20053722
Student Number

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2.

(a) Two dice are rolled, one of which is fair and the other is biased. What is the probability that the sum is 7?

Solution:

Let

$$\begin{aligned} A &= \text{the sum is 7} \\ I_A &= \begin{cases} 1, & \text{if A occurs} \\ 0, & \text{if A does not occur} \end{cases} \\ Y &= \text{the \# the fair die rolls} \\ Z &= \text{the \# the biased die rolls} \end{aligned}$$

It follows that $\mathrm{P}(A) = \mathrm{E}[I_A]$.

Then by the law of total probability,

$$\begin{aligned} \mathrm{E}[I_A] &= \mathrm{E}[\mathrm{E}[I_A|Y]] \\ &= \sum_{i=1}^{6} \mathrm{E}[I_A|Y = i]\,\mathrm{P}(Y = i) \\ &= \frac{1}{6}\sum_{i=1}^{6} \mathrm{P}(Z = i) \end{aligned}$$

where $\{Z = i\}$, for $i \in \{1, \dots, 6\}$, forms a partition of the sample space. Hence, we have that $\sum_{i=1}^{6} \mathrm{P}(Z = i) = 1$. Therefore,

$$\boxed{\mathrm{P}(A) = \frac{1}{6}}$$

(b) n dice are rolled, one of which is fair and the rest are biased. What is the probability that the sum of the rolls is even?

Solution:

Let

$$\begin{aligned} A &= \text{`the sum of the rolls is even'} \\ I_A &= \begin{cases} 1, & \text{if } A \text{ occurs} \\ 0, & \text{if } A \text{ does not occur} \end{cases} \\ Y &= \begin{cases} 1, & \text{if the fair die is even} \\ 0, & \text{if the fair die is odd} \end{cases} \\ Z &= \text{`the sum among the biased dice is even'} \end{aligned}$$

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It follows that $\mathrm{P}(A) = \mathrm{E}[I_A]$.

Then by the law of total probability,

$$\begin{aligned}
\mathrm{E}[I_A] &= \mathrm{E}[\mathrm{E}[I_A|Y]] \\
&= \mathrm{E}[I_A|Y=0]\,\mathrm{P}(Y=0) + \mathrm{E}[I_A|Y=1]\,\mathrm{P}(Y=1) \\
&= \mathrm{P}\left(Z^{\complement}\right)\left(\frac{1}{2}\right) + \mathrm{P}(Z)\left(\frac{1}{2}\right) \\
&= \frac{1}{2}(\mathrm{P}\left(Z^{\complement}\right) + 1 - \mathrm{P}\left(Z^{\complement}\right)) \\
\end{aligned}$$

$$\boxed{\implies \mathrm{P}(A) = \frac{1}{2}}$$