137438691328
Student Number

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Name

2.

$$
\begin{aligned}
f_{X_1,X_2}(x_1,x_2) &= \int_{-\infty}^{\infty} f_X(x_1,x_2,x_3)\,\mathrm{d}x_3 \\
&= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x_1-x_3)^2}{2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x_2-x_3)^2}{2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x_3^2}{2}}\,\mathrm{d}x_3 \\
&= \frac{1}{(2\pi)^{\frac{3}{2}}} \int_{-\infty}^{\infty} e^{\frac{-x_1^2+2x_1x_3-x_3^2}{2}} e^{\frac{-x_2^2+2x_2x_3-x_3^2}{2}} e^{-\frac{x_3^2}{2}}\,\mathrm{d}x_3 \\
&= \frac{1}{(2\pi)^{\frac{3}{2}}} \int_{-\infty}^{\infty} e^{\frac{-3x_3^2+2x_1x_3+2x_2x_3-x_1^2-x_2^2}{2}}\,\mathrm{d}x_3 \qquad (1)
\end{aligned}
$$

Let us try and convert the integrand of the integral in (1) into the form $e^{\frac{-(x_3-\mu)^2}{2\sigma^2}}$ so that the integrand can be treated as a form of a probability distribution function for a ~Normal(μ, σ^2) random variable.