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2. Finding $E[X_i X_j]$, for $i, j \in \{1, \dots, k\}$.

Solution:

Let $X_i = X_{i,1} + \dots + X_{i,n}$ where

$$X_{i,k} = \begin{cases} 1, & \text{if the } k\text{th multinomial experiment has outcome } i \\ 0, & \text{otherwise} \end{cases}$$

and let $X_j = X_{j,1} + \dots + X_{j,n}$ where

$$X_{j,k} = \begin{cases} 1, & \text{if the } k\text{th multinomial experiment has outcome } j \\ 0, & \text{otherwise} \end{cases}$$

We then have

$$\begin{aligned} E[X_i X_j] &= E \left[\left(\sum_{l=1}^n X_{i,l} \right) \left(\sum_{m=1}^n X_{j,m} \right) \right] \\ &= E \left[\sum_{l=1}^n \sum_{m=1}^n X_{i,l} X_{j,m} \right] && \text{by the distributivity of summation} \\ &= \sum_{l=1}^n \sum_{m=1}^n E[X_{i,l} X_{j,m}] && \text{by the linearity of expectation} \\ &= \end{aligned}$$