36. **Solution:**

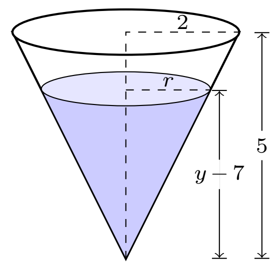

Figure 1: Diagram of the conical portion of the water tank

Let y denote the height of any given "drop" of water in the tank, where $y = 0\,\text{m}$ *is the base of the tank* and $y = 5 + 7 = 12\,\text{m}$ is the top of the tank. Then $y - 7$ for $y \in [7, 12]$ represents the height of water in the conical portion of the tank.

Let $W_S(y)$ denote the total amount of work done by letting a slice water of thickness Δy in the tank at height y drain and let W_T be the integral we want to construct. Then

$$\begin{aligned} W_S(y) &= F \cdot d \\ &= (mg)(y) \\ &= (\rho V)gy \\ &= \rho(\pi r^2 \Delta y)gy \end{aligned}$$

From Figure 1, we can represent a water slice's radius using similar triangles:

$$\begin{aligned} \frac{r}{y-7} &= \frac{2}{5} \\ r &= \frac{2}{5}(y-7) \\ \Rightarrow W_S(y) &= \rho\pi\left(\frac{2}{5}(y-7)\right)^2 gy\Delta y \\ &= \frac{4}{25}\rho g\pi y(y-7)^2\Delta y \\ &= 1568\pi y(y-7)^2\Delta y \end{aligned}$$

$\therefore$ the total amount of work done can be represented as

$$\begin{aligned} W_T &= \sum W_S(y) \\ &\boxed{= 1568\pi \int_9^{12} y(y-7)^2\,\mathrm{d}y} \end{aligned}$$

Note. If we defined y differently, then the integrand (d and r as functions of y) and the bounds of the integral would change. But the overall value of the integral would remain unchanged.

37. **Solution:**

The integrand is a rational function that can undergo partial fraction decomposition to simplify the evaluation of the integral, since $\deg(4x-2) < \deg(x^2-2x+1)$. i.e., the integrand is a proper rational function.

Let I denote the value of the indefinite integral. Then using partial fraction decomposition yields

$$\begin{aligned}
I &= \int \frac{4x-2}{x^2-2x+1}\,\mathrm{d}x \\
&= 2\int \frac{2x-1}{(x-1)^2}\,\mathrm{d}x \\
&= 2\int \frac{c_1}{x-1} + \frac{c_2}{(x-1)^2}\,\mathrm{d}x \\
\Rightarrow \frac{2x-1}{(x-1)^2} &= \frac{c_1}{x-1} + \frac{c_2}{(x-1)^2} \\
2x-1 &= c_1(x-1) + c_2 \qquad (1) \\
2x-1 &= c_1x - c_1 + c_2 \\
&\Rightarrow \begin{cases} 2x = c_1x \\ -1 = -c_1 + c_2 \end{cases} \\
&\Rightarrow \begin{cases} c_1 = 2 \\ c_2 = 1 \end{cases}
\end{aligned}$$

Note. The constants can also be solved for by substituting in certain values of x into (1).

$$\begin{aligned}
\Rightarrow I &= 2\int \frac{2}{x-1} + \frac{1}{(x-1)^2}\,\mathrm{d}x \\
&\boxed{= 2(2\ln|x-1| - (x-1)^{-1}) + C}.
\end{aligned}$$