

25.

(a) **Solution:**

Let D_{right} denote distance travelled by the car as estimated by the right sum. Then the right sum, in terms of t and $v(t)$, can be calculated using

$$\begin{aligned} D_{\text{right}} &= RIGHT(5) \\ &= \Delta t \sum_{i=1}^5 v(t_i) \end{aligned} \quad (1)$$

From the table, it can be observed that the width of each subinterval is 0.2 h. Otherwise, here's the calculation for completeness:

$$\begin{aligned} \Delta t &= \frac{b-a}{n} \\ &= \frac{1-0}{5} \\ &= 0.2 \text{ h} \end{aligned}$$

Then let's use (1) to calculate the estimated distance:

$$\begin{aligned} D_{\text{right}} &= \Delta t \sum_{i=1}^5 v(t_i) \\ &= 0.2(30 + 30 + 70 + 90 + 90) \\ &= 62 \text{ km} \end{aligned}$$

(b) **Solution:**

Let G_{right} denote the amount of gas consumed by the car as estimated by the right sum and let $f(v)$ be the efficiency of the car given a speed v .

Note that the right sum isn't as straightforward now in terms of units. Multiplying Δv by any given $f(v)$ results in a quantity with the units L/100h, which will be the average gas consumption with respect to time. Since we're looking for amount of gas consumed over an hour, we should factor this into the calculation.

$$\begin{aligned} G_{\text{right}} &= (1 \text{ h}) \cdot RIGHT(5) \\ &= (1 \text{ h}) \Delta v \sum_{i=1}^5 f(v_i) \end{aligned} \quad (2)$$

The width of each subinterval is 20 km/h:

$$\begin{aligned}\Delta v &= \frac{b-a}{n} \\ &= \frac{110-10}{5} \\ &= 20 \text{ km/h}\end{aligned}$$

Using (2) to calculate the estimated gas consumed:

$$\begin{aligned}G_{\text{right}} &= (1 \text{ h})\Delta v \sum_{i=1}^5 f(v_i) \\ &= (1 \text{ h})(20 \text{ km/h})(15 + 10 + 7 + 8 + 9)(\text{L}/100\text{km}) \\ &= (20 \text{ km})(49 \text{ L}/100\text{km}) \\ &= 9.8 \text{ L}\end{aligned}$$

26.

(a) **Solution:**

The length of each of the four subintervals is

$$\begin{aligned}\Delta x &= \frac{b-a}{n} \\ &= \frac{\frac{\pi}{2}-0}{4} \\ &= \frac{\pi}{8}\end{aligned}$$

With $f(x) = \cos(x)$ and $n = 4$, the trapezoidal rule gives us

$$\begin{aligned}\int_0^{\frac{\pi}{2}} f(x) \, dx &\approx TRAP(4) \\ &= \Delta x \left(\frac{f(x_0) + f(x_4)}{2} + \sum_{i=1}^3 f(x_i) \right) \\ &= \frac{\pi}{8} \left(\frac{\cos(0) + \cos(\frac{\pi}{2})}{2} + \cos\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{3\pi}{8}\right) \right) \\ &\approx 0.987\end{aligned}$$

(b) **Solution:**

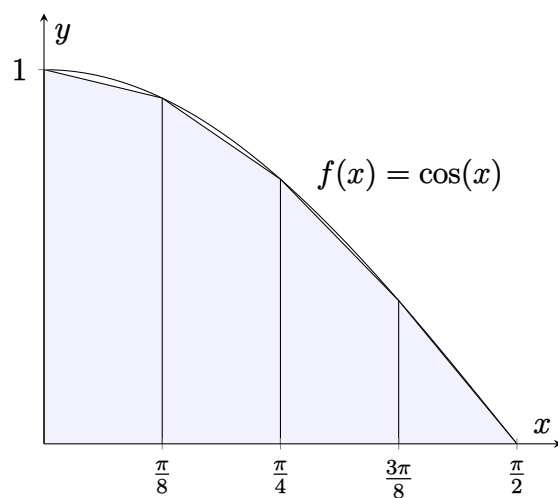


Figure 1: A sketch graph of the trapezoidal rule for approximating the integral of $\cos(x)$ from 0 to $\frac{\pi}{2}$.

From Figure 1, we can deduce that the estimate from part (a) is an underestimate of the exact integral value.