20053722
Student Number

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Name

1. For $i \in \{1, \ldots, 6\}$, let

$$X_i = \# \text{ of times the number i is rolled in ten dice rolls}$$

Then $(X_1, \ldots, X_6)$ have a multinomial distribution with parameters 10 and $(\frac{1}{6}, \ldots, \frac{1}{6})$.

(a) Finding the probability that one 1, two 2's, three 3's, and four 4's are rolled.

**Solution:**

*Note:* $10 - 1 - 2 - 3 - 4 = 0$

Using the joint marginal pmf of $(X_1, X_2, X_3, X_4)$, we can calculate the probability as follows:

$$\begin{aligned}
P(X_1 = 1, X_3 = 3, X_2 = 2, X_4 = 4) &= \frac{10!}{1!2!3!4!(0)!}\left(\frac{1}{6}\right)^{10}\left(1 - 2\left(\frac{1}{6}\right)\right)^0 \\
&= \frac{10!}{1!2!3!4!(6)^{10}} \\
&= \frac{175}{839\,808} \\
&\boxed{\approx 0.00021}
\end{aligned}$$

(b) Finding the probability that the number of 1's plus the number of 2's equals three and the number of 3's equals four.

**Solution:**

Let's calculate the probability of the above event using combinatorics.

$$\begin{aligned}
P(X_1 + X_2 = 3, X_3 = 4) &= \overbrace{\binom{10}{3}\left(\frac{1+1}{6}\right)^3}^{\text{choosing 1's or 2's}} + \overbrace{\binom{7}{4}\left(\frac{1}{6}\right)^4}^{\text{choosing 3's}} + \overbrace{\binom{3}{3}\left(\frac{1}{2}\right)^3}^{\text{choosing other numbers}} \\
&= \frac{175}{11\,664} \\
&\boxed{\approx 0.015}
\end{aligned}$$

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(c) Finding the probability that three 5's were rolled, given that exactly four of the ten rolls resulted in an outcome less than 4.

**Solution:**

Let's calculate the probability of the above event using combinatorics.

$$\begin{aligned}
P\left(X_5 = 3 \mid \text{four dice} < 4\right) &= \overbrace{\binom{10}{4}\left(\frac{1+1+1}{6}\right)^4}^{\text{choosing four dice} < 4} \cdot \overbrace{\binom{6}{3}\left(\frac{1+1}{6}\right)^3}^{\text{choosing three 3's or 4's}} \\
&\cdot \overbrace{\binom{3}{3}\left(\frac{1}{6}\right)^3}^{\text{choosing three 5's}} \\
&= \frac{175}{3\,888} \\
&\boxed{\approx 0.045}
\end{aligned}$$