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Yes or no

Question: Is $\log^k n \in \mathcal{O}(n), \forall k \geq 1$?

Answer: Yes.

Question: Is $\sqrt{n} \in \mathcal{O}(\log n)$?

Answer: No.

Question: Is $n^k \in \mathcal{O}(2^n), \forall k \geq 1$?

Answer: Yes.

Problems and algorithms

Suppose all algorithms that solve a particular problem are in $\Omega(n^3)$.

What is the strongest statement (of those below) that can be made about the fastest algorithm that solves this problem in time $f(n)$?

Question: $f(n) \in \mathcal{O}(n^3)$

Answer: Not necessarily true, since we don't know if the best algorithm will take n^3 time. The upper bound also needs to be proved separately.

Question: $f(n) \in \Theta(n^3)$

Answer: Can't claim this to be true, since we can't claim the previous statement for the reasons stated.

Question: $f(n) \in \Omega(n^3)$

Answer: True, based on the premise of the question (i.e., $f(n)$ is asymptotically lower bounded in n^3 time)

Tail recursion

Question: What is the tightest asymptotic complexity of

$$T(n) = \begin{cases} c & \text{if } n \in \mathbb{Z}_{<1}, \\ 0 & \text{otherwise.} \end{cases}$$

Answer: $T(n) \in \mathcal{O}(nf(n))$. Unless we know what $f(n)$ is, we can't say for sure if $T(n) \in \mathcal{O}(f(n))$. If $f(n)$ is some constant, then we could say that $T(n) \in \mathcal{O}(n)$.

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More general recursion

What is the tightest asymptotic complexity of

$$T(n) = \begin{cases} c & \text{if } n \in \mathbb{Z}_{\leq 1}, \\ n + aT(\frac{n}{b}) & \text{otherwise.} \end{cases}$$

Question: When $(a, b) = (2, 4)$?

Answer: $T(n) \in \mathcal{O}()$.

Question: When $(a, b) = (3, 3)$?

Answer: $T(n) \in \mathcal{O}()$.

Question: When $(a, b) = (4, 2)$?

Answer: $T(n) \in \mathcal{O}()$.

Tightest bound

Question: What is the tightest asymptotic complexity (from the list below) of

$$T(n) = \begin{cases} c & \text{if } n \in \mathbb{Z}_{\leq 2}, \\ T(n-1) + T(n-2) & \text{otherwise.} \end{cases}$$

Answer: $T(n) \in \mathcal{O}()$.