Source:
http://www.momtastic.com/webecoist/2012/10/29/the-golden-spiral-complex-geome
tries-in-nature/)

Fibonacci numbers in Nature

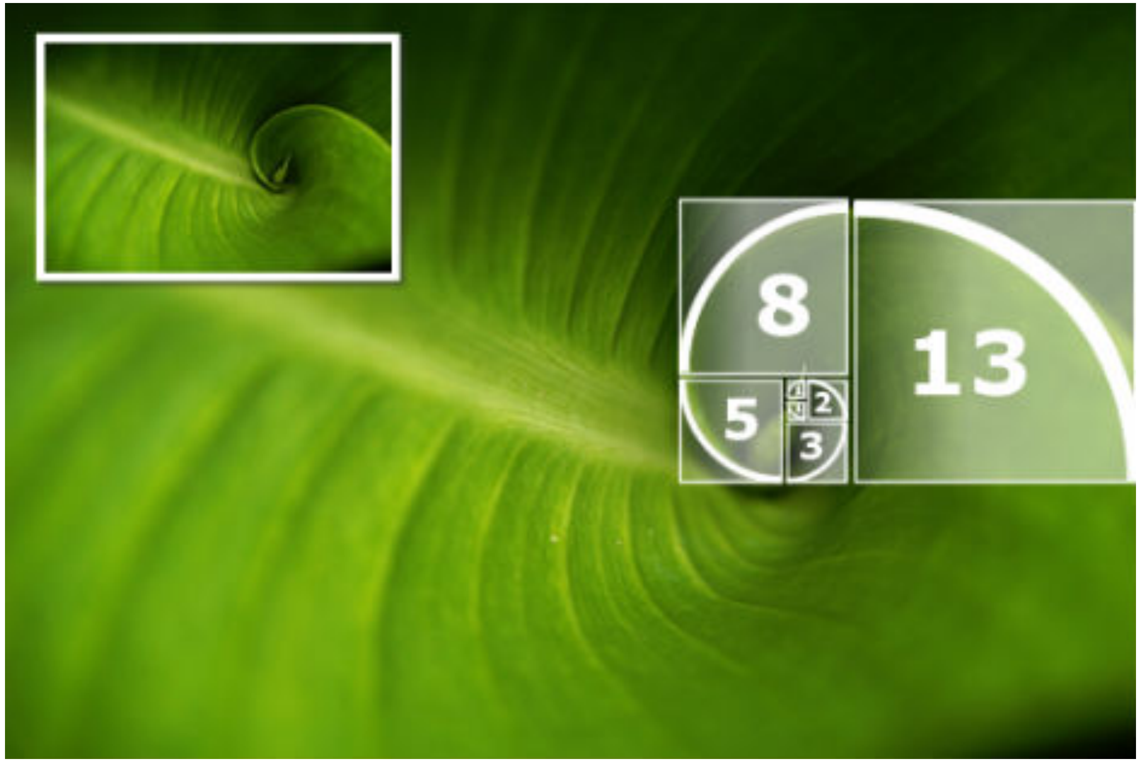


Figure 13. Highlighted image of a leaf from bromeliad plant

(Source:
http://www.momtastic.com/webecoist/2012/10/29/the-golden-spiral-complex-geo

Fibonacci numbers in Nature

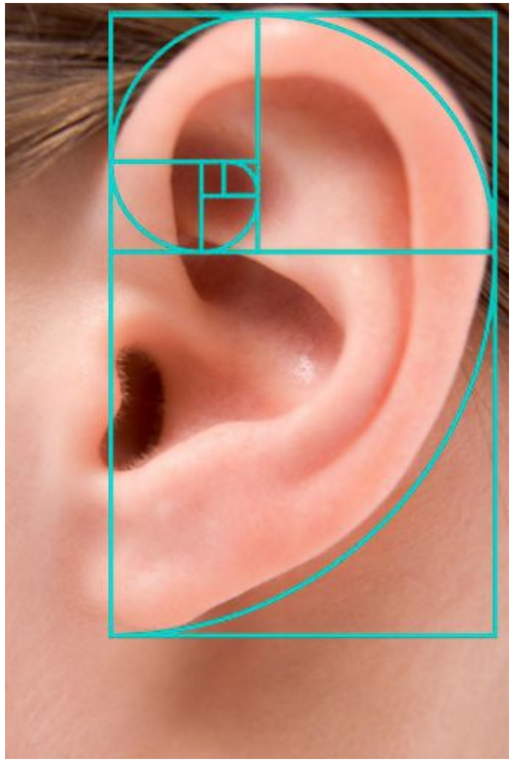
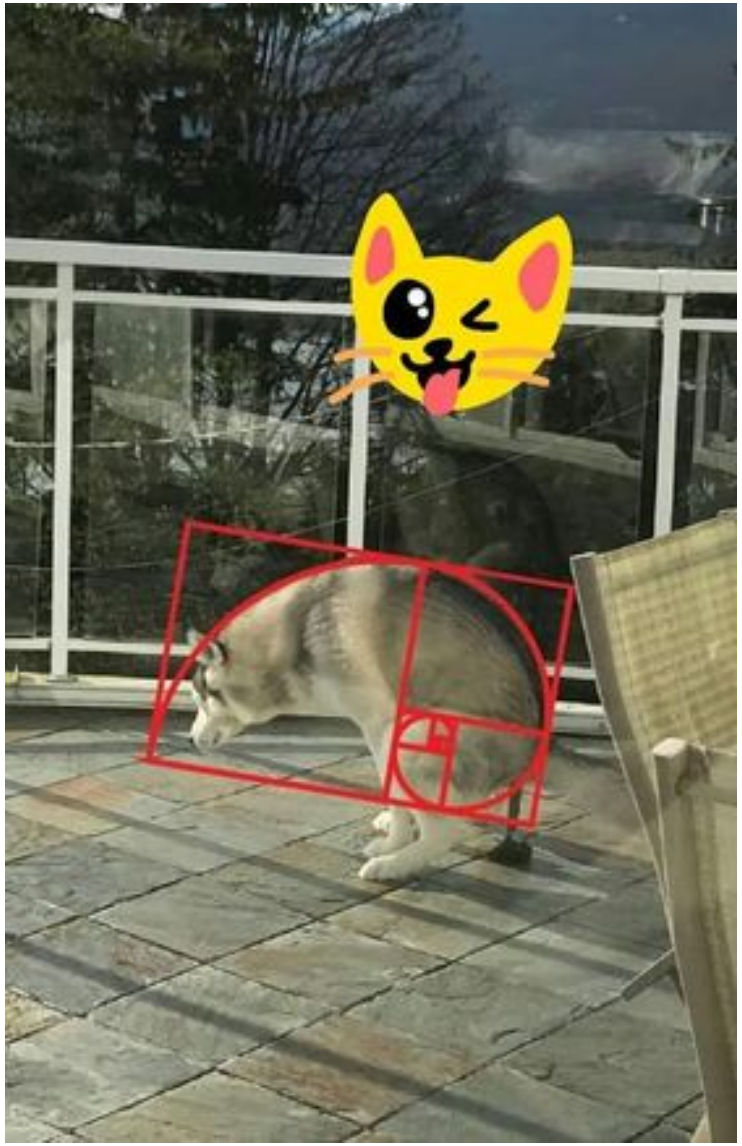


Figure 14. Human ear

(Source: https://www.pinterest.ph/pin/371406300500878451/)

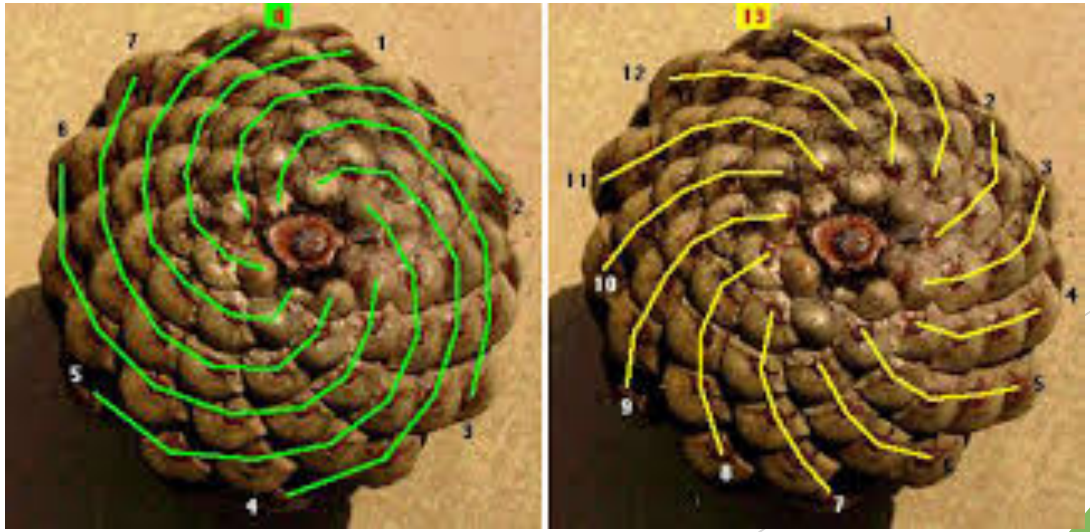
Fibonacci numbers in Nature



The Fibonacci Sequence in Nature

Spiral

Many plants grow in spirals. Often the number of spirals is a Fibonacci number and the spiral resemble the Fibonacci spiral.



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GROUP ACTIVITY: Campus Tour

- Form a group of five to six members.
- Collect pictures of patterns and regularities found inside the campus.
- Organized the collected pictures and present it to the class.

Q: What generalizations can we say about mathematics?

Generalizations:

Many patterns and occurrences exist in nature, in our world and in our life. Mathematics helps make sense of these patterns and occurrences.

Q: What generalizations can we say about mathematics?

Generalizations:

Mathematics is a tool to quantify, organize and control the world, predict phenomena and make life easier for us.

Q: What generalizations can we say about mathematics?

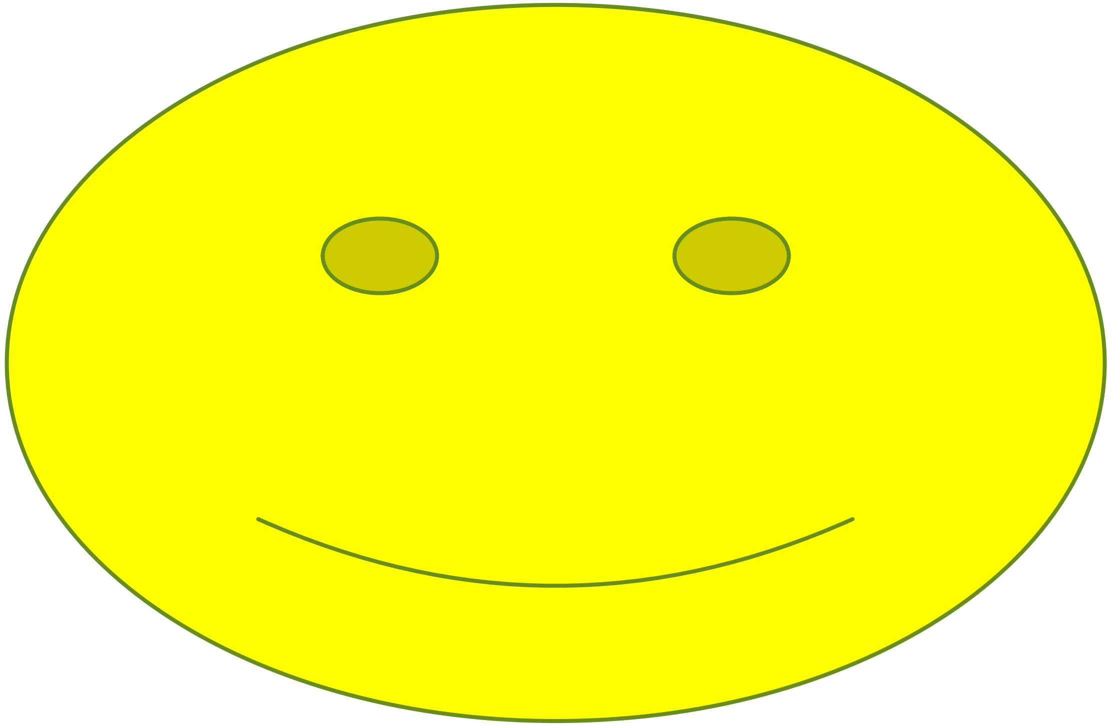
Generalizations:

Mathematics is not just solving for X, It's also figuring out Y.

Q: What generalizations can we say about mathematics?

Generalizations:

Mathematics is not just solving for X, It's also figuring out **WHY**.



Thank You!!!