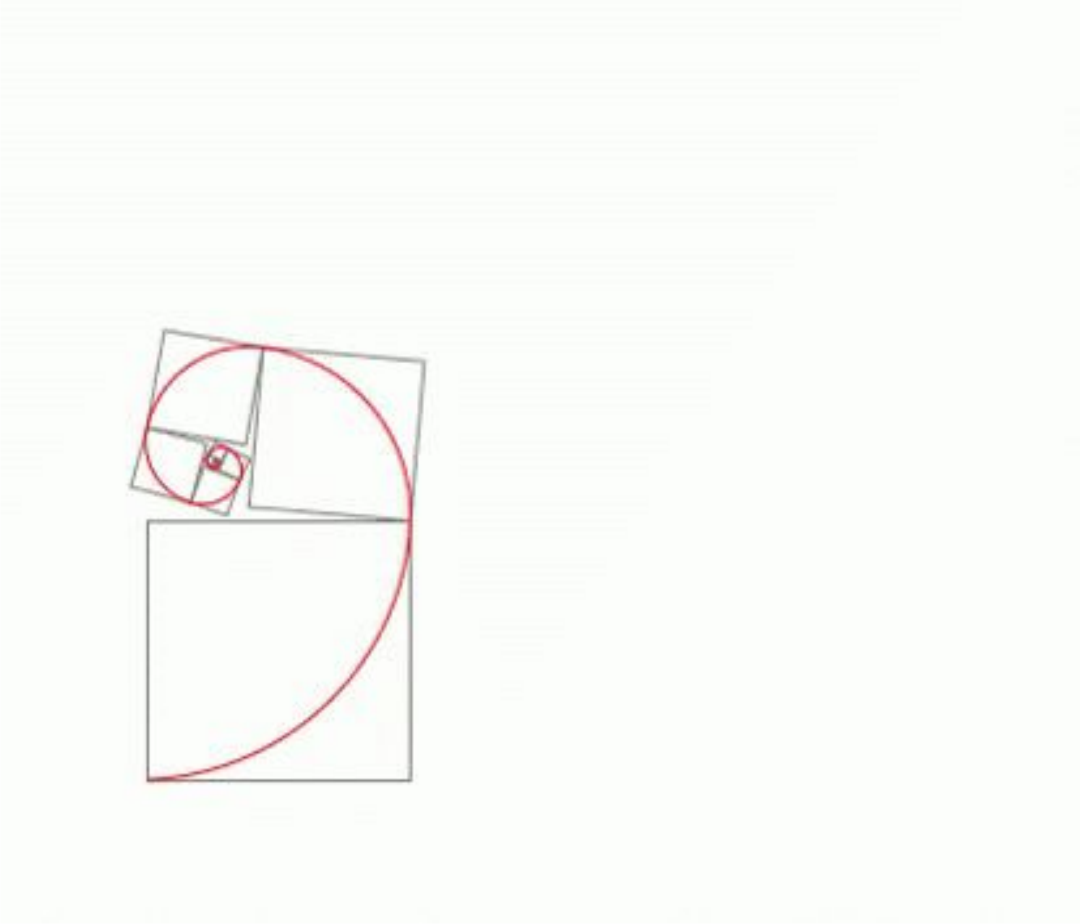
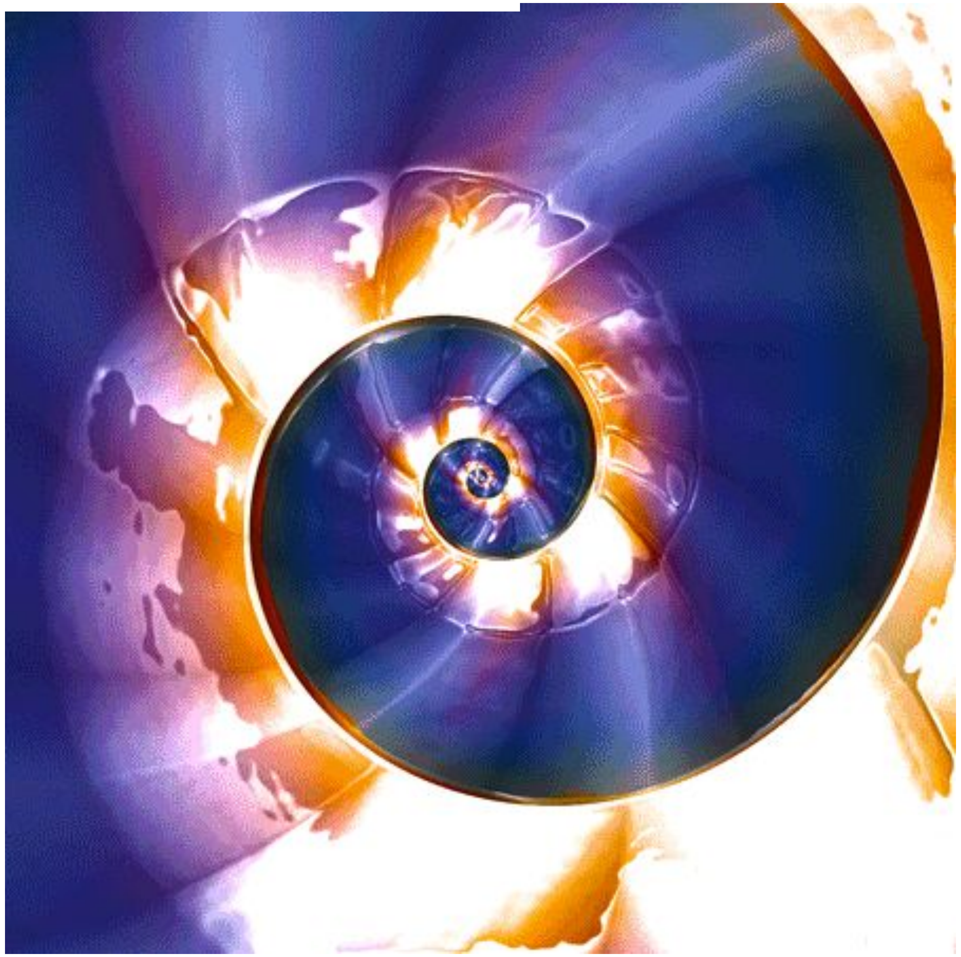
FIBONACCI





LEONARDO PISANO BIGOLLO

-lived between 1170 and 1250 in PISA (ITALY NOW).

-better known by his nickname **Fibonacci**. He was the son of Guilielmo and a member of the Bonacci family. Fibonacci himself sometimes used the name Bigollo, which may mean good-for-nothing or a traveler.

Leonard of Pisa or **Fibonacci** played an important role in reviving ancient mathematics and made significant contributions of his own. ***Liber abaci*** introduced the **Hindu-Arabic** place-valued decimal system and the use of **Arabic numerals** into Europe.

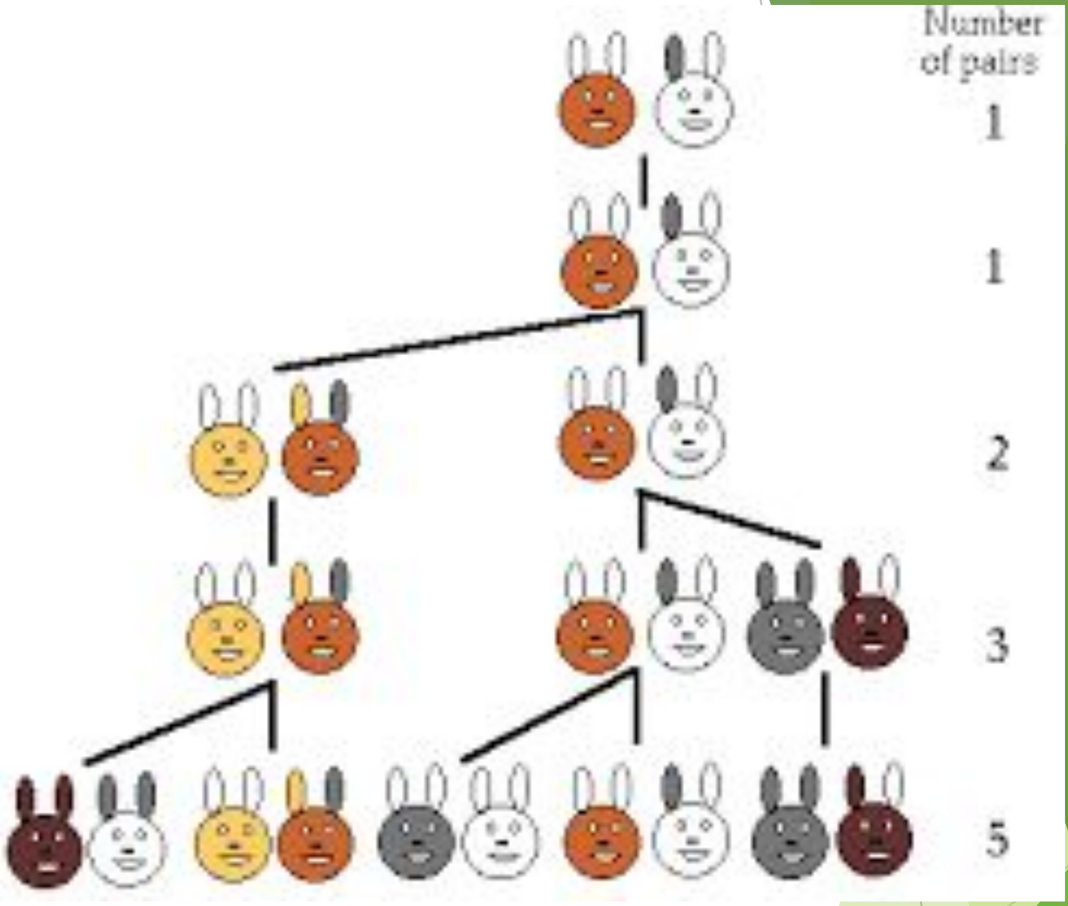
He introduced the world to such wide-ranging mathematical concepts as what is now known as the Arabic numbering system, the concept of square roots, number sequencing, and even math word problems.

Fibonacci showed the world how to use what is now our current numbering system in his book "Liber Abaci," which he published in 1202. The title translates as "The Book of Calculation."

"A certain man put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair, which from the second month on becomes productive?"

The Original Rabbit Problem

A newly-born pair of rabbit (1 male, 1 female) are put into a field. Rabbits are able to mate at the age of 1 month, which means at the end of the 2^{nd} month a female can produce another pair of rabbit. Suppose the rabbits never die and the female always produce 1 male and 1 female.



The number of pairs of rabbits at the start of each month is 1, 1, 2, 3, 5,8, 13, 21, 34, …

It was this problem that led Fibonacci to the introduction of the Fibonacci Numbers and the Fibonacci Sequence, which is what he remains famous for to this day.

The sequence is 1, 1, 2, 3, 5, 8, 13, 21, 34, 55... This sequence shows that each number is the sum of the two preceding numbers. It is a sequence that is seen and used in many different areas of mathematics and science today. The sequence is an example of a recursive sequence.

The Fibonacci Sequence defines the curvature of naturally occurring spirals, such as snail shells and even the pattern of seeds in flowering plants. The Fibonacci Sequence was actually given the name by a French mathematician Edouard Lucas in the 1870s.

The Fibonacci Sequence

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots ,$$

$$a_0, a_1, a_2, a_3, a_4, \ldots$$

Starting with 0 and 1, each term is the sum of the two previous terms.

$$a_0 = 0$$
$$a_0 = 1$$
$$a_N = a_{N-2} + a_{N-1}$$

$$a_3 = a_{3-2} + a_{3-1}$$
$$a_3 = a_1 + a_2 = 1 + 1 = 2$$

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MORE FIBONACCI NUMBERS IN NATURE

- Most of the time, the number of pedals on a flower is a Fibonacci number!

1 pedal-calla lily

2 pedals-euphorbia

3 pedals-trillium

5 pedals-columbine

8 pedals-bloodroot

13 pedals-bla eyed susan

The Fibonacci Sequence

0 1 1 2 3 5 8 13 21 34 55 . . .

0 1 1 4 9 25 64 169 441 1156 3025 . . .

$$1 + 1 = 2$$

$$1 + 4 = 5$$

$$4 + 9 = 13$$

$$9 + 25 = 34$$

so on ….

The Fibonacci Sequence

0 1 1 2 3 5 8 13 21 34 55 . . .

0 1 1 4 9 25 64 169 441 1156 3025 . . .

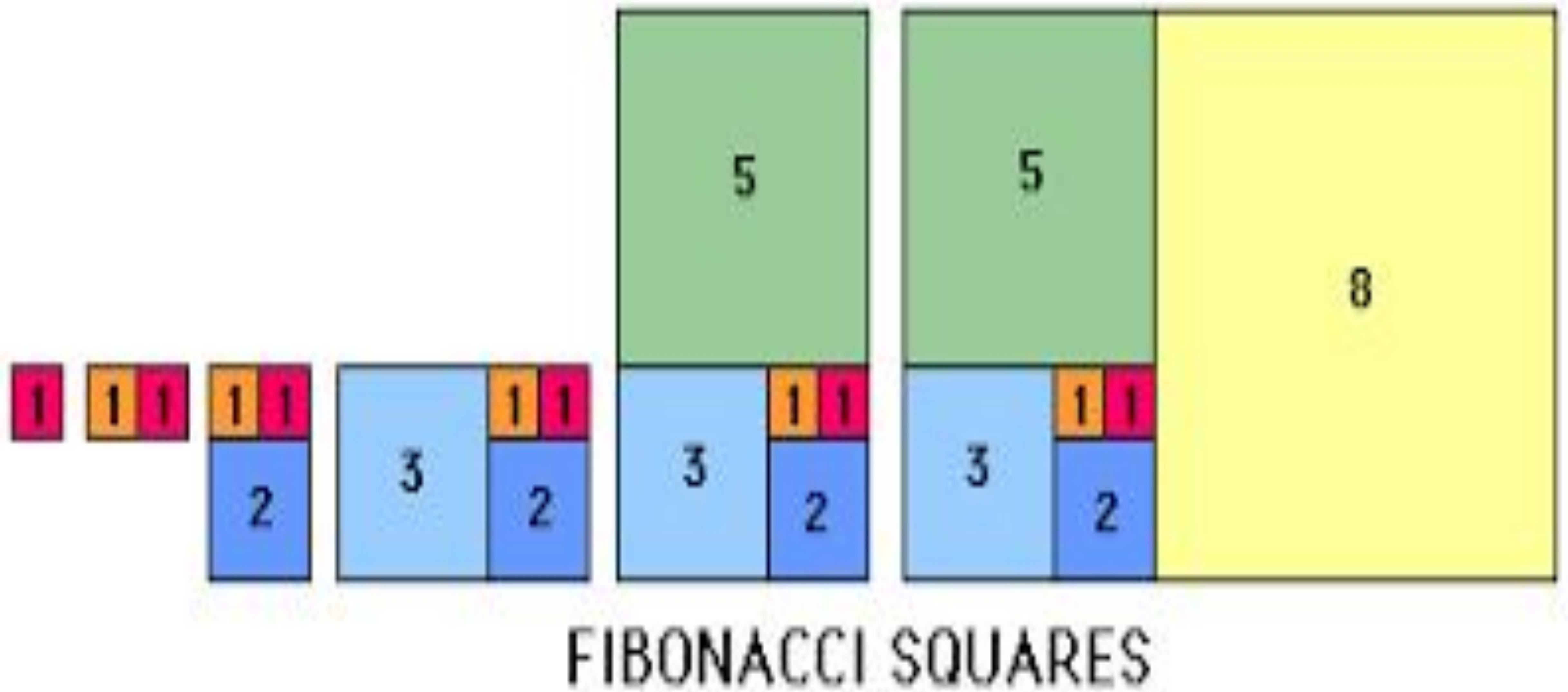
$$1+1+4=6 \quad = 2\ x\ 3$$

$$1+1+4+9=15 \quad = 3\ x\ 5$$

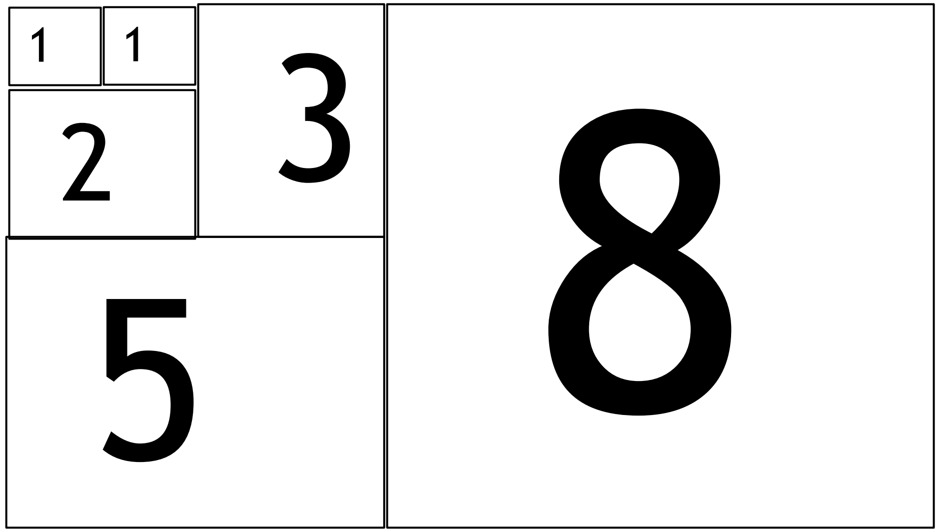
$$1+1+4+9+25=40 \quad = 5\ x\ 8$$

$$1+1+4+9+25+64=104 = 8\ x\ 13$$

$$1^2+1^2+2^2+3^2+5^2+8^2=8\ x\ 13$$


1
1 1
1 1
2
3
1 1
2
5
3
1 1
2
5
3
1 1
2
8
FIBONACCI SQUARES

The Fibonacci Sequence



What is the area of the rectangle?

$$Area = 1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 =$$

$$Area = 1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 = \mathbf{H \times B}$$

$$= 8 \text{ x } (5 + 8) = 8\, x\, 13$$

THE GOLDEN RATIO

- Golden Section

  -line segments whose ratio equal to phi (φ)

- Golden Spiral

  -a logarithmic spiral whose growth factor is phi (φ)

- Golden Triangle

  -triangle whose ratio of the sides equal to phi (φ)

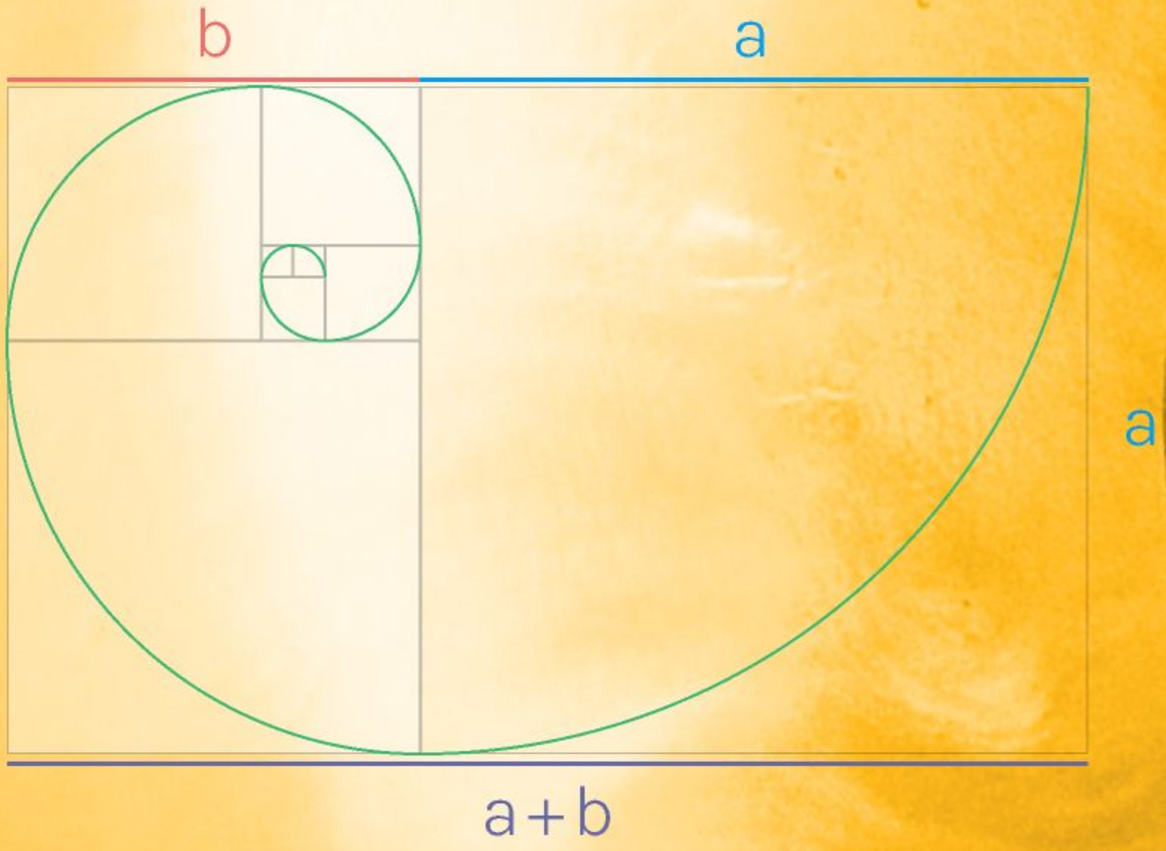
- Golden Rectangle

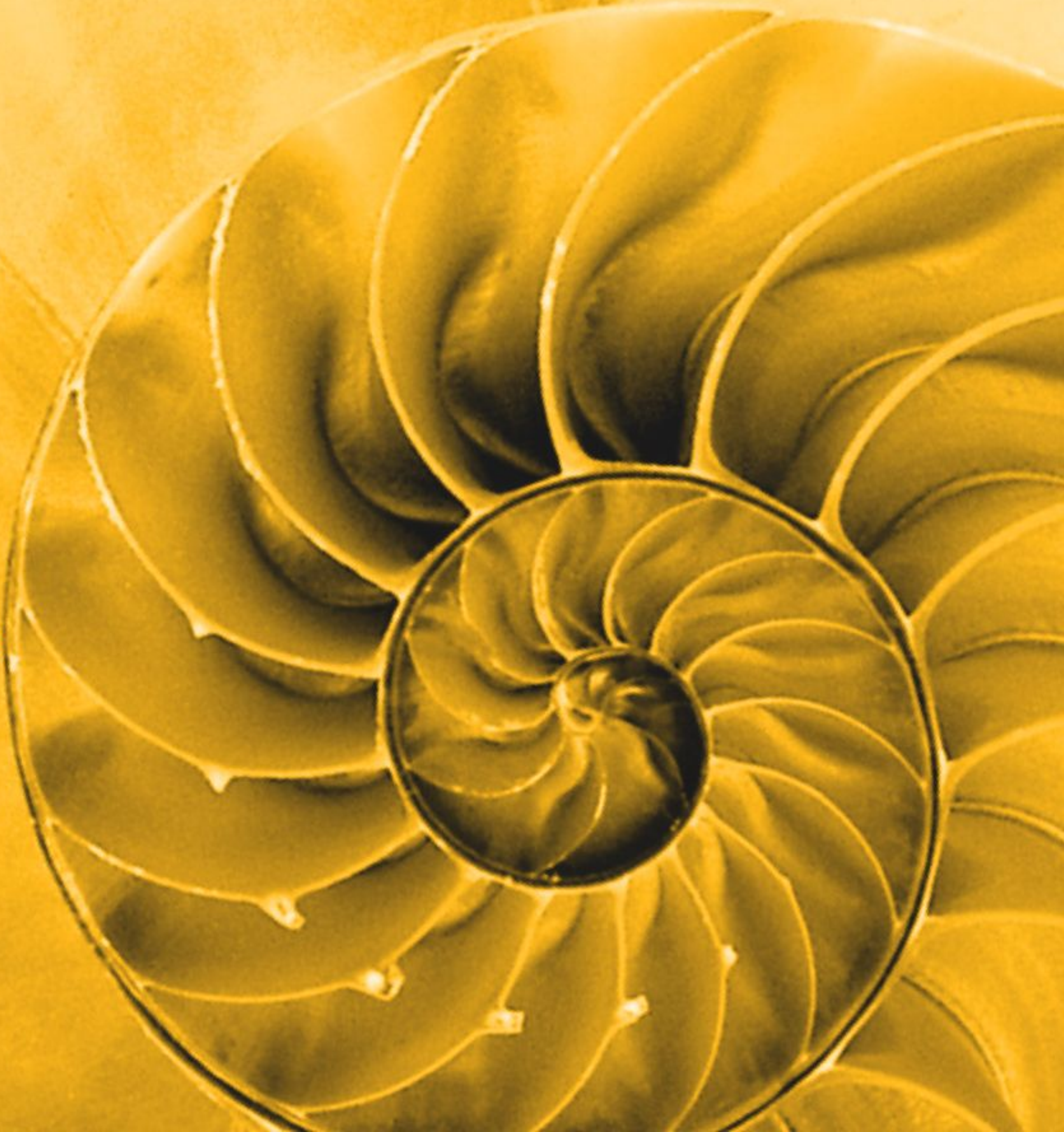
  -rectangle whose ratio of the sides equal to phi (φ)

- Golden Angle (ψ) psi

  angle (approx.137.5 degrees) whose ratio of the arc lengths equal to phi (φ)

Golden Ratio





THE GOLDEN RATIO

- The ratio of any two successive Fibonacci Numbers is very close to the Golden Ratio, referred to and represented as phi (φ) which approximately equal to 1.618034…

A	B	B/A = φ
2	3	1.5
3	5	1.666667
5	8	1.6
8	13	1.625
…	…	…
233	377	1.6180257511
196418	317811	1.6180339887

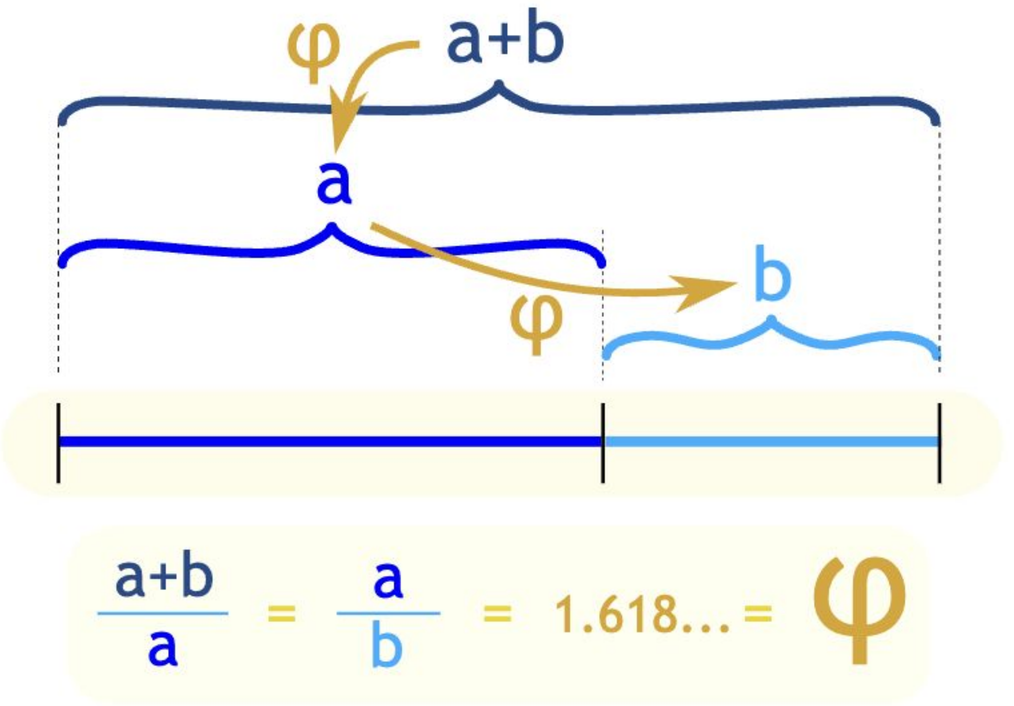
THE GOLDEN RATIO

We find the golden ratio when we divide a line into two parts so that:

the whole length divided by the long part

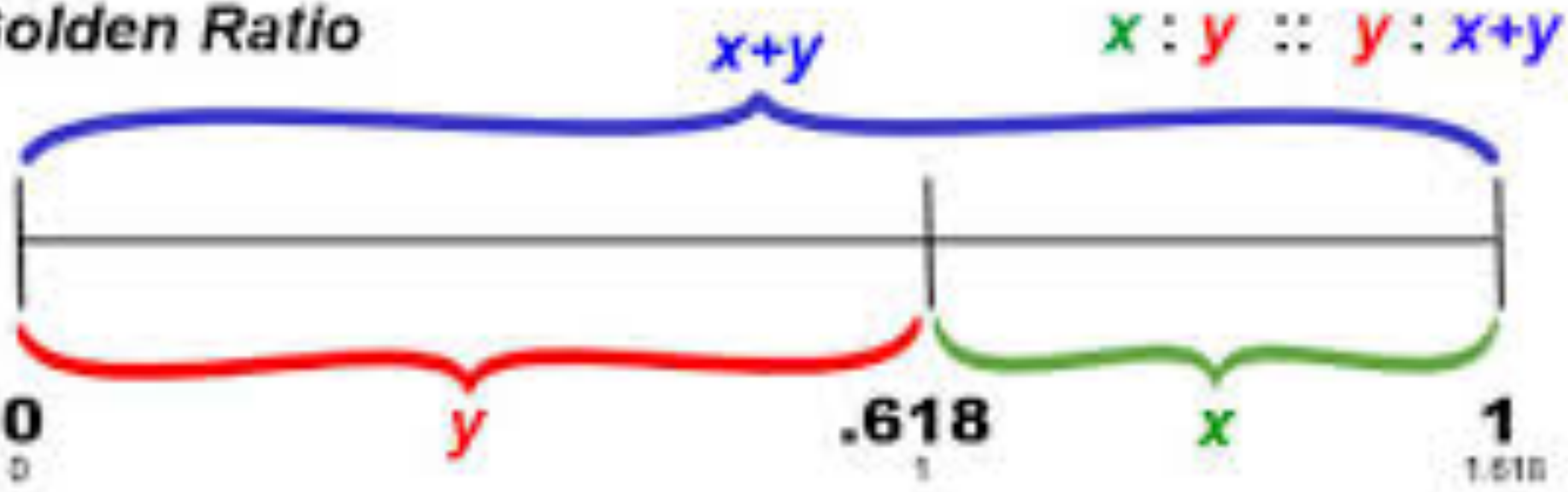
is also equal to

the long part divided by the short part



$$\varphi = \frac{(1+\sqrt{5})}{2} = 1.6180339887498948482 0\ldots \text{ (etc.)}$$

Golden Ratio

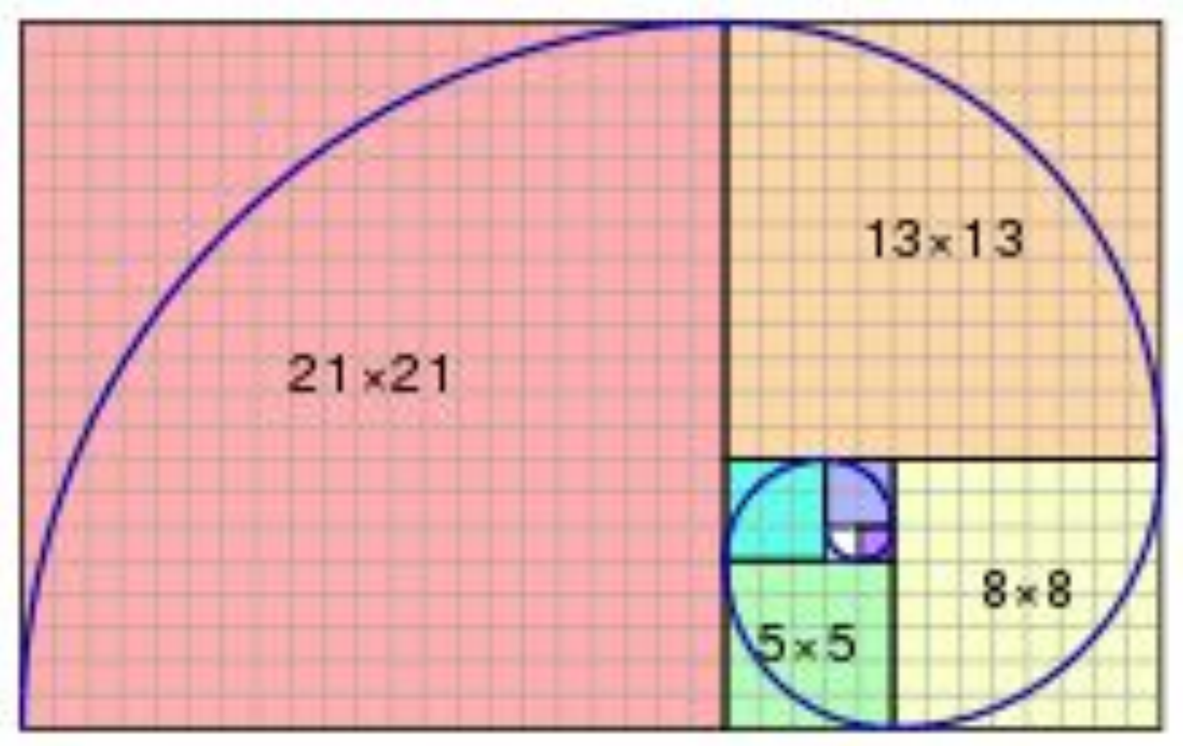


Fibonacci Sequence

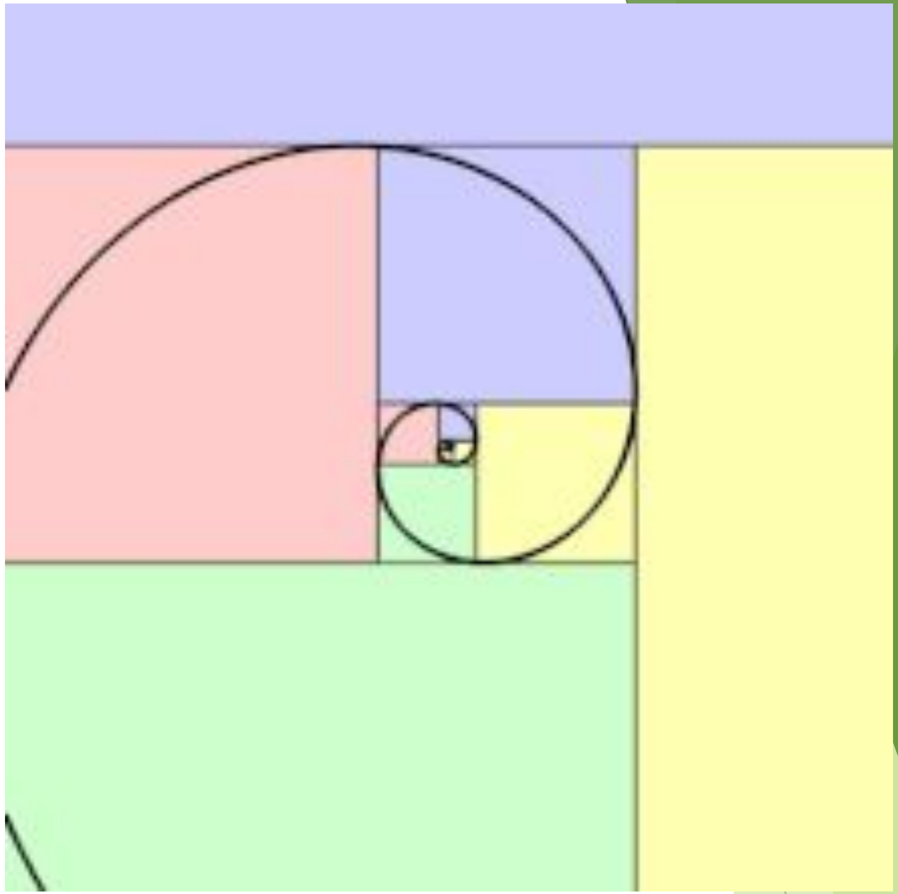
0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610...

3 x .618 ≈ 2	13 x .618 ≈ 8	55 x .618 ≈ 34
5 x .618 = 3	21 x .618 = 13	89 x .618 = 55
8 x .618 = 5	34 x .618 = 21	144 x .618 = 89

A Fibonacci spiral approximates the golden spiral using quarter circle arcs inscribed in squares of integer Fibonacci number side, shown for square sizes 1, 1, 2, 3, 5, 8, 13, and 21.



A golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio. The spiral gets wider by a factor of φ every quarter turn.

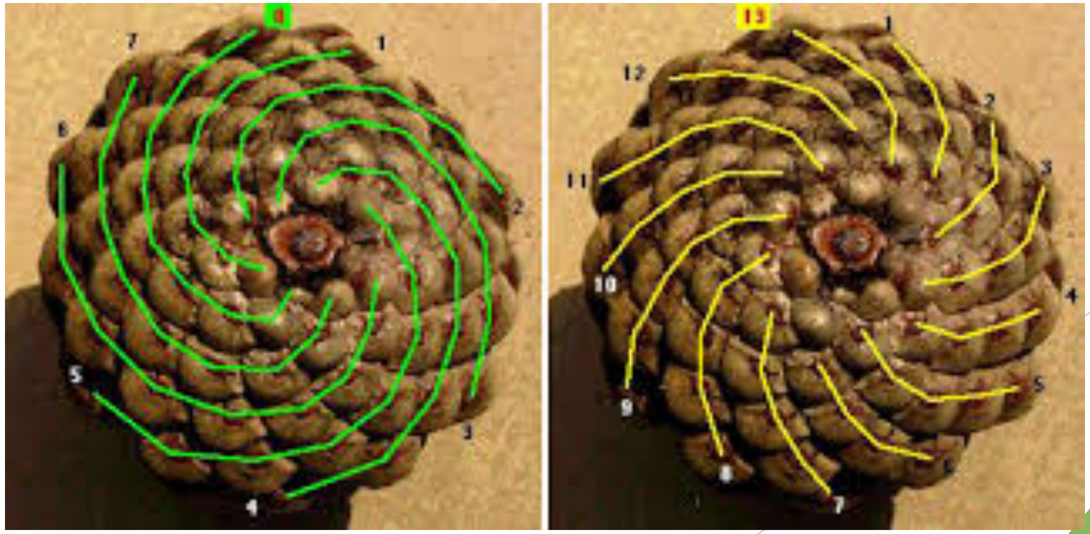


$$\varphi = \frac{(1+\sqrt{5})}{2} = 1.618034$$

The Fibonacci Sequence in Nature

Spiral

Many plants grow in spirals. Often the number of spirals is a Fibonacci number and the spiral resemble the Fibonacci spiral.



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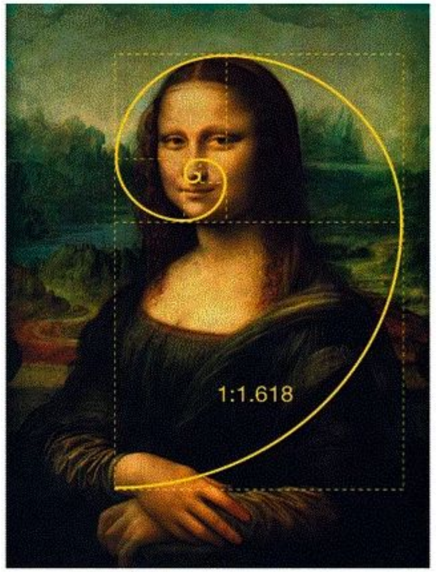

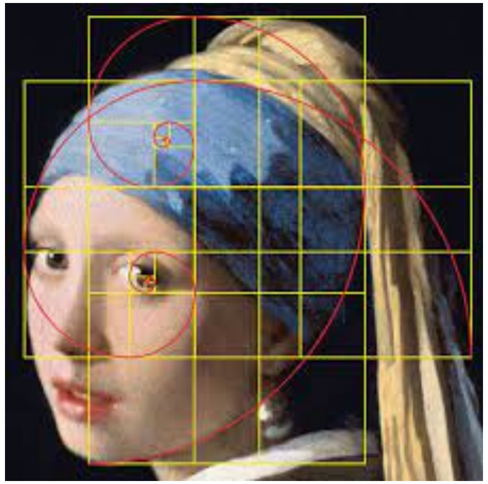

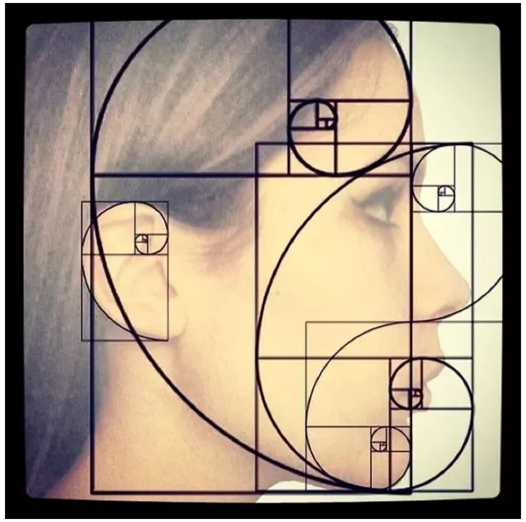

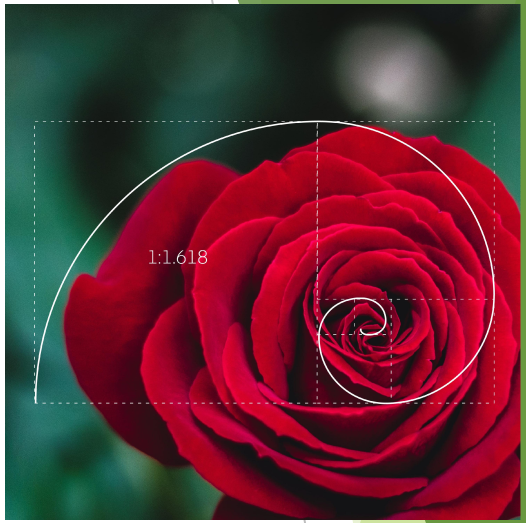

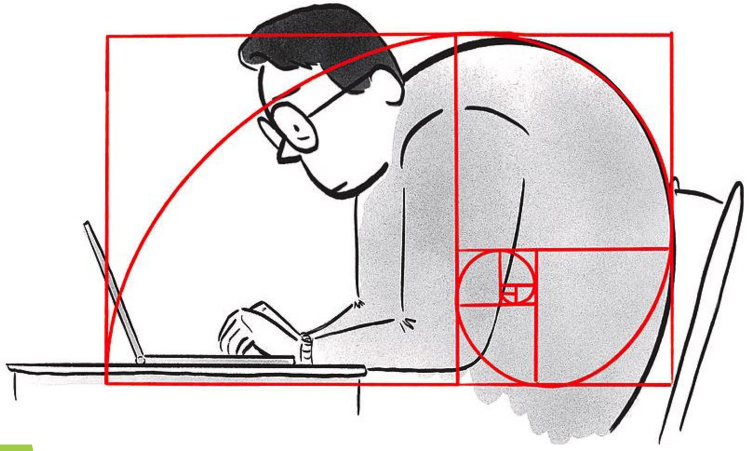

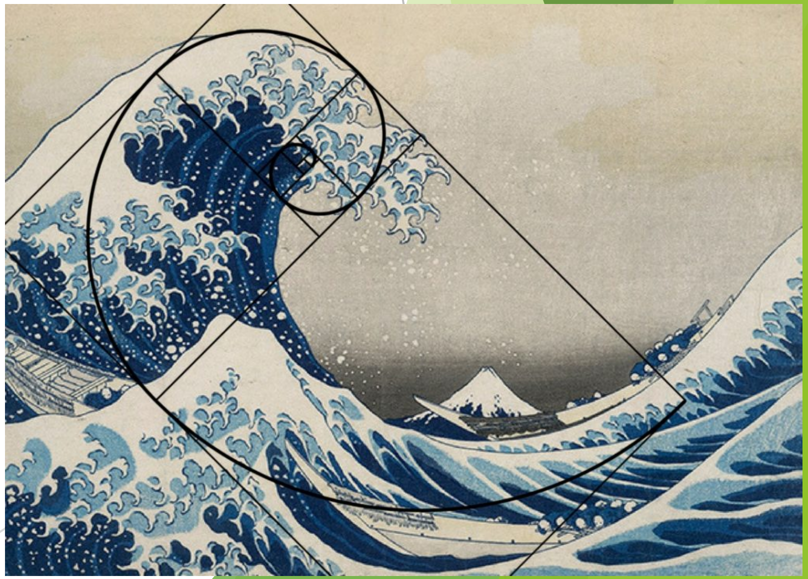

Patterns and Regularities in Nature and the World

- Scientific and mathematical principles undergrid the patterns or regularities in nature and the world.
- tree branching patterns, butterfly markings, leopard spots and tiger stripes
- snail shells, branches of tree, fractal pattern in a romanesco broccoli, spiral nautilus shell, bilateral peacock’s tail, almost perfect circular spider webs, etc.
- mud-crack patterns, water waves like waves on the surface of puddles, ponds, lakes, or oceans, rainbows, cloud formations, regular cycles of days and night, recurrence of seasons, swirling stars of a galaxy, river network, etc.
- Motion of a pendulum, reflection in a plane mirror, motion of falling object, action-reaction pair of forces, sounds and music, etc.

Q: What generalizations can we say about mathematics?

Generalizations:

Many patterns and occurrences exist in nature, in our world and in our life. Mathematics helps make sense of these patterns and occurrences.

Q: What generalizations can we say about mathematics?

Generalizations:

Mathematics is a tool to quantify, organize and control the world, predict phenomena and make life easier for us.

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Generalizations:

Mathematics is not just solving for X, It's also figuring out Y.

Q: What generalizations can we say about mathematics?

Generalizations:

Mathematics is not just solving for X, It’s also figuring out **WHY**.


ขอบคุณ~