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Student Number

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(a) **Solution:**

Proof. Since each X_i in the sequence has the mean $\mu = \frac{1}{\lambda}$, then by the strong law of large numbers, $\overline{X}_n \xrightarrow{a.s.} \frac{1}{\lambda}$. Let $f(x) = \frac{1}{x}$. Then $f(\cdot)$ is continuous on $(0, \infty)$, the support of each X_i. Then from the lectures, we have that

$$\begin{aligned} &f(\overline{X}_n) \xrightarrow{a.s.} f\left(\frac{1}{\lambda}\right) \\ \implies &Y_n \xrightarrow{a.s.} \lambda \\ \implies &Y_n \xrightarrow{p} \lambda \end{aligned}$$

□

(b) **Solution:**

Proof. We have that

$$\begin{aligned} \mathrm{E}[\overline{X}_n - \mu)^2] &= \mathrm{Var}(\overline{X}_n) \\ &= \mathrm{Var}(\frac{1}{n}\sum_{i=1}^{n} X_i) \\ &= \frac{1}{n^2}\sum_{i=1}^{n}\mathrm{Var}(X_i) \\ &= \frac{1}{n^2} \cdot n\sigma^2 \\ &= \frac{\sigma^2}{n} \end{aligned}$$

which implies that $\lim_{n\to\infty} \mathrm{E}[\overline{X}_n - \mu)^2] = 0$.

Therefore, $\overline{X}_n$ converges to μ in mean square. □