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3. **Solution:**

For the covariance between X_k and X_{k+m}, we have

$$\begin{aligned}\operatorname{Cov}(X_k, X_{k+m}) &= \operatorname{Cov}(\sum_{i=1}^{k} Y_i, \sum_{j=1}^{k+m} Y_j) \\ &= \sum_{i=1}^{k}\sum_{j=1}^{k+m} \operatorname{Cov}(Y_i, Y_j)\end{aligned}$$

Since each Y_l is independent, $\operatorname{Cov}(Y_i, Y_j)$ is only non-zero when $i = j$. Thus,

$$\begin{aligned}\operatorname{Cov}(X_k, X_{k+m}) &= \sum_{i=1}^{k} \operatorname{Cov}(Y_i, Y_i) \\ &\boxed{= k\sigma^2}\end{aligned}$$

The variances of X_k and X_{k+m} are

$$\begin{aligned}\operatorname{Var}(X_k) &= \operatorname{Var}(\sum_{i=1}^{k} Y_i) \\ &= \sum_{i=1}^{k} \operatorname{Var}(Y_i) \\ &= k\sigma^2 \\ \operatorname{Var}(X_{k+m}) &= \operatorname{Var}(\sum_{i=1}^{k+m} Y_i) \\ &= \sum_{i=1}^{k+m} \operatorname{Var}(Y_i) \\ &= (k+m)\sigma^2\end{aligned}$$

Thus, the correlation coefficient between X_k and X_{k+m} is

$$\begin{aligned}\rho(X_k, X_{k+m}) &= \frac{\operatorname{Cov}(X_k, X_{k+m})}{\sqrt{\operatorname{Var}(X_k)\operatorname{Var}(X_{k+m})}} \\ &= \frac{k\sigma^2}{\sqrt{k\sigma^2 \cdot (k+m)\sigma^2}} \\ &\boxed{= \sqrt{\frac{k}{k+m}}}\end{aligned}$$

As $m \to \infty$, we see that

$$\begin{aligned}&\lim_{m\to\infty} \operatorname{Cov}(X_k, X_{k+m}) \\ &= \lim_{m\to\infty} k\sigma^2 \\ &\boxed{= k\sigma^2}\end{aligned} \qquad \begin{aligned}&\lim_{m\to\infty} \rho(X_k, X_{k+m}) \\ &= \lim_{m\to\infty} \sqrt{\frac{k}{k+m}} \\ &\boxed{= 0}\end{aligned}$$