25.

(a) **Solution:**

Let D_{right} denote distance travelled by the car as estimated by the right sum. Then the right sum, in terms of t and $v(t)$, can be calculated using

$$\begin{aligned} D_{\text{right}} &= RIGHT(5) \\ &= \Delta t \sum_{i=1}^{5} v(t_i) \end{aligned} \tag{1}$$

From the table, it can be observed that the width of each subinterval is 0.2 h. Otherwise, here's the calculation for completeness:

$$\begin{aligned} \Delta t &= \frac{b-a}{n} \\ &= \frac{1-0}{5} \\ &= 0.2\,\text{h} \end{aligned}$$

Then let's use (1) to calculate the estimated distance:

$$\begin{aligned} D_{\text{right}} &= \Delta t \sum_{i=1}^{5} v(t_i) \\ &= 0.2(30+30+70+90+90) \\ &\boxed{= 62\,\text{km}} \end{aligned}$$

(b) **Solution:**

Let G_{right} denote the amount of gas consumed by the car as estimated by and let $g(v(t))$ be the fuel gas efficiency of the car given a speed $v(t)$.

Unit analysis: We want L from the distances (speed × time) in part (a) and the efficiency from the table.

$$\underbrace{\cancel{\text{h}}}_{t} \times \underbrace{\frac{\cancel{\text{km}}}{\cancel{\text{h}}}}_{v(t)} \times \underbrace{\frac{\text{L}}{100\,\cancel{\text{km}}}}_{g(v(t))} = \frac{1}{100}\text{L}$$

So then the terms we're summing over are $f(t_i) = v(t_i)g(v(t_i))$, which implies that the quantity that we're looking for is

$$\begin{aligned} G_{\text{right}} &= RIGHT(5) \\ &= \Delta t \sum_{i=1}^{5} f(t_i) \\ &= \Delta t \sum_{i=1}^{5} v(t_i) g(v(t_i)). \end{aligned} \tag{2}$$

With Δt as before, using (2) to calculate the estimated gas consumed yields

$$\begin{aligned} G_{\text{right}} &= \Delta t \sum_{i=1}^{5} v(t_i) g(v(t_i)) \\ &= 0.2\left(30 \cdot \frac{15}{100} + 30 \cdot \frac{15}{100} + 70 \cdot \frac{7}{100} + 90 \cdot \frac{8}{100} + 90 \cdot \frac{8}{100}\right) \\ &\boxed{= 5.66\,\mathrm{L}} \end{aligned}$$

26.

(a) **Solution:**

The length of each of the four subintervals is

$$\begin{aligned} \Delta x &= \frac{b-a}{n} \\ &= \frac{\frac{\pi}{2} - 0}{4} \\ &= \frac{\pi}{8} \end{aligned}$$

With $f(x) = \cos(x)$ and $n = 4$, the trapezoidal rule gives us

$$\begin{aligned} \int_0^{\frac{\pi}{2}} f(x)\,\mathrm{d}x &\approx TRAP(4) \\ &= \Delta x \left(\frac{f(x_0) + f(x_4)}{2} + \sum_{i=1}^{3} f(x_i) \right) \\ &= \frac{\pi}{8} \left(\frac{\cos(0) + \cos(\frac{\pi}{2})}{2} + \cos\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{3\pi}{8}\right) \right) \\ &\boxed{\approx 0.987} \end{aligned}$$

(b) **Solution:**

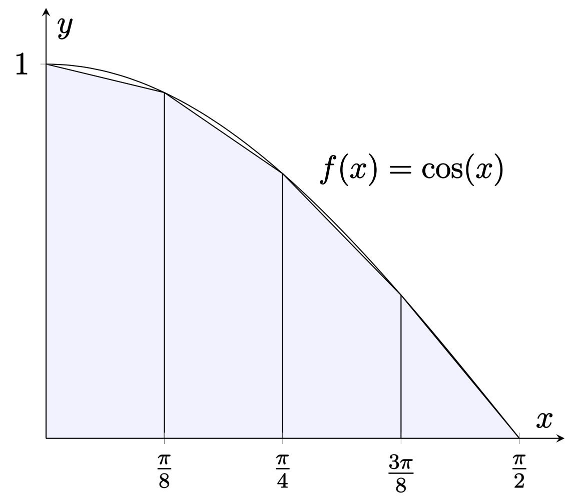

Figure 1: A ~~sketch~~ graph of the trapezoidal rule for approximating the integral of $\cos(x)$ from 0 to $\frac{\pi}{2}$.

From Figure 1, we can deduce that the estimate from part (a) is an **underestimate** of the exact integral value.