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COMPARISON OF MARKET RISK ESTIMATION FOR SINGAPORE AND CHINA STOCK MARKETS USING VALUE-AT-RISK (VaR) AND EXPECTED SHORTFALL (ES) METHODOLOGIES AND THE STUDY OF DOWNSIDE RISK SPILLOVER EFFECTS

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by

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List of Abbreviations

ACF	Autocorrelation function
ADF	Augmented Dickey Fuller test
AIC	Akaike’s Information criterion
APARCH	Asymmetric Power ARCH
AR	Autoregressive model
ARCH	Autoregressive Conditional Heteroskedasticity
ARMA	Autoregressive Moving Average model
CAViAR	Conditional Autoregressive Value-at-Risk
CDF	Cumulative Density Function
CVaR	Conditional Value-at-Risk
DF	Dickey Fuller test
EGARCH	Exponential GARCH model
ES	Expected Shortfall
EVT	Extreme Value Theory
EWMA	Exponential Weighted Moving Average
FHS	Filtered Historical Simulation
FRM	Financial Risk Management
FTA	Free Trade Agreement
FTSE	Financial Times Stock Exchange
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
GDP	Gross Domestic Product
GED	Generalized Error Distribution
GEV	Generalized Extreme Value
GJR-GARCH	Glosten, Jagannathan and Runkle GARCH model

GPD	Generalized Pareto Distribution
HQIC	Hannan-Quinn information criterion
HS	Historical Simulation
IC	Information criterion
iid	independent and identically distributed
JB	Jarque and Bera
KE	Kernel Estimator
LBQ	Ljung-Box Pierce Q-test
LM	Lagrange Multiplier test
LR	Likelihood ratio statistic
MC	Monte Carlo Simulation
MLE	Maximum Likelihood Estimation
MRC	Market Risk Capital
OLS	Ordinary Least Squares
OTC	Over-the-counter
P&L	Profit and loss
PACF	Partial Autocorrelation function
pdf	Probability Density Function
POT	Peak-over-Threshold
QMLE	Quasi Maximum Likelihood Estimation
QQ	Quantile-quantile plot
RSS	Residual sum of squares
SBIC	Schwarz's Bayesian Information criterion
SGX	Singapore Exchange
SMA	Simple Moving Average

SPH	Singapore Press Holdings
SRC	Specific Risk Charge
SSE	Shanghai Stock Exchange index
SSE	Sum of Squared Error
STI	Singapore's Straits Times Index
SZSE	Shenzen Composite Index
TGARCH	Threshold GARCH model
VaR	Value-at-Risk
VAR	Vector Autoregression
VC	Variance-covariance

Abstract

Financial risk management is important nowadays with the frequent economic downturn and increase in market turbulence. VaR has become a popular and reliable market risk measure but it lacks the sub-additivity property and ability to describe losses beyond VaR. ES is the upcoming risk measure advocated by Basel Committee recently as it can overcome these flaws. This work aims to estimate the downside market risk of the most recent Singapore's Straits Times Index (STI) and China's Shanghai Stock Exchange (SSE) composite market indices by using VaR and ES risk indicators. Composite market index is a good representation of a typical portfolio of volatile financial assets, reflecting market risk well. Various VaR and ES estimation approaches (parametric, non-parametric and semi-parametric) are used and their performances are assessed via various backtesting methodologies. The estimations were done using rolling window approach which uses initial in-sample-data from 2009-2013 to forecast daily out-of-sample VaR or ES from 2014-2016. The backtesting results demonstrated that variants of dynamic GARCH-EVT and GARCH filtered HS approaches are good VaR or ES estimators for both indices. STI index is less volatile and the risk can be described by some static models including Extreme Value Theory and Historical Simulations. This is completely opposite for SSE index. In most cases, either t-distribution or GED model performs better than normal distribution (but good for VaR of STI index) due to leptokurtic data. Excluding VaR at 99% confidence level for STI index, most risk estimator can be described accurately with one of the dynamic asymmetric GARCH (T/GJR-GARCH, EGARCH) or APARCH model with GED or t-distribution. APARCH model does not have advantage compared to other variants of GARCH despite its freedom to model in power parameter. Within each variant of GARCH, GED has the best ES estimation followed by t-distribution and normal distribution. RiskMetrics EWMA is the least preferred choice except for VaR at 99% confidence level and ES at 99% confidence level for SSE index. This also applies to exponential weighted HS in VaR estimation for SSE index at 95% confidence level and for STI index. Exponential weighted HS, however, does perform better than Historical Simulation for SSE index. With performance of ES analogous to that of VaR, the outlook of ES replacing VaR is promising. As the Chinese economy has grown influential, this dissertation also studied the linkage and interdependency between financial market of China and Singapore in terms of downside risk spillover. Engle and Granger cointegration test suggested that there is no long-run relationship between downside risks of the two markets. Conversely, Granger causality test under VAR framework has showed that there is short-run risk spillover effect from China's market to Singapore's market and vice-versa. Specifically, the spillover effect from Singapore's market to China's seems to be weaker. These results would be useful for fund managers, investors in portfolio management and for policy maker to guard against market catastrophe.

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1 Introduction

Financial Risk Management

Trading activities have grown rapidly and more complex in recent years, reflecting the growth in over-the-counter (OTC) derivatives market. The market crash events and losses from trading activities of institutions such as October 1987 Black Monday stock market crash, $1.4 billion lost by Barings Bank UK, $1.6 billion lost by Orange County US in 1994, failure of Long-Term Capital Management Fund 1998, Asian Currency crisis 1997, Internet bubble 2000 and subprime mortgage crisis 2007 have increased the regulatory demand for reliable risk management. *Risk* is referred as the volatility of unexpected outcomes which can occur in the value of assets, equities or earnings (Jorion 2007b).

Financial risk is caused by movements in financial market activities that bring losses or gains. There are several types of financial risks: credit risk, market risk, liquidity risk and operational risk. *Credit risk* is the risk of losses due to counterparties not fulfilling their contractual obligations and defaulting on loan or bond assets. *Market risk* is the risk of losses due to financial market price movement or volatilities related to interest rate, equity, exchange rate or commodity. *Liquidity risk* results from insufficient market activities or the sudden needs for liquidity by selling assets at discount due to cash flow problem. *Operational risk* is the risk of losses results from failed internal processes, systems, people, or from external events related to fraud, trading errors, legal and regulatory issues (Jorion 2007b).

Financial risk management (FRM) involves the design and implementation of procedures to identify, measure and manage financial risks. Despite being costly, a proper FRM is essential as underestimation of risk leads to incorrect decisions regarding capital requirement which caused financial distress in financial institutions such as Lehman Brothers, Bear Stearns and Merrill Lynch during financial crisis 2007. Some of the risk controlling approaches include establishing stop-loss limits, placing limit on notional amount, implementing sensitivity analysis (eg. duration analysis for bonds or delta analysis for options) and scenario analysis (eg. *stress testing*) (Jorion 2007b). Another commonly used risk measure of a financial asset is *standard deviation*. Growing significance of market risk has prompted regulators to add market risk component to bank's risk management beside credit risk.

Value-at-Risk (VaR)

Value-at-Risk (VaR) has emerged as the most prominent instrument to manage and measure *downside market risk* and has been widely used since 1993 among practitioners, namely hedge funds, pension funds, banks and financial institutions. It was made popular by US investment bank J.P. Morgan who incorporated it in their risk management model *RiskMetrics* in the late 1980s and by the risk-adjusted capital adequacy measures enforced by the Basel committee. Its popularity is also due to its simplicity and easy interpretation. VaR takes into account the correlation between risk factors and combines all the potential losses in a portfolio into a single number that can be readily used at trading desk, for regulation (setting capital requirements) and for backtesting.

VaR is the *maximum level of loss* that will *not be exceeded* over a specific time horizon (*k*-days) with *q%* level of confidence, certainty or probability. It is effectively a 100α% (α = 1-*q*) conditional quantile of the projected profit and loss (P&L) distribution over the target horizon, i.e. corresponds to the α lower tail level or probability (Jorion 2007b).

$$\alpha = \int_{-\infty}^{VaR_{t,q}} f_t \, dX$$

where f_t represents the true return distribution function (usually not known and is estimated) for date *t* on information set time *t*-*k*.

Let V_t be the market value of a financial asset on day *t*. A *k*-day VaR (negative) on day *t* is defined by:

$$P\left(V_t - V_{t-k} \leq VaR_{t,q}\right) = \alpha$$

The VaR value above is expressed in actual *dollar* amount. If r_t indicates the return value over a given time horizon, the variable $X_{q,t}$ which fulfils $\mathrm{P}\left[r_t \leq X_{q,t}\right] = \alpha$ is the VaR in *return terms* (fraction or percentage). Multiplying portfolio value V_{t-k} with $X_{q,t}$ gives the *dollar amount* VaR. This thesis will focus on VaR in *return terms* and time horizon of 1 day (k=1).

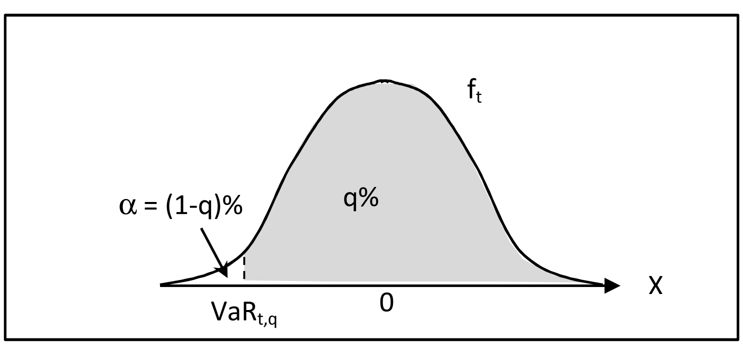

Fig 1 – illustration of Value-at-Risk (VaR)

VaR can also be viewed as the *minimum amount* that is *expected to lose* with a small probability α over a given time horizon. For example, a $\alpha = 1\%$, 1-day VaR of $10 million provides information that one out of 100 days, a minimum loss of $10 million is expected to occur. Alternatively, it could be interpreted originally that the maximum loss expected on 99 out of 100 days is $10 million.

In 1996, the Basel Committee made amendment to the market risk framework in 1988 Basel Accord and allowed banks to use *internal model* to determine their VaR and thus capital requirements provided certain criteria are met (Basel 2005). It states that VaR calculation should be on daily basis with a confidence level of 99% covering the holding period of 10 days. The observation period should be a minimum of 250 days. Basel allows bank to compute their 10-day VaR by scaling up 1-day VaR using *square root of time adjustment* rule: VaR N-days = $\sqrt{N} \times$ 1-day VaR. This rule is applicable when the assumption that returns are *iid* (independent and identically distributed) is valid. Similarly, both US Federal Reserve and European Union's Capital Adequacy Directive also allow banks to calculate their capital requirements using VaR. The use of internal models has encouraged growth and more academic research on different VaR estimating approaches.

The Basel Accord II capital requirement is simply found by multiplying the VaR by a *scaling factor* known as *hysteria factor*. It evaluates the quality of internal models through *backtesting* and checking the *violation rate*, where higher *hysteria factor* will be imposed for higher violation rate as penalty (thus higher capital charges). Higher capital charges would affect the profitability and financial institution would therefore need to choose a more stringent model to forecast their VaR. This approach is known as *Basel traffic light approach*. The Basel committee has decided that result with up to 4 violations is acceptable which defines a "green zone". Results with higher number of exceedance would fall into a "yellow" or "red" zone (Jorion 2007a). If the violation rate is more than 9 times in a year, a bank may be required to adopt the *Standardized approach*.

Table 1 – The relationship between violation rate and scaling factor for capital charge (Jorion 2007a)

Zone	Number of violations	Scaling factor (hysteria factor)
Green zone	0-4	3.00
Yellow zone	5	3.40
	6	3.50
	7	3.65
	8	3.75
	9	3.85
Red zone	> 9	4.00

The formula for market risk capital (MRC) was amended as the following, which is the higher of the previous day's VaR or the average VaR over the last 60 business days, times *k* factor. The VaR used is over 10-day horizon at 99% level of confidence.

$$MRC = \max(VaR_{t-1,0.99}\,, k \times \frac{1}{60}\sum_{i=1}^{60} VaR_{t-i,0.99}) + SRC_t$$

where SRC_t is the *specific risk charge* represents a buffer against idiosyncratic factors like default or event risk. The *k* factor is determined by local regulators, subject to absolute floor of 3 and includes the *hysteria factor* which is a penalty component (Jorion 2007a).

Two important aspects that define a VaR are the confidence level and time horizon. The level of confidence in VaR could be decided by a manager's risk appetite in which a risk-averse manager would prefer a higher confidence interval. For validation purposes, a lower confidence level would allow one to accumulate enough data with excess losses in a shorter timespan to produce reliable results. Meanwhile, the longer the time horizon, the higher the resulted VaR value assuming the same confidence level. A bank would usually be interested in a 1-day VaR since they have highly liquid currencies while longer period like 50-day VaR might be more appropriate for pension fund. A shorter time horizon also justifies the use of a normal distribution approximation.

One of the problems with VaR is that it can be estimated by different approaches which give different values. It is rather problematic to compare VaR outputs and decide which estimate to rely on. Financial institutions also have tendency to choose VaR approach that produces a lower risk estimate and therefore minimizing their capital requirement.

Expected Shortfall (ES)

VaR has been criticized because it lacks *coherence* in general and ignores losses beyond the VaR level. Artzner, Delbaen, Eber and Heath (1999) postulated that a risk measure ρ that satisfies all the following desired properties is termed as *coherent*:

- Monotonicity: If $W_1 \leq W_2$, then $\rho(W_1) \geq \rho(W_2)$. If portfolio 1 has systematically lower returns than portfolio 2, its risk must be greater
- Translation invariance: $\rho(W+k) = \rho(W)-k$. Adding amount of cash *k* to a portfolio should reduce its risk by *k*.

- Homogeneity: $\rho(bW) = b\rho(W)$. Increasing size of portfolio by b should simply scale its risk by the same factor.
- Sub-additivity: $\rho(W_1+W_2) \leq \rho(W_1) + \rho(W_2)$. The risk for two portfolios after merged should be no greater than the sum of individual risk before merged.

The fourth property above (sub-additivity) states that diversification helps reduce risk (aggregated total risk should either decrease or stay the same). VaR satisfies the first three conditions but does not always satisfy the fourth condition, penalizing diversification instead of rewarding it. See appendix for further illustration of sub-additivity. VaR does not provide information about tail of the distribution beyond VaR and cannot differentiate between two distributions with the same VaR. Consequently, Artzner et al. (1997) proposed *expected shortfall, ES* (also known as 'Conditional VaR (CVaR)', 'Mean Excess Loss', 'Expected Tail Loss', 'Beyond VaR' or 'Tail VaR') to alleviate the problems inherent in VaR. Expected shortfall (negative) is the *conditional expectation of loss* given that loss is beyond the VaR level, defined as

$$ES_{t,q} = E[X \mid X \leq VaR_{t,q}] = \frac{\int_{-\infty}^{VaR_{t,q}} X f_t \, dX}{\int_{-\infty}^{VaR_{t,q}} f_t \, dX} = \frac{1}{\alpha} \int_{-\infty}^{VaR_{t,q}} X f_t \, dX$$

The expected shortfall indicates the *average loss* when the loss exceeds the VaR level. While VaR answers question like "how bad can things get?", ES answers the part "How much is expected to lose when things do get bad". Expected shortfall satisfies all four properties above and is *coherent* measurement of risk.

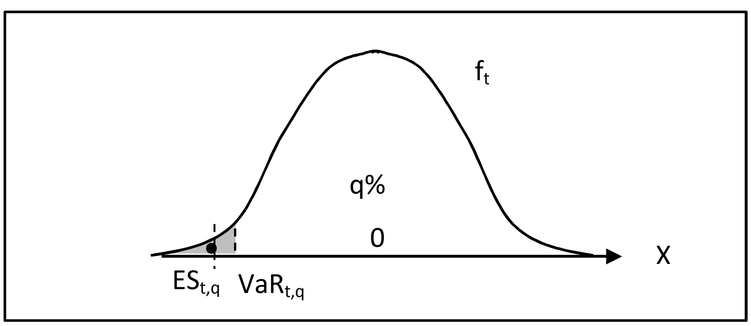

Fig 2 – illustration of Expected Shortfall (ES) as average loss beyond VaR

VaR had invited criticism and gone under scrutiny, especially after 2007 financial crisis for poor performance during market stress condition. Hence, Basel III has moved beyond VaR and endorses expected shortfall. Since May 2012, Basel Committee has promoted a switch from VaR to ES measure and a corresponding confidence level adjustment from 99% for VaR to 97.5% for ES (Basel 2012). However, the use of ES raises concern of the ability to backtest it reliably similar to VaR. In addition, Yamai and Yoshida (2005) also find that ES requires a larger sample size to provide the same level of accuracy as VaR. Whether VaR or ES is implemented, both risk measures only indicate the likely level of loss rather than the worst-case loss of individual events (Coleman 2012)

Background and Literature Review

With the country's rapid growth or progress after Deng Xiaoping's liberalization and entry into WTO, China has surpassed US and become world's largest economy in 2014 with a GDP of $17.63 million measured by *Purchasing Power* (Halbert 2015). It is undisputable that China plays a significant role nowadays influencing the world's economy. In 2015, China's economy has experienced slow down with GDP dropping to 6-7% range from 10% previously in 2010. It not uncommon to study whether there is interdependency or *spillover effect* on economies such as between stock market in China and in other countries like Singapore. If such *spillover effect* exists, historical information about risk in one market can help to forecast the current or future risk in another mark. As VaR or ES is used to measure market risk associated with equity, a reliable and accurate VaR or ES estimation technique would greatly benefit regulators and financial practitioners in the understanding of their exposed downside risks.

Composite market index is a good representation of a typical portfolio of volatile financial assets and is a proportional function of the market value of an investment portfolio. Stock market fluctuations and market risk are well reflected by composite index since it usually includes some of the biggest companies. The non-systematic risk is diversified away but systemic risk remained according to Portfolio Theory. One of the market indices used in China is *Shanghai Composite Index (SSE)* while the other is *Shenzen Composite Index (SZSE)*. The market index used in Singapore is *Straits Times Index* (STI).

Shanghai Composite Index (SSE) was established in 1990 and constitutes of all listed stocks (*A shares* and *B shares*) at Shanghai Stock Exchange which is the world's 4th largest stock market in 2014 with market capitalization of RMB 24,400 billion (SSE 2015). *A shares* are priced in the local *renminbi yuan* currency and partially opened to foreign investors after the reform in 2002. *B shares* are quoted in US dollars and are available to both domestic and foreign investors. Total number of companies on SSE by end of 2015 is 1039, including companies from diverse industries such as Air China, Bank of China, Petrochina, China Life, Daqin Railway and China Yangtze Power (SSE 2015). SSE index reached peak value in July 2015 exceeding 5000 mark but plunged subsequently to almost half the amount at the beginning of 2016. Specifically, SSE suffered an 8.5% decline, its worst one-day performance since 2007 on 24 August 2015, known as '*Black* Monday' which wiped billions off indices across the world in a day of frantic selling (McHugh 2015).

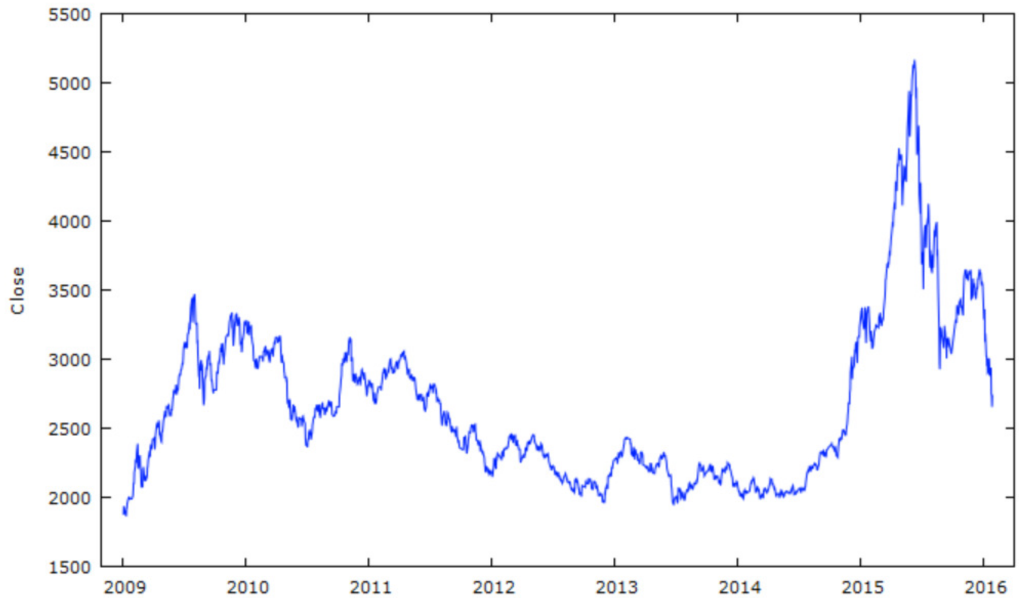

Fig 3 – Time series plot of closing price for China's SSE Index

The *Straits Times Index* (STI) is one of the globally-recognized flagship and benchmark index and serves as the national economy and market barometer for Singapore. It has long history dated back to 1966 and replaced Straits Times Industrial Index (STII). It was jointly created by FTSE Group, Singapore Exchange (SGX) and Singapore Press Holdings (SPH) and has market capitalization of SGD 230,000 million in 2016 (FTSE Russell 2016). It was relaunched in 1998 and tracks the performance of top 30 companies listed on Singapore Exchange and is therefore likely to have a heavy weightage in the trading profile of Singapore-incorporated banks. The companies are also from diverse industries and include DBS Banks, Oversea-Chinese Banks, Singapore Telecommunication, Singapore Airlines, Capitaland, Wilmar International and Keppel Corp (FTSE Russell 2016). Similar to SSE, the STI index also experienced downside risk and has since lost nearly 30% from its peak value in 2015 at the beginning of 2016.

Fig 4 – Time series plot of closing price for Singapore's STI Index

There have been numerous studies estimating market downside risk of stock index in Singapore and China using various VaR methodologies. Wong (2002) compared variants of three main VaR estimation methods: *Variance-Covariance (VC)*, *Monte Carlo (MC)* simulation and *historical simulation* (HS) using different ARCH-based volatility models and different distributions (normal, student-t and logistic distribution) on a few emerging Asia stock indices including Singapore (STI data from Jan. 1985 – Apr. 2000). His backtesting results of 1-day VaR with 99% confidence level showed that VC, MC-normal and HS underestimated the risk and produced serially dependent results, MC-logistic distribution overestimated the risk and incur high opportunity cost, EWMA volatility model with small decay factor performed relatively well, EGARCH or TGARCH asymmetric volatility model performed better than other ARCH models but he would overall recommend MC t-distribution. Lin, Li, and Tse (2006) has estimated the daily VaR using HS, VC with SMA (Simple Moving Average) and EWMA volatility models for five emerging markets in Asia including STI and SSE index (data from Jan. 1992 – Dec. 2006). They found that HS performed better than VC under normal condition and EWMA with small decay factor 0.94 performed well during financial crisis 1997 due to its fast adaptability. In addition, 99% confidence level VaR results are preferred as 95% confidence level results tend to underestimate the risk. Ong and Wang (2011) calculated VaR of STI data (different sample sizes from 2002 – 2011) using GARCH, EGARCH and TGARCH volatility models with t-distribution and alleged that EGARCH with small sample size (i.e. using more recent data) produced the least violations and generic losses (opportunity cost). For larger sample size, TGARCH model would be a better choice having less generic loss despite the slightly more violations. Guidi and Gupta (2012) pointed out that volatility modeling of STI data (from Jan. 2002 – Jan. 2012) using APARCH with t-distribution performed better than APARCH with *General Error Distribution (GED)*.

Several significant works have been made by researches on VaR estimation of STI index using EVT (*Extreme Value Theory*). Leong and Neo (2007) estimated the VaR (various confidence levels and time horizons) of stock indices of four Asian countries including Singapore (STI data from Jan. 2000 – Aug. 2007) using *block maxima* method in EVT and makes comparisons with standard deviation only without doing *backtesting*. Lu, Zhang and Lim (2001) applied EVT to stock market indices of developed countries and emerging markets including Singapore (STI data from Jan. 1986 – Mar. 2001) using *peak-over-threshold* methods to estimate whole-period VaR and studied about choosing the right threshold. Lim, Lim and Peh (2006) also applied *peak-over-threshold* method of EVT on STI index (data from 1988 – 2006), US-Japan currency exchange rate and spot gold price and found that EVT has a better performance than normal distribution. Ramaza and Faruk (2004) estimated VaR of nine emerging stock market indices including Singapore (from Jan. 1985 – Dec. 2000) using VC with normal distribution, t-distribution, HS and *peak-over-threshold* method in EVT and concluded that EVT is the best estimation method especially for VaR at high quantile.

Substantial literatures also exist on the VaR analysis of market risk in China using SSE composite index. Fan, Wei and Xu (2004) used VC with EWMA volatility model and λ=0.88 to estimate the daily VaR of Shanghai index (from Jan. 1994 – Feb. 1998) and concluded that EWMA performed well at 95% confidence level. Zou, Zhang and Qin (2003) estimated the daily VaR of SSE (from Oct. 1997 – Feb. 2001) using GARCH(1,1), SMA and EWMA volatility models and claimed that GARCH(1,1) has the best high quantile VaR performance followed by EWMA while SMA underestimated the risk. Gencer and Demiralay (2015) asserted that APARCH models are appropriate for analyzing conditional volatility for emerging stock markets including China (from Jan. 1998 – Dec. 2013). Wu (2015) studied the VaR of SSE index (from Jan. 2009 – Feb. 2014) using variants of GARCH, EGARCH and APARCH and concluded that t-distribution doesn't apply to Chinese stock market, GED is better at describing market risk than normal distribution and APARCH(1,1)-GED best at describing market risk in China. Conversely, Wang and Zhang (2010) claimed that the computation of daily VaR of SSE index (from Jan. 1997 – Apr. 2009) using EGARCH volatility model with t-distribution is accurate. Lin, Huang, Yang and WeiYu (2008) applied dynamic EVT with GARCH and GJR-GARCH volatility models to three Asia markets including Shanghai (from Dec. 1990 – Dec. 2005) and found that GJR-GARCH-EVT is a better VaR risk estimator.

Meanwhile, Huang and Tseng (2009) performed daily 99% confidence level VaR forecast on stock market indices of developed countries and emerging countries and concluded that HS performed relatively well for STI index and SSE index (data from Jan. 1980 – Sep. 2007) compared to GARCH, GJR-GARCH and MC. Chen and Yu (2002) pointed out that the daily 99% quantile VaR of SSE index (from Dec 1996 – May 2001) is more accurate using t-distribution or GED rather than normal distribution under different GARCH-M volatility models. Liu, Chen and Wu (2007) evaluated GARCH and TGARCH volatility models with normal, t-distribution and GED used in VaR computation of SSE index (from Jan. 1999- Dec. 2005) and concluded that leverage effect exists, GARCH or TGARCH with GED or t-distribution produces best result at 95% confidence level (but underestimates at 90% confidence level) while normal distribution produces best estimate at 99% quantile. Chen and Yang (2004) carried out the VaR and ES estimation of SSE index (from Feb. 1997 – Apr. 2003) using APARCH volatility model with normal, t-distribution, GED and concluded that GED produced accurate results, normal distribution underestimated the risk, whereas t-distribution overestimated the risk. In addition, they alleged that the average value of ES reflected the average value of VaR exceedance well and is a good indicator of actual loss beyond VaR.

Several authors also have accomplished research and findings of risk dependency between markets like China and Singapore and some results are contradictory. Chen, David and Feng (2010) estimated the whole period VaR and ES of China stock market (SSE) and a few international stock markets including Singapore (data from Jan. 2000 – Apr. 2010) using dynamic GARCH filtered EVT with *peak-over-threshold* approach and discovered that VaR and ES of Chinese market index is the highest and there is hardly any extreme downside risk dependency of Chinese market to other markets except Canada. Hong, Cheng, Liu, and Wang (2004) investigated the Chinese stock market downside risk spillover effect (data from Feb. 1995 – Apr. 2003) on some other international stock markets and reported that B shares in Shanghai stock market has spillover effect on Singapore stock market under 90% or 95% quantile. Hooi, Penm and Terrell (2003) reported that there exists co-integration relationship between China stock market and three regional counterparts: Taiwan, Hong Kong and Singapore especially after Asian financial crisis and individual market like Singapore unilaterally Granger causes Chinese stock market but not in the opposite case.

Objective and Structure of Study

The purpose and contribution of this dissertation is twofold. Firstly, it aims to estimate the market downside risk of China Shanghai stock market and Singapore market using various univariate Value-at-Risk (VaR) and Expected Shortfall (ES) models or methodologies with the latest data (STI and SSE index from Jan 2014 – Jan 2016). As there are only a few studies and prior works that focused on using ES, this study will fill the gap in the growing literature of evaluating the effectiveness of ES as market risk estimator especially after advocacy by Basel Committee in 2012. The assessment on which model is the best estimator will be based on a few VaR and ES backtesting methodologies introduced later.

Secondly, this dissertation will also extend the study and answer the question whether the recent slowdown of China's economy pose downside market risk and has spillover effect to Singapore stock market. As Singapore is a small open economy and highly interconnected with the global economy, it would be a great interest for financial institutions to be able to predict adverse effects from China markets. In order to test this hypothesis, the concept of co-integration and Granger causality in risk estimator can be used.

This dissertation consist of 5 main chapters and is outlined as the following: First chapter, as seen, introduced the concept of financial risk management, Value-at-Risk (VaR) and Expected Shortfall (ES) as market risk indicator, Singapore's and China's STI and SSE market indices and all previous research works on VaR and ES done related to these indices. The second chapter will analyze the returns distribution of the two market indices, check for stationarity and heteroskedasticity (ARCH effect) and test the GARCH model fitting. The third chapter will introduce the theoretical framework which consists of various VaR and ES estimation methodologies (parametric, non-parametric and semi-parametric), their parameters fitting, VaR and ES backtesting methodologies and the risk spillover test. The fourth chapter will present and analyze the empirical results of VaR and ES backtesting to determine the performance of estimation models and also decipher the downside risk spillover test results. The fifth chapter will summarize the results with some discussions and draw meaningful conclusions from the findings. References and appendices are also attached at the end.

2 Data Analysis

Distribution Statistical and Normality Test

This study gathered stock index data of Singapore (STI) and Shanghai (SSE) for the period 2 Jan. 2009 – 29 Jan. 2016 and 5 Jan. 2009 – 29 Jan. 2016 respectively from *Yahoo Finance* database online, and computed total of 1796 (log) returns and 1747 (log) returns respectively. Definition of log returns is explained below.

Denoting the closing price of an asset at day *t* as S_t. One-day raw returns for day *t* is defined by

$$r_t = \frac{S_t - S_{t-1}}{S_{t-1}} = \frac{S_t}{S_{t-1}} - 1$$

Since $\ln(1+x) \approx x$ for small x, the one-day raw returns can be approximated by log returns:

$$r_t = \frac{S_t}{S_{t-1}} - 1 \approx \ln\left(1 + \frac{S_t}{S_{t-1}} - 1\right) = \ln(\frac{S_t}{S_{t-1}})$$

The (log) returns data are then fed into *Gretl* software to generate statistics summaries, plots and undergo normality test:

STI index (Singapore)	SSE index (China)
Summary Statistics: *Mean* 0.00022297 *Median* 0.00014877 *Minimum* -0.043905 *Maximum* 0.057684 *Standard deviation* 0.0097560 *Skewness* 0.22988 *Ex. kurtosis* 4.5079	*Summary Statistics:* *Mean* 0.00023343 *Median* 0.00050902 *Minimum* -0.088729 *Maximum* 0.074123 *Standard deviation* 0.015839 *Skewness* -0.78569 *Ex. kurtosis* 4.4833
Test for normality of r: *Doornik-Hansen test = 614.36, with* ***p-value 3.92131e-134*** *:* *Jarque-Bera test = 1536.54, with* ***p-value 0***	*Test for normality of r:* *Doornik-Hansen test = 614.36, with* ***p-value 3.92131e-134*** *:* *Jarque-Bera test = 1536.54, with* ***p-value 0***

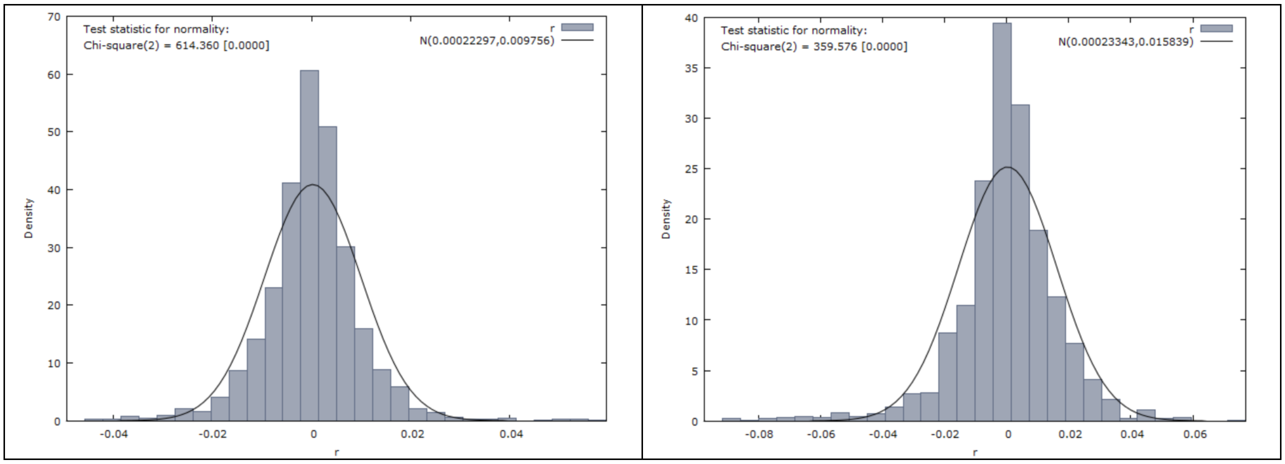

Fig 5 – Frequency plots of STI and SSE indices with summary statistics and normality test results

The frequency plot (with superimposed normal plot) and normality test results clearly show that the returns distribution of Singapore and Shanghai stock index is not normal. For both indices, the null hypothesis of normality based on *Doornik-Hansen*'s test was rejected at a reasonable level of significance (the p-value is very small or test statistics is large). The *Doornik-Hansen* test statistic has a chi-square distribution χ^2 if the null hypothesis of normality is true (Lee 2014).

Another popular normality test is by *Jarque and Bera* (JB), relying on the fact that *skewness* and *excess kurtosis* are both zero for normal distribution (Jondeau, Ser-Huang and Rockinger 2007). The JB test statistics involves *standardized skewness* and *excess kurtosis* and is asymptotically distributed as chi-square with two degree of freedom, $\chi^2(2)$.

$$JB = T\left[\frac{(skewness)^2}{6} + \frac{(excess\ kurtosis)^2}{24}\right]$$

where excess kurtosis = kurtosis – 3

$$skewness = E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{X-\mu}{\sigma}\right)^3$$

$$kurtosis = E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{X-\mu}{\sigma}\right)^4$$

JB test's limitation is that asymptotic distribution holds only for very large samples. The p-values of JB test for both markets are small and thus null hypothesis of normality is rejected. Furthermore, QQ-plots (quantile-quantile plot) against normal distribution of STI and SSE indices (fig 6) also showed the departure from linearity at both ends and reaffirmed that the (log) returns distributions are not normal.

STI and SSE indices showed typical excess kurtosis or *fat tail (leptokurtic)* which signifies a higher frequency of extreme losses. SSE exhibited negative skewness while STI exhibited positive skewness. SSE index has a higher maximum positive return than STI index (+7.4% vs. +5.8%) but also a higher negative return or loss than STI index (-8.9% vs. -4.4%). This is consistent with the higher standard deviation in SSE index than STI index (1.5% vs. 0.98%), showing that investing in Shanghai market has higher risk but also higher reward.

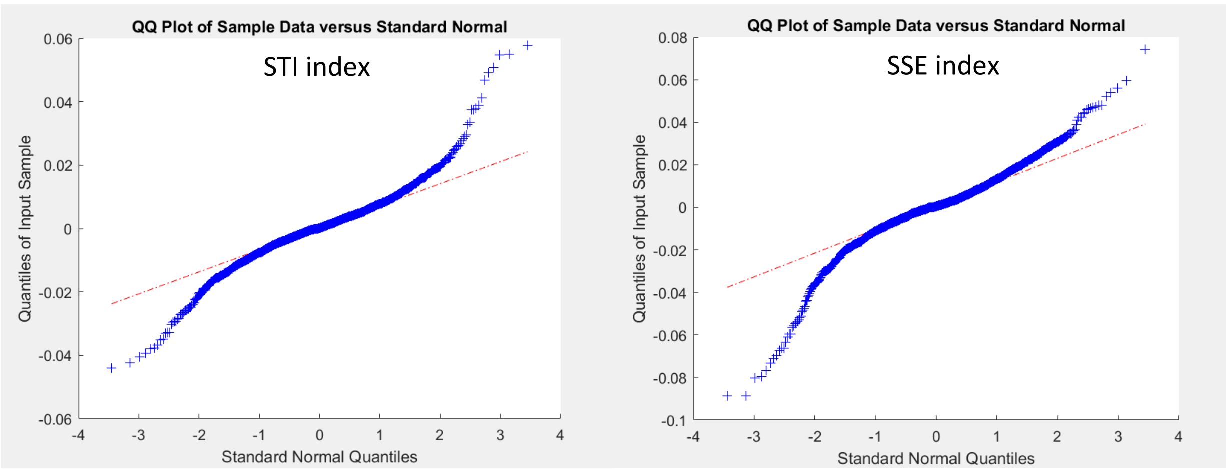

Fig 6 – QQ-plot of STI and SSE indices returns against normal distribution

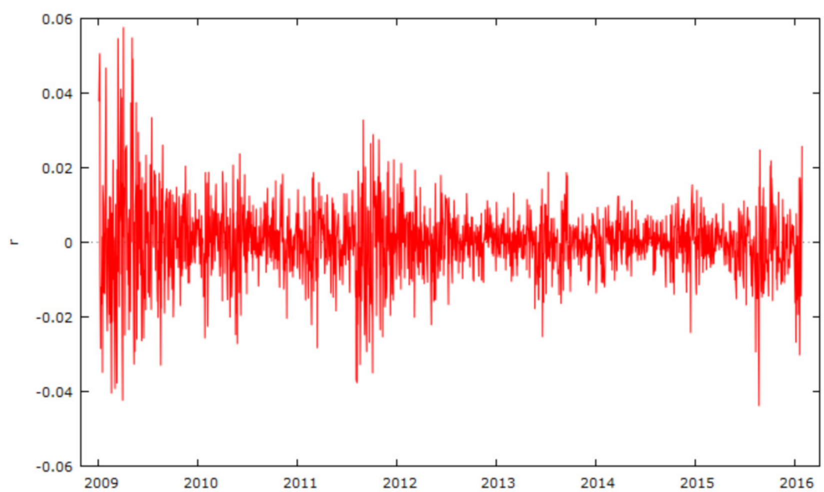

Fig 7 – Plot of daily (log) returns of STI Index

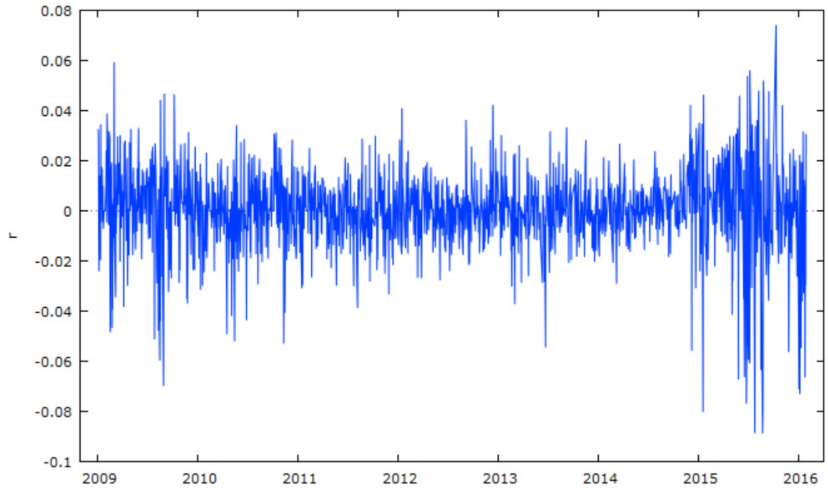

Fig 8 – Plot of daily (log) returns of SSE Index

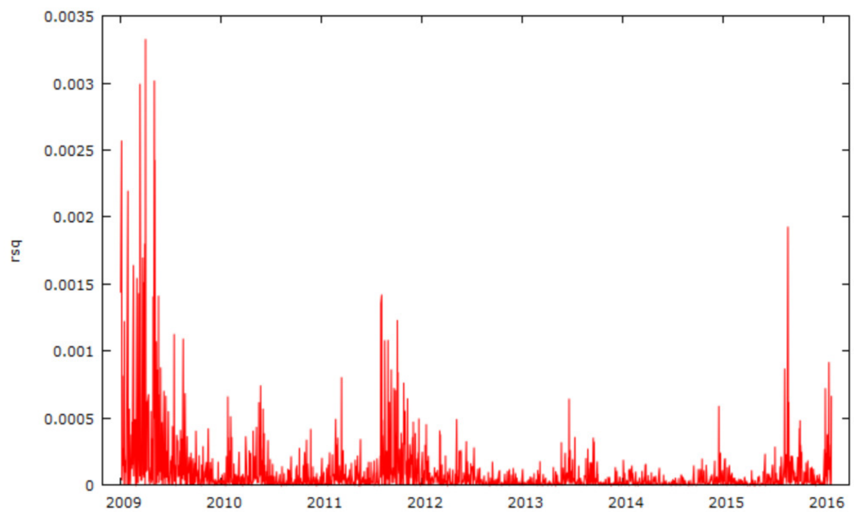

Fig 9 – Plot of squared (log) returns of STI Index

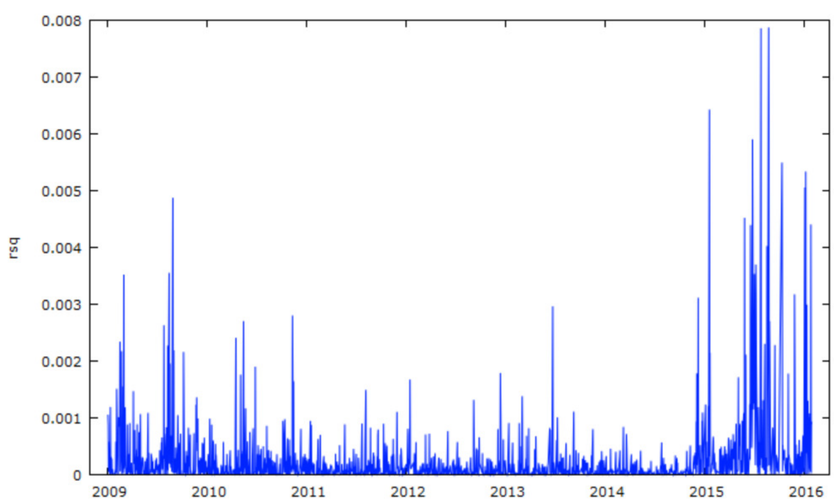

Fig 10 – Plot of squared (log) returns of SSE Index

The time series plot of daily (log) returns for STI and SSE indices show that there is no clear discernible pattern of behavior and the values vary around zero which can be modeled as:

$$r_t = \mu + \epsilon_t \qquad \text{where } \epsilon_t = \sigma_t z_t$$

where the mean μ is approximately zero and ϵ_t is the residual, innovation or error (Kuester, Mittnik and Paolella 2006; Yong, Tae-Hwy and Burak 2006). The residual is made up of standard deviation σ_t and sequence of *iid* (*independent and identically distributed*) random variable (white noise), z_t with zero mean and unit variance. The plots of squared (log) returns, represent the *variance* or *volatility* as explained in the following, demonstrated the evidence of *volatility clustering* (i.e. low values of volatility followed by low values and high values of volatility followed by high values).

The expected value of squared returns is approximately the expected value of squared residual and variance of residual since the expected value of residual is zero, $E[\epsilon_t] = E[\sigma_t z_t] = 0$.

$$E[{r_t}^2] = E[(\mu + \epsilon_t)^2] \approx E[{\epsilon_t}^2] = E[(\epsilon_t - E[\epsilon_t])^2] = var[\epsilon_t]$$

$$E[{r_t}^2] \approx E[(r_t - E[r_t])^2] = E[(r_t - \mu)^2] = var[r_t]$$

Hence, squared returns represent the *variance of the residual*. It is also approximately the *variance of returns* since $[r_t] = E[\mu] + E[\epsilon_t] = \mu \approx 0$. Note that returns is essentially the same as residuals and used interchangeably when $\mu \approx 0$. As seen from the squared return plot, the variance of residuals is not constant and changes with time, exhibiting *heteroskedasticity* or *volatility clustering* characteristic.

Stationarity Test

Before further analysis, the STI and SSE (log) returns time-series are tested for non-stationary using *unit root test*. It is important to check for stationarity because when a unit root exists in a non-stationary series, persistence of shocks will be infinite and traditional asymptotic normality will be invalid. One of the frequently used models to characterize non-stationary is the *random walk model with drift* given by:

$$y_t = \mu + y_{t-1} + u_t$$

while another $y_t = \alpha + \beta t + u_t$ describes with *trend process* and in both cases u_t is *iid* (Brooks 2008). Let $\Delta y_t = y_t - y_{t-1},\ \Delta y_t = \mu + u_t$ is said to induce stationary by "differencing one". The y_t series is said to be an I(1) series containing one unit root and Δy_t series is said to be a stationary series, I(0). One of the early work for stationary or unit root test is introduced by Dickey and Fuller (Dickey and Fuller 1979; Fuller 1976). The main objective of the test is to test null hypothesis that $\emptyset = 1$ in the series

$$y_t = \emptyset y_{t-1} + u_t$$

against the one-sided alternative $\emptyset$<1. This is equivalent to testing $\psi = 0$ for the regression $\Delta y_t = \psi y_{t-1} + u_t$. The null hypothesis is *H_o: series contains a unit root*, *non-stationary, $y_t \sim I(1)$* and *H_1: series is stationary, $y_t \sim I(0)$*. For higher orders testing, accepting presence of unit root and null hypothesis means one has to further test if *H_o: $y_t \sim I(2)$* vs. *H_1: $y_t \sim I(1)$* and so on until we reject *H_o*. This Dicker and Fuller (DF) test is only valid if u_t is white noise (Brooks 2008).

An Augmented Dickey Fuller (ADF) solves the problem of u_t having autocorrelation in the dependent variable, with alternative model $\Delta y_t = \psi y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-1} + u_t$ in which the test is augmented using *p* lags. The ADF uses the same critical values as original DF test which is used to reject null hypothesis if the DF test statistic is more negative than critical values (Brooks 2008).

This study performed the stationary test using ADF test (using lags of 12) in *Gretl* software for both STI and SSE (log) returns and the results indicate both the returns are stationary after rejecting the null hypothesis of unit root as the test statistics are more negative than critical values (-2.86 for the case with constant but no trend; -3.41 for the case with constant and trend at 5% significance level) or p-value < 0.05.

STI Index: *Augmented Dickey-Fuller test for r* *including 9 lags of (1-L)r* *(max was 12, criterion modified AIC)* *sample size 1786* *unit-root null hypothesis: a = 1* ***test without constant*** *model: (1-L)y = (a-1)*y(-1) + ... + e* *:* *test statistic: tau_nc(1) =* ***-12.1211*** *asymptotic* ***p-value 3.755e-025*** ***test with constant*** *model: (1-L)y = b0 + (a-1)*y(-1) + ... + e* *:* *test statistic: tau_c(1) =* ***-12.1533*** *asymptotic* ***p-value 3.305e-026***	***SSE index:*** *Augmented Dickey-Fuller test for r* *including 12 lags of (1-L)r* *(max was 12, criterion modified AIC)* *sample size 1734* *unit-root null hypothesis: a = 1* ***test without constant*** *model: (1-L)y = (a-1)*y(-1) + ... + e* *:* *test statistic: tau_nc(1) =* ***-10.4411*** *asymptotic* ***p-value 1.206e-020*** ***test with constant*** *model: (1-L)y = b0 + (a-1)*y(-1) + ... + e* *:* *test statistic: tau_c(1) =* ***-10.4444*** *asymptotic* ***p-value 1.227e-020***

with constant and trend *model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e* *:* *test statistic: tau_ct(1) =* ***-12.4076*** *asymptotic* ***p-value 2.696e-030***	***with constant and trend*** *model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e* *:* *test statistic: tau_ct(1) =* ***-10.4546*** *asymptotic* ***p-value 1.176e-021***

Fig 11 – Results of stationary test using ADF for (log) returns of STI and SSE indices

Heteroskedasticity Check Using Correlogram

In order to check for heteroskedasticity, this study plotted the autocorrelation function (ACF) and partial autocorrelation (PACF) of the (log) return in a *correlogram* using *MATLAB* software. Autocovariance (self-covariance) of a variable y determines how a variable is related to its previous value (lags), which is the covariance between y_t and y_{t-s}, i.e. $\gamma_s = E[(y_t - E[y_t])(y_{t-s} - E[y_{t-s}])]$. Notice that autocovariance at lag s=0 is equivalent to variance. *Autocorrelation* is defined as the normalized autocovariance divided by the variance, i.e. $\tau_s = \frac{\gamma_s}{\gamma_0}$ (Brooks 2008). ACF is a plot of autocorrelation τ_s against *s* values (lags). The partial autocorrelation function (PACF) measures the correlation between an observation *k* periods ago y_{t-k} and the current observation y_t , after controlling observations (removing effects) at intermediate lags $y_{t-k+1}, y_{t-k+2}, \dots, y_{t-1}$.

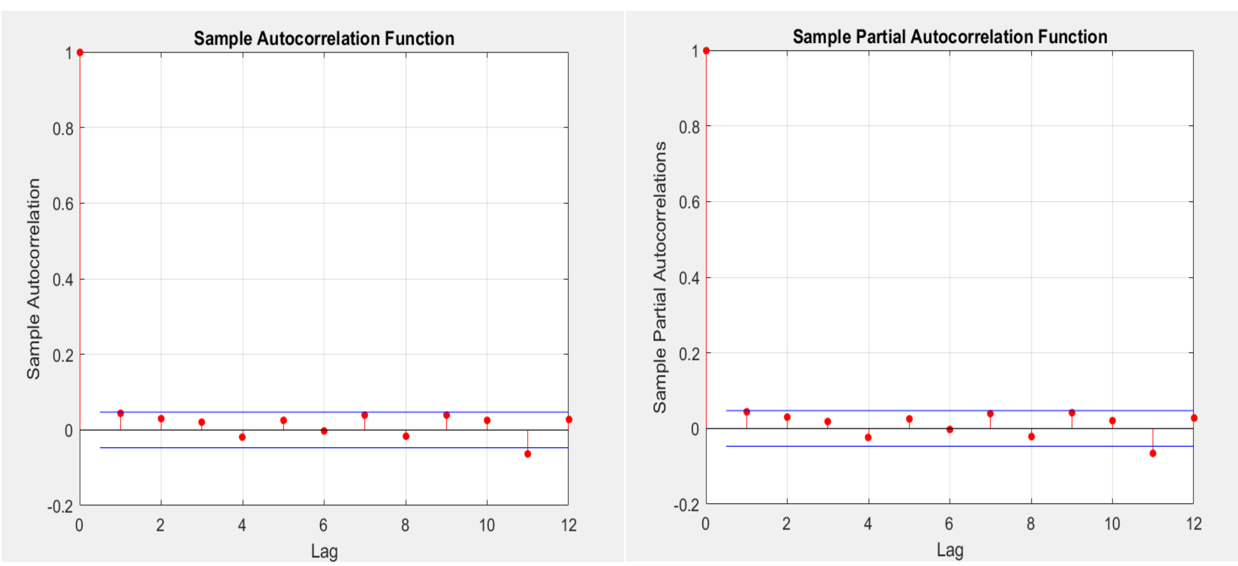

Fig 12 – ACF and PACF plot (correlogram) for (log) returns of STI index

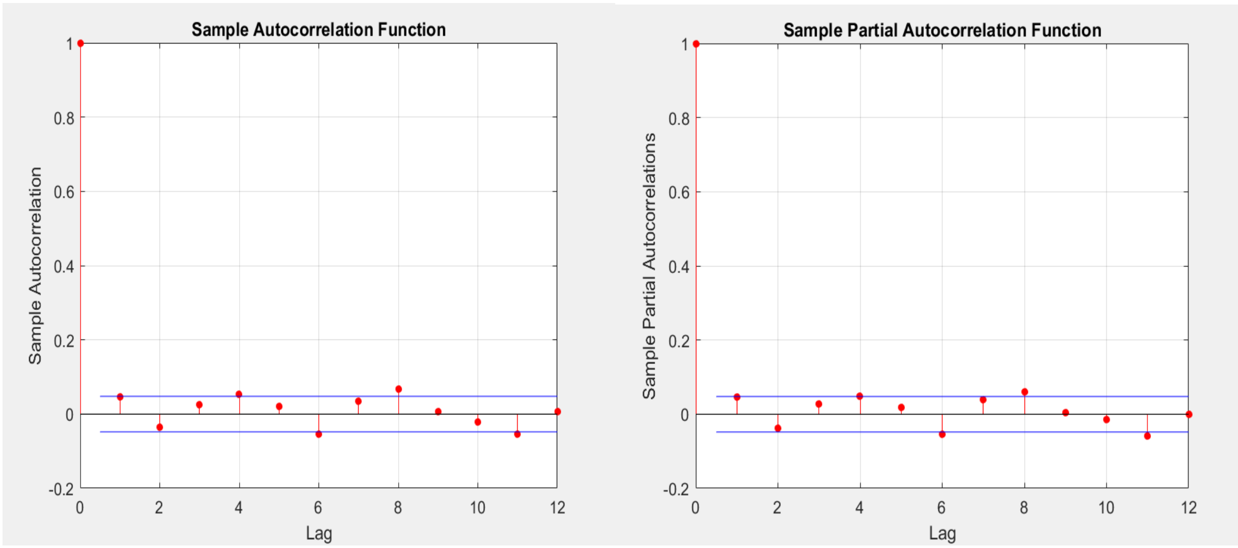

Fig 13 – ACF and PACF plot (correlogram) for (log) returns of SSE index

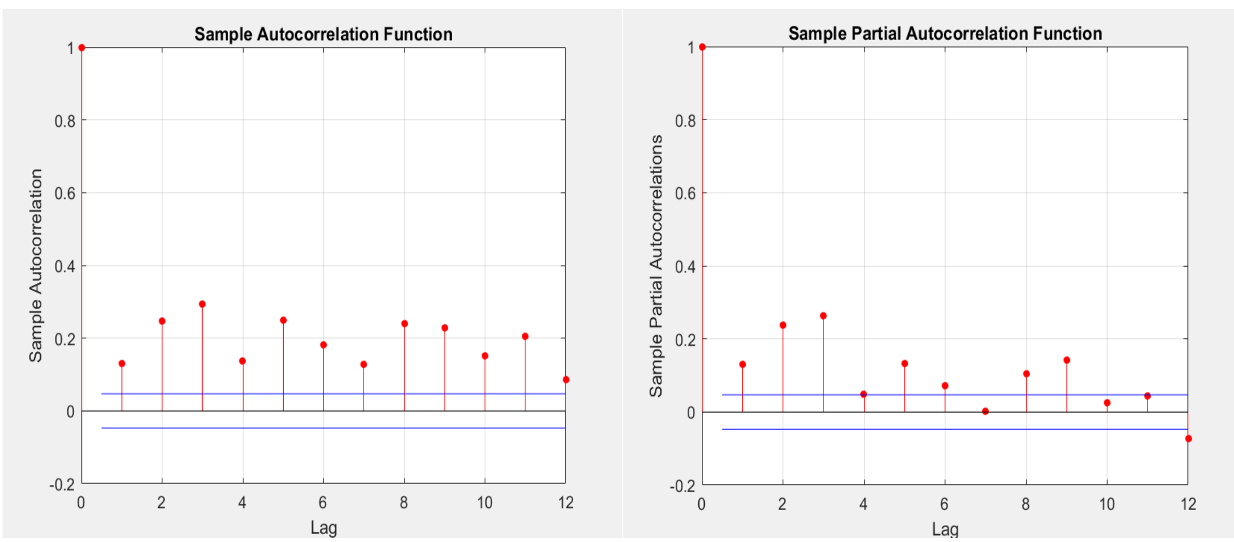

Fig 14 – ACF and PACF plot (correlogram) for squared (log) returns of STI index

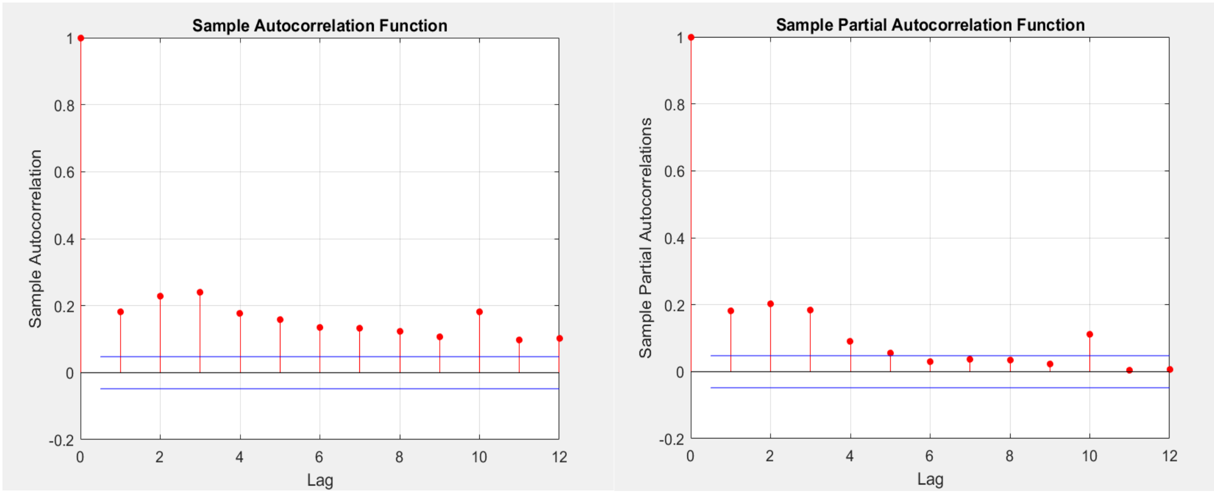

Fig 15 – ACF and PACF plot (correlogram) for squared (log) returns of SSE index

ACF and PACF values that exceed the blue band (*Box-Pierce confidence interval*) are considered significant (to be explained later). ACF and PACF values are 1 for lag 0 ($\tau_0 = \frac{\gamma_0}{\gamma_0} = 1$) but are small for lags greater than 1. The ACF and PACF plot of (log) returns of STI index (fig 12) show that there is no significant autocorrelation for the returns data (assuming the spike at lag 11 is spurious). For SSE index (fig 13), there is overall no significant autocorrelation for returns data except a few spikes (6,8 and 11) that are assumed to be spurious.

The dependency is clearer in ACF and PACF plot of squared (log) returns (fig 14 & 15). There is autocorrelation for the *square of the returns* for both STI and SSE indices. The slowly decaying nature of the ACF and PACF for square of returns signify the presence of higher lag orders of AR (autoregressive) and MA (moving average) respectively or a combined ARMA (autoregressive-moving average) effect, related to variance of returns or residuals. Note that the following general representation for AR(p): $y_t = \mu + \sum_{i=1}^{p} \alpha_i \, y_{t-i} + \epsilon_t$, for MA(q): $y_t = \mu + \sum_{i=1}^{q} \beta_i \, \epsilon_{t-i} + \epsilon_t$ and for ARMA(p,q): $y_t = \mu + \sum_{i=1}^{p} \alpha_i \, y_{t-i} + \sum_{j=1}^{q} \beta_j \, \epsilon_{t-j} + \epsilon_t$. In fact, this means there is ARCH effect (*autoregressive conditional heteroskedasticity*) in the residuals where ARCH(m) is represented by the expression ${\sigma_t}^2 = \omega_o + \sum_{i=1}^{m} \alpha_i \epsilon_{t-i}^2$ where ${\sigma_t}^2$ is known as the *conditional variance* . It is therefore proper to model the time series using GARCH (*general autoregressive conditional heteroskedasticity*). The general GARCH(p,q) modeling equation is given by: ${\sigma_t}^2 = \omega_0 + \sum_{i=1}^{p} \beta_i {\sigma_{t-i}}^2 + \sum_{j=1}^{q} \alpha_j \epsilon_{t-j}^2$. Notice that how ARCH resembles MA and GARCH resembles ARMA.

Heteroskedasticity Check Using LBQ Test

ARCH effect could also be tested using *Ljung-Box Pierce Q-Test* (LBQ Test) and Engle's ARCH test (Nasir 2007). Box and Pierce developed a *Q-statistic* by testing a joint null hypothesis that all *m* of the τ_s correlation coefficients are simultaneously equal to zero (Brooks 2008).

$Q = T \sum_{s=1}^{m} \hat{\tau}_s{}^2$ where T =sample size and $\hat{\tau}_s$ = sample autocorrelation coefficient

It tests whether autocorrelation coefficient is significant different from zero by constructing a 95% non-rejection region (confidence interval) given by $\pm 1.96 \times \frac{1}{\sqrt{T}}$ assuming sample autocorrelation is approximately normally distributed $(0, \frac{1}{T})$. The Q-statistic is asymptotically distributed as a $\chi_m{}^2$ under the null hypothesis.

As Box-Pierce test is poor for small samples, a modified statistics known as *Ljung-Box statistics* is introduced which is statistically equivalent to Box-Pierce as $T \to \infty$ (Brooks 2008).

$$Q^* = T(T+2) \sum_{s=1}^{m} \frac{\hat{\tau}_s{}^2}{T-s} \sim \chi_m{}^2$$

The Ljung-Box statistics of squared returns of STI Index are listed below (using *lbqtest* in *MATLAB* software). The test on squared returns is essentially the ARCH test by testing the autocorrelations of squared residuals. ARCH effect exist as p-values for squared returns <0.05 or Q*>critical value (H=1). Therefore, autocorrelations of squared return (or residual) is significant which proves the presence of ARCH effect.

LBQ-test for ***squared returns*** *of STI*

Lags	*H*	*p-value*	*Q*-Stat*	*Critical Value*
1.0000	*1.0000*	*0*	*30.7815*	*3.8415*
2.0000	*1.0000*	*0*	*140.7297*	*5.9915*
3.0000	*1.0000*	*0*	*297.0698*	*7.8147*
4.0000	*1.0000*	*0*	*330.8740*	*9.4877*
5.0000	*1.0000*	*0*	*443.6982*	*11.0705*
6.0000	*1.0000*	*0*	*503.7948*	*12.5916*
7.0000	*1.0000*	*0*	*533.5909*	*14.0671*
8.0000	*1.0000*	*0*	*637.7583*	*15.5073*
9.0000	*1.0000*	*0*	*732.3935*	*16.9190*
10.0000	*1.0000*	*0*	*773.7326*	*18.3070*
11.0000	*1.0000*	*0*	*850.4935*	*19.6751*
12.0000	*1.0000*	*0*	*863.5609*	*21.0261*

LBQ-test for ***squared returns*** *of SSE*

Lags	*H*	*p-value*	*Q*-Stat*	*Critical Value*
1.0000	*1.0000*	*0*	*57.1742*	*3.8415*
2.0000	*1.0000*	*0*	*148.2609*	*5.9915*
3.0000	*1.0000*	*0*	*249.5466*	*7.8147*
4.0000	*1.0000*	*0*	*304.8523*	*9.4877*
5.0000	*1.0000*	*0*	*348.4730*	*11.0705*
6.0000	*1.0000*	*0*	*380.1654*	*12.5916*
7.0000	*1.0000*	*0*	*410.7104*	*14.0671*
8.0000	*1.0000*	*0*	*437.1231*	*15.5073*
9.0000	*1.0000*	*0*	*457.3627*	*16.9190*
10.0000	*1.0000*	*0*	*515.0374*	*18.3070*
11.0000	*1.0000*	*0*	*531.6294*	*19.6751*
12.0000	*1.0000*	*0*	*550.1076*	*21.0261*

Fig 16 – LBQ-test results for squared (log) returns of STI and SSE indices.

Heteroskedasticity Check Using LM Test

Engle proposed *Lagrange multiplier* (LM) test to test for ARCH effect by squaring the residuals, regress them on a constant and q lagged values (i.e. an AR(q): $\epsilon_t{}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, \epsilon^2{}_{t-i} + \delta_t$) and test for null hypothesis that all lagged coefficients are zero (i.e. $\alpha_1 = \alpha_2 = \cdots = \alpha_q = 0$) which is no autocorrelation in the squared residuals. In a sample of T residuals and R^2 value obtained from the regression, the test statistics TR^2 under null hypothesis of no ARCH effect has an asymptotic χ^2 distribution with q degree of freedom (Brooks 2008).

The test statistics of returns of STI index are listed below (using *archtest* in *MATLAB* software). ARCH effect exist as p-values <0.05 or stat>critical value (H=1).

*ARCH-test for **returns** of STI*

Lags	*H*	*p-value*	*Stat*	*Critical Value*
1.0000	1.0000	0	31.3386	3.8415
2.0000	1.0000	0	127.8122	5.9915
3.0000	1.0000	0	257.2205	7.8147
4.0000	1.0000	0	269.1646	9.4877
5.0000	1.0000	0	289.7861	11.0705
6.0000	1.0000	0	299.0624	12.5916
7.0000	1.0000	0	301.8646	14.0671
8.0000	1.0000	0	322.4333	15.5073
9.0000	1.0000	0	356.3407	16.9190
10.0000	1.0000	0	344.1403	18.3070
11.0000	1.0000	0	347.1864	19.6751
12.0000	1.0000	0	357.1744	21.0261

*ARCH-test for **returns** of SSE*

Lags	*H*	*p-value*	*Stat*	*Critical Value*
1.0000	1.0000	0	57.1311	3.8415
2.0000	1.0000	0	125.9220	5.9915
3.0000	1.0000	0	181.3643	7.8147
4.0000	1.0000	0	193.6186	9.4877
5.0000	1.0000	0	198.2456	11.0705
6.0000	1.0000	0	199.6072	12.5916
7.0000	1.0000	0	201.6024	14.0671
8.0000	1.0000	0	203.4632	15.5073
9.0000	1.0000	0	204.3220	16.9190
10.0000	1.0000	0	223.1355	18.3070
11.0000	1.0000	0	223.1397	19.6751
12.0000	1.0000	0	223.2595	21.0261

Fig 17 – Engle's ARCH-test results for (log) returns of STI and SSE indices

GARCH Fitting

Deciding the appropriate model orders for GARCH(p,q) could be done by choosing the model order that minimizes the value of an *information criterion* (IC). Information criteria embody two competing effects, one is a function of residual sum of squares (RSS) and another is some penalty for the loss of degrees of freedom from additional new variable or lag (Brooks 2008). Using higher lag order reduces the RSS but increases the value of penalty term. The value of the criteria will fall if fall of RSS outweighs increase in penalty. Three most popular information criteria are *Akaike's information criterion* (AIC), *Schwarz's Bayesian information criterion* (SBIC) and *Hannan-Quinn criterion* (HQIC). SBIC has a stiffer penalty term than AIC while HQIC is somewhere in between (Brooks 2008).

The corresponding SBIC values are listed in the table below when GARCH(p,q) parameters are estimated using *MLE (maximum likelihood estimation)*. Notice that for STI index, GARCH(1,1) has the smallest SBIC value (most negative). For SSE index, GARCH(3,2) has the smallest SBIC value but it has poor fitting for parameter α_1 (p-value=0.28 >0.05). For this reason and for simplicity, the second smallest SBIC model, GARCH(1,1) is a good pick for SSE index.

Table 2 - Schwarz's Bayesian information criterion of different GARCH(p,q) fitting for STI index

STI index	SBIC values for GARCH(p,q)			
q p	1	2	3	4
1	**−15490.95**	−15469.72	−15459.23	−15452.34
2	−15469.66	−15463.56	−15454.20	−15444.86
3	−15461.79	−15457.64	−15459.35	−15439.18
4	−15453.20	−15448.61	−15452.60	−15446.07

Table 3 - Schwarz's Bayesian information criterion of different GARCH(p,q) fitting for SSE index

SSE index	SBIC values for GARCH(p,q)			
q p	1	2	3	4
1	**−13148.25**	−13141.79	−13137.72	−13132.95
2	−13147.84	−13136.11	**−13159.04**	−13131.08
3	−13142.47	−13141.06	−13132.09	−13136.57
4	−13136.12	−13131.23	−13112.38	−13141.64

The effectiveness of GARCH(1,1) could be verified by fitting the data to the model and checking autocorrelations of *squared standardized residuals* ($(\frac{\epsilon_t}{\sigma_t})^2 = z_t^2$). The correlogram of the *squared standardized residuals* of both STI and SSE indices showed that ACF and PACF values are significantly reduced and most fall within the Box-Pierce confidence interval. Hence, there is no autocorrelation and GARCH(1,1) has successfully filtered and removed majority of the heteroskedasticity or *volatility clustering* and produced *iid* standardized residuals. GARCH(1,1) is appropriate to describe the volatility model and would be used. This result is consistent with the LBQ-test of squared standardized residuals for both indices (fig 20).

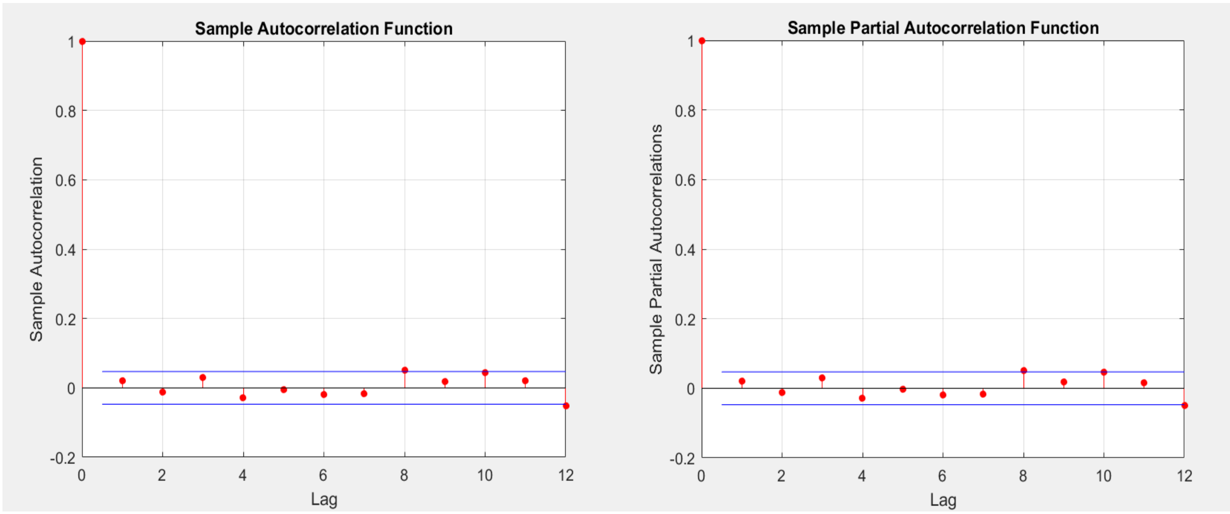

Fig 18 – ACF and PACF plot (correlogram) for squared standardized residuals of STI index

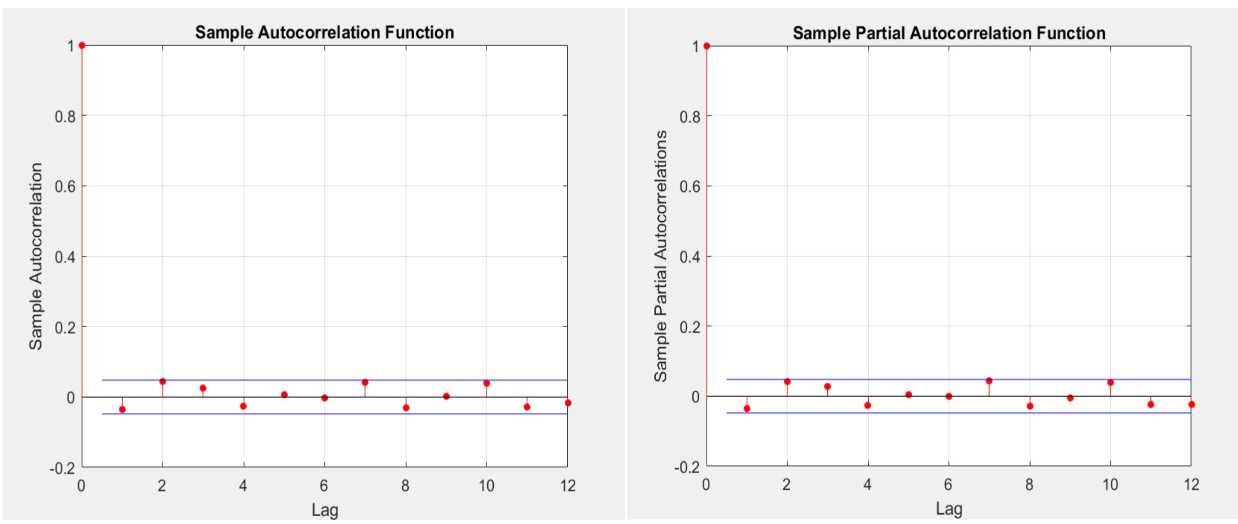

Fig 19 – ACF and PACF plot (correlogram) for squared standardized residuals of SSE index

LBQ-test for ***squared std. residual*** *of STI*

Lags	*H*	*p-value*	*Stat*	*Critical Value*
1.0000	*0*	*0.3626*	*0.8288*	*3.8415*
2.0000	*0*	*0.5734*	*1.1125*	*5.9915*
3.0000	*0*	*0.4272*	*2.7776*	*7.8147*
4.0000	*0*	*0.3803*	*4.1949*	*9.4877*
5.0000	*0*	*0.5154*	*4.2401*	*11.0705*
6.0000	*0*	*0.5662*	*4.8268*	*12.5916*
7.0000	*0*	*0.6122*	*5.3925*	*14.0671*
8.0000	*0*	*0.2576*	*10.1066*	*15.5073*
9.0000	*0*	*0.2986*	*10.6751*	*16.9190*
10.0000	*0*	*0.1601*	*14.2907*	*18.3070*
11.0000	*0*	*0.1779*	*15.1018*	*19.6751*
12.0000	*0*	*0.0715*	*19.7724*	*21.0261*

LBQ-test for ***squared std. residual*** *of SSE*

Lags	*H*	*p-value*	*Stat*	*Critical Value*
1.0000	*0*	*0.1324*	*2.2646*	*3.8415*
2.0000	*0*	*0.0647*	*5.4774*	*5.9915*
3.0000	*0*	*0.0854*	*6.6094*	*7.8147*
4.0000	*0*	*0.0973*	*7.8483*	*9.4877*
5.0000	*0*	*0.1591*	*7.9479*	*11.0705*
6.0000	*0*	*0.2413*	*7.9561*	*12.5916*
7.0000	*0*	*0.1323*	*11.1468*	*14.0671*
8.0000	*0*	*0.1207*	*12.7507*	*15.5073*
9.0000	*0*	*0.1740*	*12.7546*	*16.9190*
10.0000	*0*	*0.1147*	*15.5052*	*18.3070*
11.0000	*0*	*0.1079*	*16.9999*	*19.6751*
12.0000	*0*	*0.1321*	*17.4883*	*21.0261*

Fig 20 – LBQ-test results for squared standardized residuals of STI and SSE indices

3 Theoretical Framework and Methodologies:

VaR and ES ESTIMATION

There are different ways to calculate Value-at-Risk (VaR) and Expected Shortfall (ES). The calculation methods can be generally divided into *parametric*, *non-parametric* and *semi-parametric* approaches. Parametric (or analytic) approach is based on statistical model of a *full distribution* while non-parametric approach uses historical data to forecast VaR or ES. A semi-parametric approach does not assume statistical model on full distribution but only on *part of distribution* such as at the tail. Each approach can be further divided into *static* and *dynamic* approach. *Monte Carlo* approach is also a parametric method that generates random movement in risk factors from estimated parametric distributions (Jorion 2007b). Monte Carlo simulation is not applied here as it does not offer much, the data used is univariate and VaR estimated is only over 1-day horizon.

Parametric approach

Fitting the right distribution to the data is crucial for accurate estimation in the parametric or semi-parametric approach. This approach requires parameters like average value and variance (or standard deviation) of a distribution. The larger the variance, the higher the occurrence of large swing in value and hence the larger is the VaR or ES. The parametric approach is often called the *variance-covariance* (VC) method or *delta-normal* method when it assumes a *normal distribution* and the correlation between risk factors are constant. The VaR value is estimated from the multiplication of standard deviation by a coefficient determined by the confidence level (explained later). When this approach is used in a portfolio, the standard deviation is calculated from variance equation that involves the correlation and covariance between the assets in the portfolio:

$$\sigma_p^2 = \sum_i \omega_i^2\, \sigma_i^2 + \sum_i \sum_{j \neq i} \omega_i\, \omega_j \rho_{ij} \sigma_i \sigma_j = \boldsymbol{w}\, \boldsymbol{\Sigma}\, \mathbf{w}^{\boldsymbol{T}}$$

where ω_i and ω_j represents the weights of asset *i* and *j*; ρ_{ij} represents the correlation coefficient between the returns on asset *i* and *j*; $\boldsymbol{w}$ is the weight matrix and $\boldsymbol{\Sigma}$ is the variance-covariance matrix (Dowd 1999).

STATIC MODELS

In *parametric static* model, the data is assumed to come from a fully parametric distribution, i.e. *r ~ iid F(·)*. The model is static because the parameters specifying the distribution are constant over the specific time horizon. The only source of change in parameter values is the change in time horizon in the rolling window forecasting approach. In other words, the static model actually assumes a *moving average volatility model* (a variance value calculated that is based on equal weight of past returns data, explained later) for $r_t = \mu + \sigma_t z_t$. The volatility clustering behavior has shown that *r* not really independent (thus not *iid*). Nonetheless, the VaR (negative) with (1-α)% level of confidence can be calculated as the following:

$$VaR_q\,(r) = \mu - \sigma\, F^{-1}(q)$$

where $F^{-1}(q)$ is the *q*th quantile ($q = 1 - \alpha$) value of a standard distribution function F(·) (i.e inverse cumulative distribution function) (Yong et al. 2006; Kjellson 2013). The window of sample data up to *t-1* is fitted to the distribution to forecast VaR value for day *t*. The parameters μ and σ belong to the location and scale parameter respectively of a location-scale family of probability distribution function (Kuester et al. 2006; Kjellson 2013). μ is usually very small for returns data and considered zero for simplicity. The concept of location-scale family is described in the figure below. Notice that the scale parameter may not be necessarily equivalent to the standard deviation of a distribution (σ^2 may not $\neq \sigma_t^2$ or variance).

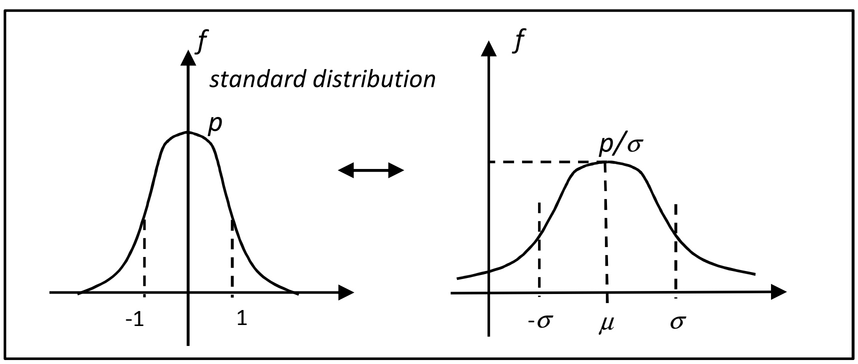

Fig 21 – Location-scale family of probability distribution

Similarly, the Expected Shortfall (ES) or Conditional VaR at level *q* (negative) can be expressed as the following*:*

$$ES_q(r) = \mu - \sigma G(q)$$

where *G(q)* is a specific function derived for different distribution (Kjellson 2013).

There are three types of distribution used here: normal distribution, student-t distribution and Generalized Error Distribution (GED).

Normal Distribution

Normal distribution has a symmetrical bell-shaped curve with more weight at the center and tails tapering off to zero and is first proposed two centuries ago by Karl F. Gauss (hence also known as *Gaussian distribution*) (Jorion 2007b; Jorion 2007a). It is the most popular distribution and can be characterized by first two moments only, the mean and variance $N(\mu, \sigma^2)$. The probability density function has the expression:

$$pdf = f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

The scale parameter is the same as standard deviation for normal distribution ($\sigma = \sigma_t$). It has skewness of 0 and kurtosis of 3. When $E[X] = \mu = 0$ and $Var[X] = {\sigma_t}^2 = 1$, it is known as the *standard normal distribution*. Data from normal distribution will become standard normal distribution when it is standardized ($\frac{r-\mu}{\sigma}$).

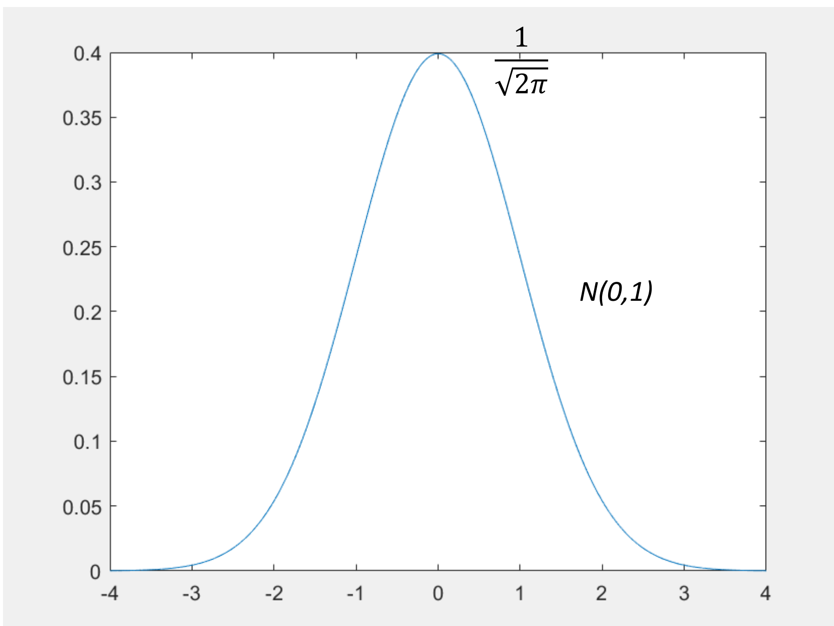

Fig 22 – Probability density function of Standard Normal distribution

The VaR value (negative) is given by:

$$VaR_q(r) = \mu - \sigma\, \Phi^{-1}(q) = \mu - \sigma_t\, \Phi^{-1}(q)$$

where $\Phi^{-1}(\cdot)$ is the inverse cumulative distribution function of a standard normal distribution. The expected shortfall (negative) has the expression:

$$ES_q(r) = \mu - \sigma \frac{1}{\alpha\sqrt{2\pi}}\, e^{-\frac{1}{2}(\Phi^{-1}(q))^2} = \mu - \sigma \frac{\phi(\Phi^{-1}(q))}{1-q}$$

where $\phi(\cdot)$ is the standard normal probability density function (Kjellson 2013; Eldar and Torstin 2011). One of the serious drawback with normal distribution is the financial data usually exhibit heavier tail (fat tail) and normal distribution underestimates the risk (Hull 2012).

Student t-distribution

Unlike normal distribution, *student's t-distribution* or *t-distribution* exhibit a heavier tail and excess kurtosis as the probability of tail event is higher. Many studies have reported that t-distribution fit the returns data better as there is higher chance of higher losses. The probability density function is described by:

$$pdf = f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\upsilon\pi}\sigma} (1 + \frac{\left(\frac{x-\mu}{\sigma}\right)^2}{\nu})^{-\frac{\nu+1}{2}}$$

where $\Gamma(\cdot)$ is the gamma function defined by $\Gamma(t) = \int_0^\infty x^{t-1} e^{-x}\, dx$ (Nicklas and Daniel 2010).

The t-distribution is also symmetry about μ and has zero skewness. Besides location and scale parameter, the distribution is described by an extra parameter υ known as the degree of freedom which describes different levels of leptokurtic behavior. The smaller the value of υ, the fatter the tail. Moment of order k (i.e. $E[X^k]$) exists only for $k < \upsilon$ (Ming Chen et al. 2015). Therefore, finite mean μ only exist if $\upsilon > 1$. Finite variance exists only if $\upsilon > 2$ and is given by $r[X] = \sigma_t^2 = \sigma^2 \frac{\nu}{\nu-2}$. Notice that variance is not equal to the squared scale parameter. As degree of freedom becomes larger $\upsilon \to \infty$, t-distribution becomes a normal distribution with variance σ^2 (Goorbergh and Vlaar 1999). Finite kurtosis exist for $\upsilon > 4$ and is given by $\kappa = \frac{6}{\nu-4} + 3$. A t-distribution with location parameter $\mu = 0$ and scale parameter $\sigma = 1$ is called a *standard t-distribution*.

The VaR value (negative) is given by:

$$VaR_q\,(r) = \mu - \sigma\, {T_\nu}^{-1}(q) = \mu - \sigma_t \sqrt{\frac{\nu-2}{\nu}} T_\nu^{-1}(q)$$

where $T_\nu^{-1}(\cdot)$ is the inverse cumulative distribution function of a standard t-distribution. The expected shortfall (negative) is described by (Eldar and Torstin 2011):

$$ES_q(r) = \mu - \sigma \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\alpha\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{\nu}{\nu-1} (1 + \frac{(T_\nu^{-1}(q))^2}{\nu})^{\frac{1-\nu}{2}}$$

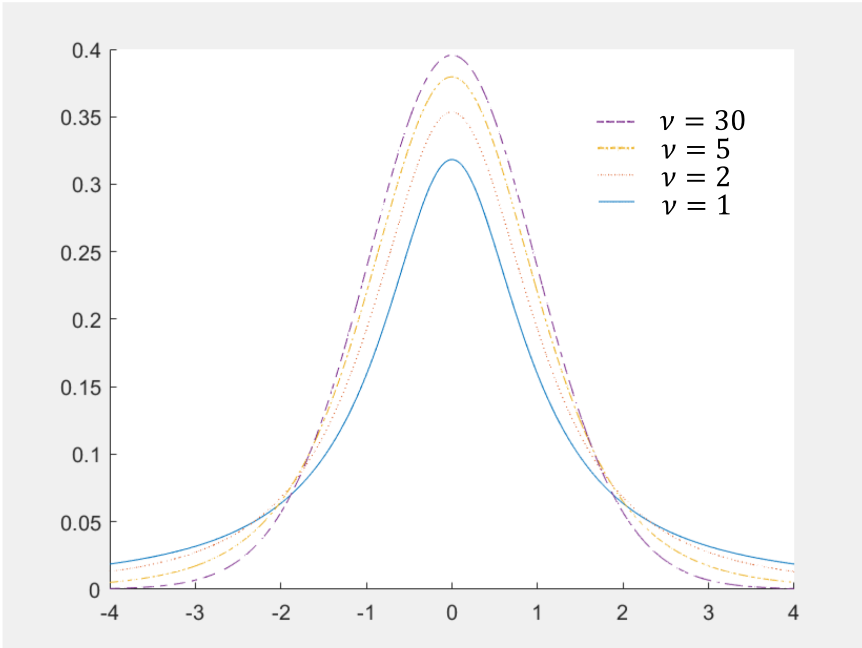

Fig 23 - Probability density function of Standard t-distribution with different degree of freedom (ν)

Generalized Error Distribution (GED)

Generalized Error distribution (GED) is another distribution that can describe fat tail behavior. It has probability density function defined by:

$$pdf = f(x) = \frac{\nu}{\lambda 2^{\left(1+\frac{1}{\nu}\right)} \Gamma(\frac{1}{\upsilon})\sigma} e^{-\frac{1}{2}\left|\frac{x-\mu}{\sigma\lambda}\right|^{\nu}} \quad with \quad \lambda = \sqrt{2^{-\left(\frac{2}{\nu}\right)} \frac{\Gamma(\frac{1}{\upsilon})}{\Gamma(\frac{3}{\upsilon})}}$$

where $\Gamma(\cdot)$ is the gamma function and ν is the shape or tail-thickness parameter. The standard GED has zero location ($\mu = 0$) and unity scale (σ=1) parameter. When ν =2, the distribution becomes a normal distribution. When $\nu < 2$, the GED has tail thicker than normal distribution. As $\nu \to \infty$, a standard GED becomes a uniform distribution between interval $(-\sqrt{3}, \sqrt{3})$ (Angelidis, Benos and Degiannakis 2004). GED is symmetrical, has zero skewness and higher kurtosis. A standard GED distribution has unit variance and the standard deviation is the same as scale parameter $Var[X] = \sigma_t^2 = \sigma^2$.

The VaR value (negative) is given by (Eldar and Torstin 2011):

$$VaR_q\,(r) = \mu - \sigma\, G_\nu{}^{-1}(q) = \mu - \sigma_t\, G_\nu^{-1}(q)$$

where $G_\nu^{-1}(\cdot)$ is the inverse cumulative distribution function of a standard general error distribution. The expected shortfall (negative) is described by:

$$ES_q(r) = \mu - \sigma \frac{\lambda 2^{(\frac{1}{\nu}-1)}}{\alpha\Gamma\left(\frac{1}{\nu}\right)} \Gamma(\frac{2}{\nu}, \frac{1}{2}(\frac{G_\nu^{-1}(q)}{|\lambda|})^\nu)$$

where $\Gamma(\mathrm{t},\mathrm{u}) = \int_u^\infty x^{t-1}e^{-x}\,dx$ is the *incomplete gamma function* (Eldar and Torstin 2011).

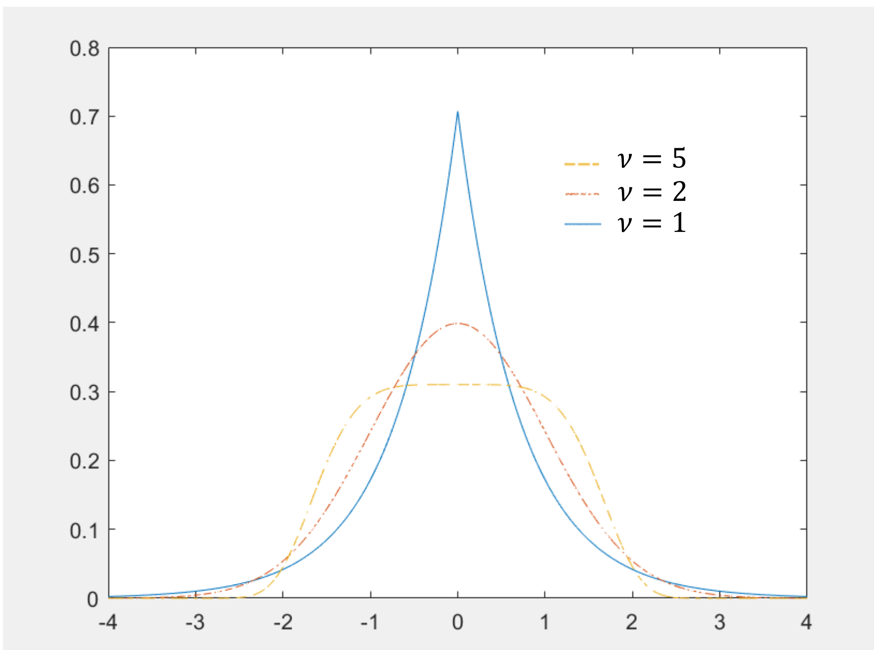

Fig 24 - Probability density function of standard GED with different shape parameter (ν)

Parameters estimation

This study used the rolling window approach with *in-sample* data (initial window size is: 2nd Jan. 2009 – 31st Dec. 2013 for STI index; 5th Jan. 2009 – 31 Dec. 2013 for SSE index) to estimate *out-of-sample* VaR or ES (2nd Jan. 2014 – 29 Jan. 2016 for both indices). When the next daily VaR or ES is estimated, the whole in-sample window moves or shifts forward by one. The parameters within the out-of-sample window period are estimated and assumed to be fixed.

The following (fig 25) shows the returns data of the initial rolling window with the estimated standard deviation and fitting it to normal distribution (*pdf* normalized for total area of 1). Notice the higher peak and fatter tails (leptokurtic) exhibited in the actual return data.

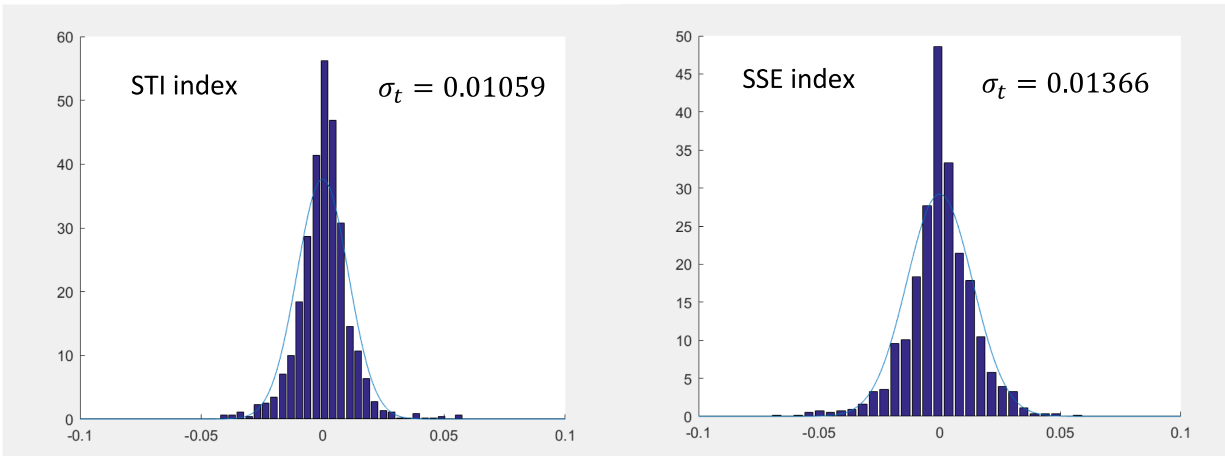

Fig 25 – the initial rolling window data fit to normal distribution for static model approach.

The following (fig 26) shows the same returns data from initial rolling window fitted to student t-distribution (*pdf* normalized for total area of 1) using *MATLAB* software (*fitdist* function). Notice the better fit for peak and the tail. The small value of ν signifies a fatter tail than normal distribution. For t-distribution, $\sigma_t = \sigma\sqrt{\frac{\nu}{\nu-2}}$.

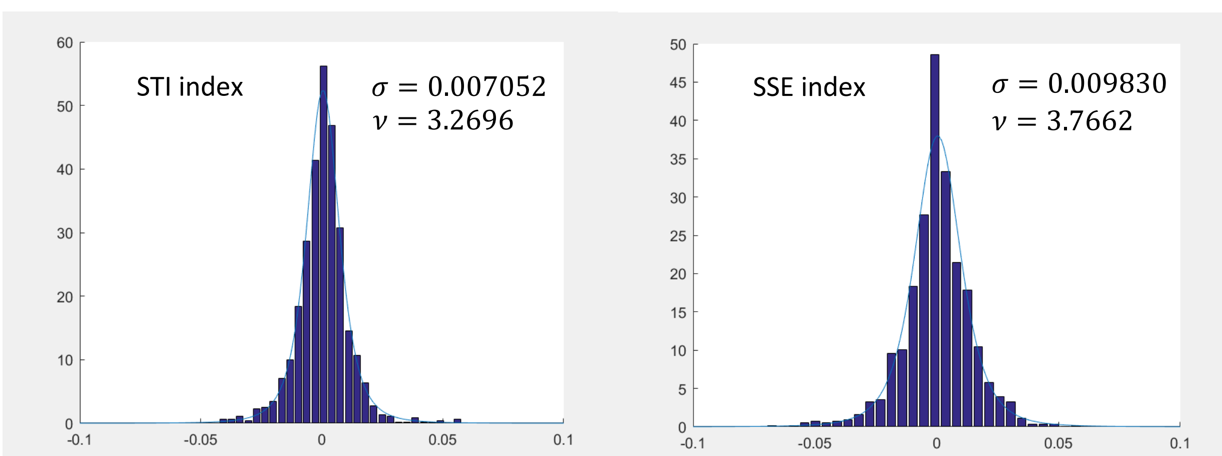

Fig 26 – the initial rolling window data fit to student t-distribution for static model approach.

The following (fig 27) shows fitting of returns data from initial rolling window to Generalized Error Distribution (*pdf* normalized for total area of 1) using *MATLAB* software (using *MLE*). The peak has the best fit and the tail fit is quite similar to student t-distribution. $\nu < 2$ signifies a fatter tail than normal distribution. Notice σ is (almost) the same as σ_t (of normal distribution).

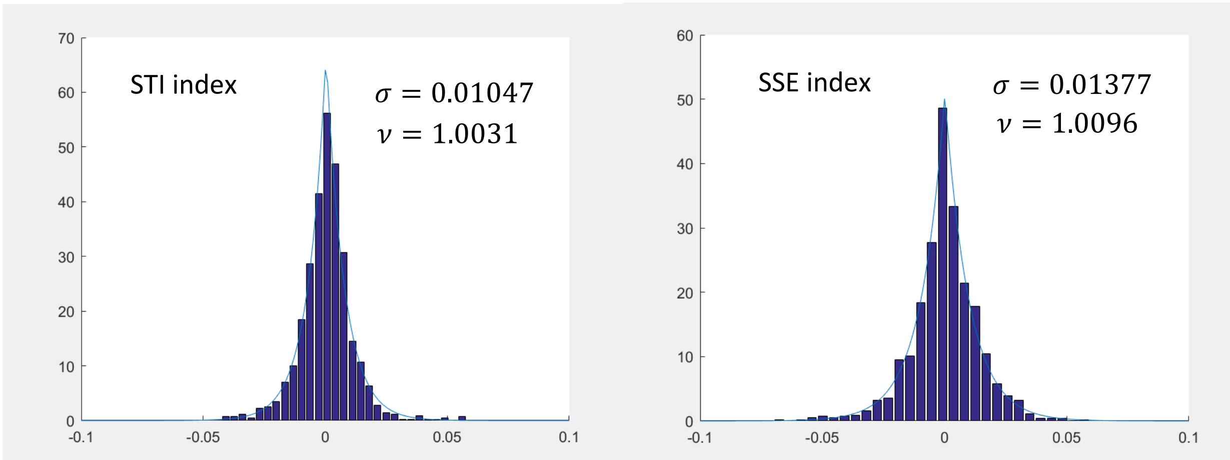

Fig 27 – the initial rolling window data fit to GED for static model approach.

DYNAMIC MODELS

All static models assumed that returns are independent and identically distributed (*iid*). However, financial returns exhibit strong serial dependence, especially in their second moment. This study has previously demonstrated how GARCH models incorporate *volatility clustering* and persistency behavior well. Engle (2001) improved the estimation of VaR by allowing standard deviation to change with time (heteroskedasticity) using ARCH and GARCH, which is the basic of dynamic approach. The following describes the volatility models used in dynamic approach in details.

Moving Average model

Assuming returns observed over *m* days, the volatility estimate is constructed similar to a standard deviation equation and from a moving average (MA), i.e.

$$\sigma_t^2 = \frac{1}{m}\sum_{i=1}^{m} r_{t-i}^2$$

where *m* has replaced (*m*-1) for unbiased estimate and $E[r_t] = \mu = 0$ as expected return in short one day is very small (Jorion 2007b; Hull 2012). This model suffers serious drawback as older observations which may no longer be relevant receive the same weights as recent information. The modified model gives more weights to recent data:

$\sigma_t^2 = \sum_{i=1}^{m} \alpha_i r_{t-i}^2$ where the weights sum to unity ($\sum_{i=1}^{m} \alpha_i = 1$) and $\alpha_i < \alpha_{i-1}$

EWMA model (RiskMetrics)

Using modified volatility model and $\alpha_i = (1-\lambda)\lambda^{i-1}$ where $0 < \lambda < 1$

$$\sigma_t^2 = (1-\lambda)\sum_{i=1}^{m}\lambda^{i-1}r_{t-i}^2 \qquad \text{and} \qquad \sigma_{t-1}^2 = (1-\lambda)\sum_{i=1}^{m}\lambda^{i-1}r_{t-1-i}^2$$

Extending *m* to *m+1* in σ_t^2 has negligible effect since extra term $\lambda^m r_{t-(m+1)}^2 \to 0$ as *m* is large

$$\sigma_t^2 = (1-\lambda)\sum_{i=1}^{m+1}\lambda^{i-1}r_{t-i}^2$$

$$= (1-\lambda)\sum_{i=2}^{m+1}\lambda\,.\,\lambda^{i-2}r_{t-i}^2 + (1-\lambda)r_{t-1}^2$$

Letting *j=i-1* and rewriting:

$$\sigma_t^2 = \lambda(1-\lambda)\sum_{j=1}^{m}\lambda^{j-1}r_{t-1-j}^2 + (1-\lambda)r_{t-1}^2$$

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)r_{t-1}^2$$

The last line is called Exponentially Weighted Moving Average (EWMA) model where weights α_i decrease exponentially as one move back through time, i.e. $\alpha_i = \lambda\alpha_{i-1}$(Hull 2012). The value λ governs how responsive the estimation of volatility to recent daily returns. A high value means daily volatility respond relatively slow to new information.

RiskMetrics created by J.P. Morgan in 1994 had used EWMA model with $\lambda = 0.94$ for daily data and $\lambda = 0.97$ for monthly data as it gave forecast of the variance rate that matches realized variance rate. This study used $\lambda = 0.94$ for all daily VaR and ES estimations. In 2006, RiskMetrics switched to using long memory model (Hull 2012; Jorion 2007a). The initial value of volatility is taken as standard deviation of recent *N* returns of *N+1* day in the rolling window period (Ivolatility 2014).

GARCH model

Extending the modified volatility model to include long-run average variance rate:

$$\sigma_t^2 = \gamma V_L + \sum_{i=1}^{m}\alpha_i r_{t-i}^2 \qquad \text{where weights sum to unity } (\gamma + \sum_{i=1}^{m}\alpha_i = 1)$$

This is known as ARCH(m) model, first introduced by Engle (1982).

$${\sigma_t}^2 = \omega_o + \sum_{i=1}^{m}\alpha_i\, r_{t-i}^2$$

Bollerslev (1986) introduced an extension of ARCH(m) model and parsimonious form which is GARCH(1,1). GARCH (*Generalised Autoregressive Conditional Heteroskedasticity*) captures the volatility clustering assuming *conditional variance* of residual or innovations depends on latest past innovations. The GARCH(1,1) model can be written as:

$$\sigma_t{}^2 = \omega_o + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad \text{where } \omega_o = \gamma V_L \text{ ;} \qquad \gamma + \alpha_1 + \beta_1 = 1$$

Sometimes, it is written in conditional variance format

$$h_t = \omega_o + \alpha_1 r_{t-1}^2 + \beta_1 h_{t-1}$$

The average, *unconditional variance* is found by setting $E[r_{t-1}^2] = h_t = h_{t-1} = h$ and solving it yields

$$h = \frac{\omega_0}{1-\alpha_1-\beta_1} \qquad \text{or} \qquad h = \frac{\gamma V_L}{1-\alpha_1-\beta_1} = \frac{(1-\alpha_1-\beta_1)\, V_L}{(1-\alpha_1-\beta_1)} = V_L$$

The GARCH(1,1) model will be stationary if the sum $\alpha_1 + \beta_1 < 1$ (Jorion 2007a). The restrictions $\omega_o > 0, \alpha_1 > 0, \beta_1 > 0$ are imposed to ensure conditional variance $\sigma_t{}^2 > 0$.

Rewriting GARCH(1,1) equation using $\omega_o = (1 - \alpha_1 - \beta_1)\, V_L$

$$\sigma_t{}^2 = V_L + \alpha_1 (r_{t-1}^2 - V_L) + \beta_1 (\sigma_{t-1}^2 - V_L)$$

Using $E[r_t^2] = \sigma_t^2$ and setting expected values on both sides

$$E[\sigma_t{}^2 - V_L] = (\alpha_1 + \beta_1) E[\sigma_{t-1}^2 - V_L]$$

For future time *t+n* and replacing $E[\sigma_{t+n-1}^2 - V_L]$ recursively

$$E[\sigma_{t+n}^2] = V_L + (\alpha_1 + \beta_1)^n (\sigma_t^2 - V_L)$$

The sum $\alpha_1 + \beta_1$ is called the *persistence* as it defines the speed at which shocks to variance revert to their long-run values. The last term becomes progressively smaller over time as $\alpha_1 + \beta_1 < 1$. Typical financial series have *persistence* around 0.95 to 0.99 for daily data (Jorion 2007b).

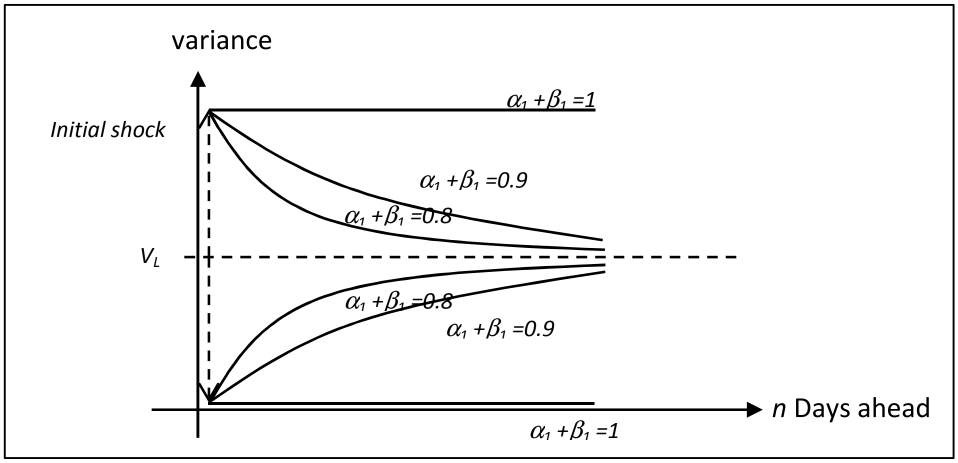

Fig 28 – Shocks to variance of GARCH process

EWMA is a special case of GARCH process where $\omega_o = 0, \alpha_1 = 1 - \lambda, \beta_1 = \lambda$ and $\alpha_1 + \beta_1 = 1$ (permanent persistence and no mean reversion). Over 1-day horizon, however, EWMA is quite similar to GARCH (Jorion 2007b).

GJR-GARCH (TGARCH) model

The GARCH model has limitation in enforcing a symmetric response of volatility to positive and negative shock since the conditional variance depends on the magnitude and not on the sign of r_t or ϵ_t (squared values). Black (1976, cited in Brooks 2008) first observed negative correlations between stock return and the changes in return volatility. This is called *leverage effect* where volatility tends to rise in response to bad news (loss) (r_t or ϵ_t <0) and fall in response to good news (r_t or ϵ_t >0). The fall in stock causes firm's debt to equity ratio to rise and leads shareholders who bear residual risk of the firm to perceive higher risk (Brooks 2008).

GJR-GARCH model is one of the popular asymmetric GARCH models proposed by Glosten, Jagannathan and Runkle (1993). It is a simple extension of GARCH with an additional term accounted for asymmetric behavior. GJR-GARCH is used interchangeably and sometimes known as Threshold GARCH (TGARCH) but the difference is TGARCH expresses the equation in conditional standard deviation (Zakoian 1994). The conditional variance of GJR-GARCH is given by:

$$\sigma_t^2 = \omega_0 + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma_1 r_{t-1}^2 I_{t-1}$$

where indicator $I_{t-1} = 1$ if $r_{t-1} < 0$ and $I_{t-1} = 0$ otherwise.

When $r_t > 0$, its contribution on σ_t^2 is α_1, otherwise is $(\alpha_1 + \gamma_1)$ when $r_t < 0$. Leverage effect exists if $\gamma_1 > 0$, otherwise positive shock has larger effect on volatility if $\gamma_1 < 0$. The condition for non-negativity applies where $\omega_0 > 0$, $\alpha_1 > 0$, $\beta_1 \geq 0$ and $\alpha_1 + \gamma_1 \geq 0$. γ_1 can be negative provided $\alpha_1 + \gamma_1 \geq 0$.

EGARCH model

Exponential GARCH is another popular asymmetric GARCH model proposed by Nelson (1991). The conditional variance is given by:

$$\ln(\sigma_t^2) = \omega_0 + \beta_1 \ln(\sigma_{t-1}^2) + \gamma_1 z_t + \alpha_1[|z_t| - |E(z_t)|] \qquad ; z_t = \frac{r_{t-1}}{\sqrt{\sigma_{t-1}^2}}$$

$$\ln(\sigma_t^2) = \omega_0 + \beta_1 \ln(\sigma_{t-1}^2) + \gamma_1 \frac{r_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha_1 \left[\frac{|r_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}}\right]$$

where $|E(z_t)| = \sqrt{\frac{2}{\pi}}$ for conditional normal errors (Nelson originally proposed GED errors) and standardized form $z_t = \frac{r_{t-1}}{\sqrt{\sigma_{t-1}^2}}$ is used instead of direct form ϵ_t or r_t. Since the use of logarithm form $\ln(\sigma_t^2)$ ensures variance σ_t^2 will be positive, there is no positive restriction on model parameters. The logarithm also implies leverage effect is exponential rather than quadratic. Asymmetries are allowed since if the relationship between volatility and returns is negative, γ_1 will be negative (Brooks 2008). The standardized effect on $\ln(\sigma_t^2)$ is $(\gamma_1 + \alpha_1)$ when $r_t > 0$, otherwise is $(\gamma_1 - \alpha_1)$. If $r_t < 0$ and $\gamma_1 < 0$, $(\gamma_1 - \alpha_1)$ has a larger magnitude (leverage effect). Converserly, if $r_t > 0$ and $\gamma_1 > 0$, $(\gamma_1 + \alpha_1)$ has larger magnitude (opposite leverage effect). Therefore, it can capture effects in both ways.

APARCH model

Asymmetric Power ARCH (APARCH) is a model that captures asymmetric effects and proposed by Ding, Granger and Engle (1993). It has the following expression:

$$\sigma_t^\delta = \omega_0 + \alpha_1(|r_{t-1}| - \gamma r_{t-1})^\delta + \beta_1 \sigma_{t-1}^\delta$$

where $\omega_0, \delta \geq 0, \alpha_1 \geq 0, \beta_1 \geq 0$ and $-1 < \gamma_1 < 1$. A significant positive asymmetry parameter $\gamma_1$indicates past negative shocks have deeper effect on conditional variance and vice-versa. APARCH model imposes a Box-Cox power transformation via power parameter δ and can linearize otherwise nonlinear models. APARCH estimates δ freely by modeling distribution characteristics and 'long memory' behavior more accurately than Bollerslev's GARCH. Long memory effect presents when $|r_t|^d$ for $d > 0$ has quite high autocorrelations over long lags. APARCH is touted as one of the most promising ARCH model as it encompasses other GARCH models such as Bollerslev's GARCH model when $\delta = 2, \gamma_1 = 0$ and GJR-GARCH when $\delta = 2$ (Ding 2011; Erhan 2010).

Dynamic approach

Recall that the returns model is $r_t = \mu + \epsilon_t$ where $\epsilon_t = \sigma_t z_t$ is the residuals, innovation or error. The mean μ is approximately zero, σ_t is the standard deviation and z_t is *iid* random variable with mean zero and unit variance (standardized residual). In dynamic approach, time varying σ_t or conditional variance is described by volatility models such as GARCH(1,1) and z_t is described by a conditional distribution such as *normal distribution*, *student-t distribution*, or *Generalized Error distribution (GED)*. For example, a dynamic approach with GARCH(1,1)-n (or equivalently GARCH(1,1)) means the return data is conditional normally distributed based on past information on day *t-1* or before and $z_t \sim$ *iid N(0,1).* The time varying σ_t which models volatility clustering is fitted to the distribution. The advantage of dynamic approach is that z_t is *iid* (proven previously when GARCH filtered squared standardized residual $z_t^2 = (\frac{\epsilon_t}{\sigma_t})^2$ has no autocorrelation).

The GARCH model parameters $(\omega_o, \alpha_1, \beta_1)$ can be estimated using *maximum likelihood method* (MLE). The maximum likelihood method involves choosing values for parameters that maximize the probability (or likelihood) of the data occurring (Hull 2012). The likelihood of *T* observations occurring is equal to multiplication of *T* probability density functions (i.e. $pdf \times pdf \times \ldots = \prod_{t=1}^{T} pdf$). Maximizing this probability term is equivalent to maximizing the logarithm of this probability term which simplifies to the summation of the logarithm of individual probability term (i.e. $\sum_{t=1}^{T} \ln(pdf)$). This summation of logarithm terms is known as the log-likelihood function, l_t (Brooks 2008). GARCH model parameters are usually estimated using MLE by *assuming a normal distribution* for ϵ_t, which is maximizing

$$\prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\sigma_t^{\,2}}} e^{-\frac{1}{2}(\frac{\epsilon_t}{\sigma_t})^2} \Rightarrow l_t = \sum_{t=1}^{T}(-\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln(\sigma_t^2) - \frac{1}{2}\frac{r_t^2}{\sigma_t^2})$$

where $\sigma_t^{\,2}$ is given for example by GARCH(1,1) equation $\sigma_t^{\,2} = \omega_o + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1}^2$ with ϵ_t and r_t used interchangeably since $\mu \approx 0$. Using the log-likelihood function and GARCH equation, the parameters $\omega_o, \alpha_1, \beta_1$ that maximize the logarithm sums is determined (by trying different combinations of $\omega_o, \alpha_1, \beta_1$) . This could be done in *Excel* spreadsheet using *Solver* function, in *MATLAB* software or *Gretl* software using *MLE function*. Notice that $\frac{r_t^2}{\sigma_t^2} = \frac{\epsilon_t^2}{\sigma_t^2} = \frac{(\sigma_t z_t)^2}{\sigma_t^2} = z_t^2$ where $z_t = \frac{\epsilon_t}{\sigma_t}$ is the standardized residual.

The GARCH model parameters with different conditional distributions can be estimated with different log-likelihood function as the following (Angelidis et al. 2004):

- For normal distribution:

$$l_t = -\frac{1}{2} T \ln(2\pi) - \frac{1}{2} \sum_{t=1}^{T} (\ln(\sigma_t^2) + z_t^2)$$

which is derived from $\prod_{t=1}^{T} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(\frac{\epsilon_t - \mu}{\sigma})^2}$ where $\sigma = \sigma_t;\ \mu = 0; \epsilon_t = \sigma_t z_t$

- For student t-distribution:

$$l_t = T\left(ln\,\Gamma\left(\frac{v+1}{2}\right) - ln\,\Gamma\left(\frac{v}{2}\right) - \frac{1}{2}\ln\big(\pi(v-2)\big) \right) - \frac{1}{2}\sum_{t=1}^{T} (\ln(\sigma_t^2) + (1+\nu)\ln(1 + \frac{z_t^2}{\nu - 2}))$$

which is derived from $\prod_{t=1}^{T} \frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)} \frac{1}{\sqrt{v\pi}\sigma} (1 + \frac{\left(\frac{\epsilon_t - \mu}{\sigma}\right)^2}{v})^{-\frac{v+1}{2}}$ where $\sigma = \sqrt{\frac{v-2}{v}}\sigma_t; \mu = 0; \epsilon_t = \sigma_t z_t$

- For Generalized Error Distribution (GED):

$$l_t = \sum_{t=1}^{T} (\ln(\frac{v}{\lambda}) - \left(1 + \frac{1}{\nu}\right)\ln(2) - \ln\Gamma\left(\frac{1}{v}\right) - \frac{1}{2}\left|\frac{z_t}{\lambda}\right|^{\nu} - \frac{1}{2}\ln(\sigma_t^2)) \ ; \qquad \lambda = \sqrt{2^{-(\frac{2}{v})} \frac{\Gamma(\frac{1}{v})}{\Gamma(\frac{3}{v})}}$$

which is derived from $\prod_{t=1}^{T} \frac{\nu}{\lambda 2^{\left(1+\frac{1}{\nu}\right)} \Gamma(\frac{1}{v})\sigma} e^{-\frac{1}{2}|\frac{\epsilon_t - \mu}{\sigma\lambda}|^{\nu}}$ where $\sigma = \sigma_t;\ \mu = 0; \epsilon_t = \sigma_t z_t$

After determination of GARCH model parameters ($\omega_o, \alpha_1, \beta_1$) and distribution parameters (ν) using window data sample up to t-1 and MLE method, the conditional variance σ_t^2 (different value for each distribution) could be estimated and used to calculate the one-day ahead VaR and ES forecast for day t. The VaR and ES estimation uses exactly the same equations for different distribution previously seen in static model (reinstated in table below with a different form where $VaR_q(z_t)$ and $ES_q(z_t)$ represent the positive VaR and ES of z_t distribution, with zero-mean and unit variance, respectively) (Kjellson 2013).

Table 4 – The VaR and ES equations for different conditional distribution in dynamic approach

Value-at-Risk	Expected Shortfall
Normal distribution: $$VaR_q\,(r) = \mu - \sigma_t\, VaR_q(z_t)$$ where $VaR_q(z_t) = \Phi^{-1}(q)$	$$ES_q(r) = \mu - \sigma_t\, ES_q(z_t)$$ where $ES_q(z_t) = \frac{\phi(\Phi^{-1}(q))}{1-q} = \frac{1}{\alpha\sqrt{2\pi}}\, e^{-\frac{1}{2}(\Phi^{-1}(q))^2}$
Student t-distribution: $$VaR_q\,(r) = \mu - \sigma_t\, VaR_q(z_t)$$ where $VaR_q(z_t) = \sqrt{\frac{\nu-2}{\nu}}\, T_\nu^{-1}(q)$	$$ES_q(r) = \mu - \sigma_t\, ES_q(z_t)$$ where $ES_q(z_t) = \sqrt{\frac{\nu-2}{\nu}}\, \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\alpha\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{\nu}{\nu-1} (1 + \frac{(T_\nu^{-1}(q))^2}{\nu})^{\frac{1-\nu}{2}}$
GED distribution: $$VaR_q\,(r) = \mu - \sigma_t\, VaR_q(z_t)$$ where $VaR_q(z_t) = G_\nu^{-1}(q)$	$$ES_q(r) = \mu - \sigma_t\, ES_q(z_t)$$ where $ES_q(z_t) = \frac{\lambda 2^{(\frac{1}{\nu}-1)}}{\alpha\Gamma\left(\frac{1}{\nu}\right)} \Gamma(\frac{2}{\nu}, \frac{1}{2}(\frac{G_\nu^{-1}(q)}{\lvert\lambda\rvert})^\nu)$

The dynamic approach mentioned above is a *joint single step* estimation approach that estimates GARCH parameters and distribution parameters simultaneously. It depends a lot on the data generating process, sensitive to misspecification and loses its flexibility. If the residuals of GARCH process are not really of a particular assumed conditional distribution, the parameter estimates would be biased or corrupted. It was found that the *quasi maximum likelihood estimation* (QMLE) of GARCH model with assumed normal distributed residuals is consistent and unbiased (Ergen 2010). QMLE, by definition, is a likelihood estimation method in which *likelihood function* has misspecified models, treating certain data values as independent. In other words, *QMLE with misspecified normal residuals* is essentially the same as the MLE method assuming normal distribution.

With the unbiased estimates of QMLE, McNeil and Frey (2000) introduced a *two steps* dynamic approach. The first step involves fitting a GARCH model to the returns data using MLE method assuming normal distribution, finding parameters $\omega_o, \alpha_1, \beta_1$ and σ_t^2. The second step involves generating the implied or filtered standardized residuals ($z_t = \frac{\epsilon_t}{\sigma_t}$) using σ_t estimates up to *t-1* and fit them to a distribution to find parameters like ν. McNeil and Frey have used this approach in their GARCH-EVT estimation and it has proven to produce good results. Attempts on GARCH-St (*skewed t-distribution*) using two steps approach also have outperformed the joint single step (Ergen 2010).

This study will use the *two steps* dynamic approach. The one day ahead VaR or ES forecast for day *t* is still estimated using the VaR and ES equations above. However, all conditional distributions use the σ_t values estimated using GARCH assuming normal distribution via MLE. This *two step*s approach is applied to other variants of GARCH(1,1) models: GJR-GARCH, EGARCH and APARCH with different conditional distributions: Normal, Student t-distribution and GED. For EWMA of RiskMetrics, the one day ahead VaR and ES forecast assumes a normal distribution (RiskMetrics 1996).

Parameters estimation

The GARCH(1,1) parameters that were estimated assuming a normal distribution for returns data of the initial rolling window using *MATLAB* or *Gretl* software (*garch* function) are shown as below. All parameters are significant since p-values<0.05. Notice that the *persistence* $\alpha_1 + \beta_1 < 1$ for both indices and the GARCH model is stationary.

Table 5 – GARCH(1,1) estimated parameters assuming normal distribution

	ω_0	α_1	β_1	$\alpha_1 + \beta_1$
STI index	0.8897e-06 [0.01]	0.08129 [0.00]	0.9089 [0.00]	0.9902
SSE index	4.3339e-06 [0.02]	0.03829 [0.00]	0.9370 [0.00]	0.9753

*p-values in square bracket

The following shows the standardized residuals ($z_t = \frac{\epsilon_t}{\sigma_t}$) from the returns data of the initial rolling window fitted to a standardized normal distribution (*pdf* normalized for total area of 1). Notice that normal distribution underestimated the fat tails and peak. Standardized residuals have variance close to 1.

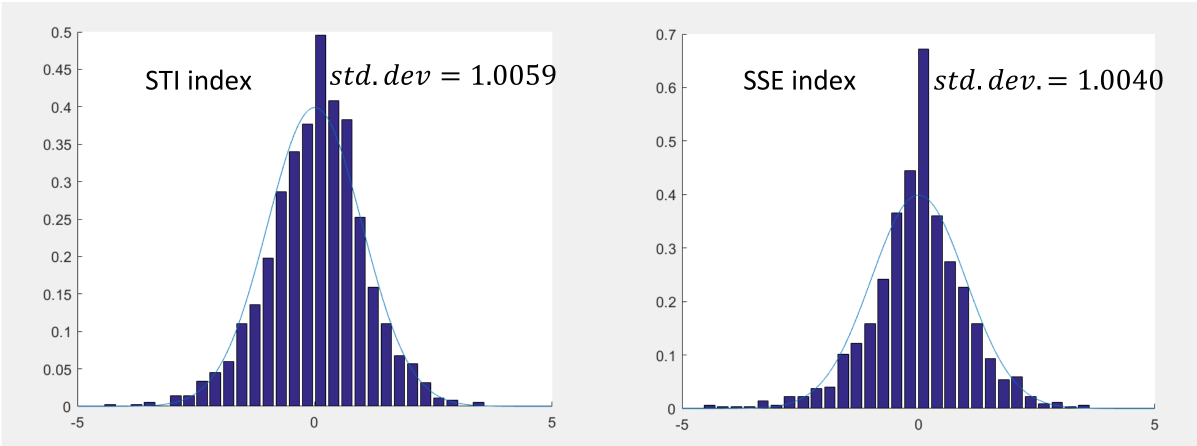

Fig 29 – the standardized residuals fit to standard normal distribution for dynamic approach.

The following shows the fitting of standardized residuals ($z_t = \frac{\epsilon_t}{\sigma_t}$) from the returns data of the initial rolling window to a student t-distribution (not standard t-distribution as $\sigma \neq 1$; *pdf* normalized for total area of 1) . Notice that both have fatter tails than normal distribution (SSE index having smaller ν and fatter). $\sigma\sqrt{\frac{\nu}{\nu-2}}$ calculation gives variance values of close to 1.

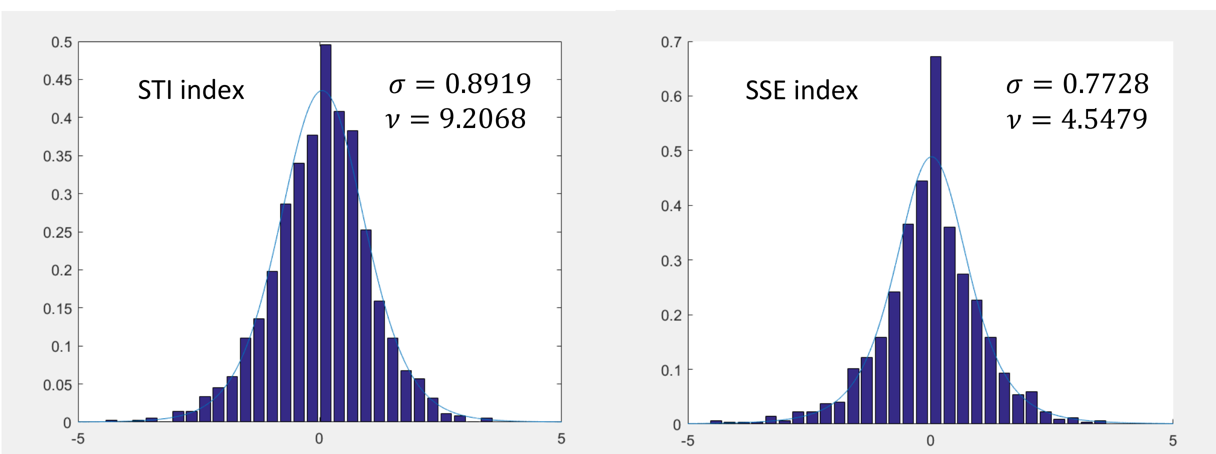

Fig 30 - the standardized residuals fit to student t-distribution for dynamic approach

The standardized residuals ($z_t = \frac{\epsilon_t}{\sigma_t}$) from returns data of the initial rolling window is also fitted to the standard Generalized Error Distribution (GED) as shown below (*pdf* normalized for total area of 1).

The GED fits the peak and tail well and both indices have fatter tail than normal distribution as $\nu < 2$. Notice that the variance (scale parameter) is close to 1.

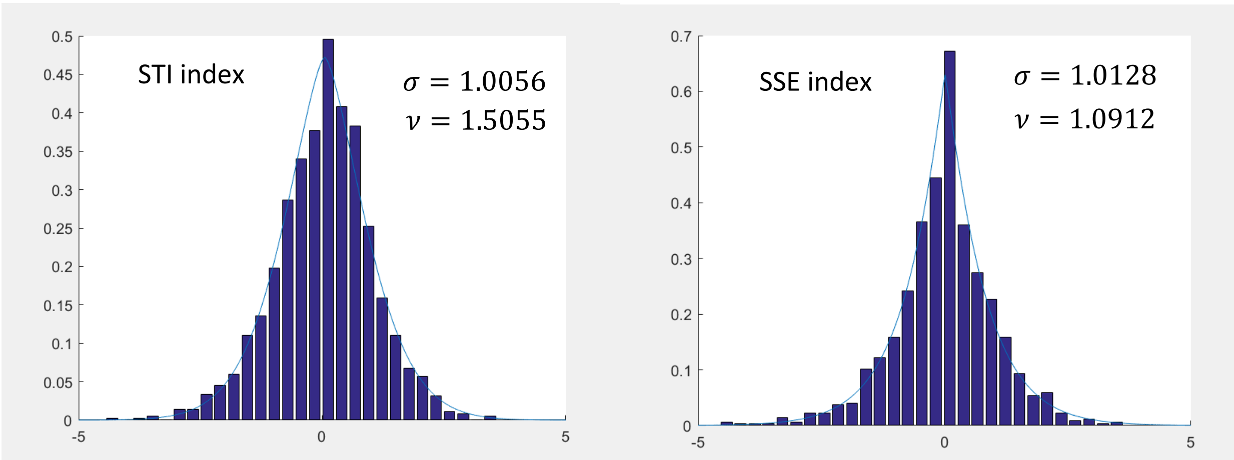

Fig 31 - the standardized residuals fit to standard GED for dynamic approach

The GJR-GARCH or TGARCH parameters are also estimated using *MATLAB* software (*MFE* toolbox) or *Gretl* software (*MLE function*) using conditional normal distribution and listed as below.

Table 6 – T/GJR-GARCH estimated parameters assuming normal distribution

	ω_0	α_1	β_1	γ_1
STI index	0.8108e-06 [0.01]	0.04643 [0.00]	0.9140 [0.00]	0.060238 [0.00]
SSE index	4.4657e-06 [0.00]	0.03625 [0.00]	0.9359 [0.00]	0.004474 [0.62]

*p-values in square bracket

The p-value<0.05 shows that γ_1 is significant in STI index (asymmetric effect) but not so in SSE index for the initial rolling window time horizon. The positive value γ_1 in GJR-GARCH shows the presence of leverage effect in which volatility increases with negative shocks.

The EGARCH parameters estimated assuming a conditional normal distribution using *MATLAB* software (*egarch function*) or GRETL (*MLE function*) are shown as below.

Table 7 – EGARCH estimated parameters assuming normal distribution

	ω_0	α_1	β_1	γ_1
STI index	-0.09132 [0.01]	0.1596 [0.00]	0.9901 [0.00]	-0.05294 [0.00]
SSE index	-0.15063 [0.01]	0.08387 [0.00]	0.9820 [0.00]	-0.003184 [0.65]

*p-values in square bracket

The p-value<0.05 shows that γ_1 is significant in STI index (asymmetric effect is prominent) but not so in SSE index in the initial rolling window time horizon. The negative value γ_1 in EGARCH shows the presence of leverage effect in which volatility increases with negative returns.

The model parameters estimation for APARCH model assuming conditional normal distribution using *MATLAB* software (*MFE toolbox*) is shown as the following.

Table 8 – APARCH estimated parameters assuming normal distribution

	ω_0	α_1	β_1	γ_1	δ
STI index	0.6861e-06	0.059530	0.9259	0.28230	2.0195
SSE index	0.0006e-06	0.005712	0.9505	0.07870	3.9822

The significant positive value γ_1 for STI index again shows the presence of asymmetric effect and leverage effect where negative shocks have deep effects on conditional variance, compared to SSE index in the initial rolling window time horizon. The δ value of close to 2 for STI index shows conditional variance has linear relationship with its lagged term and squared lagged returns (similar to normal GJR-GARCH), whereas a δ value close to 4 for SSE index shows conditional variance has a non-linear relationship with its lagged terms.

The standardized residuals ($z_t = \frac{\epsilon_t}{\sigma_t}$) of the return data using σ_t from GJR-GARCH, EGARCH or APARCH would be quite similar to those obtained from GARCH and the fitting to different conditional distributions are not repeated here.

Non-parametric approach

STATIC METHOD

Historical Simulation (HS)

The non-parametric approach is also called the *Historical Simulation* (HS) method as it uses the historical data with its empirical distribution without making any assumptions on distribution of the risk factors. It is simple to implement as no estimation of distribution parameters is required and there is no risk of parameters misspecification such as kurtosis or skewness. It is a full valuation method which goes back in time and apply the percentage changes in the variable values (eg. stock price) on past day *i* to the current day *n* to create a scenario *i* (Hull 2012):

Value under scenario *i*, $v_s = v_n \frac{v_i}{v_{i-1}} = v_n \left(\frac{v_{i-1}+(v_i-v_{i-1})}{v_{i-1}}\right) = v_n \left(1 + \frac{v_i-v_{i-1}}{v_{i-1}}\right) = v_n(1+R_i)$

The return of scenario *i* or percentage change of the value due to scenario *i* is $= \frac{v_s-v_n}{v_n} = R_i$, which is essentially the return on past day *i*. With *M* historical return data, *M* scenario returns would be generated and this forms a distribution of possible returns. The VaR value (negative) forecast for day *n* is obtained from the α-percentile of the *sorted M scenario returns* (in ascending order starting from most negative values), i.e. $R_{s1} \leq R_{s2} \leq \cdots \leq R_{sM}$ (Harmantzis, Miao and Chien 2006).

$$VaR_q\,(r) = R_{sn} \qquad \text{where } \alpha \in (\tfrac{n-1}{M}, \tfrac{n}{M})$$

The expected shortfall (negative) is the average loss or negative returns value beyond VaR:

$$ES_q\,(r) = E[R_i \mid R_i < VaR] = \frac{\sum_{k=1}^{n} R_{sk}}{n}$$

The HS approach assumes that returns are *iid* (independent and identically distributed) and does not allow for time-varying volatility. It puts the same weight on all observation in the chosen window and it is often dominated by data of single crisis which makes it difficult to test other assumptions. If market emerges from a volatile period entering a tranquil period, VaR would most likely be overestimated and vice-versa. The drop out of a crisis data from the window period may alter the risk measurement abruptly and this method is also sensitive to window size (Marrison 2002). A short window will not have sufficient historical observation whereas a long window will place emphasis on stale data and insufficiently sensitive to new information. This method also relies on the fact that past data is sufficient to give us reliable and plausible future prediction which may not be true. What did not happen in the past may happen in the future. It also fails to reflect permanent changes in risk factors such as a policy change that could affect the risk measurement (Dowd 1999).

DYNAMIC METHOD

Exponential Weighted Historical Simulation (Hybrid)

Boudoukh, Richardson and Whitelaw (1998) overcame the weakness of historical simulation by putting more weights to recent information using the exponential declining weights with a decay factor λ. It is a *hybrid approach* from the combination of RiskMetrics and historical simulation. Exponential weight of $w_i = \lambda^{i-1} \frac{(1-\lambda)}{(1-\lambda^M)}$ is paired with the returns of scenario *i* (i.e. returns of past day *i*, where *i*=1,2,3… *M* and *i*=1 is the present). All *M* exponential weights sums to 1 (i.e. $\sum_{i=1}^{M} w_i = 1$). For large rolling window or large *M* samples and $-1 \leq \lambda \leq 1$, the exponential weight is approximately $\lambda^{i-1}\,(1-\lambda)$ which is similar to exponential weight used in EWMA of RiskMetrics. For example, the weights when $\lambda = 0.94$ would be 0.06, (0.94)(0.06), $(0.94)^2(0.06)$ … $(0.94)^{M-1}(0.06)$ for *i*=1,2,3 … *M*. By sorting the scenario returns from most negative to positive (in ascending order, $R_{s1} \leq R_{s2} \leq \cdots \leq R_{sM}$), its *paired* or *corresponding exponential weight* is aggregated until the sum reaches the α-percentile. VaR value (negative) is the scenario return corresponds to the exponential weight last used in the sum (Boudoukh et al. 1998).

$$VaR_q\,(r) = R_{sn} \qquad \text{where} \quad \sum_{k=1}^{n} w_k = \alpha$$

The expected shortfall (negative) is the average loss or negative returns value beyond VaR:

$$ES_q\,(r) = E[R_i \mid R_i < VaR] = \frac{\sum_{k=1}^{n} w_k\, R_{sk}}{w_k}$$

This method should improve the performance of normal Historical Simulation method where it is just a case where $w_k = \frac{1}{M}$. Similar to EWMA, this study used $\lambda = 0.94$ for VaR and ES estimations.

Filtered Historical Simulation (FHS)

Hull and White (1998), Barone-Adesi, Giannopoulous and Vosper (2000; 1999) introduced the *Filtered Historical Simulation* (FHS) approach that combines Historical Simulation and conditional volatility from GARCH model. It inherited the advantage of non-parametric HS approach of not making any distributional assumption and variance forecast of varying volatility model. Barone-Adesi and Giannopoulos (2001) demonstrated the superiority of the Filtered Historical Simulation over Historical Simulation in VaR forecast.

FHS uses GARCH-based conditional volatility computed from past samples $\{R_i\}_{t=1}^{T}$ or rolling window of size *T* to standardize the residuals (or returns) and then scaled them by the forecasted volatility, i.e. $R_t = \frac{\sigma_t}{\sigma_i} R_i$. Such scaling normalizes the historical residuals to reflect current market condition. Subsequently, the VaR and ES are derived from the quantile of the scaled returns distribution similar to HS approach. Besides GARCH model, GJR-GARCH is also used to compute conditional volatility for FHS to account and test for asymmetric response.

Semi-parametric approach

STATIC METHOD

While the parametric approach specifies the entire distribution, the Extreme Value Theory (EVT) approach focuses only the tails of the return distribution. The interpretation here would be described for the *right tail* first which can be easily transform to the left tail by negating. Similar to central limit theorem which studies the behavior of the sum and average of a sequence of random variables from large observation regardless of underlying variable distribution, EVT studies maxima characteristic which is at the tail of a distribution. There are two approaches to EVT, one that considers the maximum of sequence of random variables (known as the *Block maxima method*) and the other considers the exceedance at some specified high level (Jorion 2007b; Coleman 2012). *Block maxima* method postulated that the normalized maxima of a sequence of random variables converges to a *Generalized Extreme Value (GEV*) distribution $H(x)$. The details are described in appendix. As asymptotic characterization of the maxima by GEV uses only the maxima and is wasteful of data, threshold exceedance also known as *Peaks over threshold* (POT) method is more practical (Coleman 2012). Suppose $X_1, X_2, \ldots, X_n$ from an unknown distribution with CDF $F(X)$ and the probability of X exceedance over high threshold u (i.e. $X - u$) is less than x is given by *Excess Distribution*:

$$F_u(x) = P(X - u \le x \mid X > u) = \frac{P(u<X\le u+x)}{P(X>u)} = \frac{F(x+u)-F(u)}{1-F(u)} = \frac{F(X)-F(u)}{1-F(u)};\ x \ge 0$$

Balkema and De Haan (1974) and Pickands (1975) showed that as threshold u rises to the right endpoint, the excess distribution $F_u(x)$ converges to a *Generalized Pareto Distribution (GPD)* with a shape parameter ξ (same as GEV's) that governs tail behavior. This can be proven by substituting $H(x)$ of GEV into $\frac{F(x+u)-F(u)}{1-F(u)}$ (Ruey 2009). The GPD is given by:

$$G_{\xi,\beta}(x) = 1 - \left(1 + \xi\frac{x}{\beta}\right)^{-\frac{1}{\xi}};\quad \xi \neq 0 \qquad \Leftrightarrow g(x) = \frac{1}{\beta}(1 + \xi\frac{x}{\beta})^{-\frac{1}{\xi}-1}$$

$$G_{\xi,\beta}(x) = 1 - exp\left(-\frac{x}{\beta}\right);\quad \xi = 0 \qquad \Leftrightarrow g(x) = \frac{1}{\beta}\ exp(-\frac{x}{\beta})$$

where $\beta > 0, x \ge 0$ for $\xi \ge 0$ and $0 \le x \le -\frac{\beta}{\xi}$ for $\xi < 0$. $g(x)$ is the pdf (differentiating CDF on the left) and β is the scale parameter. One important property of GPD is the excess distribution over an arbitrary threshold $u' > u$ is also a GPD (Ruey 2009).

Excess distribution (over u') for $G_{\xi,\beta}(x)$ = $G_{\xi,\beta(u')}(x)$; $\beta(u') = \beta + \xi(u' - u)$

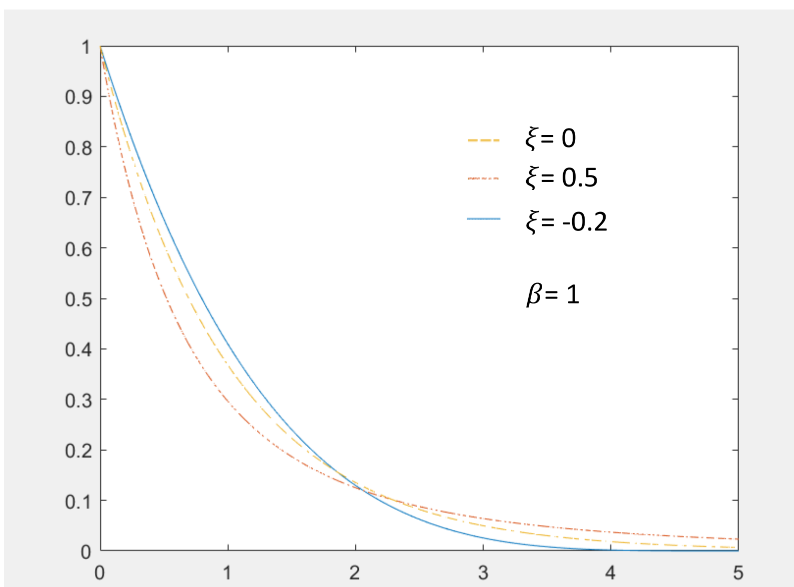

Fig 32 – GPD distribution with ξ= -0.2 (Pareto II; $x < 5$), ξ= 0.5 (Pareto; $x > 0$) and ξ=0 (Exponential)

Similar to GEV, if $\xi > 0$, $G_{\xi,\beta}(x)$ is of an ordinary Pareto distribution and is heavy-tailed. $\xi = 0$ corresponds to the exponential distribution (thinner tail) and $\xi < 0$ correspond to a short-tailed, Pareto type II distribution. Estimates of ξ are typical around 0.2 to 0.4 for financial data and it can be related to degree of freedom of t-distribution, approximately $\nu = \frac{1}{\xi}$ (Jorion 2007b). As $F_u(x) = \frac{F(X)-F(u)}{1-F(u)} \Rightarrow$ $F(X) = \big(1 - F(u)\big)F_u(x) + F(u)$, substituting $G_{\xi,\beta}(x)$ into $F_u(x)$ gives the following:

$$F(X) = 1 - [1 - F(u)]\left(1 + \xi\frac{x}{\beta}\right)^{-\frac{1}{\xi}} = 1 - \frac{N_u}{N}\left(1 + \xi\frac{X-u}{\beta}\right)^{-\frac{1}{\xi}} \quad ; \xi \neq 0$$

$$F(X) = 1 - [1 - F(u)]\exp\left(-\frac{x}{\beta}\right) = 1 - \frac{N_u}{N}\exp\left(-\frac{X-u}{\beta}\right) \quad ; \ \xi = 0$$

where $[1 - F(u)] \ = P(X > u)$ is approximated by fraction of number of exceedances $\frac{N_u}{N}$. For a small upper tail probability α where $\alpha = 1 - q$, the *q-th* quantile is obtained by setting $F(X) = q$ and solving for X which yields the VaR (positive) value (Jorion 2007b):

$$VaR_q = u + \frac{\beta}{\xi}\left\{[\frac{N}{N_u}(1-q)]^{-\xi} - 1\right\} \quad ; \ \xi \neq 0$$

It can be proven that the *mean excess* over the threshold u is $(X - u \mid X > u) = \frac{\beta}{1-\xi}$. Since $\beta(u') = \beta + \xi(u' - u)$, the *mean excess function* is $e(u') = E(X - u'|X > u') = \frac{\beta+\xi(u'-u)}{1-\xi}$. The expected shortfall is defined by $ES_q = E(X \mid X > VaR_q) = VaR_q + E\big(X - VaR_q \mid X > VaR_q\big)$ where $\big(X - VaR_q \mid X > VaR_q\big) = \frac{\beta+\xi(VaR_q-u)}{1-\xi}$. Therefore, the expected shortfall is given by (Ruey 2009; Singh, Allen and Powell 2011):

$$ES_q = \frac{VaR_q}{1-\xi} + \frac{\beta-\xi u}{1-\xi} \quad ; \ 0 < |\xi| < 1$$

The parameters of GPD can be estimated by *maximum likelihood (MLE)* once the threshold u has been chosen. The log-likelihood function (based on $pdf, g(x)$) is

$$l_t = -N_u ln\beta - (\frac{1}{\xi} + 1)\sum_{i=1}^{N_u} ln(1 + \frac{\xi}{\beta}(X - u)) \quad ; \ \xi \neq 0, X > u$$

$$l_t = -N_u ln\beta - \frac{1}{\beta}\sum_{i=1}^{N_u}(X - u) \quad ; \ \xi = 0, X > u$$

Other ways to determine the shape parameter is using *hill estimator* where ξ is the stable value of $\frac{1}{k}\sum_{i=1}^{k}(\,lnX_{N-i+1} - lnX_{N-k})$ plot against k, with order statistic $X_1 \leq X_2 \leq \cdots \leq X_N$ (Ruey 2009; Ramazan and Faruk 2004). Determining the threshold u is important as a low value may include more observations and producing biased estimate while a high value means less observations and high variance. The threshold u could also be determined by method such as *mean access function*. Since $e(u')$ is a linear function of $u' - u$, u is the starting point of linearity in the plot of *empirical mean access function* $e(u') = \frac{1}{N_u}\sum_{i=1}^{N_u}(X - u')$ against u'. Threshold value can also be seen from QQ-plot (quantile-quantile) against exponential distribution (thin tail) at the point where convex departure from straight line signifies fat tail (Ramazan and Faruk 2004).

As determining threshold u from these plots is somewhat difficult, subjective and the rolling window approach for in-sample data is used, it is impractical to determine individual threshold for each window. Therefore, the 90th quantile (upper 10 percent) of each window sample was chosen as threshold u which also included the 95th quantile for VaR and ES determination.

Parameters estimation

The EVT parameters that were estimated using *MATLAB* software (*gpfit function* which is based on *MLE*) are shown as below. The negative returns of the initial window period have been transformed temporarily into absolute positive values and the exceedance over threshold u values (i.e. $-u$) are shown (*pdf* normalized for total area of 1). As STI index returns have GPD shape parameter of $\xi < 0$, they are thinner tail than SSE index returns which have $\xi > 0$. Notice that QQ-plot (against exponential distribution) depart from straight line at around 0.02 for STI index and at around 0.03 for SSE index, relatively close to the 90th quantile threshold value u picked.

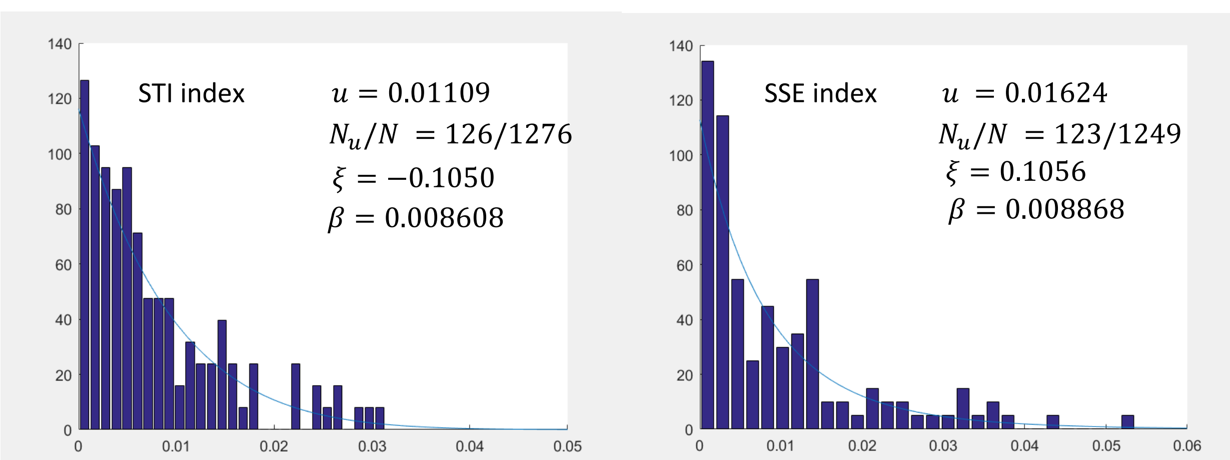

Fig 33 - the returns exceedance over threshold u fits to GPD distribution for EVT approach.

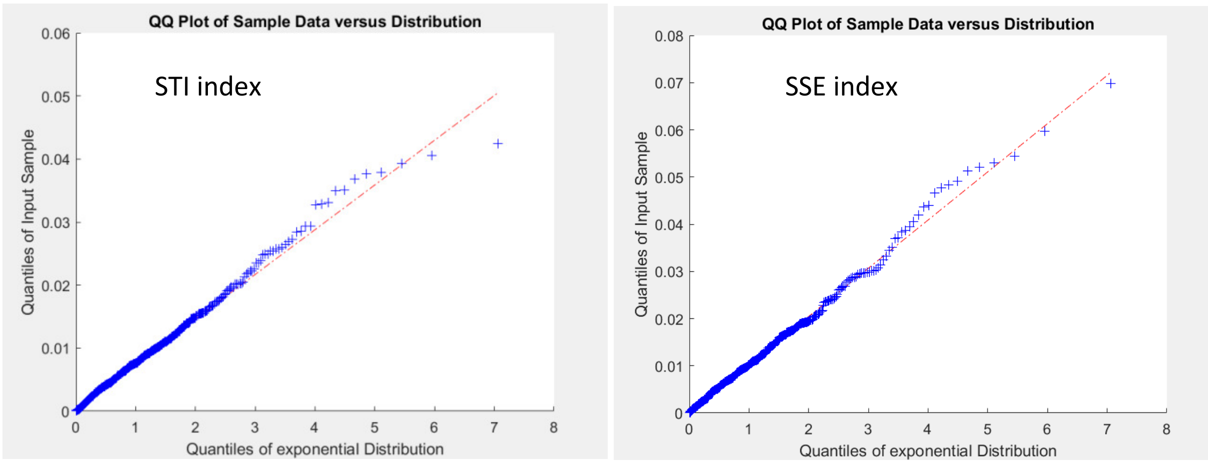

Fig 34 – QQ-plot of negated negative returns against exponential distribution

DYNAMIC METHOD

The dynamic method is similar to the two steps method of the dynamic parametric approach. It involves fitting a time-varying volatility model like GARCH using QMLE or equivalently MLE assuming normal distribution (which gives GARCH parameters and σ_t^2) and estimating the tail of the filtered or *iid* standardized residuals ($z_t = \frac{\epsilon_t}{\sigma_t}$; using σ_t estimates up to *t-1*) by an EVT model. This method has been used by McNeil and Frey (2000) and produced good results.

The one day ahead VaR and ES forecast for day *t* are estimated using the VaR and ES equations below. $VaR_q(z_t)$ and $ES_q(z_t)$ represent the positive VaR and ES respectively of z_t distribution with zero-mean and unit-variance.

Table 9 – The VaR and ES equations for EVT in dynamic approach

Value-at-Risk	Expected Shortfall
EVT-tail distribution: $VaR_q\,(r) = \mu - \sigma_t\, VaR_q(z_t)$ where $VaR_q(z_t) = u + \frac{\beta}{\xi}\left\{[\frac{N}{N_u}(1-q)]^{-\xi} - 1\right\}$	$ES_q(r) = \mu - \sigma_t\, ES_q(z_t)$ where $ES_q(z_t) = \frac{VaR_q(z_t)}{1-\xi} + \frac{\beta - \xi u}{1-\xi}$

The same dynamic approach is also applied with GJR-GARCH in place of GARCH(1,1) to test for asymmetric response and the results would not be repeated here as they are quite similar.

Parameters estimation

The EVT parameters for the GARCH filtered standardized residuals that were estimated using *MATLAB* software (*gpfit function* which is based on *MLE*) are shown as below. The standardized residuals from the initial window period have been transformed temporarily into absolute positive values and the exceedance over threshold u values are shown (*pdf* normalized for total area of 1). As STI index standardized residuals have GPD shape parameter of $\xi < 0$, they have thinner tail than SSE index standardized residuals who have $\xi > 0$. Notice that QQ-plot (against exponential distribution) depart from straight line at around 2 for STI index and at around 1.8 for STI index, relatively close to the 90th quantile threshold value u picked.

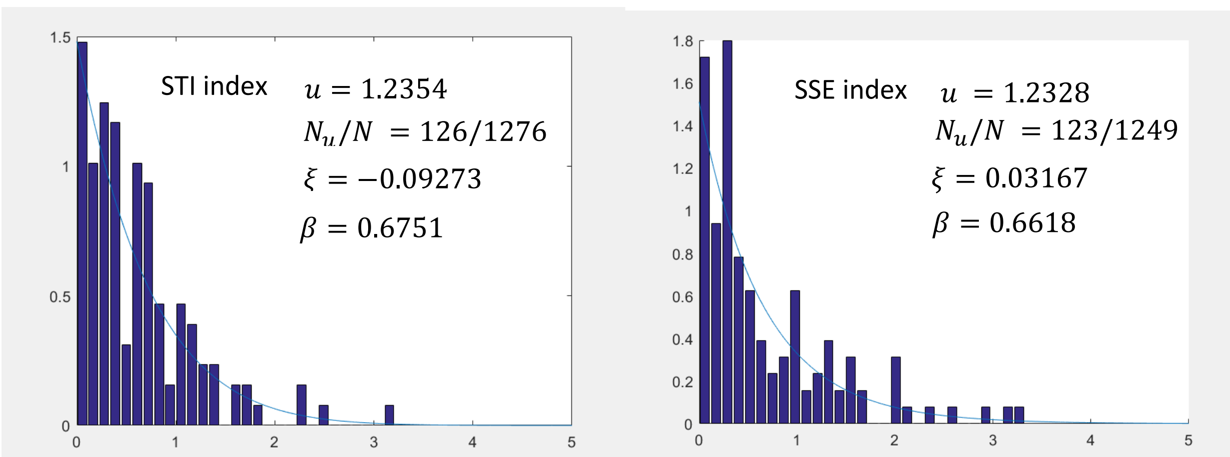

Fig 35 – GARCH filtered standardized residuals exceedance over threshold u fits to GPD distribution for dynamic EVT approach.

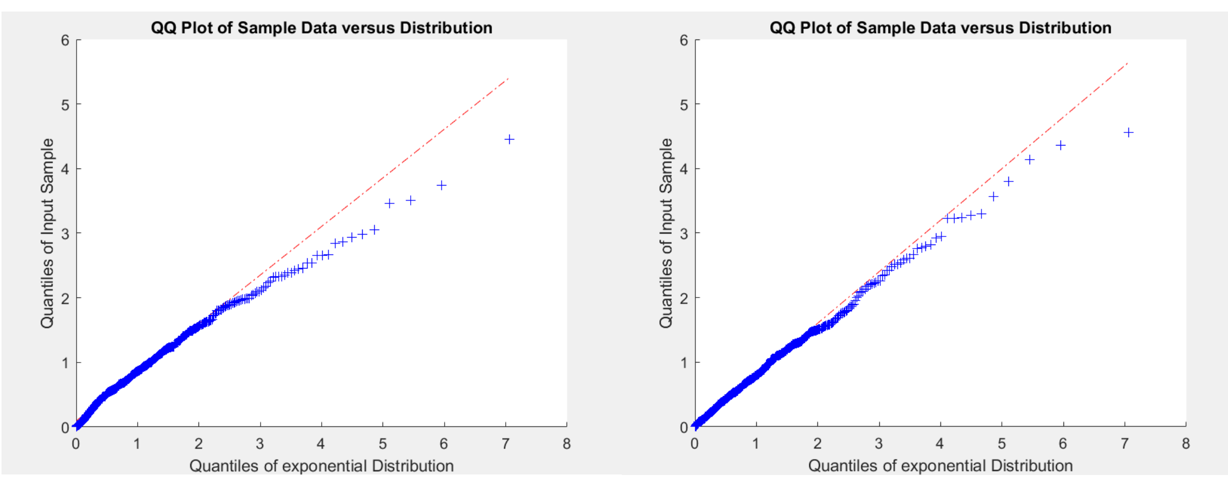

Fig 36 – QQ-plot of GARCH filtered standardized residuals against exponential distribution

The same dynamic approach is also applied with GJR-GARCH in place of GARCH(1,1) to test for asymmetric response and the results would not be repeated here as they are quite similar.

VaR BACKTESTING AND EVALUATION

In a risk management environment, it is essential to evaluate the accuracy and quality of the out-of sample VaR estimation. VaR values have to be checked if statistically adequate and consistent with subsequently realized returns (or loss) given the confidence interval on which the VaR values were constructed in the first place. This verification process is known as *backtesting* and is central to Basel Committee's ground-breaking decision to allow internal VaR models used for capital requirement calculation (Jorion 2007b). With rigorous backtesting mechanism, Basel Committee can prevent banks from understating their risk or avoid unduly penalization of banks whose VaR is exceeded simply because of bad luck.

Failure Rate

Days when actual loss exceeds the forecasted VaR are referred as *exceptions* or *violations*. The *number of exceptions* in the sample is given by $N = \sum_{t=1}^{T} I_t$ where

$$I_t = \begin{cases} 1 & if\ r_t < VaR_{t,q} \\ 0 & if\ r_t > VaR_{t,q} \end{cases}$$

When VaR is reported with confidence level of *q* percent (e.g. 99%), we expect α=1-*q* percent (e.g. 1%) of exceedance observed. The simplest method to verify the accuracy of VaR model is to compare the *failure rate* (number of exceptions over total number of predicted or out-of-sample days, *N/T*) with α. Ideally, the failure rate should give an unbiased measure of α, i.e. under null hypothesis *N/T* = α. If the failure rate *N/T* exceeds α (e.g. 7%>1%), the VaR estimation is likely to be underestimated. Conversely, if failure rate is well below α (e.g. 0.3%<1%), the VaR estimation is likely to be overestimated. In fact, the number of exceptions *N* follows a *binomial probability distribution*, $f(N) = \frac{T!}{N!(T-N)!} . \alpha^N (1-\alpha)^T$ with the *probability of an exception* on any given day of α, *expected value* of αT and *variance* of $\alpha(1-\alpha)T$. Whether *N* is too big or too small under null hypothesis could be evaluated by approximating the binomial distribution with normal distribution (using *central limit theory* as *T* is large), and compare the test statistics $z = \frac{(N-\alpha T)}{\sqrt{\alpha(1-\alpha)T}} \approx N(0,1)$ with critical value 1.96. If it exceeds the critical value, null hypothesis *N/T* = α will be rejected with 95 percent confidence in the two-tailed test (Jorion 2007b). This test is also known as the *Wald Test*.

Unconditional Coverage

Kupiec (1995) has proposed a relatively powerful two-tailed test. It is an unconditional coverage test that measures whether the number of exceptions or violation is consistent with the chosen confidence level. The number of exceptions follow a binomial distribution $N \sim B(T, \alpha)$ and the appropriate likelihood ratio (LR) statistic, under the null hypothesis that the expected exception frequency $N/T = \alpha$, equals

$$LR_{uc} = -2\ln[(1-\alpha)^{T-N}\alpha^{N}] + 2\ln[\left(1-\frac{N}{T}\right)^{T-N}\left(\frac{N}{T}\right)^{N}]$$

which is asymptotically distributed chi-square with one degree of freedom $\chi^2(1)$. The further N/T deviates from α, the larger is LR_{uc}. The null hypothesis will be rejected with 95 percent confidence if $LR_{uc} >$ 3.841 and VaR model is said to be inaccurate or not adequate. This test is equivalent to Wald Test using normal approximation as a chi-square variable is the square of a normal variable ($1.96^2=3.84$). This test may fail to detect VaR measures that exhibit correct unconditional coverage but exhibit VaR dependency.

Conditional Coverage

Occurrence of exceptions should evenly spread over time and randomly. Observed exceptions that cluster or happen closely in time suggest the losses on successive days are not independent, the VaR model not adequately responsive to market changes and this invalidates the unconditional coverage. A verification system should be designed to test for serially independence and measure proper conditional coverage, i.e. conditional on previous conditions. Such a test was developed by Christoffersen (1998).

With indicator where state of 1 represents *occurrence of exception* and 0 represent *absence of exception*, n_{ij} represents the number of days in which state *j* occurred in one day after state *i* in the previous day. For example, n_{01} denotes number of days with exception following a non-exception. There is also π_{i1} which is the probability of observing an exception conditional on state *i* in the previous day. For example, π_{01} denotes probability of observing an exception given non-exception previously.

Table 10 - Exception Table for conditional coverage

	$I_{t-1}=0$	$I_{t-1}=1$	
$I_t=0$	n_{00}	n_{10}	$n_{00} + n_{10}$
$I_t=1$	n_{01}	n_{11}	$n_{01} + n_{11}$
total	$n_{00} + n_{01}$	$n_{10} + n_{11}$	T

Christoffersen (1998) suggested the likelihood ratio (LR) statistics:

$$LR_{ind} = -2\ln[(1-\pi)^{n_{00}+n_{10}}\pi^{n_{01}+n_{11}}] + 2\ln[(1-\pi_{01})^{n_{00}}\pi_{01}{}^{n_{01}}(1-\pi_{11})^{n_{10}}\pi_{11}{}^{n_{11}}]$$

where $\pi = \frac{n_{01}+n_{11}}{n_{00}+n_{01}+n_{10}+n_{11}}$; $\pi_{01} = \frac{n_{01}}{n_{00}+n_{01}}$ and $\pi_{11} = \frac{n_{11}}{n_{10}+n_{11}}$

$n_{01} + n_{11} = N$ and $n_{00} + n_{10} = T - N$

Here, the first logarithm term represents the maximized likelihood under the null hypothesis that exceptions are independent across days, or $\pi_{01} = \pi_{11} = \pi = \frac{n_{01}+n_{11}}{T} = \frac{N}{T}$

A joint or combined test statistics for conditional coverage is then given by

$$LR_{cc} = LR_{uc} + LR_{ind}$$

Each component is independently distributed as chi-square with one degree of freedom $\chi^2(1)$ asymptotically. The sum is distributed as chi-square with two degree of freedom $\chi^2(2)$. Thus conditional coverage will be rejected at 95 percent test confidence level if $LR_{cc} > 5.991$. Independence of exceptions will be rejected alone if $LR_{ind} > 3.841$.

Loss Function

Kupiec's and Christoffersen's test cannot compare if an "adequate" VaR estimation is more accurate than another. Lopez (1998) suggested comparing VaR based on *loss function* which measures the distance between observed returns and forecasted VaR values. He defined a penalty variable using quadratic loss function:

$$\Psi_t = \begin{cases} 1 + (r_t - VaR_{t,q})^2 & if\ r_t < VaR_{t,q} \\ 0 & if\ r_t > VaR_{t,q} \end{cases}$$

This measurement is negative orientated as VaR model is penalized when an exception takes place. A model which yields a lower total loss value, defined as the sum of these penalty variables: $\Psi = \sum_{t=1}^{T} \Psi_t$ is preferred. The VaR forecast must not overestimate or underestimate the true VaR value as in both cases more or less capital than necessary would be allocated. This function incorporates both cumulative number (sum of ones) and magnitude of exceptions (the larger the failure the higher the penalty added).

ES BACKTESTING AND EVALUATION

V-Test

Backtesting expected shortfall (ES) is not as well established as the case for Value-at-Risk (VaR). There are two general measures used for backtesting which was introduced by Embrechts, Kaufmann and Patie (2005) known as the *V-test*. The first method calculates the average difference between the realized returns and forecasted ES, conditional on the returns (loss) exceeding the VaR value (negative). At confidence level *q*, the test statistics for *T* number of estimates is defined as the following

$$V_{ES1} = \frac{\sum_{t=1}^{T} (r_t - ES_{t,q}) I_{1t}}{\sum_{t=1}^{T} I_{1t}} \quad \text{where indicator function } I_{1t} = \mathbf{1}_{\{r_t < VaR_{t,q}\}}$$

Ideally, a value close to zero indicates an accurate ES measure. A negative value of V indicates an underestimation of expected risk losses while a positive value indicates an overestimation. This method is the standard method for ES backtesting and sticks closely to the theoretical definition of ES. However, it has weakness as it depends strongly on VaR estimation. This method may takes the mean over subsample of size different from *T(1-q)* depending how accurate is the VaR estimated (Kjellson 2013).

The second measure looks for the average difference that occur in a 1/(1-*q*) or 1/α events and not reliant on VaR. The test statistics is defined by

$$\mathrm{V}_{ES2} = \frac{\sum_{t=1}^{T} (r_t - ES_{t,q}) I_{2t}}{\sum_{t=1}^{T} I_{2t}} \quad \text{where indicator function } I_{2t} = \mathbf{1}_{\{D_t < D_q\}}$$

where the $D_t = (r_t - ES_{t,q})$ and D_q is the empirical (1-*q*) quantile or α-percentile of D_t. The measure can be represented in the figure below (ES is negative).

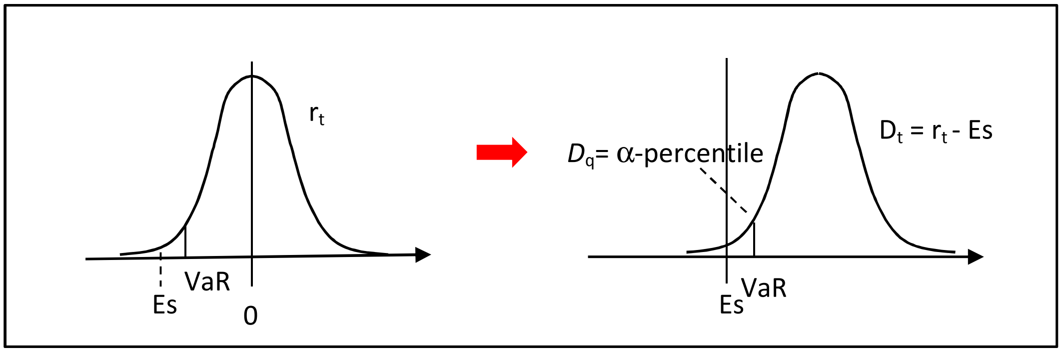

Fig 37 – D_t distribution in second measure of V-test for ES backtesting

Similarly, a good estimated ES should give value close to zero and D_t is expected to be negative in less than one out of 1/α cases. We can combine the two measures into a third measure that strikes a balance between the two by taking the absolute values and averaging them (Kjellson 2013):

$$V_{ES} = \frac{|V_{ES1}|+|V_{ES2}|}{2}$$

ASSESSING DOWNSIDE RISK SPILLOVER EFFECTS

One of the important methods to assess the downside risk spillover effects is to use Granger causality test. Granger (1980; 1969) developed a method to examine the causal effect of one time-series behavior on another. The main idea of the Granger causality is to check whether the past value of X can help predict the future value of Y besides Y's own past values, i.e. to test if X Granger-causes Y. Consider two stationary time series $X = \{x_t\}$ and $Y = \{y_t\}$ and let $I_{t-1} \equiv \{I_{X(t-1)}, I_{Y(t-1)}\}$ denotes all of the information sets available at time t-1, $I_{Y(t-1)} = \{Y_{(t-1)}, \ldots, Y_1\}$ and $I_{X(t-1)} = \{X_{(t-1)}, \ldots, X_1\}$ represent each information sets. The future values of Y given I_{t-1} and $I_{Y(t-1)}$ should be the same if unique information of X does not contribute to the immediate future value of Y (X does not Granger-cause Y)

$$H_0: \quad P(Y_t|I_{t-1}) = P(Y_t|I_{Y(t-1)})$$

More specifically, the test X does not Granger-cause Y can be expressed in two regression models below:

$$H_0: \quad y_t = \alpha_o + \sum_{i=1}^{k} \alpha_i \, y_{t-i} + u_{1t} \qquad \text{(restricted conditional regression)}$$

$$H_{1:} \quad y_t = \alpha_o + \sum_{i=1}^{k} \alpha_i \, y_{t-i} + \sum_{i=1}^{k} \beta_i \, x_{t-i} + u_{2t} \quad \text{(unrestricted conditional regression)}$$

where *k* is the maximal number of time lags. The null hypothesis is a joint hypothesis that lagged values of X are not statistically significant, i.e. $H_0: \beta_1 = \beta_2 = \cdots = \beta_k = 0$ and the alternative hypothesis is $H_1: \beta_i \neq 0$ for at least one value of *i*. F-test can be carried out to compare the *Sum of Squared Error* (or residuals) from the restricted model (SSE_r) with the *Sum of Squared Errors* of the unrestricted model (SSE_u). If SSE_r is statistically different from SSE_u then the null hypothesis is not valid. The F-statistic is given by

$$F = \frac{(SSE_r - SSE_u)/M}{SSE_u/(N-K)}$$

where *M* is the number of restrictions, i.e. number of lagged X values that have been omitted from the unrestricted regression. *N-K* is the total number of degrees of freedom in the unrestricted regression. *N* is the number of observations in the sample and *K* is the total number of parameters estimated in the unrestricted model (including constant). The F-statistic is compared to appropriate critical value from table to decide whether to accept or reject the null hypothesis of X does not Granger-cause Y.

We could use time series of VaR or ES downside risk estimator directly as X or Y depending on which stock market it represents. More discussion on this method is in empirical results section. Note that this Granger casuality test is different from Granger casuality in risk methodology provided by Hong (2001) and Hong et al. (2004) which takes VaR indicator function $Z_t = I$ where $\mathbf{1}_{\{r_t < VaR_{t,q}\}}$ in calculation of sample cross-correlation function and then used in determining test statistics of a kernel-based Granger causality test.

4 Empirical Results

VaR and ES of STI Index at 99% Confidence Level

Fig 38-46 show the STI index returns with the estimated VaR and ES at 99% confidence level using various methodologies where one can observe the number of VaR exceedance and compare actual exceedance loss with expected average loss in ES.

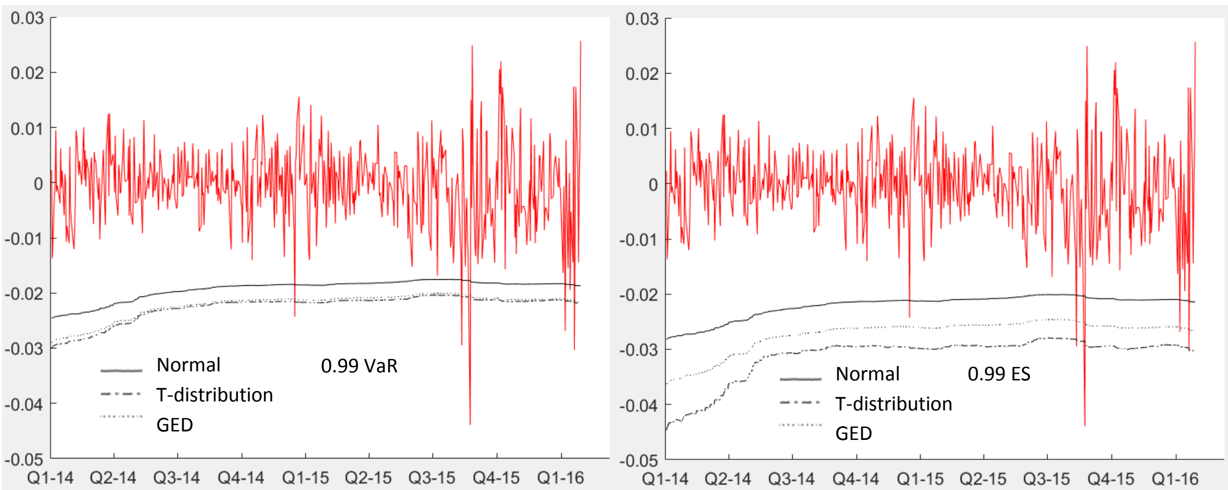

Fig 38 – STI returns with VaR and ES est. using VC/parametric static models at q=99%

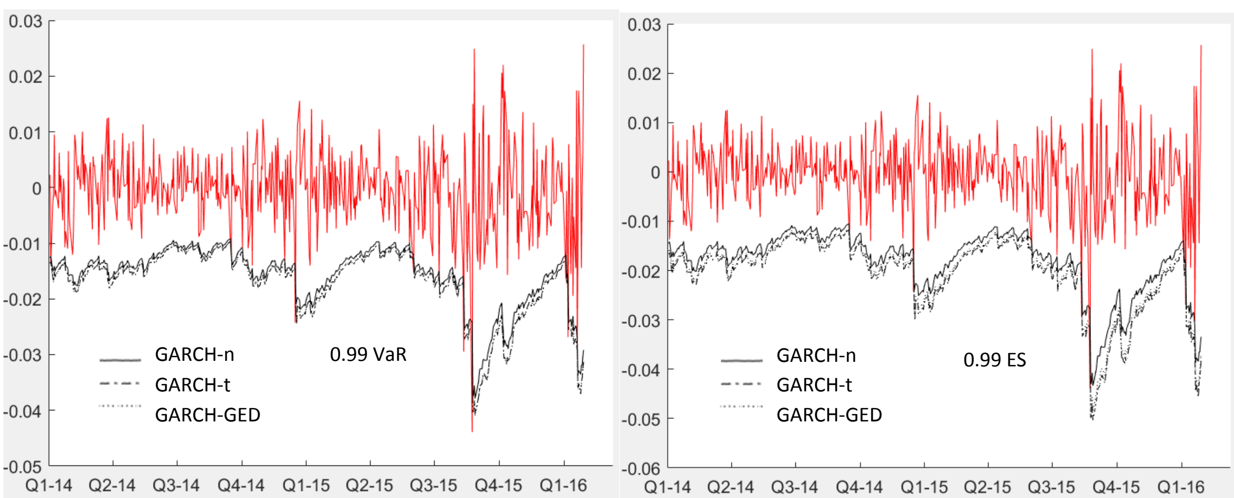

Fig 39 – STI returns with VaR and ES est. using variants of GARCH at q=99%

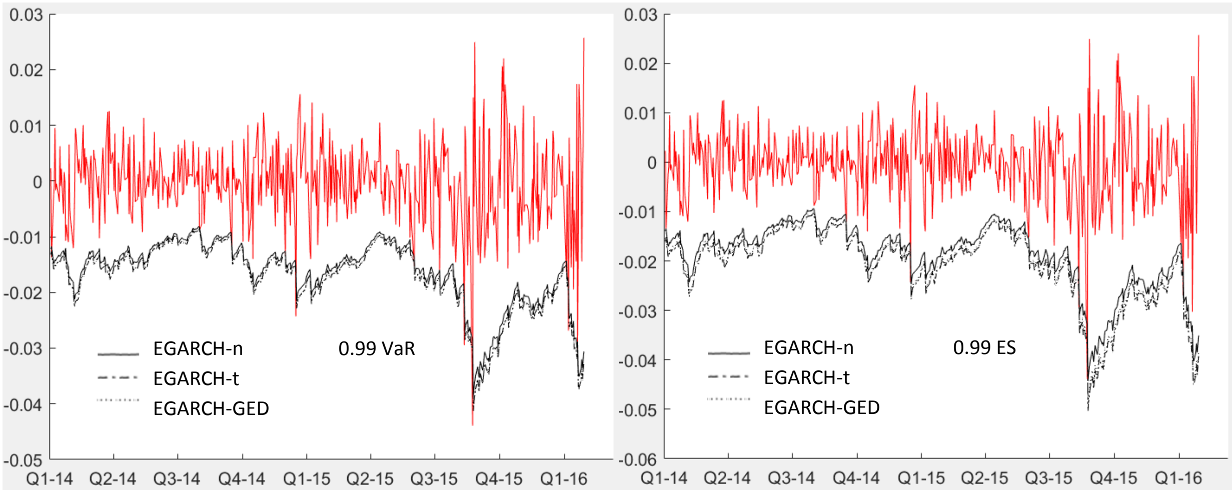

Fig 40 – STI returns with VaR and ES est. using variants of EGARCH at q=99%

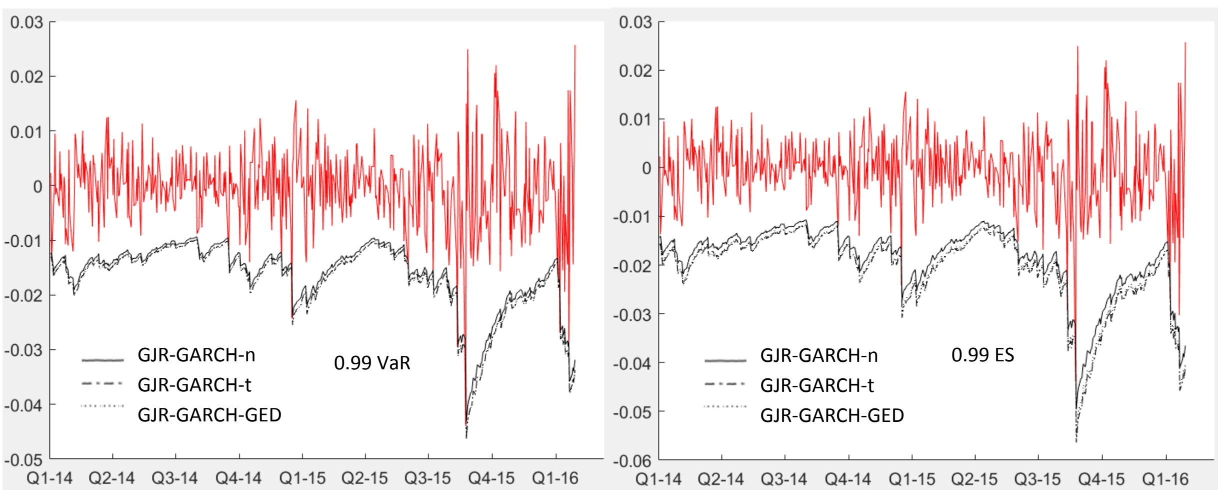

Fig 41 – STI returns with VaR and ES est. using variants of T/GJR-GARCH at q=99%

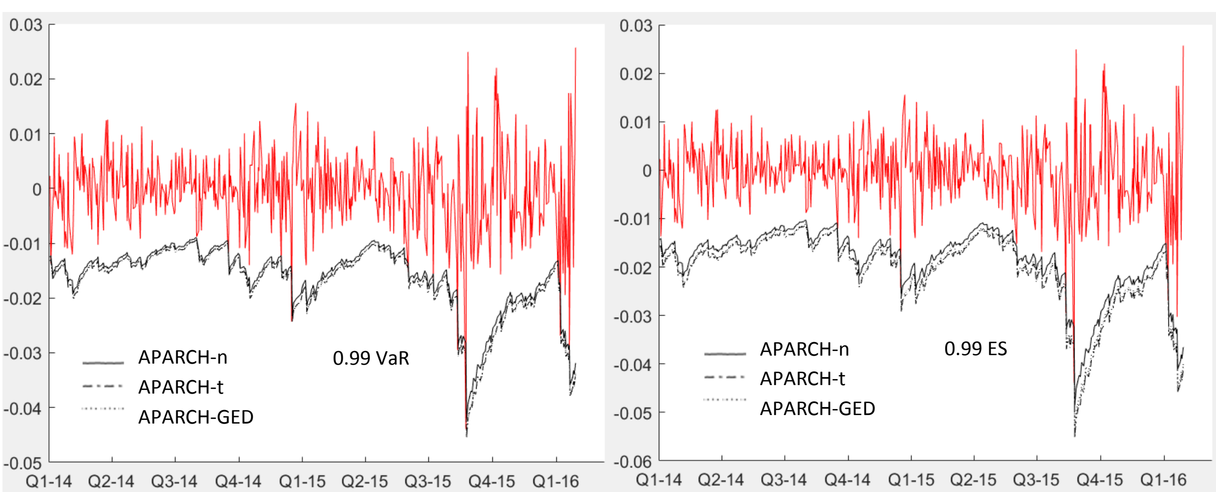

Fig 42 – STI returns with VaR and ES est. using variants of APARCH at q=99%

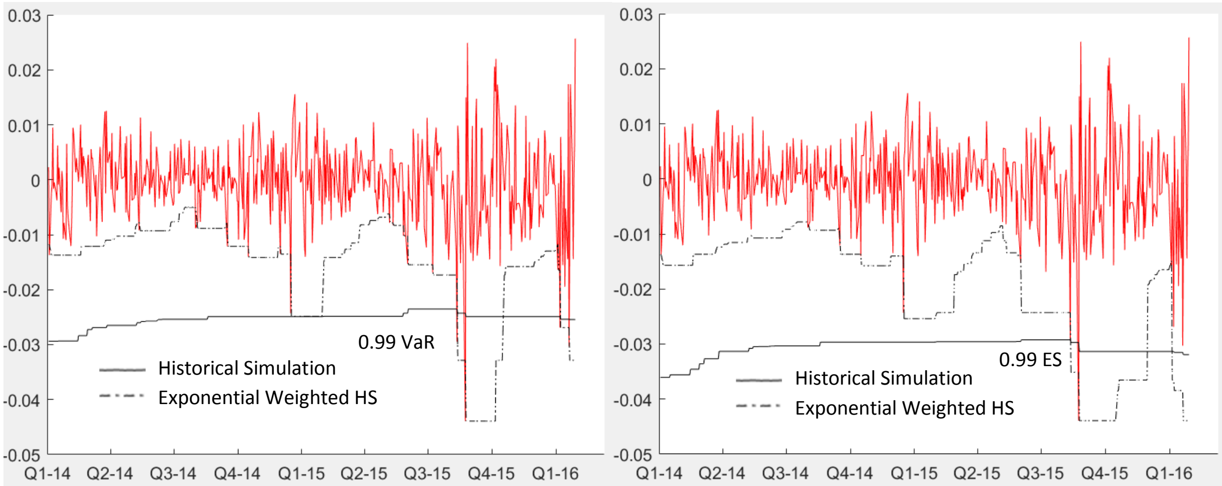

Fig 43 – STI returns with VaR and ES est. using HS and Exponential Weighted HS at q=99%

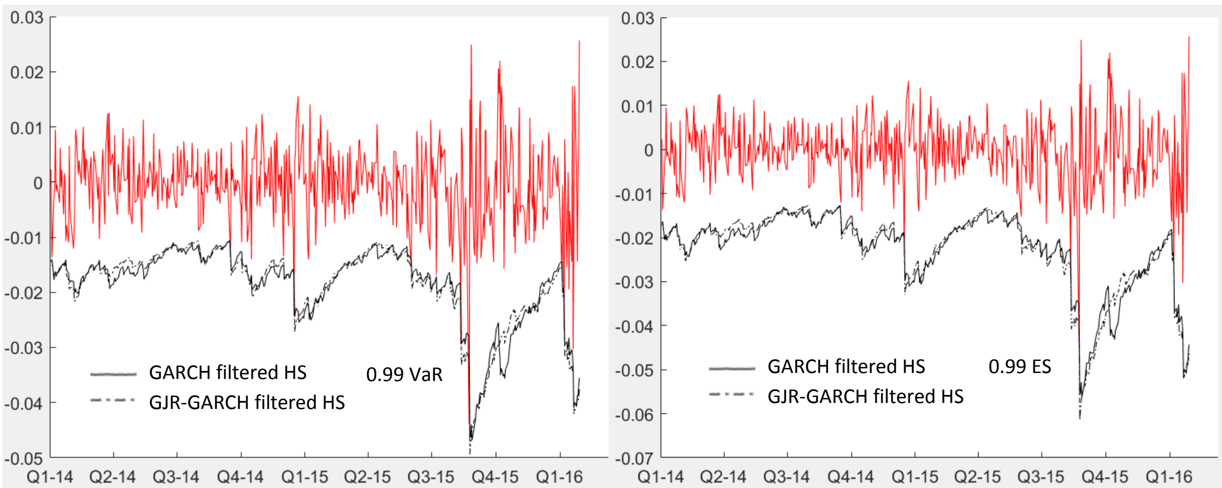

Fig 44 – STI returns with VaR and ES est. using variants of GARCH filtered HS at q=99%

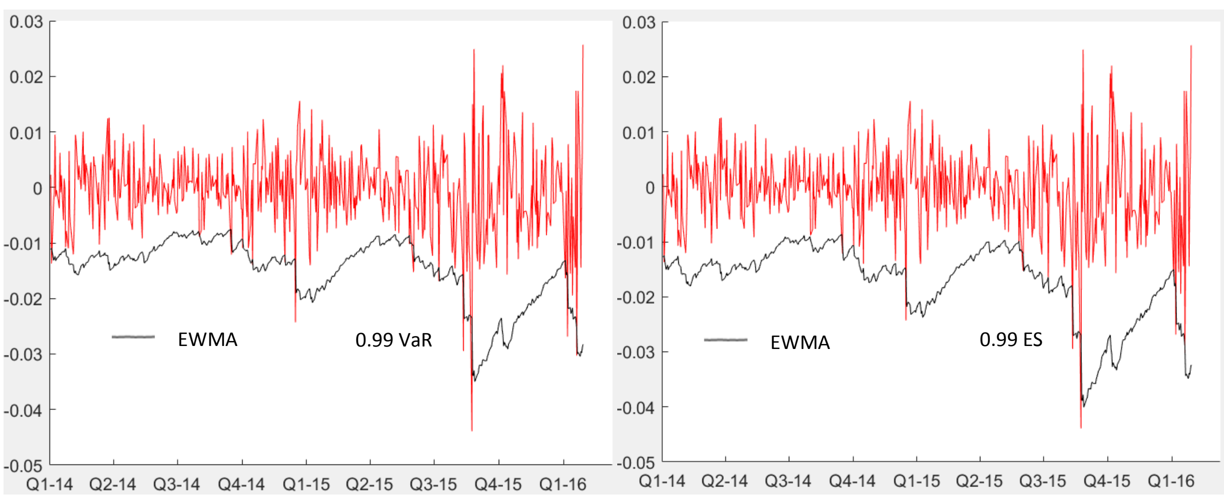

Fig 45 – STI returns with VaR and ES est. using RiskMetrics EWMA at q=99%

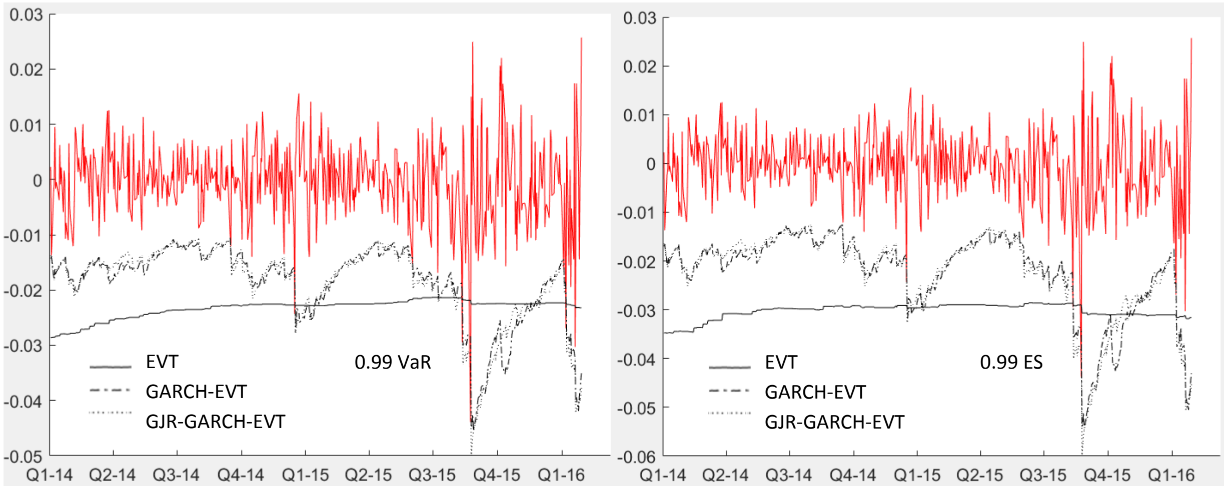

Fig 46 – STI returns with VaR and ES est. using static EVT and variants of dynamic GARCH-EVT at q=99%

Table 11 and 12 shows the backtesting results of VaR and ES respectively of STI index at 99% confidence level. It can be seen that the parametric approach using static model t-distribution and GED have failure rate 0.96% almost equal to desired 1% and both have one of the smallest z-statistic, conditional and unconditional coverage LR statistics. The static normal distribution slightly underestimated the VaR but the result is still good. All GARCH, EGARCH, GJR-GARCH, APARCH and EWMA underestimated VaR badly with high lost function, high unconditional and conditional LR statistics or high z statistic confirming a rejection, except for GJR-GARCH-GED and APARCH-GED which have moderate performance. Exponential weighted HS has the worst VaR forecast among all. On the other hand, GJR-GARCH filtered HS has the best VaR performance with failure rate close to 1% and lower loss function than the case in static t-distribution and GED. Extreme Value Theory also has similar VaR performance, second to GJR-GARCH filtered HS. GARCH-EVT, GARCH filtered HS and GJR-GARCH-EVT have good estimation too followed by Historical Simulation which slightly overestimated the VaR but still good.

In terms of ES estimation, Extreme Value Theory has the best performance with the lowest V_{ES} close to zero followed by GJR-GARCH filtered HS and Historical Simulation. It can be observed that ES estimation using GJR-GARCH-EVT, EGARCH-t, static t-distribution and followed by APARCH-t has good performance too. Normal distribution has the worst ES prediction (underestimated) and followed by RiskMetrics EWMA, GARCH-n, GED, GJR-GARCH-n, GARCH-GED and APARCH-n. The rest of the models such as GARCH-EVT, GARCH filtered HS, GJR-GARCH-GED, APARCH-GED and exponential weighted HS have mediocre performance. Within each variant of dynamic GARCH, t-distribution performs better than

GED and then followed by normal distribution. Comparing both VaR and ES estimation, both agreeable that GJR-GARCH Filtered HS, Extreme Value Theory, t-distribution, GJR-GARCH-EVT and historical simulation are good risk estimating approaches for STI index at 99% confidence level.

Table 11 – Backtesting results of VaR at q=99% for STI index

T=520	Failure rate	z	LR_{uc}	LR_{ind}	LR_{cc}	Ψ
Parametric approach						
Static (Variance-covariance, VC)						
Normal distribution	0.0115	0.3526	0.1185	0.1401	0.2585	6.001072
Student (t-distribution)	0.0096	-0.0881	0.0079	0.0971	0.1050	5.000733
GED	0.0096	-0.0881	0.0079	0.0971	0.1050	5.000764
Dynamic						
EWMA (RiskMetrics)	0.0308	4.7600	14.5939	0.4361	15.0300	16.000897
GARGH-n	0.0269	3.8785	10.2825	0.7639	11.0464	14.000781
GARCH-t	0.0212	2.5563	4.9488	1.4947	6.4435	11.000625
GARCH-GED	0.0212	2.5563	4.9488	1.4947	6.4435	11.000614
EGARCH-n	0.0250	3.4378	8.3423	0.9720	9.3143	13.000465
EGARCH-t	0.0231	2.9970	6.5602	1.2140	7.7742	12.000350
EGARCH-GED	0.0192	2.1155	3.5234	0.3922	3.9156	10.000340
T/GJR-GARCH-n	0.0212	2.5563	4.9488	0.4755	5.4243	11.000507
T/GJR-GARCH-t	0.0192	2.1155	3.5234	0.3922	3.9156	10.000387
T/GJR-GARCH-GED	0.0173	1.6748	2.3023	0.3170	2.6193	9.000377
APARCH-n	0.0231	2.9970	6.5602	1.2140	7.7742	12.000496
APARCH-t	0.0192	2.1155	3.5234	0.3922	3.9156	10.000378
APARCH-GED	0.0173	1.6748	2.3023	0.3170	2.6193	9.000367
Non-parametric approach						
Static						
Historical Simulation (HS)	0.0077	-0.5289	0.3039	0.0620	0.3659	4.000448
Dynamic						
Exponential weighted HS	0.0346	5.6414	19.4226	0.2091	19.6317	18.000594
Filtered HS (GARCH)	0.0115	0.3526	0.1185	0.1401	0.2585	6.000447
Filtered HS (T/GJR-GARCH)	0.0096	-0.0881	0.0079	0.0971	0.1050	5.000260
Semi-parametric approach						
Static						
Extreme Value Theory (EVT)	0.0096	-0.0881	0.0079	0.0971	0.1050	5.000628
Dynamic						
GARCH-EVT	0.0115	0.3526	0.1185	0.1401	0.2585	6.000425
T/GJR-GARCH-EVT	0.0115	0.3526	0.1185	0.1401	0.2585	6.000250

Table 12 – Backtesting results of ES at q=99% for STI index

	V_{ES1}	V_{ES2}	V_{ES}
Parametric approach			
Static (Variance-covariance, VC)			
Normal distribution	-0.008208	-0.010191	0.009199
Student (t-distribution)	-0.00189	-0.00189	0.001890
GED	-0.005355	-0.005355	0.005355
Dynamic			
EWMA (RiskMetrics)	-0.003299	-0.009471	0.006385
GARGH-n	-0.002728	-0.008757	0.005742
GARCH-t	-0.00131	-0.006	0.00365
GARCH-GED	-0.001884	-0.006608	0.004246
EGARCH-n	-0.001601	-0.005728	0.003664
EGARCH-t	0.00012	-0.00304	0.00158
EGARCH-GED	-0.000877	-0.003556	0.002216
T/GJR-GARCH-n	-0.00236	-0.00632	0.00434
T/GJR-GARCH-t	-0.00031	-0.00371	0.00201
T/GJR-GARCH-GED	-0.001485	-0.004218	0.002852
APARCH-n	-0.00197	-0.00623	0.0041
APARCH-t	-0.00023	-0.00362	0.00193
APARCH-GED	-0.001389	-0.004119	0.002754
Non-parametric approach			
Static			
Historical Simulation (HS)	-0.00219	-0.00068	0.00143
Dynamic			
Exponential weighted HS	0.00017	-0.00542	0.00279
Filtered HS (GARCH)	-0.00265	-0.00364	0.00315
Filtered HS (T/GJR-GARCH)	-0.00143	-0.00143	0.00143
Semi-parametric approach			
Static			
Extreme Value Theory (EVT)	-0.00113	-0.00113	0.00113
Dynamic			
GARCH-EVT	-0.003158	-0.004153	0.003656
T/GJR-GARCH-EVT	-0.001158	-0.00188	0.001519

VaR and ES of STI Index at 95% Confidence Level

Fig 47-55 show the STI index returns with the estimated VaR and ES at 95% confidence level using various methodologies where one can observe the number of VaR exceedance and compare actual exceedance loss with expected average loss in ES.

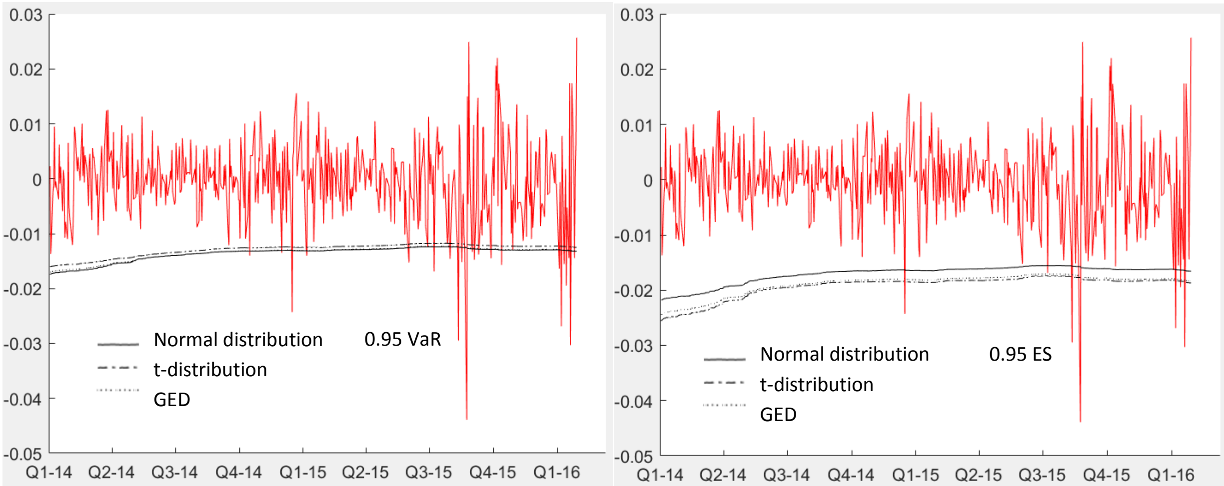

Fig 47 – STI returns with VaR and ES est. using VC/parametric static models at q=95%

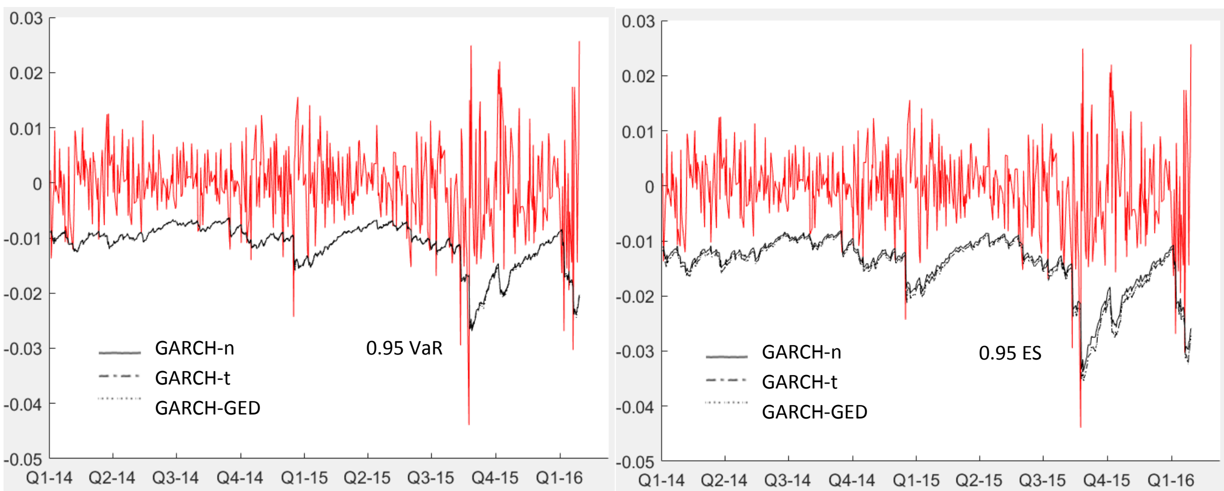

Fig 48 – STI returns with VaR and ES est. using variants of GARCH at q=95%

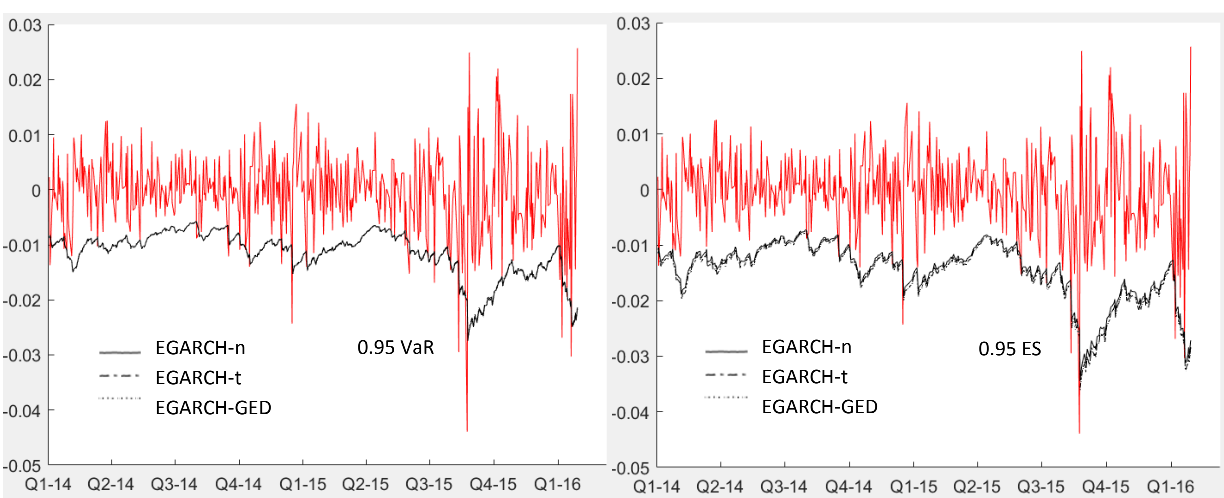

Fig 49 – STI returns with VaR and ES est. using variants of EGARCH at q=95%

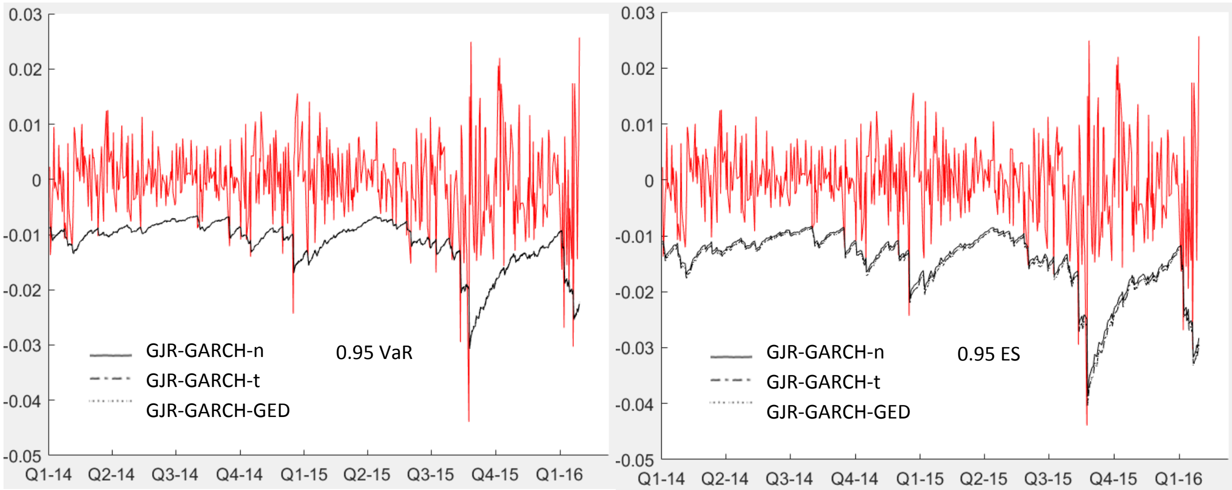

Fig 50 – STI returns with VaR and ES est. using variants of T/GJR-GARCH at q=95%

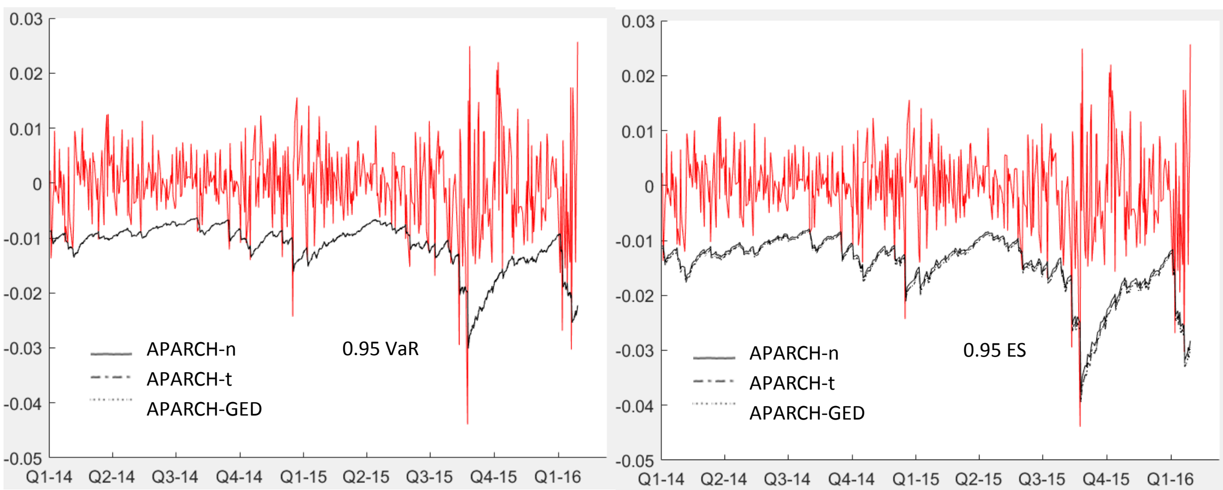

Fig 51 – STI returns with VaR and ES est. using variants of APARCH at q=95%

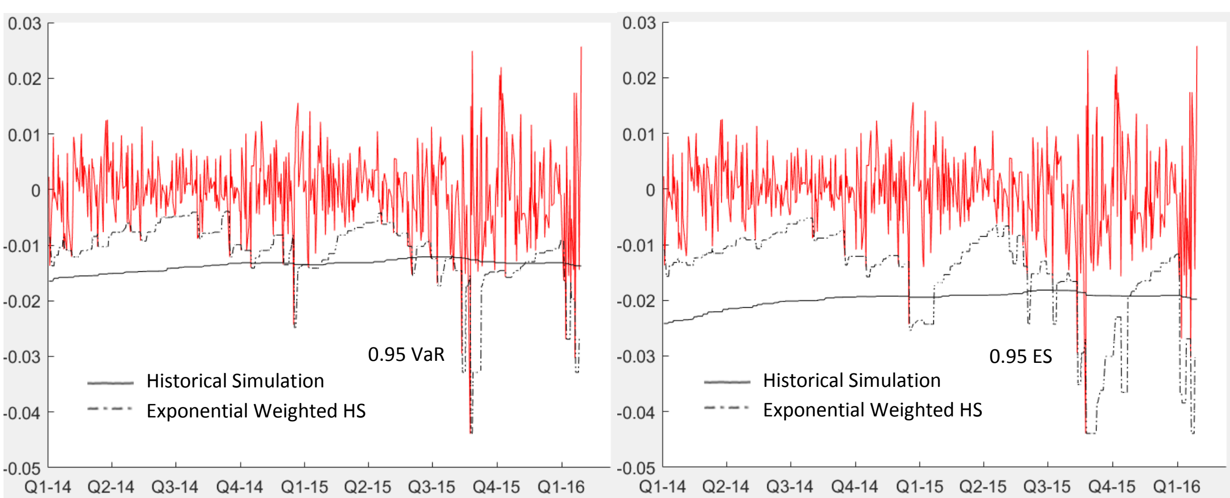

Fig 52 – STI returns with VaR and ES est. using HS and Exponential Weighted HS at q=95%

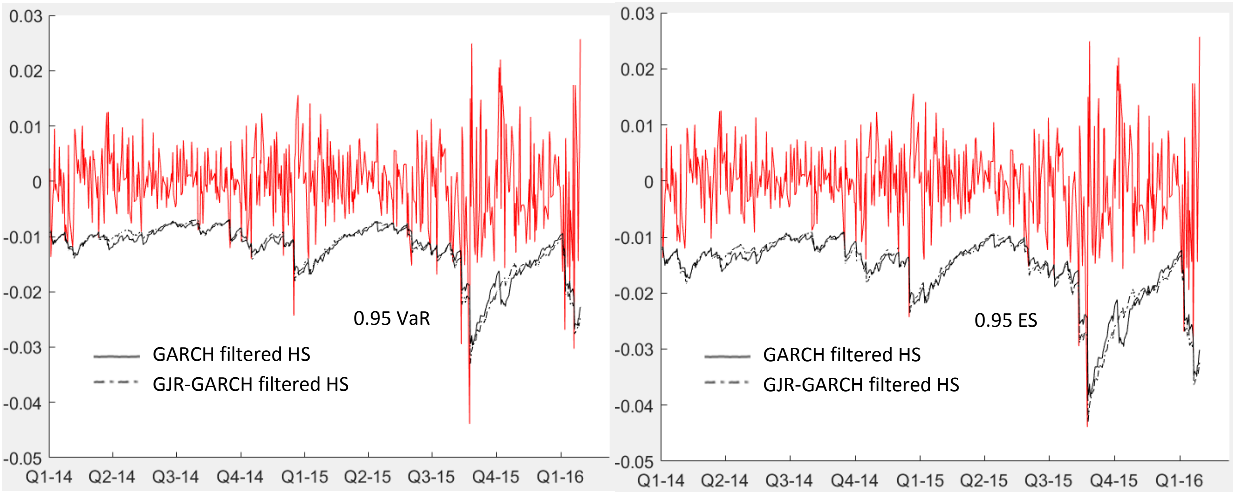

Fig 53 – STI returns with VaR and ES est. using variants of GARCH filtered HS at q=95%

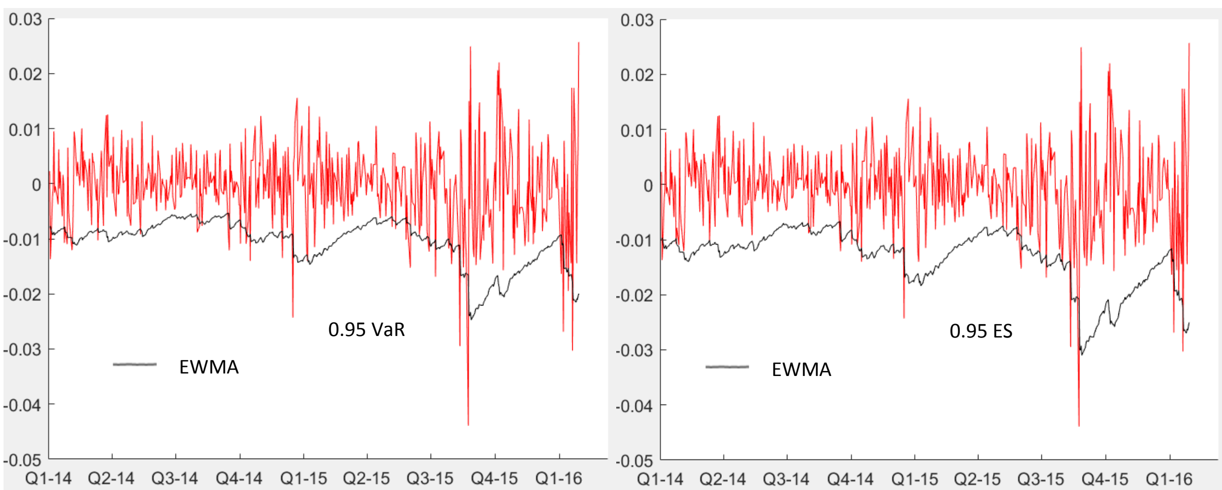

Fig 54 – STI returns with VaR and ES est. using RiskMetrics EWMA at q=95%

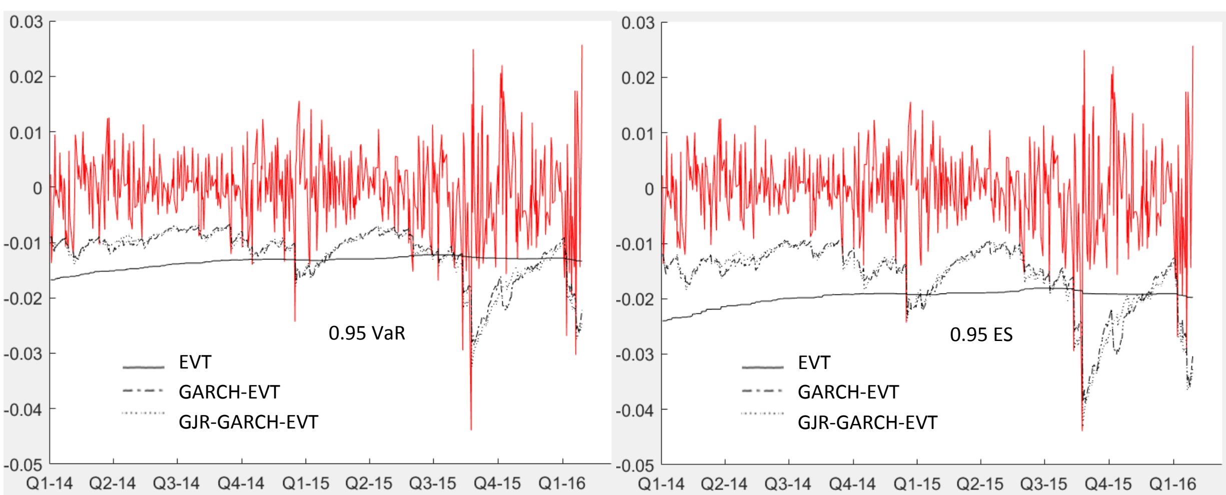

Fig 55 – STI returns with VaR and ES est. using static EVT and variants of dynamic GARCH-EVT at q=95%

Table 13 and 14 shows the backtesting results of VaR and ES respectively of STI index at 95% confidence level. At this lower confidence level, static model GED clearly stands out having failure rate the same as 5% and passed the Wald test, conditional and unconditional coverage test (small value close to zero). The second best in line for VaR estimation (slightly overestimated) are GJR-GARCH-EVT, Historical Simulation, Extreme Value Theory and followed by GARCH filtered HS with comparable small conditional and unconditional LR statistics. Among these four, Extreme Value Theorem approach has the largest loss function followed by Historical Simulation. In addition, normal distribution slightly overestimated the VaR but performed relatively well in the estimation too. On the slightly underestimated side, GARCH-EVT, GJR-GARCH-GED and t-distribution methods have good performance too. Although EGARCH-GED have comparable failure rate, it suffered from poorer LR_{cc} statistics. The estimation methods that are least accurately in terms of failure rate are EWMA (high loss function and high LR_{cc}) and exponential weighted HS. Estimation method like GJR-GARCH-n, GARCH-GED, APARCH-GED have moderately well performance while the rest are mediocre.

Examining the ES estimation, static model GED with the smallest V_{ES} close to zero has the best ES estimation followed by t-distribution. The second best in line for ES estimation are GJR-GARCH-EVT, Extreme Value Theory and Historical Simulation. Estimation approach of GJR-GARCH filtered HS, APARCH-t, EGARCH-t and APARCH-GED have accurate ES estimation too. Unlike VaR, exponential weighted HS have relative good ES performance and share similar performance with GARCH-EVT and GJR-GARCH-t. The ES backtesting results indicate that EWMA has least satisfying result followed by GARCH-n. EGARCH-GED, GJR-GARCH-GED, GARCH filtered HS and normal distribution have moderately well estimation and the rest are mediocre. Within each variant of dynamic GARCH, t-distribution performs better than GED and then followed by normal distribution. Juxtaposing the VaR and ES results, both results are consistent in reporting that GED, GJR-GARCH-EVT, Extreme Value Theory, historical simulation and t-distribution are good risk estimating methods for STI index at 95% confidence level.

Table 13 – Backtesting results of VaR at q=95% for STI index

T=520	Failure rate	z	LR_{uc}	LR_{ind}	LR_{cc}	Ψ
Parametric approach						
Static (Variance-covariance, VC)						
Normal distribution	0.0462	-0.4024	0.1660	0.6510	0.8171	24.002008
Student (t-distribution)	0.0538	0.4024	0.1582	1.3197	1.4779	28.002189
GED	0.0500	0.0000	0.0000	0.3642	0.3642	26.002029
Dynamic						
EWMA (RiskMetrics)	0.0654	1.6097	2.3722	1.3549	3.7271	34.002073
GARGH-n	0.0577	0.8048	0.6185	0.0452	0.6637	30.001857
GARCH-t	0.0577	0.8048	0.6185	0.0452	0.6637	30.001915
GARCH-GED	0.0558	0.6036	0.3518	0.0952	0.4470	29.001843
EGARCH-n	0.0558	0.6036	0.3518	2.9183	3.2701	29.001462
EGARCH-t	0.0596	1.0061	0.9560	0.6981	1.6541	31.001504
EGARCH-GED	0.0538	0.4024	0.1582	3.3188	3.4769	28.001444
T/GJR-GARCH-n	0.0558	0.6036	0.3518	0.0952	0.4470	29.001504
T/GJR-GARCH-t	0.0577	0.8048	0.6185	0.0452	0.6637	30.001543
T/GJR-GARCH-GED	0.0538	0.4024	0.1582	0.1646	0.3227	28.001485
APARCH-n	0.0577	0.8048	0.6185	0.8810	1.4995	30.001492
APARCH-t	0.0615	1.2073	1.3621	0.5382	1.9003	32.001529
APARCH-GED	0.0558	0.6036	0.3518	1.0879	1.4397	29.001472
Non-parametric approach						
Static						
Historical Simulation (HS)	0.0481	-0.2012	0.0410	0.0395	0.0805	25.001972
Dynamic						
Exponential weighted HS	0.0596	1.0061	0.9560	0.0138	0.9698	31.001766
Filtered HS (GARCH)	0.0481	-0.2012	0.0410	0.4962	0.5372	25.001522
Filtered HS (T/GJR-GARCH)	0.0442	-0.6036	0.3785	0.0003	0.3788	23.001258
Semi-parametric approach						
Static						
Extreme Value Theory (EVT)	0.0481	-0.2012	0.0410	0.0395	0.0805	25.002010
Dynamic						
GARCH-EVT	0.0519	0.2012	0.0400	0.2540	0.2940	27.001649
T/GJR-GARCH-EVT	0.0481	-0.2012	0.0410	0.0395	0.0805	25.001317

Table 14 – Backtesting results of ES at q=95% for STI index

	V_{ES1}	V_{ES2}	V_{ES}
Parametric approach			
Static (Variance-covariance, VC)			
Normal distribution	-0.002135	-0.001718	0.001927
Student (t-distribution)	0.000701	0.000307	0.000504
GED	-0.000048	-0.000048	0.000048
Dynamic			
EWMA (RiskMetrics)	-0.002863	-0.004195	0.003529
GARGH-n	-0.002477	-0.003221	0.002849
GARCH-t	-0.00185	-0.0026	0.002223
GARCH-GED	-0.002011	-0.002543	0.002277
EGARCH-n	-0.00183	-0.00234	0.002083
EGARCH-t	-0.00088	-0.00178	0.001328
EGARCH-GED	-0.001301	-0.001713	0.001507
T/GJR-GARCH-n	-0.00179	-0.00235	0.00207
T/GJR-GARCH-t	-0.00102	-0.00181	0.001419
T/GJR-GARCH-GED	-0.001312	-0.001750	0.001532
APARCH-n	-0.00159	-0.00232	0.001953
APARCH-t	-0.00071	-0.00178	0.001243
APARCH-GED	-0.001085	-0.001711	0.001398
Non-parametric approach			
Static			
Historical Simulation (HS)	0.000996	0.001201	0.001098
Dynamic			
Exponential weighted HS	-0.0006	-0.00229	0.001445
Filtered HS (GARCH)	-0.00173	-0.00159	0.001661
Filtered HS (T/GJR-GARCH)	-0.00147	-0.00096	0.001216
Semi-parametric approach			
Static			
Extreme Value Theory (EVT)	0.000916	0.001122	0.001019
Dynamic			
GARCH-EVT	-0.001307	-0.001538	0.001422
T/GJR-GARCH-EVT	-0.000951	-0.000824	0.000887

VaR and ES of SSE Index at 99% Confidence Level

Fig 56-64 show the SSE index returns with the estimated VaR and ES at 99% confidence level using various methodologies where one can observe the number of VaR exceedance and compare actual exceedance loss with expected average loss in ES.

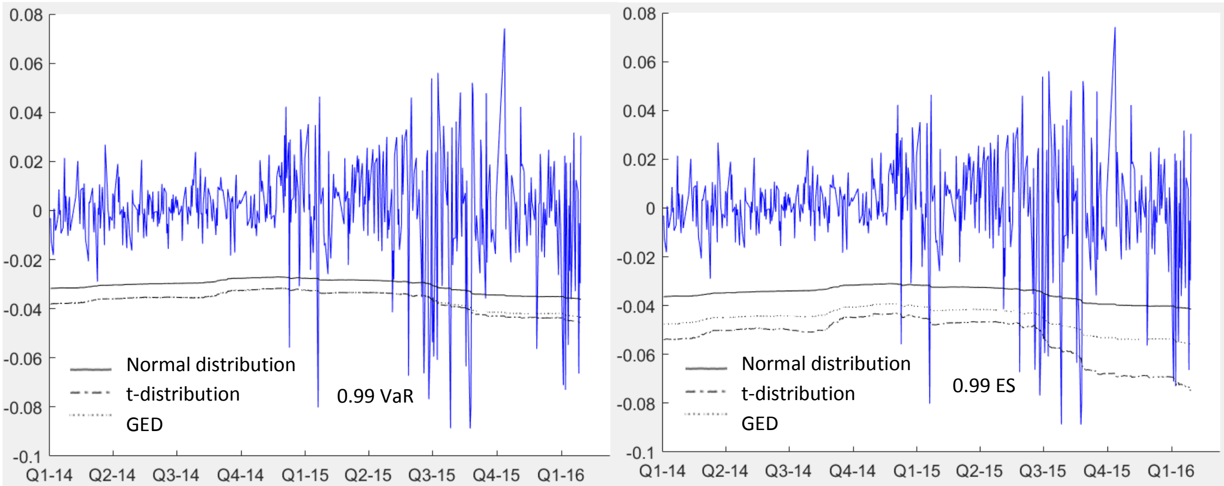

Fig 56 – SSE returns with VaR and ES est. using VC/parametric static models at q=99%

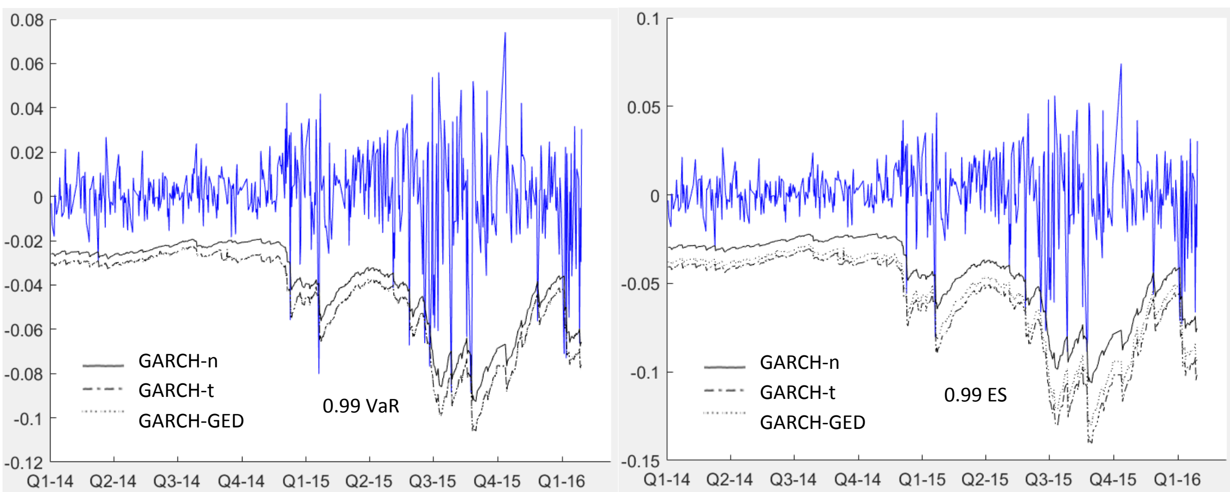

Fig 57 – SSE returns with VaR and ES est. using variants of GARCH at q=99%

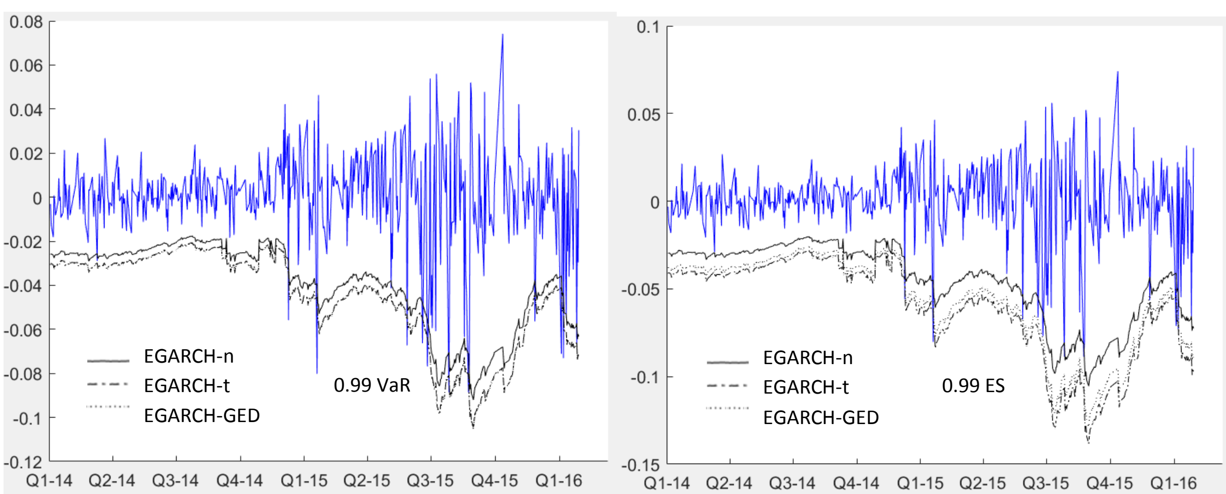

Fig 58 – SSE returns with VaR and ES est. using variants of EGARCH at q=99%

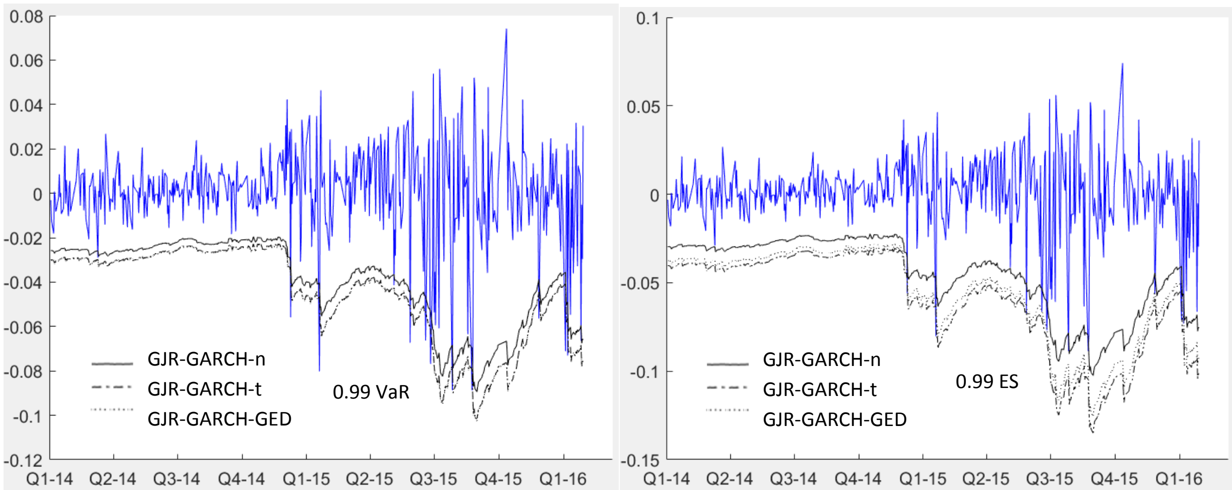

Fig 59 – SSE returns with VaR and ES est. using variants of T/GJR-GARCH at q=99%

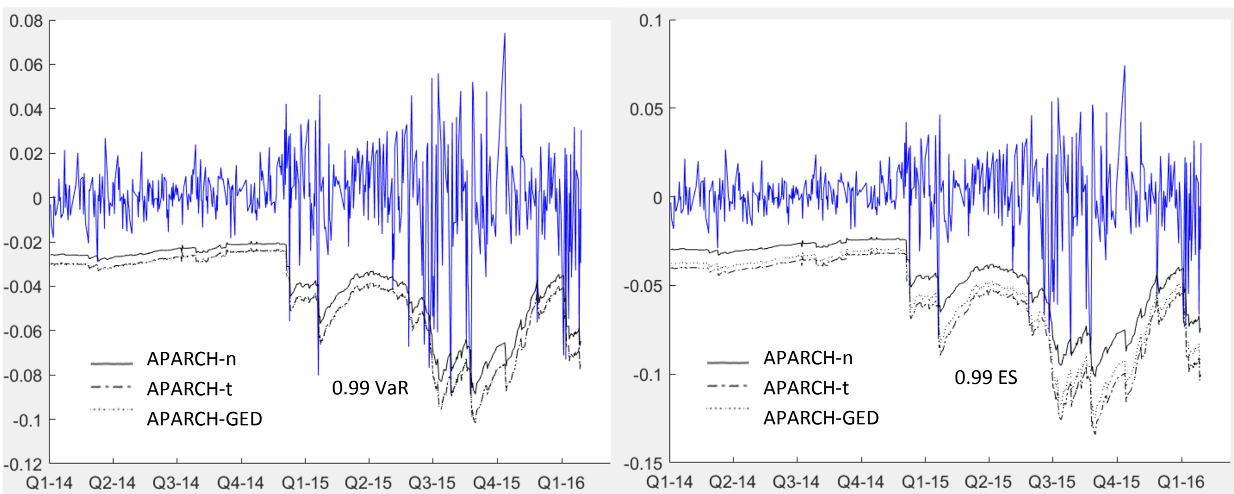

Fig 60 – SSE returns with VaR and ES est. using variants of APARCH at q=99%

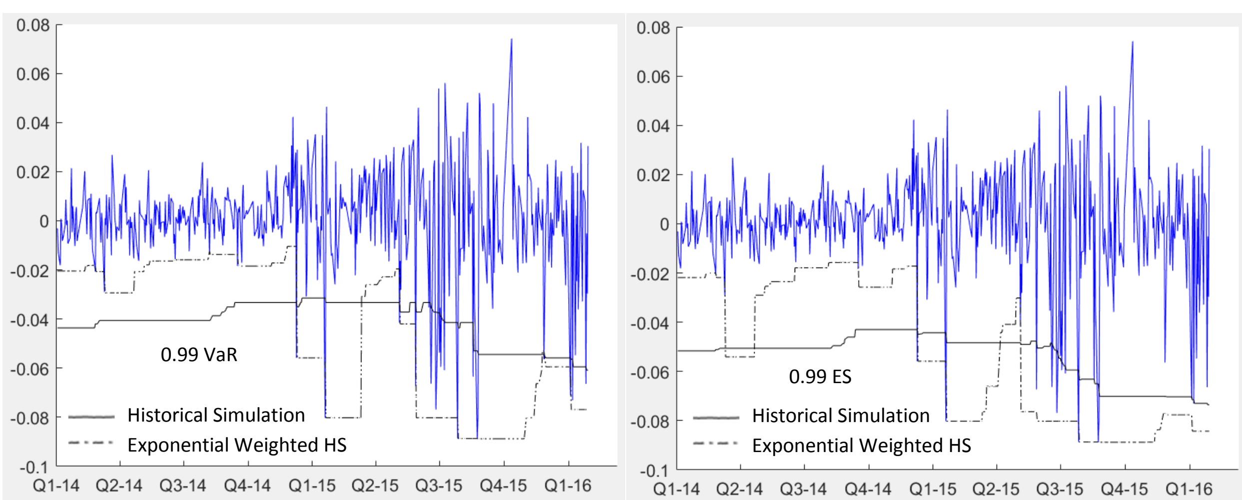

Fig 61 – SSE returns with VaR and ES est. using HS and Exponential Weighted HS at q=99%

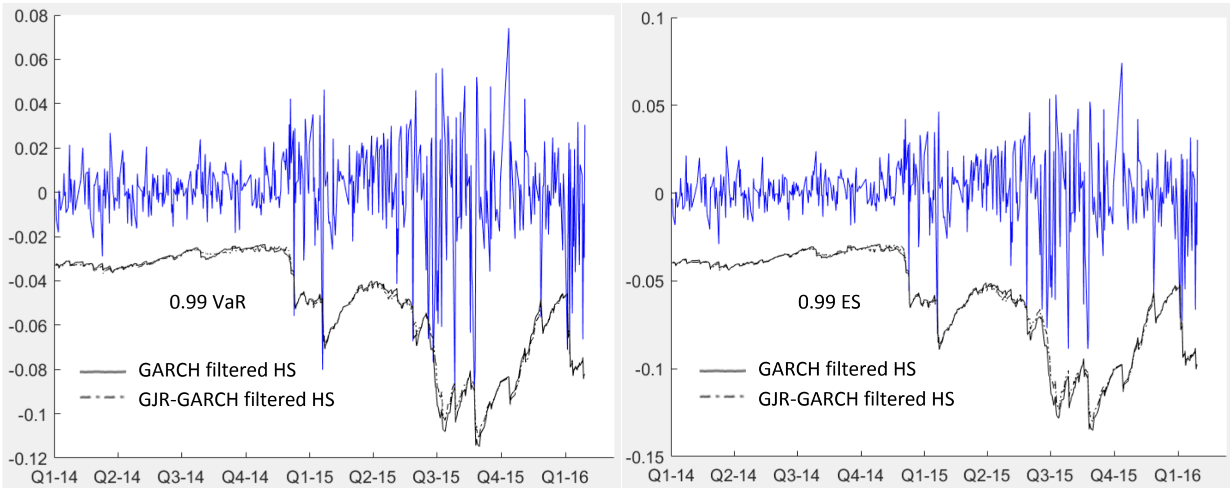

Fig 62 – SSE returns with VaR and ES est. using variants of GARCH filtered HS at q=99%

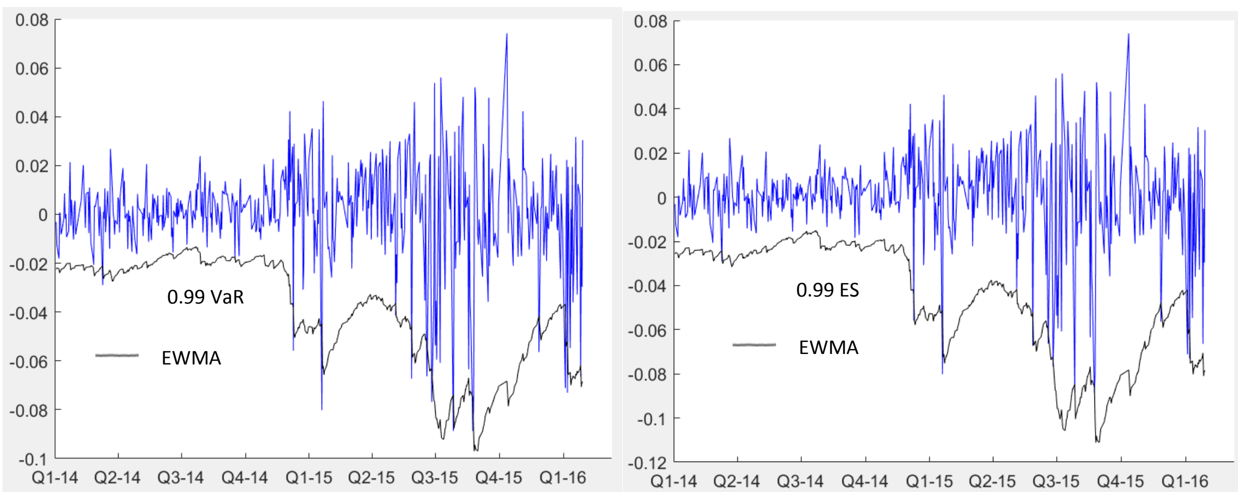

Fig 63 – SSE returns with VaR and ES est. using RiskMetrics EWMA at q=99%

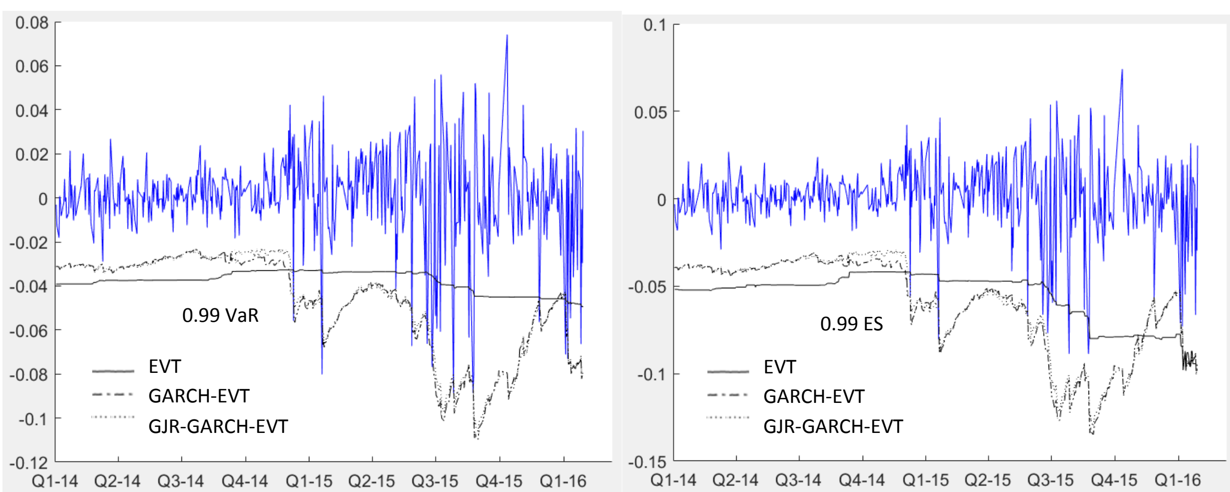

Fig 64 – SSE returns with VaR and ES est. using static EVT and variants of dynamic GARCH-EVT at q=99%

Table 15 and 16 provides the backtesting results of VaR and ES respectively of SSE index at 99% confidence level. Looking across the table, none of the methods provide accurate modeling of failure rate as 1% (all underestimated) and passed the Wald test, conditional and unconditional coverage test except for GARCH filtered HS method which has a closest failure rate of 1.8%. This could indicate a more extreme volatile and fatter tail (leptokurtic) nature of SSE index returns. Normal distribution has the worst underestimation, followed by static model GED, t-distribution and Extreme Value Theory. It's surprising to see Extreme Value Theory has poor performance, but situation improved in the dynamic GARCH-EVT and GJR-GARCH-EVT approach. Although with violations, the second best in line after GARCH filtered HS besides GJR-GARCH-EVT approach are GJR-GARCH filtered HS, GJR-GARCH-GED and GJR-GARCH-t.

Turning to the ES estimation, GJR-GARCH-t has the best result with the smallest V_{ES} close to zero despite the value not as small as the better ones observed so far. This is followed by estimation method using APARCH-t, GJR-GARCH filtered HS, GARCH-EVT, GARCH filtered HS and GJR-GARCH-EVT. Conversely, the results also confirmed that all static models particularly with normal distribution being the worst followed by GED, all variants of GARCH with normal distribution, Historical Simulation, Extreme Value Theory, t-distribution and RiskMetrics EWMA have weak ES estimation. Within each variant of dynamic GARCH, t-distribution performs better than GED and then followed by normal distribution. Both the VaR and ES backtesting suggest that GARCH filtered HS, GJR-GARCH-EVT and GJR-GARCH filtered HS are reasonable risk estimating method at high quantile 99% for SSE index.

Table 15 – Backtesting results of VaR at q=99% for SSE index

T=498	Failure rate	z	LR_{uc}	LR_{ind}	LR_{cc}	Ψ
Parametric approach						
Static (Variance-covariance, VC)						
Normal distribution	0.0562	10.3675	51.7514	16.6658	68.4171	28.025262
Student (t-distribution)	0.0442	7.6653	31.9214	6.1996	38.1210	22.017245
GED	0.0442	7.6653	31.9214	6.1996	38.1210	22.018092
Dynamic						
EWMA (RiskMetrics)	0.0281	4.0623	11.0676	0.8100	11.8776	14.004654
GARGH-n	0.0261	3.6120	9.0387	0.6970	9.7357	13.006389
GARCH-t	0.0221	2.7112	5.4680	0.4970	5.9650	11.003424
GARCH-GED	0.0221	2.7112	5.4680	0.4970	5.9650	11.003364
EGARCH-n	0.0281	4.0623	11.0676	0.8100	11.8776	14.006745
EGARCH-t	0.0221	2.7112	5.4680	0.4970	5.9650	11.003839
EGARCH-GED	0.0221	2.7112	5.4680	0.4970	5.9650	11.003750
T/GJR-GARCH-n	0.0261	3.6120	9.0387	0.6970	9.7357	13.006689
T/GJR-GARCH-t	0.0201	2.2608	3.9544	0.4099	4.3643	10.003779
T/GJR-GARCH-GED	0.0201	2.2608	3.9544	0.4099	4.3643	10.003684
APARCH-n	0.0281	4.0623	11.0676	0.7078	11.7754	14.007014
APARCH-t	0.0221	2.7112	5.4680	0.4970	5.9650	11.004036
APARCH-GED	0.0221	2.7112	5.4680	0.4970	5.9650	11.003939
Non-parametric approach						
Static						
Historical Simulation (HS)	0.0382	6.3142	23.2449	1.6950	24.9399	19.012988
Dynamic						
Exponential weighted HS	0.0241	3.1616	7.1679	0.5927	7.7605	12.004085
Filtered HS (GARCH)	0.0181	1.8105	2.6452	0.3313	2.9765	9.002442
Filtered HS (T/GJR-GARCH)	0.0201	2.2608	3.9544	0.4099	4.3643	10.002438
Semi-parametric approach						
Static						
Extreme Value Theory (EVT)	0.0442	7.6653	31.9214	3.1200	35.0414	22.016222
Dynamic						
GARCH-EVT	0.0221	2.7112	5.4680	0.4970	5.9650	11.002912
T/GJR-GARCH-EVT	0.0201	2.2608	3.9544	0.4099	4.3643	10.003268

Table 16 – Backtesting results of ES at q=99% for SSE index

	V_{ES1}	V_{ES2}	V_{ES}
Parametric approach			
Static (Variance-covariance, VC)			
Normal distribution	-0.019665	-0.048663	0.034164
Student (t-distribution)	-0.004977	-0.030042	0.017510
GED	-0.014460	-0.038120	0.026290
Dynamic			
EWMA (RiskMetrics)	-0.008534	-0.021385	0.014959
GARGH-n	-0.012892	-0.026085	0.019488
GARCH-t	0.003703	-0.00949	0.006594
GARCH-GED	-0.00189	-0.014163	0.008026
EGARCH-n	-0.012056	-0.027238	0.019647
EGARCH-t	0.002321	-0.01226	0.007289
EGARCH-GED	-0.003109	-0.016848	0.009978
T/GJR-GARCH-n	-0.01349	-0.02538	0.019437
T/GJR-GARCH-t	0.000074	-0.01075	0.005414
T/GJR-GARCH-GED	-0.005178	-0.014987	0.010082
APARCH-n	-0.01233	-0.026338	0.019334
APARCH-t	0.00054	-0.01116	0.005848
APARCH-GED	-0.004503	-0.015537	0.010020
Non-parametric approach			
Static			
Historical Simulation (HS)	-0.00692	-0.02861	0.017763
Dynamic			
Exponential weighted HS	-0.00457	-0.02068	0.012622
Filtered HS (GARCH)	-0.0012	-0.01211	0.006654
Filtered HS (T/GJR-GARCH)	-0.00127	-0.01174	0.006505
Semi-parametric approach			
Static			
Extreme Value Theory (EVT)	-0.00029	-0.02785	0.014067
Dynamic			
GARCH-EVT	0.001835	-0.011212	0.006523
T/GJR-GARCH-EVT	-0.00157	-0.0118	0.006682

VaR and ES of SSE Index at 95% Confidence Level

Fig 65-73 show the SSE index returns with the estimated VaR and ES at 95% confidence level using various methodologies where one can observe the number of VaR exceedance and compare actual exceedance loss with expected average loss in ES.

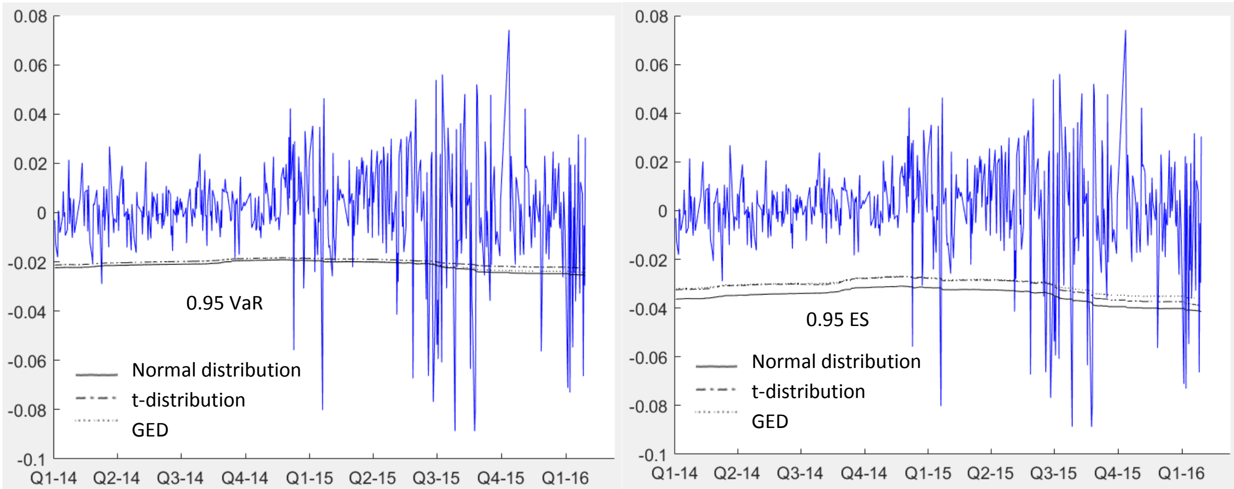

Fig 65 – SSE returns with VaR and ES est. using VC/parametric static models at q=95%

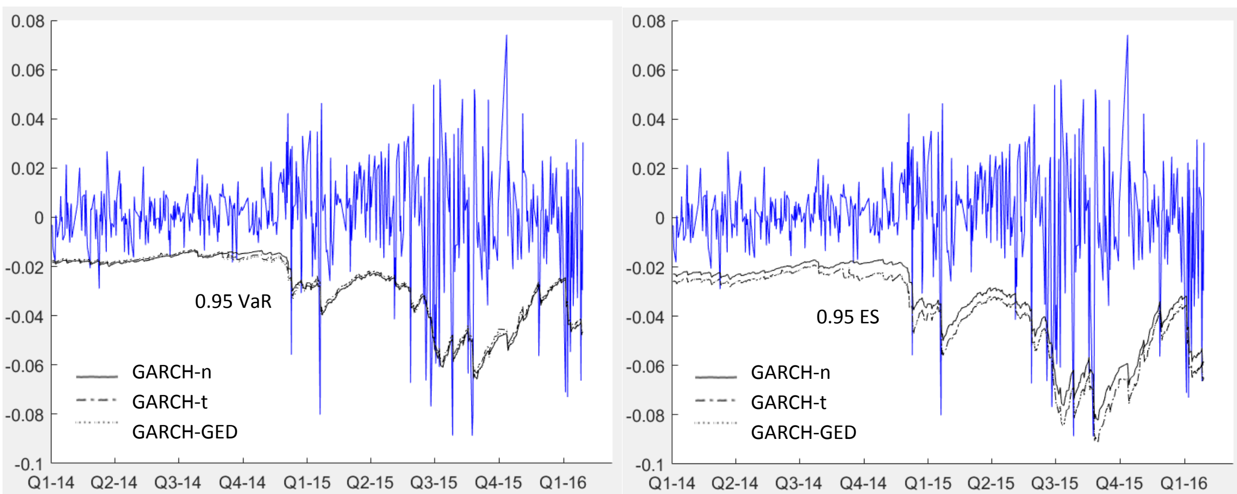

Fig 66 – SSE returns with VaR and ES est. using variants of GARCH at q=95%

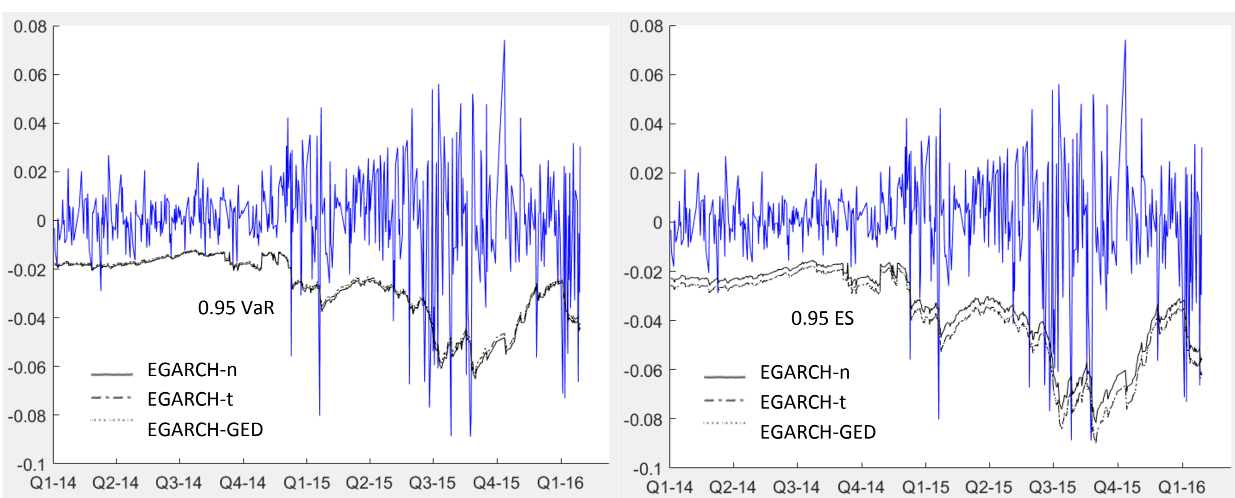

Fig 67 – SSE returns with VaR and ES est. using variants of EGARCH at q=95%

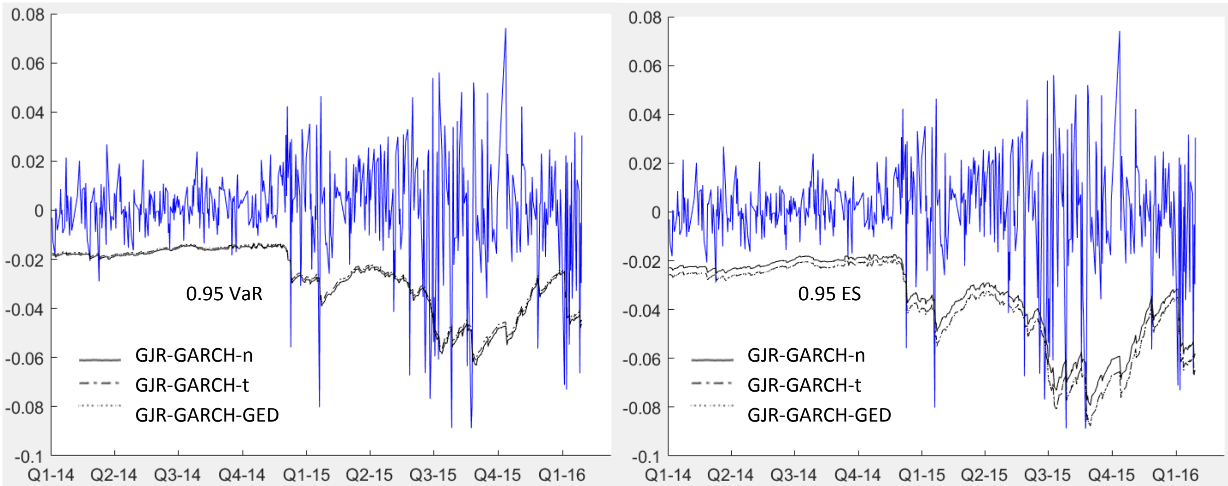

Fig 68 – SSE returns with VaR and ES est. using variants of T/GJR-GARCH at q=95%

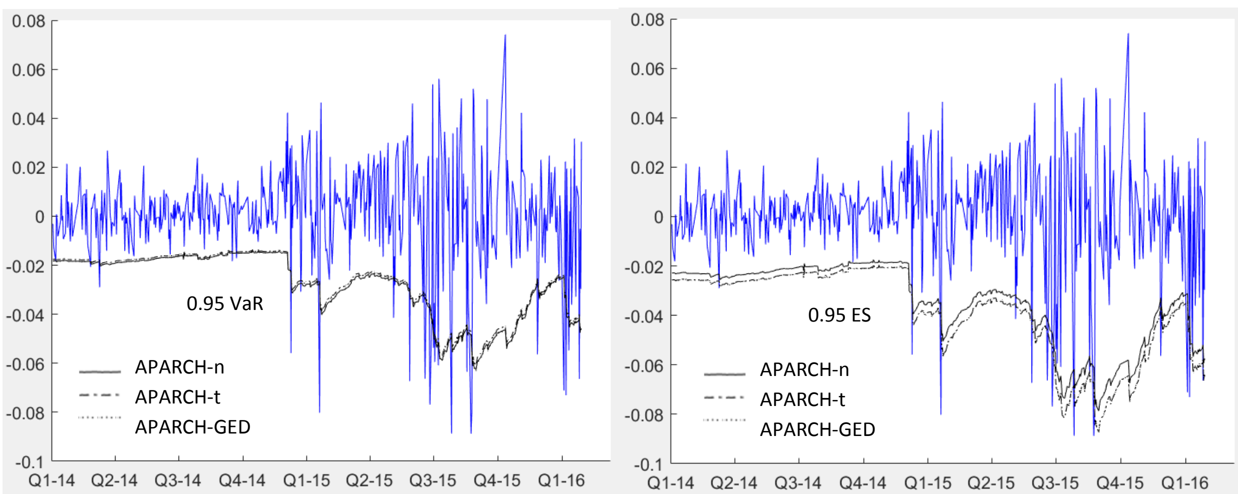

Fig 69 – SSE returns with VaR and ES est. using variants of APARCH at q=95%

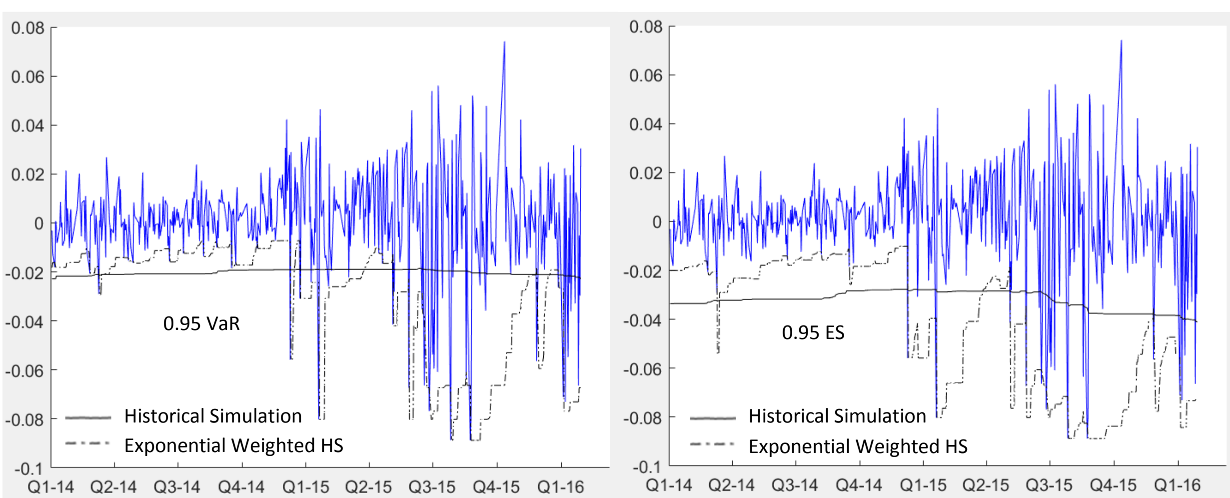

Fig 70 – SSE returns with VaR and ES est. using HS and Exponential Weighted HS at q=95%

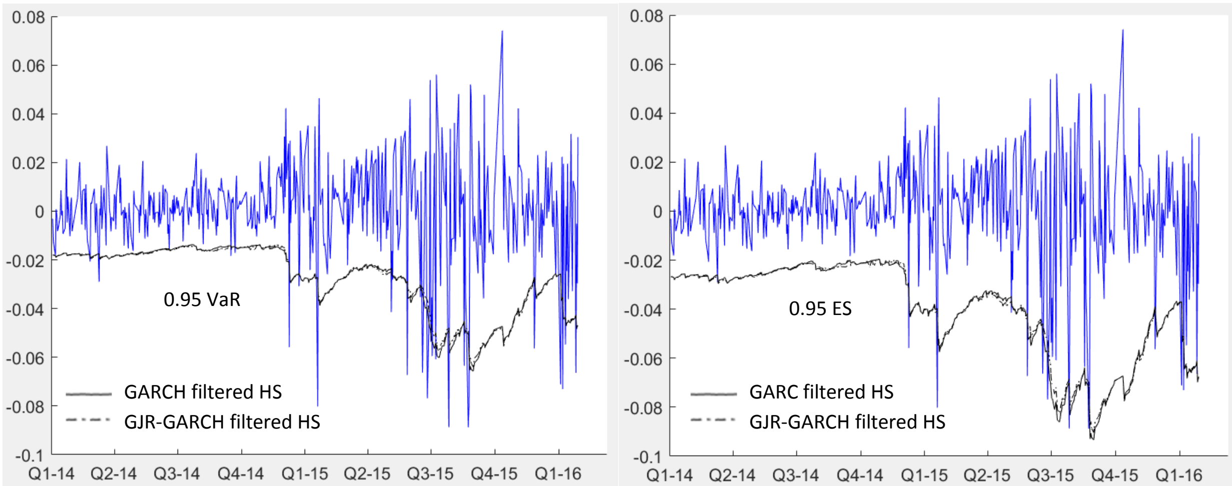

Fig 71 – SSE returns with VaR and ES est. using variants of GARCH filtered HS at q=95%

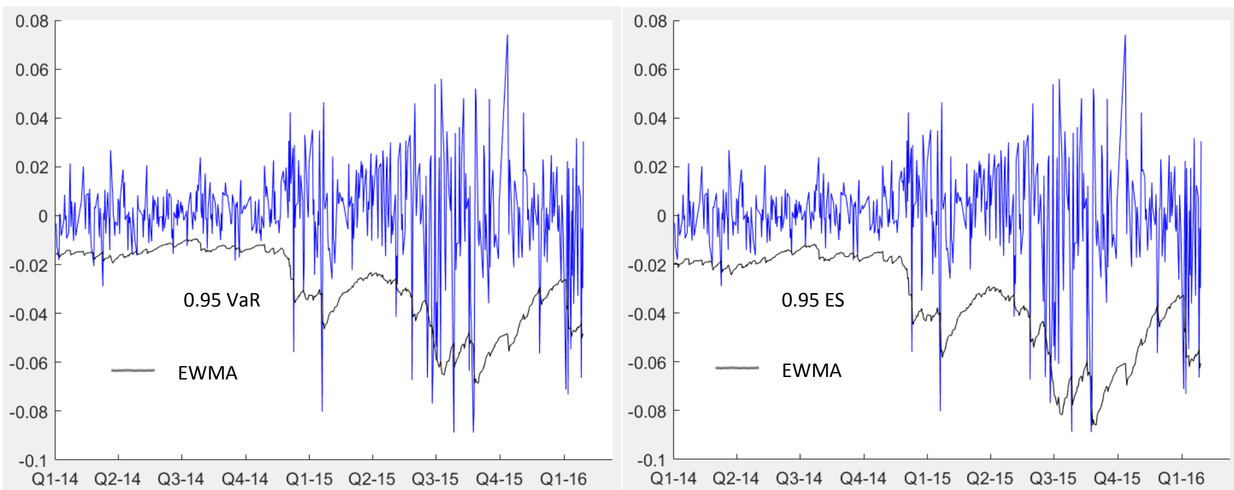

Fig 72 – SSE returns with VaR and ES est. using RiskMetrcs EWMA at q=95%

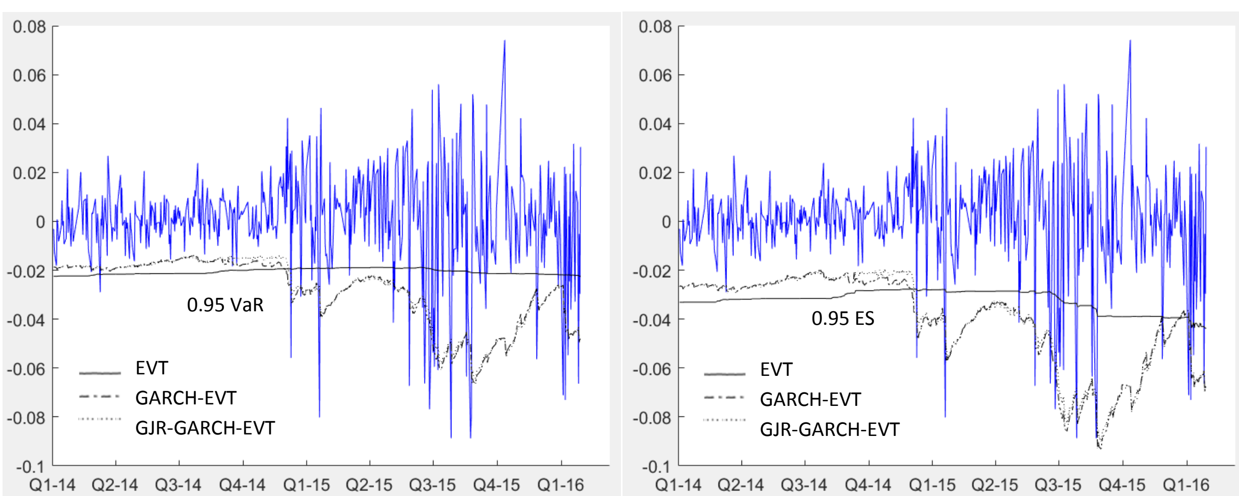

Fig 73 – SSE returns with VaR and ES est. using static EVT and variants of dynamic GARCH-EVT at q=95%

Table 17 and 18 summarized the backtesting results of VaR and ES respectively of SSE index at 95% confidence level. All estimation methods show VaR underestimation with violation rate more than 5%. Based on failure rate closest to 5% and least loss function, dynamic GARCH-EVT approach is the best with violation rate of 5.22% followed by EGARCH-GED and EGARCH-n. All three have small z-statistic and small unconditional or conditional LR statistic. The next best estimation approaches with smallest loss function first are GARCH-GED, GARCH-n, GJR-GARCH-GED, GJR-GARCH-n, GJR-GARCH-EVT, APARCH-GED, APARCH-n and GJR-GARCH filtered HS. APARCH-t and GARCH filtered HS have moderately good estimation too. On the other hand, the estimation methods that have way-off and most violation rate of over 8% are Historical simulation, Extreme Value Theory and all parametric static models. A similar sentiment applies to exponential weighted HS and EWMA which didn't fare quite as well with failure rate of 6.43% and 6.22% respectively while the rest of the methods have mediocre performance.

In the ES estimation, exponential weight HS on the contrary has the best result with the smallest V_{ES} close to zero (magnitude similar to those at 99% quantile) followed by GARCH filtered HS and GARCH-EVT. The second in line are GARCH-t, GARCH-GED and GJR-GARCH filtered HS. The results also reveal that methods like GJR-GARCH-EVT, GJR-GARCH-t, EGARCH-t, GJR-GARCH-GED, APARCH-t and APARCH-GED have relatively well ES estimation. Conversely, the results also confirmed that all static models with particularly GED being the worst, t-distribution, Extreme Value Theory, Historical Simulation and normal distribution have weakest ES estimation. Within each variant of dynamic GARCH, t-distribution performs better than GED and then followed by normal distribution. Both the VaR and ES predictions seem to suggest that GARCH-EVT and GJR-GARCH filtered HS are good risk estimating approaches for SSE index at 95% confidence interval. A table of summary of all results can be found in appendix.

Table 17 – Backtesting results of VaR at q=95% for SSE index

T=498	Failure rate	z	LR_{uc}	LR_{ind}	LR_{cc}	Ψ
Parametric approach						
Static (Variance-covariance, VC)						
Normal distribution	0.0803	3.1047	8.2081	11.9175	20.1256	40.040559
Student (t-distribution)	0.0884	3.9271	12.6820	11.4653	24.1473	44.044226
GED	0.0823	3.3103	9.2480	11.0591	20.3070	41.041530
Dynamic						
EWMA (RiskMetrics)	0.0622	1.2542	1.4644	0.5893	2.0537	31.014522
GARGH-n	0.0542	0.4318	0.1817	0.1993	0.3810	27.016913
GARCH-t	0.0602	1.0486	1.0350	0.0226	1.0575	30.018023
GARCH-GED	0.0542	0.4318	0.1817	0.1993	0.3810	27.016571
EGARCH-n	0.0522	0.2262	0.0505	0.2998	0.3503	26.017711
EGARCH-t	0.0582	0.8430	0.6765	0.0615	0.7379	29.019047
EGARCH-GED	0.0522	0.2262	0.0505	0.2998	0.3503	26.017561
T/GJR-GARCH-n	0.0542	0.4318	0.1817	0.1993	0.3810	27.017635
T/GJR-GARCH-t	0.0602	1.0486	1.0350	0.0226	1.0575	30.019013
T/GJR-GARCH-GED	0.0542	0.4318	0.1817	0.1993	0.3810	27.017520
APARCH-n	0.0542	0.4318	0.1817	0.1993	0.3810	27.018074
APARCH-t	0.0562	0.6374	0.3912	0.1201	0.5114	28.019389
APARCH-GED	0.0542	0.4318	0.1817	0.1993	0.3810	27.017900
Non-parametric approach						
Static						
Historical Simulation (HS)	0.0924	4.3383	15.2225	9.8466	25.0691	46.045815
Dynamic						
Exponential weighted HS	0.0643	1.4598	1.9626	0.4411	2.4037	32.014683
Filtered HS (GARCH)	0.0562	0.6374	0.3912	0.1201	0.5114	28.017216
Filtered HS (T/GJR-GARCH)	0.0542	0.4318	0.1817	0.1993	0.3810	27.018108
Semi-parametric approach						
Static						
Extreme Value Theory (EVT)	0.0884	3.9271	12.6820	11.4653	24.1473	44.044883
Dynamic						
GARCH-EVT	0.0522	0.2262	0.0505	0.2998	0.3503	26.016671
T/GJR-GARCH-EVT	0.0542	0.4318	0.1817	0.1993	0.3810	27.017676

Table 18 – Backtesting results of ES at q=95% for SSE index

	V_{ES1}	V_{ES2}	V_{ES}
Parametric approach			
Static (Variance-covariance, VC)			
Normal distribution	-0.010974	-0.023414	0.017194
Student (t-distribution)	-0.012096	-0.026927	0.019512
GED	-0.014887	-0.027977	0.021432
Dynamic			
EWMA (RiskMetrics)	-0.007413	-0.01212	0.009766
GARGH-n	-0.010524	-0.013109	0.011816
GARCH-t	-0.00393	-0.00825	0.00609
GARCH-GED	-0.005237	-0.008319	0.006778
EGARCH-n	-0.01202	-0.01357	0.012796
EGARCH-t	-0.00561	-0.0093	0.007454
EGARCH-GED	-0.007303	-0.009323	0.008313
T/GJR-GARCH-n	-0.011456	-0.013642	0.012549
T/GJR-GARCH-t	-0.00542	-0.00928	0.007351
T/GJR-GARCH-GED	-0.006855	-0.009287	0.008071
APARCH-n	-0.011766	-0.013909	0.012838
APARCH-t	-0.00663	-0.00955	0.008092
APARCH-GED	-0.007155	-0.009559	0.008357
Non-parametric approach			
Static			
Historical Simulation (HS)	-0.010386	-0.026620	0.018503
Dynamic			
Exponential weighted HS	-0.00269	-0.00882	0.005755
Filtered HS (GARCH)	-0.00413	-0.00763	0.00588
Filtered HS (T/GJR-GARCH)	-0.00574	-0.00817	0.006952
Semi-parametric approach			
Static			
Extreme Value Theory (EVT)	-0.011005	-0.026119	0.018562
Dynamic			
GARCH-EVT	-0.004651	-0.007238	0.005945
T/GJR-GARCH-EVT	-0.005907	-0.008321	0.007114

Downside Risk Spillover

In order to investigate the downside risk spillover effects from China market to Singapore market or vice versa, one of the good risk estimating methodologies based on analyzed results is chosen for individual stock index at the particular confidence level for analysis. For both VaR and ES at 99% confidence level, GJR-GARCH filtered HS and GARCH filtered HS are chosen for STI index and SSE index respectively. At 95% confidence level, GJR-GARCH-EVT and GARCH-EVT are picked for both VaR and ES of STI index and SSE index respectively. The VaR and ES plots below show that China market has a larger downside risk than Singapore market.

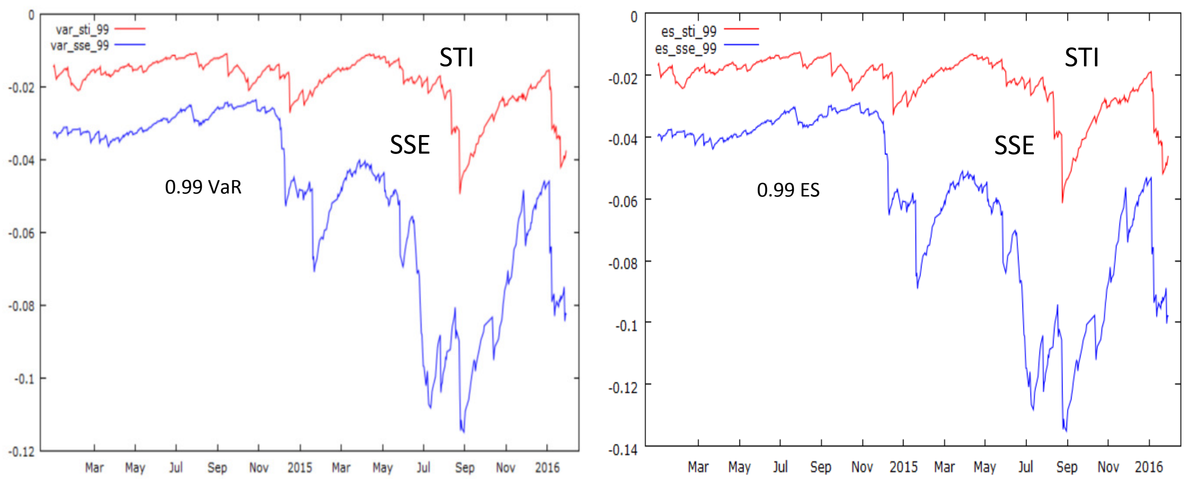

Fig 74 – Chosen VaR and ES estimate of STI and SSE indices returns at q=99%

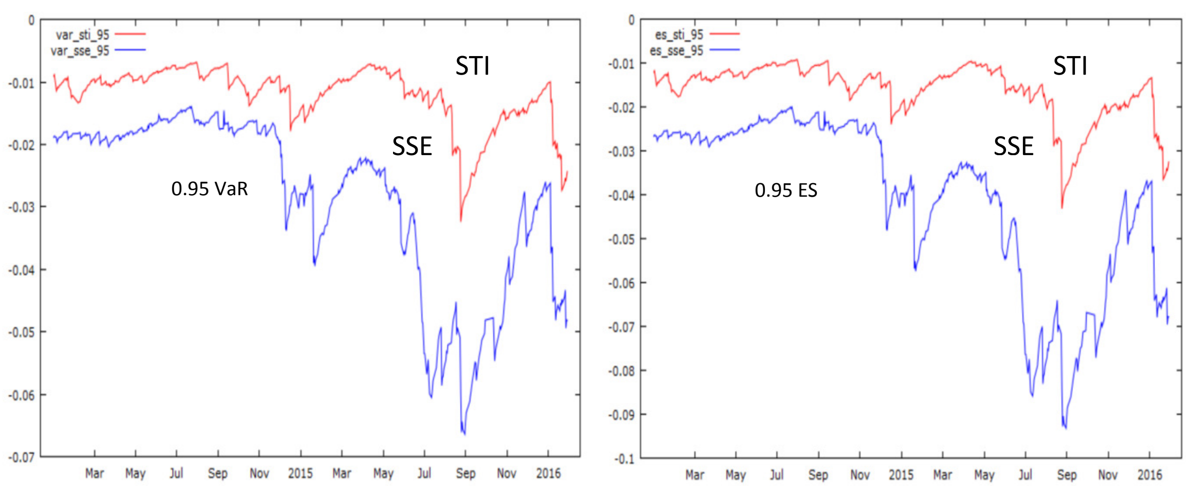

Fig 75 – Chosen VaR and ES estimate of STI and SSE indices returns at q=95%

As Singapore and China stock market operate at different location and trading days may differ due to reasons such as public holidays, only VaR or ES data that have common trading days are used. This is one of the most direct methods compared to other methods like interpolation or holding previous day value for missing data. Nonetheless, all these methods would create undesired bias in the results and there is no perfect solution. Before further analysis, the chosen VaR and ES time-series are tested for non-stationary using ADF unit root test. The test results are summarized as below:

Table 19 – Results of stationary test (ADF unit root test) on VaR and ES of picked est. methods

VaR of STI index			VaR of SSE index		
q=99%			q=99%		
GJR-GARCH filtered HS	Accept H_o (pval=0.6221; 0.6078)	Unit root, Non-S.	GARCH filtered HS	Accept H_o (pval=0.6682; 0.6631)	Unit root, Non-S.
q=95%			q=95%		
GJR-GARCH-EVT	Accept H_o (pval=0.6156; 0.5955)	Unit root, Non-S.	GARCH-EVT	Accept H_o (pval=0.6762; 0.7286)	Unit root, Non-S.

ES of STI index			ES of SSE index		
q=99%			q=99%		
GJR-GARCH filtered HS	Accept H_o (pval=0.6154; 0.6269)	Unit root, Non-S.	GARCH filtered HS	Accept H_o (pval=0.6711; 0.6661)	Unit root, Non-S.
q=95%			q=95%		
GJR-GARCH-EVT	Accept H_o (pval=0.6177; 0.5952)	Unit root, Non-S.	GARCH-EVT	Accept H_o (pval=0.6745; 0.7162)	Unit root, Non-S.

*p-val = first value: test without constant; second value: test with constant

As all the test results accept null hypothesis that there is a unit root, the VaR and ES time series are non-stationary. With VaR or ES being I(1), the long term, stable equilibrium or *cointegrating relationship* between the VaR or ES of one market and another market could be examined to check for long term risk spillover effect. *Cointegrating relationship* exists if the time series are non-stationary but "move together" over time. If variables are cointegrated, linear combination of them will be stationary I(0) (Brooks 2008). For a OLS regression model $y_t = \beta_1 + \beta_2 x_t + u_t$ where y_t and x_t are cointegrated, u_t should be stationary I(0). Engle and Granger (1987) used the *stationary test of residual* to check for cointegration relationship. The stationary test of residual was done using DF or ADF (test H_o : *unit root, I(1)*) but the critical values have changed as it is a test on residuals. The cointegrating relationship between the downside risk estimator VaR or ES of two markets are tested using this Engle and Granger test in *Gretl* software (at maximum lag of 12) and the results are shown below:

Table 20 – Results of cointegration test between downside risk estimator of STI and SSE indices

Downside risk estimator of markets	Residual (*uhat*) test results	Long-run relationship results
VaR of STI and SSE at q=99%	Accept H_o (p-val=0.2071; 0.2994)	Unit root, Not co-integrated
VaR of STI and SSE at q=95%	Accept H_o (p-val=0.2235; 0.3325)	Unit root, Not co-integrated
ES of STI and SSE at q=99%	Accept H_o (p-val=0.1286; 0.2249)	Unit root, Not co-integrated
ES of STI and SSE at q=95%	Accept H_o (p-val=0.1323; 0.2155)	Unit root, Not co-integrated

*p-val = first value: STI's being dependent variable; second value: SSE's being dependent variable.

```
Step 1: testing for a unit root in var_sti_99

Augmented Dickey-Fuller test for var_sti_99
:
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  :
  test statistic: tau_c(1) = -1.40076
  asymptotic p-value 0.5836

Step 2: testing for a unit root in var_sse_99

Augmented Dickey-Fuller test for var_sse_99
:
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  :
  test statistic: tau_c(1) = -1.20179
  asymptotic p-value 0.6761

Step 3: cointegrating regression

Cointegrating regression -
OLS, using observations 2014-01-02:2016-01-29 (T = 484)
Dependent variable: var_sti_99


             coefficient   std. error    t-ratio    p-value
  --------------------------------------------------------------
  const       −0.00759765   0.000501443   −15.15    1.08e-042 ***
  var_sse_99   0.219592     0.00916086     23.97    3.50e-084 ***
  :

Step 4: testing for a unit root in uhat
```

```
Augmented Dickey-Fuller test for uhat
:
unit-root null hypothesis: a = 1

  model: (1-L)y = (a-1)*y(-1) + ... + e
  :
  test statistic: tau_c(2) = -2.67915
  asymptotic p-value 0.2071
```

Fig 76 – Sample Engle and Granger test for cointegration between VaR at q=99% of STI and SSE indices

The results above showed that there is no long term risk spillover, stable equilibrium or cointegrating relationship between the two markets. With that and VaR or ES satisfying non-stationary pre-requisite, Granger causality test under VAR (*Vector Autoregression*) framework is used to check for *short term relationship or risk spillover* between the two markets. Granger causality test using VAR framework and F-test that jointly test for significance of the lags on the explanatory variables have been introduced previously. The advantage of using VAR model is that it does not rely on an underlying theory and does not require any assumptions regarding the values of the exogenous variables. Granger causality test was performed in *Gretl* software using 'F-test of zero restriction' under VAR.

Table 21 – Results of Granger causality test between downside risk estimator of STI and SSE indices

Downside risk estimator of markets	F-test under VAR	short-run relationship results
VaR of STI $\Leftarrow$ VaR of SSE at q=99%	Reject H_o (p-val=0.0079)	Granger Causal
VaR of STI $\Rightarrow$ VaR of SSE at q=99%	Accept H_o (p-val=0.0589)*	Non-Granger Causal
VaR of STI $\Leftarrow$ VaR of SSE at q=95%	Reject H_o (p-val=0.0022)	Granger Causal
VaR of STI $\Rightarrow$ VaR of SSE at q=95%	Reject H_o (p-val=0.0188)	Granger Causal
ES of STI $\Leftarrow$ ES of SSE at q=99%	Reject H_o (p-val=0.0046)	Granger Causal
ES of STI $\Rightarrow$ ES of SSE at q=99%	Accept H_o (p-val=0.0647)*	Non-Granger Causal
ES of STI $\Leftarrow$ ES of SSE at q=95%	Reject H_o (p-val=0.0038)	Granger Causal
ES of STI $\Rightarrow$ ES of SSE at q=95%	Reject H_o (p-val=0.0236)	Granger Causal

X $\Rightarrow$ Y: X Granger cause Y; X $\Leftarrow$ Y: Y Granger cause X

```
VAR system, lag order 12
OLS estimates, observations 2014-01-20-2016-01-29 (T = 472)
Log-likelihood = 4570.7916
:

Equation 1: var_sti_99

             coefficient    std. error    t-ratio     p-value
  ----------------------------------------------------------------
  const         −0.000505225   0.000202678   −2.493     0.0130     **
  var_sti_99_1    0.956982      0.0491371     19.48      1.28e-061 ***
  var_sti_99_2   −0.0127802     0.0667713    −0.1914    0.8483
```

```
 :
 var_sti_99_12  −0.0497717    0.0519418    −0.9582   0.3385
 var_sse_99_1   −0.00924474   0.0252351    −0.3663   0.7143
 :
 var_sse_99_12   0.0341292    0.0259133     1.317    0.1885
 :
```

F-tests of zero restrictions:

All lags of var_sti_99 F(12, 447) = 414.97 [0.0000]
All lags of var_sse_99 F(12, 447) = 2.2888 [0.0079]
All vars, lag 12 F(2, 447) = 1.0270 [0.3589]

Equation 2: var_sse_99

```
               coefficient    std. error    t-ratio    p-value
  ---------------------------------------------------------------
  const        −0.000499823   0.000394965   −1.265     0.2064
  var_sti_99_1  0.215030      0.0957548      2.246     0.0252     **
  var_sti_99_2 −0.283614      0.130119      −2.180     0.0298     **
  :
  var_sti_99_12 0.0935895     0.101220       0.9246    0.3557
  var_sse_99_1  0.968142      0.0491764     19.69      1.37e-062 ***
  :
  var_sse_99_12 0.00606088    0.0504979      0.1200    0.9045

:
```

F-tests of zero restrictions:

All lags of var_sti_99 F(12, 447) = 1.7247 [0.0589]
All lags of var_sse_99 F(12, 447) = 1245.2 [0.0000]
All vars, lag 12 F(2, 447) = 0.52442 [0.5923]

Fig 77 – Sample F-test under VAR for Granger causality between VaR at q=99% of STI and SSE indices

The F-test results under VAR showed that downside market risk indicator VaR or ES of SSE index at both confidence level 99% and 95% Granger cause that of STI index at respective confidence level. These results are based on rejection of null hypothesis (that lagged values of casual variables are not significant) at convention 5% level of significance. Conversely, downside market risk indicator VaR or ES of STI index at 95% confidence level only Granger cause that of SSE index. VaR or ES of STI index at 99% confidence level does not Granger cause that of SSE index because the p-value slightly exceeded 0.05 (i.e. 0.0589 and 0.0647) and null hypothesis is accepted. However, this null hypothesis acceptance is rather weak. We can fully reject the null hypothesis at 10% level of significance and conclude that there is Granger causality from VaR or ES of STI index at 99% confidence level to that of SSE index. Overall, the results tend to show that there is short-run risk spillover effect from SSE index to STI index and vice-versa. However, the risk spillover effects from STI index to SSE index seems to be weaker as suggested by the case of VaR or ES at 99% confidence level. The overall results confirm the findings of Hong, Cheng, Liu, and Wang (2004) and part of the results of unilateral Granger causality from Singapore to China by Hooi, Penm and Terrell (2003).

There is casual effect between downside market risk of china stock market and Singapore stock market due to the close relationship between the two countries. The close relationship could be generally related to geographical proximity, partnerships in trade and cultural similarity. Since the establishment of diplomatic relation in 1990, Singapore and China have developed strong links in the areas of trade, finance and investment. Trade and investment has been growing steadily and rapidly over the years. The total trade between China and Singapore in 2013 amounted to S$115.2 billion. In 2014, Singapore has become China's third largest trading partner in ASEAN and China has also become Singapore largest trading partner with bilateral trade of S$121.5 billion (Narendra 2015). Bilateral trade in services has grown significantly with more Singapore companies venturing into service sectors in China to help boost modern service which also has contributed to Singapore companies' revenue. Singapore banks including DBS, OCBC and UOB, property developers such as Capitaland and Keppel, consumer goods and services such as Q&M Dental, BreadTalk and OSIM have made successful forays into China market. The development of Suzhou Industrial Park, Singapore-Hangzhou Science & Technology Park, Tianjin Eco-City and Sino-Singapore Guangzhou Knowledge City also saw involvement of Singapore companies like Ascendas (Narendra 2015).

Singapore is a traditional market for Chinese export of textile, clothing, metals, agriculture and petrochemical products, electromechanical, shipping, communication and electronics equipments. Singapore also exports food/beverages, chemical and pharmaceutical products, electronics, machines and oils to China besides services. A series of agreement were also signed to promote bilateral trade, such as China-Singapore FTA signed in 2008 which allows China goods to be duty free when exported to Singapore and vice versa (KPMG 2009). Singapore has become China's largest foreign investor for the second time with investment amounting to US$5.8 billion over 700 projects in 2014. At the same time, Singapore has become one of China's top investment destinations in Asia, signifying strong business relation and interaction (Narendra 2015). With unceasingly improvement over trading and investment activities, the economic relationship between the China and Singapore has become closer and hence the linkage between Singapore stock market and Chinese stock market.

5 Summary and Conclusion

Financial risk management has become more important nowadays with the shorter economic downturn cycle and increase in market turbulence such as the recent subprime mortgage crisis. VaR has become a popular, reliable and standard market risk measure associated with stock price or index but has drawbacks as it lacks the coherence (the sub-additivity property) and does not describe the losses beyond the VaR level. ES has been touted as the upcoming influential risk measurement metric as it can overcome these weaknesses. The purpose of this work is to estimate the downside market risk of the most recent Singapore (STI) and China Shanghai (SSE) stock indices by using VaR and ES risk estimators via various models or approaches and assess which approaches yield accurate performance. Before that, the indices returns are checked for the stationarity and heteroskedasticity using correlogram, LBQ test and LM test and were verified to be stationary and contain ARCH effects in the residuals. The GARCH fitting test also shown that GARCH(1,1) is suitable to describe a volatility model. Various VaR and ES estimation techniques (parametric approaches such as static t-distribution, dynamic GJR-GARCH or EGARCH, non-parametric approaches such as Historical Simulation and GARCH filtered HS and semi-parametric approaches such as static Extreme Value Theory or dynamic GARCH-EVT) have been introduced, discussed and applied to the STI and SSE indices. The estimations were done using rolling window approach which takes data from 2009 to 2013 as initial in-sample-data to forecast daily out-of-sample VaR or ES from 2014 to 2016 but rolling forward. In order to assess the performance of VaR and ES, various backtesting methods had been introduced: failure rate, Kupiec's unconditional coverage test and Christoffersen's independence or conditional coverage test for VaR and Embrechts's V-test for ES.

The backtesting results show that VaR of STI and SSE indices at 99% confidence level can be described well with both variants of GARCH filtered HS, i.e. GJR-GARCH filtered HS (best method for STI) and GARCH filtered HS (best method for SSE although overall results for SSE is less satisfying at 99% due to larger volatility). They can also be described well with variants of dynamic GARCH-EVT, i.e. GARCH-EVT (for STI only) and GJR-GARCH-EVT. This is consistent with the analysis of Lin, Huang, Yang and WeiYu (2008) about GJR-GARCH-EVT on SSE index. STI index probably less volatile (less leptokurtic) than SSE and can be estimated well also with static models also like t-distribution, GED, normal distribution, Historical Simulation and Extreme Value Theory. In fact, Extreme Value Theory is better than normal distribution, t-distribution and Historical simulation which confirms the findings of Ramaza and Faruk (2004). Dynamic GARCH, EGARCH and GJR-GARCH didn't do so well for STI index unlike those claimed by Ong and Wang (2011). Meanwhile, VaR of SSE index can be estimated well with dynamic model GJR-

GARCH-t and GJR-GARCH-GED, similar to those claimed by Chen and Yu (2002). For STI index, RiskMetrics EWMA and Exponential weighted HS have poor performance and strongly underestimated the risk. Normal distribution, t-distribution, GED and Extreme Value Theory estimated badly for SSE index.

At 95% confidence level, the backtesting results show that VaR of STI and SSE index can be predicted well with variants of dynamic GARCH-EVT, i.e. GARCH-EVT (best method for SSE) and GJR-GARCH-EVT. In addition, they can be described well with variants of GARCH filtered HS, i.e. GARCH filtered HS (for STI only) and GJR-GARCH filtered HS (for SSE only). STI index can be estimated well also with static models also like normal, t-distribution, GED (best method for STI), Historical Simulation, Extreme Value Theory and dynamic models like GJR-GARCH-GED. SSE index can also be estimated no less with all variants of dynamic GARCH models with normal or GED distribution: EGARCH, GARCH, GJR-GARCH, APARCH. This confirms the findings of Wu (2015) and Chen and Yang (2004) that dynamic models (including APARCH) with GED distribution describe SSE better than t-distribution. Extreme Value Theory and Historical Simulation do not fare well for SSE index. The performance of RiskMetrics EWMA is poor besides Exponential Weighted HS. This contradicts the findings by Lin, Li, and Tse (2006) and Fan, Wei and Xu (2004) about EWMA.

Turning to ES for STI and SSE index at confidence level 99%, backtesting results indicate that ES has good estimation under variants of GARCH filtered HS, i.e. GARCH filtered HS (for SSE only) and GJR-GARCH filtered HS. They can also be predicted with variant of dynamic GARCH-EVT, i.e. GARCH-EVT (for SSE only) and GJR-GARCH-EVT. STI index can be estimated well also with static models also like t-distribution, Historical Simulation, Extreme Value Theory (best method for STI), and dynamic model like EGARCH-t and APARCH-t. On the other hand, ES for SSE index could be estimated well also with dynamic models GJR-GARCH-t (best method for SSE) and APARCH-t. Normal distribution, GED and all variants of GARCH with normal distribution (except EGARCH-n for STI) and RiskMetrics EWMA have poor ES estimation for both STI and SSE index. In addition, GARCH-GED has poor ES estimation for STI while Historical Simulation and Extreme Value Theory have poor ES estimation for SSE.

The backtesting of ES for STI and SSE index at 95% confidence level demonstrated that good estimation is achieved using variant of dynamic GARCH-EVT, i.e. GJR-GARCH-EVT (for STI only) and GARCH-EVT. Similarly, they can also be estimated well with variants of GARCH filtered HS, i.e. GARCH filtered HS (for SSE only) and GJR-GARCH filtered HS. STI index can be estimated well also with static models also like GED (best method for STI), Historical Simulation and Extreme Value Theory and dynamic models like EGARCH-t, APARCH-GED and APARCH-t. On the other hand, SSE index can be estimated well with dynamic model like exponential weighted HS (best method for SSE), GARCH-GED and GARCH-t. Conversely, RiskMetrics EWMA and GARCH-n has poor ES estimation for STI index. All static models, Historical Simulation and Extreme Value Theory have poor performance for ES estimation of SSE index.

It is notably that either one of the variants of dynamic GARCH-EVT and GARCH filtered HS approaches is a good model for estimating VaR or ES of both STI and SSE. Both dynamic maxima tail characteristic of Extreme Value Theory and volatility scaled historical data move away from full distribution assumptions and proven useful in evaluating downside risk of market especially when it experiences more downward pressure and turbulence in recent years. STI index is less volatile and can be described by some static models including Extreme Value Theory and Historical Simulations. This is completely opposite for SSE index. Either one of the t-distribution or GED method will perform better than normal distribution except in the case for SSE index at 95% confidence level. However, normal distribution estimated VaR of STI index relatively well. With the exception of risk measure VaR at 99% confidence level for STI index, most can be described well with either one of the dynamic asymmetric GARCH model (T/GJR-GARCH, EGARCH) or APARCH model with GED or t-distribution. APARCH model does not seem to have better estimation and advantage than other variants of GARCH despite the ability to model power parameter other than 2 especially for SSE index. For ES estimation and within each variant of dynamic GARCH, t-distribution performs the best, followed by GED and normal distribution. RiskMetrics EWMA is the least preferred choice for VaR or ES estimation except for ES estimation at 95% confidence level and VaR at 99% confidence level for SSE index. The same sentiment applies to exponential weighted HS when it comes to VaR estimation for STI index and for SSE index at 95% confidence level. However, exponential weighted HS does perform better than Historical Simulation for SSE index. With performance of ES analogous to that of VaR (close resemblance of good estimating methods too), the possibility of ES overtaking or replacing VaR as the de facto market risk estimator is promising.

During the last two decades, the Chinese economy has grown significantly and its economic ties with the world and countries such as Singapore has become closer and integrated. This is the result of cultural similarity, geographical proximity, on-going bilateral trading, investments and FTA signed between China and Singapore. Consequently, this study investigated the linkage and interdependency between two financial markets in terms of downside risk spillover. The time-series of VaR and ES (selected based on accurate estimation models) were verified to be non-stationary (contain unit root) using ADF test and the Engle and Granger test has shown that there is no long-run or cointegration relationship between downside risk of the two stock markets indices. Subsequently, Granger causality test under VAR framework is used to check for short-term relationship or risk spillover between the two markets. It is a F-test that jointly test for significance of the lags on the explanatory variables (null hypothesis: lagged values of casual variables are not significant). The results showed that there is causality from downside risk indicator VaR or ES of SSE index to that of STI index (rejection of null hypothesis) and vice-versa. However, the causality of downside risk indicator VaR or ES from STI index to that of SSE index appeared to be weaker due to rejection of null hypothesis at 10% level of significance for the case of 99% confidence level VaR or ES. Overall, there is short-run downside risk spillover effect between the two markets based on recent data. This analysis result would serve useful for fund managers and investors in their portfolio selection and for policy maker in guarding against market spillover catastrophe.

This dissertation has limited the study and backtest to one-day ahead VaR and ES. Future work could explore the 10 day holding day period VaR which is more in tune with Basel Committee capital requirement calculation. The lesser than ideal performance of VaR estimation methods (major underestimation) for SSE index at 99% confidence level could open up opportunities for further improvement in the estimation methods such as GARCH filtered HS. One possible direction is reducing the rolling window size for in-sample observation (currently using nearly 1200 data) which may be too large to accommodate and incorporate new events or changes quickly. It would be interesting to see how smaller rolling window size affects other estimation methods as well. Alternatively, other more complex and advanced non-parametric approach such as Quantile Regression (CAViAR or *Conditional Autoregressive Value at Risk* by Engle and Manganelli (2004)) or *Kernel Estimator* (KE) by Huang (2009) could be explored. Other promising, reliable and well established ES backtesting mechanisms could also be used to complement the V-test.

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7 Appendices

Sub-additivity of VaR and ES

Diversification is a key concept in risk measurement and a risk measure should reflect diversification: the risk of a portfolio should be the same or less than the sum of the risk of subcomponents. In other words, $\rho(W_1+W_2) \leq \rho(W_1) + \rho(W_2)$ and this is technically called *sub-additivity*. VaR does not always satisfy this condition. The following examples illustrate the case for VaR and ES (Hull 2012).

Consider a trader invested in two independent stocks, each with the same characteristic having probability 0.03 of losing \$10 million and probability 0.97 of losing \$1 million over one-day period. Since the probability of losing \$1 million is greater than 95%, the one-day VaR at 95% confidence level is \$1 million. The sum of these two VaRs amounts to \$2 million. When the two stocks are placed in the same portfolio, there is a 0.03 x 0.03 = 0.0009 probability of losing \$20 million, a 2 x 0.03 x 0.97 = 0.0582 probability of losing \$11 million and a 0.97 x 0.97 = 0.9409 probability of losing \$2 million. Here, the probability of losing (maximum) \$11 million or less is 0.9409 + 0.0582 = 0.9991. The one-day VaR at 95% confidence level for the portfolio is therefore \$11 million, which is the lowest amount such that probability exceeds 95%. Since VaR of the portfolio (\$11 million) is greater than the sum of the VaRs (\$2 million), the sub-additivity property is violated.

Turning to the calculation of the expected shortfall at 95% confidence level, note that for the 5% tail of the loss distribution, 3% corresponds to \$10 million loss and 2% corresponds to \$1 million loss. Conditional in the 5% tail of the loss distribution, there is therefore a 60% (i.e. 3/5) probability of a loss of \$10 million and 40% (i.e. 2/5) probability of a loss of \$1 million. The expected loss or expected shortfall is 0.6 x 10 + 0.4 x 1 = \$6.4 million. When considering the two stocks in the portfolio, for the 5% tail of the loss distribution, 0.09% corresponds to a loss of \$20 million and 4.91% corresponds to a loss of \$11 million. Similarly conditional in the 5% tail of the loss distribution, the expected loss or expected shortfall is (0.09/5) x 20 + (4.91/5) x 11 = \$11.162 million. Because 11.162 < 6.4 + 6.4, expected shortfall measure satisfy the sub-additivity condition.

Extreme Value Theory (Block Maxima Approach)

The following describes the other approach of Extreme Value Theory, i.e. the *block maxima* method which considers the maximum of sequence of random variables. Let $M_n = \max\{R_1, R_2, \dots, R_n\}$ where *iid* random variable $\{R_i\}_{i=1}^{n}$ has common CDF $F_R(x) = P(R < x)$ but usually unknown. Thus M_n has CDF given by $F_{M_n}(x) = P(M_n < x) = P(R_1 < x, \dots, R_n < x) = [P(R < x)]^n = [F_R(x)]^n$ which degenerated to zero when $F_R(x)$ <1 and $n \to \infty$. Fisher and Tippett (1928) showed that if normalization exist such that $\frac{M_n - c_n}{d_n}$ converges to a non-degenerated distribution $H(x)$ as $n \to \infty$, then $H(x)$ is governed by Generalized Extreme Value (GEV) distribution of Jenkinson (1955):

$$H(x) = \exp(-(1+\xi x)^{-\frac{1}{\xi}}) \quad ; \xi \neq 0 \qquad \Leftrightarrow h(x) = (1+\xi x)^{-\frac{1}{\xi}-1}\exp(-(1+\xi x)^{-\frac{1}{\xi}})$$

$$H(x) = \exp(-\exp(-\mathrm{x})) \quad ; \xi = 0 \qquad \Leftrightarrow h(x) = \exp(-x - \exp(-x))$$

where $1 + \xi x > 0$ and $h(x)$ is the pdf obtained easily by differentiating $H(x)$. ξ is the shape parameter (its inverse is called *tail index*) that governs the tail behavior and $\xi = 0$ is regarded as the limit when $\xi \to 0$.

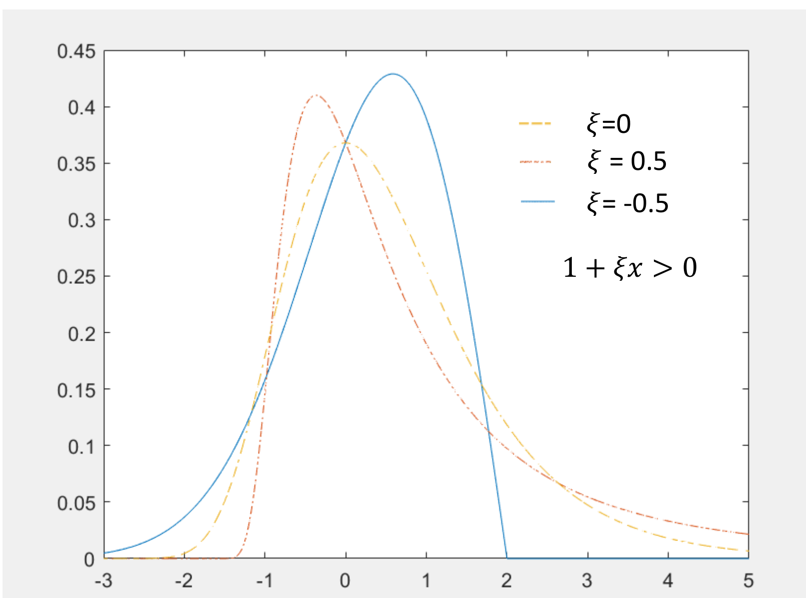

Fig A1 – GEV distribution with ξ= -0.5 (Weibull; x <2), ξ= 0.5 (Fréchet; $x > -2$) and ξ=0 (Gumbel)

GEV encompasses three limit distributions: Gumbel, Fréchet and Weibull distributions. Fréchet distribution ($\xi > 0$) has fat tails, include distribution such as F-distribution, inverse gamma and t-distribution and has finite *k*-th moment ($E(X^k) < \infty$) for $k \le \frac{1}{\xi}$. Gumbel distribution ($\xi = 0$) has tails that decay faster (exponentially and thinner), finite *k*-th moment for *k>0* and include distributions such as normal, lognormal and exponential distribution. Weibull distribution ($\xi < 0$) is of least interest as it has short tail and finite endpoint (Coleman 2012).

Assuming that there are *T* returns $\{r_j\}_{j=1}^{T}$ and they are divided into *g* non-overlapping subsamples each with *n* observations (i.e. *T=ng*). Let $r_{n,i}$ be the maximum of *i*th subsample. When *n* is sufficiently large, $x_{n,i} = \frac{(r_{n,i}-\beta_n)}{\alpha_n}$ should follow the extreme value distribution. The pdf of $r_{n,i}$ can be found by substituting $x_{n,i} = \frac{(r_{n,i}-\beta_n)}{\alpha_n}$ into $h(x)$ and after simple transformation:

$$f(r_{n,i}) = \frac{1}{\alpha_n}(1+\xi\frac{(r_{n,i}-\beta_n)}{\alpha_n})^{-\frac{1}{\xi}-1}\exp(-(1+\xi\frac{(r_{n,i}-\beta_n)}{\alpha_n})^{-\frac{1}{\xi}}) \qquad ;\ \xi \neq 0$$

$$f(r_{n,i}) = \frac{1}{\alpha_n}\exp\left(-\frac{(r_{n,i}-\beta_n)}{\alpha_n} - exp\left(-\frac{(r_{n,i}-\beta_n)}{\alpha_n}\right)\right) \qquad ;\ \xi = 0$$

where $1+\xi\frac{(r_{n,i}-\beta_n)}{\alpha_n} > 0$ for $\xi \neq 0$ (Ruey 2009). The parameters of GEV distribution ξ, β_n and α_n could be estimated via *MLE* using likelihood function for subperiod maxima $\{r_{n,i}\}$:

$$l_t(r_{n,1}, \dots, r_{n,g}|\xi, \beta_n, \alpha_n) = \prod_{i=1}^{g} f(r_{n,i})$$

After estimating the parameters, $x = \frac{(r-\beta_n)}{\alpha_n}$ can be plugged into $H(x)$ to obtain the quantile of a given probability of the GEV distribution (Ruey 2009). Let α^* be a small upper tail probability that has potential loss and ${r_n}^*$ be the $(1-\alpha^*)$th quantile of the subperiod maxima $\{r_{n,i}\}$ under GEV distribution:

$$1-\alpha^* = \exp(-(1+\xi\frac{({r_n}^*-\beta_n)}{\alpha_n})^{-\frac{1}{\xi}}) \qquad ;\xi \neq 0$$

$$1-\alpha^* = \exp(-\exp\left(-\frac{({r_n}^*-\beta_n)}{\alpha_n}\right)) \qquad ;\xi = 0$$

Thus, the quantile of subperiod maxima $\{r_{n,i}\}$:

$${r_n}^* = \beta_n - \frac{\alpha_n}{\xi}\{1-(-\ln(1-\alpha^*))^{-\xi}\} \qquad ;\xi \neq 0$$

$${r_n}^* = \beta_n - \alpha_n \ln(-\ln(1-\alpha^*)) \qquad ;\xi = 0$$

The next step is bridging the relationship between subperiod maxima $\{r_{n,i}\}$ and observed r_t series. It was mentioned previously that $P(M_n < x) = [P(R < x)]^n$, hence $1 - \alpha^* = P\left(r_{n,i} \leq r_n{}^*\right) = [P(r_t \leq r_n{}^*)]^n$ (Ruey 2009). If $(1-\alpha)$th quantile of r_t is $r_n{}^*$, i.e. $P(r_t \leq r_n{}^*) = (1-\alpha)$, a close expression of VaR can be found since $1 - \alpha^* = (1-\alpha)^n$ and is given by:

$$VaR = \beta_n - \frac{\alpha_n}{\xi}\{1 - (-n\ln(1-\alpha))^{-\xi}\} \qquad ;\xi \neq 0$$

$$VaR = \beta_n - \alpha_n \ln(-n\ln(1-\alpha)) \qquad ;\xi = 0$$

For financial data, the VaR for the case $\xi \neq 0$ would be of interest.

MATLAB/Gretl Codes

Sample Gretl code – Parametric static estimation– Normal distribution (q=0.99)

```
smpl full
num_data=$nobs
series var_v=NA
series es_v=NA

smpl 2009-01-02 2013-12-31
num_sample=$nobs

scalar num_est=num_data-num_sample
matrix var_data=zeros(1,num_est)

smpl 2009-01-02 2013-12-31

loop for j=1..num_est
    var_data[j]=var(r)
        smpl +1 +1
    smpl
endloop

smpl -(num_est-num_sample) -(num_est-1)

loop j=1..num_est --quiet
    var_v=-2.3263*sqrt(var_data[j])
    es_v=-(sqrt(var_data[j])/((1-0.99)*sqrt(2*3.1416)))*exp(-0.5*((2.3263)^2))
    if j<num_est
        smpl +1 +1
    endif
endloop
smpl full
```

Sample MATLAB code – Parametric static estimation – t-distribution (q=0.99)

```
load c:\Users\USER\Documents\sti_matlab_r.txt

r=sti_matlab_r(:,end)
num_data=length(r)
num_sample=1276
num_est=num_data-num_sample

for i=1:num_est
    pd=fitdist(r(i:num_sample+i-1),'tLocationScale');
    store_mu(i)=pd.mu;
    store_sigma(i)=pd.sigma;
    store_nu(i)=pd.nu;
    store_tinv(i)=tinv(0.99,pd.nu);
    var_v(i)=-store_tinv(i)*pd.sigma;
    es_v(i)=-pd.sigma*gamma((pd.nu+1)/2)/((1-0.99)*gamma(pd.nu/2)*sqrt(pd.nu*pi))*
(pd.nu/(pd.nu-1))*(1+((store_tinv(i)^2)/pd.nu))^(0.5*(1-pd.nu));
end

var_v2=var_v'
es_v2=es_v'
save ('sti_fit_st_var.txt','var_v2','-ascii')
save ('sti_fit_st_es.txt','es_v2','-ascii')
```

Sample MATLAB code – Parametric static estimation – GED distribution (q=0.95)

```
load c:\Users\USER\Documents\sti_matlab_r.txt

r=sti_matlab_r(:,end)
num_data=length(r)
num_sample=1276
num_est=num_data-num_sample

for i=1:num_est
    phat=gedfit(r(i:num_sample+i-1));
    store_mu(i)=phat(2);
    store_sigma(i)=phat(3);
    store_nu(i)=phat(1);
    store_ginv(i)=gedcustinv(0.95,store_nu(i));
    lda=(2^(-2/store_nu(i))*gamma(1/store_nu(i))/gamma(3/store_nu(i)))^0.5;
    var_v(i)=-store_ginv(i)*store_sigma(i);
    es_v(i)=-store_sigma(i)*lda*2^(1/store_nu(i)-1)/((1-0.95)*gamma(1/store_nu(i)))*
gamma(2/store_nu(i))*(1-gammainc(0.5*(store_ginv(i)/abs(lda))^store_nu(i),
2/store_nu(i)));
end

var_v2=var_v'
es_v2=es_v'
save ('sti_fit_ged_var.txt','var_v2','-ascii')
save ('sti_fit_ged_es.txt','es_v2','-ascii')

function phat=gedfit(r)
custlogpdf=@(r,df,mu,scale) log(df/gedlambda(df))-0.5*abs((r-mu)/(scale*
gedlambda(df))).^df-(1+1/df)*log(2)-log(gamma(1/df))-log(scale)
[phat,pci]=mle(r,'logpdf',custlogpdf,'start',[0.1 0.01 0.01])
End

function lambda=gedlambda(df)
lambda=sqrt((2.^(-2/df))*gamma(1/df)/gamma(3/df));
end
```

Sample MATLAB code – Parametric dynamic estimation – EWMA of RiskMetrics (q=0.95)

```
load c:\Users\USER\Documents\sti_matlab_r.txt

num_data=length(sti_matlab_r)
num_sample=1276
num_est=num_data-num_sample
a=sti_matlab_r(:,end)
lambda=0.94

for m=0:num_est-1

    r=a(1+m:num_sample+m);
    h(1)=var(r(2:num_sample));

    for i=2:num_sample
        h(i)=(1-lambda)*(r(i-1)^2)+lambda*h(i-1);
    end

    h_predict=(1-lambda)*(r(num_sample)^2)+lambda*h(num_sample);

    var_v(m+1)=-1.6449*sqrt(h_predict);
    es_v(m+1)=-(sqrt(h_predict)/((1-0.95)*sqrt(2*3.1416)))*exp(-0.5*((1.6449)^2));

end
```

```
var_v2=var_v'
es_v2=es_v'
save ('ewma_var_v1.txt','var_v2','-ascii')
save ('ewma_es_v1.txt','es_v2','-ascii')
```

Sample Gretl code – Parametric dynamic estimation– GARCH-Normal distribution (q=0.95)

```
smpl full
num_data=$nobs
series var_v=NA
series es_v=NA

smpl 2009-01-02 2013-12-31
num_sample=$nobs

scalar num_est=num_data-num_sample

smpl 2009-01-02 2013-12-31

loop for j=0..num_est-1

garch 1 1;r --nc
scalar omega=$coeff[1]
scalar alpha1=$coeff[2]
scalar beta1=$coeff[3]
genr u=$uhat
genr u2=$uhat^2
genr h=$h
series h_predict=NA

smpl +(num_sample) +1

smpl
h_predict = omega+alpha1*u2(-1)+beta1*h(-1)

    var_v=-1.6449*sqrt(h_predict)
    es_v=-(sqrt(h_predict)/((1-0.95)*sqrt(2*3.1416)))*exp(-0.5*((1.6449)^2))
    smpl -(num_sample) -1

    if j<num_est-1
        smpl +1 +1
    endif

endloop
smpl full
```

Sample MATLAB code – Parametric dynamic estimation– T/GJR-GARCH-t-distribution (q=0.95)

```
load c:\Users\USER\Documents\sti_matlab_r.txt

num_data=length(sti_matlab_r)
num_sample=1276
num_est=num_data-num_sample;
a=sti_matlab_r(:,end);

r=a(1:num_sample);

for i=1:num_est
    r1=a(1+i-1:num_sample+i-1);
```

```
    [fit,logl,h]=tarch(r1,1,1,1);

    omega=fit(1);
    alpha1=fit(2);
    beta1=fit(4);
    gam1=fit(3);

    h_predict=omega+alpha1*(r1(num_sample).^2)+gam1*(r1(num_sample).^2)*
(r1(num_sample)<0)+beta1*h(num_sample);

    r1_std=r1./sqrt(h);

    pd=fitdist(r1_std,'tLocationScale');
    store_mu(i)=pd.mu;
    store_sigma(i)=pd.sigma;
    store_nu(i)=pd.nu;
    store_tinv(i)=tinv(0.95,pd.nu);
    var_v(i)=-sqrt(h_predict)*store_tinv(i)*pd.sigma;
    es_v(i)=-sqrt(h_predict)*pd.sigma*gamma((pd.nu+1)/2)/((1-0.95)*gamma(pd.nu/2)*
sqrt(pd.nu*pi))*(pd.nu/(pd.nu-1))*(1+((store_tinv(i)^2)/pd.nu))^(0.5*(1-pd.nu));
end

var_v2=var_v'
es_v2=es_v'
save ('tgarch_st_t_var_v1.txt','var_v2','-ascii')
save ('tgarch_st_t_es_v1.txt','es_v2','-ascii')
```

Sample MATLAB code – Parametric dynamic estimation– EGARCH-GED distribution (q=0.99)

```
load c:\Users\USER\Documents\sti_matlab_r.txt

num_data=length(sti_matlab_r)
num_sample=1276
num_est=num_data-num_sample;
a=sti_matlab_r(:,end);

r=a(1:num_sample);

for i=1:num_est
    r1=a(1+i-1:num_sample+i-1);
    spec=egarch(1,1)
    fit=estimate(spec,r1);
    h=infer(fit,r1);
    omega=fit.Constant
    alpha1=fit.ARCH{1}
    beta1=fit.GARCH{1}
    gam1=fit.Leverage{1};
    h_predict=exp(omega+alpha1*(abs(r1(num_sample))/sqrt(h(num_sample))-
sqrt(2/pi))+gam1*r1(num_sample)/sqrt(h(num_sample))+beta1*log(h(num_sample)));

    r1_std=r1./sqrt(h);

    phat=gedfit(r1_std);
    store_mu(i)=phat(2);
    store_sigma(i)=phat(3);
    store_nu(i)=phat(1);
    store_ginv(i)=gedcustinv(0.99,store_nu(i));
    lda=(2^(-2/store_nu(i))*gamma(1/store_nu(i))/gamma(3/store_nu(i)))^0.5;
    var_v(i)=-sqrt(h_predict)*store_ginv(i)*store_sigma(i);
    es_v(i)=-sqrt(h_predict)*store_sigma(i)*lda*2^(1/store_nu(i)-1)/((1-0.99)*
gamma(1/store_nu(i)))*gamma(2/store_nu(i))*(1-gammainc(0.5*(store_ginv(i)/abs(lda))
^store_nu(i),2/store_nu(i)));
```

```
end

var_v2=var_v'
es_v2=es_v'
save ('egarch_ged_var_v1.txt','var_v2','-ascii')
save ('egarch_ged_es_v1.txt','es_v2','-ascii')

function phat=gedfit(r)
custlogpdf=@(r,df,mu,scale) log(df/gedlambda(df))-0.5*abs((r-mu)/(scale*
gedlambda(df))).^df-(1+1/df)*log(2)-log(gamma(1/df))-log(scale)
[phat,pci]=mle(r,'logpdf',custlogpdf,'start',[0.1 0.01 0.01])
end

function lambda=gedlambda(df)
lambda=sqrt((2.^(-2/df))*gamma(1/df)/gamma(3/df));
end

function yinv=gedcustinv(u,V)
    lda = (2^(-2/V)*gamma(1/V)/gamma(3/V))^0.5;
    if (u<0.5)
        yinv=-lda*((2*gammaincinv(1-2*u,1/V))^(1/V));
    elseif (u>=0.5)
        yinv=lda*((2*gammaincinv(2*u-1,1/V))^(1/V));
    end
end
```

Sample MATLAB code – Parametric dynamic estimation– APARCH-Normal distribution (q=0.95)

```
load c:\Users\USER\Documents\sti_matlab_r.txt

num_data=length(sti_matlab_r)
num_sample=1276
num_est=num_data-num_sample;
a=sti_matlab_r(:,end);

r=a(1:num_sample);

for i=1:num_est
    r1=a(1+i-1:num_sample+i-1);

    [fit,logl,h]=aparch(r1,1,1,1);

    omega=fit(1);
    alpha1=fit(2);
    beta1=fit(4);
    gam1=-fit(3);
    delta1=fit(5);

    h_predict=(omega+alpha1*(abs(r1(num_sample))-gam1*r1(num_sample))^delta1+beta1*
(h(num_sample)^(delta1/2)))^(2/delta1);

    var_v(i)=-1.6449*sqrt(h_predict);
    es_v(i)=-(sqrt(h_predict)/((1-0.95)*sqrt(2*3.1416)))*exp(-0.5*((1.6449)^2));
end

var_v2=var_v'
es_v2=es_v'
save ('aparch_norm_var_v1.txt','var_v2','-ascii')
save ('aparch_norm_es_v1.txt','es_v2','-ascii')
```

Sample MATLAB code – Parametric dynamic estimation– APARCH-t-distribution (q=0.99)

```
load c:\Users\USER\Documents\sti_matlab_r.txt

num_data=length(sti_matlab_r)
num_sample=1276
num_est=num_data-num_sample;
a=sti_matlab_r(:,end);

r=a(1:num_sample);

for i=1:1 %num_est
    r1=a(1+i-1:num_sample+i-1);

    [fit,logl,h]=aparch(r1,1,1,1);

    omega=fit(1);
    alpha1=fit(2);
    beta1=fit(4);
    gam1=-fit(3);
    delta1=fit(5);

    h_predict=(omega+alpha1*(abs(r1(num_sample))-gam1*r1(num_sample))^delta1+beta1*
(h(num_sample)^(delta1/2)))^(2/delta1);
    r1_std=r1./sqrt(h);

    pd=fitdist(r1_std,'tLocationScale');
    store_mu(i)=pd.mu;
    store_sigma(i)=pd.sigma;
    store_nu(i)=pd.nu;
    store_tinv(i)=tinv(0.99,pd.nu);
    var_v(i)=-sqrt(h_predict)*store_tinv(i)*pd.sigma;
    es_v(i)=-sqrt(h_predict)*pd.sigma*gamma((pd.nu+1)/2)/((1-0.99)*gamma(pd.nu/2)*
sqrt(pd.nu*pi))*(pd.nu/(pd.nu-1))*(1+((store_tinv(i)^2)/pd.nu))^(0.5*(1-pd.nu));

end

var_v2=var_v'
es_v2=es_v'
save ('aparch_st_t_var_v1.txt','var_v2','-ascii')
save ('aparch_st_t_es_v1.txt','es_v2','-ascii')
```

Sample Gretl code – Non-parametric static estimation– Historical Simulation (q=0.99)

```
smpl full
num_data=$nobs

smpl 2009-01-05 2013-12-31
num_sample=$nobs
matrix ref_data=zeros(1,num_data)

smpl 2009-01-05 2009-01-05

loop for i=1..num_data
    ref_data[i]=r
    if i<num_data
        smpl +1 +1
    endif
endloop

scalar num_est=num_data-num_sample
matrix hist_data=zeros(1,num_sample)
```

```
matrix hist_data_sort=zeros(1,num_sample)
series var_v=NA
series es_v=NA
scalar sum_E=0
scalar sum_w=0

smpl 2014-01-02 2014-01-02

scalar c_level=0.01
scalar q_index=num_sample*c_level

loop for j=0..num_est-1 --quiet
    sum_E=0
    sum_w=0

    loop i=1..num_sample --quiet
        hist_data[i]=ref_data[i+j]
    endloop

    hist_data_sort=sort(hist_data)
    var_v=hist_data_sort[q_index]

    loop for i=1..floor(q_index)
        sum_E=sum_E+hist_data_sort[i]
        sum_w=sum_w+1
    endloop
    es_v=sum_E/sum_w

    if j<num_est-1
        smpl +1 +1
    endif
endloop
smpl full
```

Sample Gretl code – Non-parametric dynamic estimation– Exponential Weighted HS (q=0.99)

```
smpl full
num_data=$nobs

smpl 2009-01-05 2013-12-31
num_sample=$nobs
matrix ref_data=zeros(1,num_data)

smpl 2009-01-05 2009-01-05
loop for i=1..num_data

    ref_data[i]=r
    if i<num_data
        smpl +1 +1
    endif
endloop

scalar num_est=num_data-num_sample
matrix hist_data=zeros(num_sample,2)
matrix hist_data_sort=zeros(num_sample,2)
matrix cum_weight=zeros(num_sample,1)
scalar cum_w=0
scalar var_index=0
scalar exit_flag=0
scalar sum_w=0
scalar sum_E=0
scalar var_calc=0
scalar es_calc=0
```

```
series var_v=NA
series es_v=NA


smpl 2014-01-02 2014-01-02
scalar c_level=0.01
scalar q_index=num_sample*c_level
scalar lambda=0.94

loop for j=0..num_est-1 --quiet

    cum_w=0
    exit_flag=0
    sum_w=0
    sum_E=0
    es_calc=0

    loop for i=1..num_sample --quiet
        hist_data[i,2]=(lambda^(num_sample-i))*((1-lambda)/(1-lambda^num_sample))
        hist_data[i,1]=ref_data[i+j]
    endloop

    hist_data_sort=msortby(hist_data,1)

    loop for i=1..num_sample --quiet
        cum_w=cum_w+hist_data_sort[i,2]
        cum_weight[i]=cum_w
        if (exit_flag==0)
            if cum_w>=c_level
                var_index=i
                exit_flag=1
            endif
        endif
    endloop

    if var_index>1
        var_calc=hist_data_sort[var_index-1,1]
        loop for i=1..var_index-1 --quiet
            sum_E=sum_E+hist_data_sort[i,1]*hist_data_sort[i,2]
            sum_w=sum_w+hist_data_sort[i,2]
        endloop
        es_calc=sum_E/sum_w
    endif

    if var_index=1
        var_calc=hist_data_sort[1,1]
        es_calc=hist_data_sort[1,1]
    endif

    var_v=var_calc
    es_v=es_calc

    if j<num_est-1
        smpl +1 +1
    endif

endloop
smpl full
```

Sample Gretl code – Non-parametric dynamic estimation– GARCH filtered HS (q=0.99)

```
smpl full
num_data=$nobs
series var_v=NA
series es_v=NA
scalar sum_E=0
scalar sum_w=0

smpl 2009-01-02 2013-12-31
num_sample=$nobs

scalar num_est=num_data-num_sample
matrix hist_data=zeros(1,num_sample)
matrix hist_data_sort=zeros(1,num_sample)
matrix e_data=zeros(1,num_sample)

scalar c_level=0.01
scalar q_index=num_sample*c_level

smpl 2009-01-02 2013-12-31

loop for j=0..num_est-1

garch 1 1;r --nc
scalar omega=$coeff[1]
scalar alpha1=$coeff[2]
scalar beta1=$coeff[3]
genr u=$uhat
genr u2=$uhat^2
genr h=$h
series h_predict=NA

sum_E=0
sum_w=0

smpl ; -(num_sample-1)

loop i=1..num_sample --quiet
    e_data[i]=u/sqrt(h)
    if i<num_sample
        smpl +1 +1
    endif
endloop

h_predict = omega+alpha1*u2+beta1*h

loop i=1..num_sample --quiet
        hist_data[i]=e_data[i]*sqrt(h_predict)
endloop

    hist_data_sort=sort(hist_data)

    loop for i=1..floor(q_index)
        sum_E=sum_E+hist_data_sort[i]
        sum_w=sum_w+1
    endloop

    if j<num_est
        smpl +1 +1
        var_v=hist_data_sort[q_index]
        es_v=sum_E/sum_w
        smpl -1 -1
```

```
    endif

    smpl -(num_sample-1) ;

    if j<num_est-1
        smpl +1 +1
    endif
endloop
smpl full
```

Sample MATLAB code – Non-parametric dynamic estimation– T/GJR-GARCH filtered HS (q=0.95)

```
load c:\Users\USER\Documents\sti_matlab_r.txt

num_data=length(sti_matlab_r)
num_sample=1276
num_est=num_data-num_sample;
a=sti_matlab_r(:,end);
q_index=num_sample*0.05;

r=a(1:num_sample);

for i=1:num_est
    r1=a(1+i-1:num_sample+i-1);

    [fit,logl,h]=tarch(r1,1,1,1);

    omega=fit(1);
    alpha1=fit(2);
    beta1=fit(4);
    gam1=fit(3);

h_predict=omega+alpha1*(r1(num_sample).^2)+gam1*(r1(num_sample).^2)*(r1(num_sample)<0)
+beta1*h(num_sample);
    r1_std=r1./sqrt(h);

    hist_data=r1_std*sqrt(h_predict);
    hist_data_sort=sort(hist_data);

    sum_E=0
    sum_w=0
    for j=1:floor(q_index)
        sum_E=sum_E+hist_data_sort(j);
        sum_w=sum_w+1;
    end

    var_v(i)=hist_data_sort(floor(q_index))
    es_v(i)=sum_E/sum_w

end

var_v2=var_v'
es_v2=es_v'
save ('tgarch_hist_filt_var_v1.txt','var_v2','-ascii')
save ('tgarch_hist_filt_es_v1.txt','es_v2','-ascii
```

Sample MATLAB code – Semi-parametric static estimation– Extreme Value Theory (q=0.99)

```
load c:\Users\USER\Documents\sse_matlab_r.txt

num_data=length(sse_matlab_r)
num_sample=1249
num_est=num_data-num_sample
a=sse_matlab_r(:,end)
u_index=floor(num_sample*(1-0.90))

j=0
for m=0:num_est-1
   a_sort=sort(a(1+m:num_sample+m));
   threshold_u(m+1)=a_sort(u_index);
   for i=1+m:num_sample+m
      if a(i)<threshold_u(m+1)
         j=j+1
         b(j)=abs(a(i)-threshold_u(m+1))
      end
   end
   gpfit_par=gpfit(b)
   shape1=gpfit_par(1)
   scale1=gpfit_par(2)
   store_shape1(m+1)=shape1
   store_scale1(m+1)=scale1
   var_v(m+1)=-1*(abs(threshold_u(m+1))+(scale1/shape1)*((num_sample*(1-0.99)/j)^(-
1*shape1)-1))
   es_v(m+1)=-1*(abs(var_v(m+1))/(1-shape1)+(scale1-shape1*abs(threshold_u(m+1)))/(1-
shape1))
   clear b
   j=0
end

var_v2=var_v'
es_v2=es_v'
save ('evt_sse_var_v.txt','var_v2','-ascii')
save ('evt_sse_es_v.txt','es_v2','-ascii')
```

Sample MATLAB code – Semi-parametric dynamic estimation– GARCH-EVT (q=0.95)

```
load c:\Users\USER\Documents\sse_matlab_r.txt

num_data=length(sse_matlab_r)
num_sample=1249
num_est=num_data-num_sample;
a=sse_matlab_r(:,end);
u_index=floor(num_sample*(1-0.90));

r=a(1:num_sample);
j=0;
for i=1:num_est
    r1=a(1+i-1:num_sample+i-1);

    spec=garch(1,1)
    fit=estimate(spec,r1)
    h=infer(fit,r1);
    omega=fit.Constant
    alpha1=fit.ARCH{1}
    beta1=fit.GARCH{1}
    h_predict=omega+alpha1*(r1(num_sample)^2)+beta1*h(num_sample);
    r1_std=r1./sqrt(h);
```

```
    r1_sort=sort(r1_std);
    threshold_u(i)=r1_sort(u_index);

    for k=1:num_sample
      if r1_std(k)<threshold_u(i)
         j=j+1
         b(j)=abs(r1_std(k)-threshold_u(i))
      end
    end
    gpfit_par=gpfit(b)
    shape1=gpfit_par(1)
    scale1=gpfit_par(2)
    store_shape1(i)=shape1
    store_scale1(i)=scale1

    var_vz(i)=(abs(threshold_u(i))+(scale1/shape1)*((num_sample*(1-0.95)/j)^(-1*
shape1)-1));
    es_vz(i)=(abs(var_vz(i))/(1-shape1)+(scale1-shape1*abs(threshold_u(i)))/(1-
shape1));
    var_v(i)=-1*sqrt(h_predict)*var_vz(i)
    es_v(i)=-1*sqrt(h_predict)*es_vz(i)
    clear b
    j=0
end

var_v2=var_v'
es_v2=es_v'
save ('evt_garch_sse_var_v1.txt','var_v2','-ascii')
save ('evt_garch_sse_es_v1.txt','es_v2','-ascii')
```

Sample MATLAB code – Semi-parametric dynamic estimation– T/GJR-GARCH-EVT (q=0.95)

```
load c:\Users\USER\Documents\sse_matlab_r.txt

num_data=length(sse_matlab_r)
num_sample=1249
num_est=num_data-num_sample;
a=sse_matlab_r(:,end);
u_index=floor(num_sample*(1-0.90));

r=a(1:num_sample);
j=0;
for i=1:num_est
    r1=a(1+i-1:num_sample+i-1);

    [fit,logl,h]=tarch(r1,1,1,1);

    omega=fit(1);
    alpha1=fit(2);
    beta1=fit(4);
    gam1=fit(3);

h_predict=omega+alpha1*(r1(num_sample).^2)+gam1*(r1(num_sample).^2)*(r1(num_sample)<0)
+beta1*h(num_sample);
    r1_std=r1./sqrt(h);

    r1_sort=sort(r1_std);
    threshold_u(i)=r1_sort(u_index);

    for k=1:num_sample
      if r1_std(k)<threshold_u(i)
         j=j+1
```

```
          b(j)=abs(r1_std(k)-threshold_u(i))
      end
    end
    gpfit_par=gpfit(b)
    shape1=gpfit_par(1)
    scale1=gpfit_par(2)
    store_shape1(i)=shape1
    store_scale1(i)=scale1

    var_vz(i)=(abs(threshold_u(i))+(scale1/shape1)*((num_sample*(1-0.95)/j)^(-1*
shape1)-1))
    es_vz(i)=(abs(var_vz(i))/(1-shape1)+(scale1-shape1*abs(threshold_u(i)))/(1-shape1))
    var_v(i)=-1*sqrt(h_predict)*var_vz(i)
    es_v(i)=-1*sqrt(h_predict)*es_vz(i)
    clear b
    j=0
end

var_v2=var_v'
es_v2=es_v'
save ('tgarch_sse_var_v1.txt','var_v2','-ascii')
save ('tgarch_sse_es_v1.txt','es_v2','-ascii')
```

Summary of VaR and ES estimation method results

Table A1 - VaR and ES estimation method results for STI and SSE indices (*in order left to right*)

VaR at 99% confidence level for STI index	**ES at 99% confidence level for STI index**
Good: GJR-GARCH filtered HS, Extreme Value Theory, t-dist., GED, GJR-GARCH-EVT, GARCH-EVT, GARCH filtered HS, normal dist., Historical Simulation Bad: Exponential Weighted HS, EWMA, GARCH-n	Good: Extreme Value Theory, GJR-GARCH filtered HS, Historical Simulation, GJR-GARCH-EVT, EGARCH-t, t-dist., APARCH-t Bad: normal dist., EWMA, GARCH-n, GED, GJR-GARCH-n, GARCH-GED, APARCH-n
VaR at 95% confidence level for STI index	**ES at 95% confidence level for STI index**
Good: GED, GJR-GARCH-EVT, Historical Simulation, Extreme Value Theory, GARCH filtered HS, GARCH-EVT, GJR-GARCH-GED, t-dist., normal dist. Bad: EWMA, Exponential Weighted HS	Good: GED, t-dist., GJR-GARCH-EVT, Extreme Value Theory, Historical Simulation, GJR-GARCH filtered HS, APARCH-t, EGARCH-t, APARCH-GED, GJR-GARCH-t, GARCH-EVT, Exponential Weighted HS Bad: EWMA, GARCH-n
VaR at 99% confidence level for SSE index	**ES at 99% confidence level for SSE index**
Good: GARCH filtered HS, GJR-GARCH filtered HS, GJR-GARCH-EVT, GJR-GARCH-GED, GJR-GARCH-t Bad: Normal dist., GED, t-dist., Extreme Value Theory	Good: GJR-GARCH-t, APARCH-t, GJR-GARCH filtered HS, GARCH-EVT, GARCH filtered HS, GJR-GARCH-EVT Bad: Normal dist., GED, (EGARCH/GARCH/GJR-GARCH/APARCH)-n, Historical Simulation, Extreme Value Theory, t-dist., EWMA
VaR at 95% confidence level for SSE index	**ES at 95% confidence level for SSE index**
Good: GARCH-EVT, EGARCH-GED, EGARCH-n, GARCH-GED, GARCH-n, GJR-GARCH-GED, GJR-GARCH-n, GJR-GARCH-EVT, APARCH-GED, APARCH-n, GJR-GARCH filtered HS Bad: Historical Simulation, Extreme Value Theory, t-dist., GED, normal dist., Exponential Weighted HS, EWMA	Good: Exponential Weighted HS, GARCH filtered HS, GARCH-EVT, GARCH-t, GARCH-GED, GJR-GARCH filtered HS Bad: GED, t-dist., Extreme Value Theory, Historical Simulation, normal dist.