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Student Number

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Name

2.

$$\begin{aligned}
 f_{X_1, X_2}(x_1, x_2) &= \int_{-\infty}^{\infty} f_X(x_1, x_2, x_3) dx_3 \\
 &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x_1-x_3)^2}{2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x_2-x_3)^2}{2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x_3^2}{2}} dx_3 \\
 &= \frac{1}{(2\pi)^{\frac{3}{2}}} \int_{-\infty}^{\infty} e^{\frac{-x_1^2+2x_1x_3-x_3^2}{2}} e^{\frac{-x_2^2+2x_2x_3-x_3^2}{2}} e^{-\frac{x_3^2}{2}} dx_3 \\
 &= \frac{1}{(2\pi)^{\frac{3}{2}}} \int_{-\infty}^{\infty} e^{\frac{-3x_3^2+2x_1x_3+2x_2x_3-x_1^2-x_2^2}{2}} dx_3
 \end{aligned} \tag{1}$$

Let's try and convert the integrand of the integral in (1) into the form $e^{\frac{-(x_3-\mu)^2}{2\sigma^2}}$ so that the integrand can be treated as a form of a p.d.f. for a Normal(μ, σ^2) random variable. Mainly, let's rearrange the integrand's exponent, **by adding 0**:

$$\begin{aligned}
 &\frac{(-3x_3^2 + 2x_1x_3 + 2x_2x_3 - x_1^2 - x_2^2 + \frac{2}{3}x_1^2 + \frac{2}{3}x_2^2 - \frac{2}{3}x_1x_2) - \frac{2}{3}x_1^2 - \frac{2}{3}x_2^2 + \frac{2}{3}x_1x_2}{2} \\
 &= \frac{-3(x_3^2 - 2x_3(\frac{1}{3}x_1 + \frac{1}{3}x_2) + (\frac{1}{9}x_1^2 + 2(\frac{1}{9}x_1x_2) + \frac{1}{9}x_2^2))}{2} - \frac{\frac{2}{3}x_1^2 + \frac{2}{3}x_2^2 - \frac{2}{3}x_1x_2}{2} \\
 &= \frac{-(x_3 - (\frac{1}{3}x_1 + \frac{1}{3}x_2))^2}{2(\frac{1}{\sqrt{3}})^2} - \frac{2x_2^2 - 2x_1x_2 + 2x_1^2}{2(3)}
 \end{aligned} \tag{2}$$

$$(2) \Rightarrow f_{X_1, X_2}(x_1, x_2) = \frac{1}{(2\pi)^{\frac{3}{2}}} e^{-\frac{2x_2^2-2x_1x_2+2x_1^2}{2(3)}} \underbrace{\int_{-\infty}^{\infty} e^{\frac{-(x_3 - (\frac{1}{3}x_1 + \frac{1}{3}x_2))^2}{2(\frac{1}{\sqrt{3}})^2}} dx_3}_I$$

\Rightarrow The integrand of I is related to the p.d.f. of a Normal($\frac{1}{3}x_1 + \frac{1}{3}x_2, \frac{1}{3}$) r.v.

$$\Rightarrow I = \sqrt{2\pi} \frac{1}{\sqrt{3}}$$

$$\Rightarrow f_{X_1, X_2}(x_1, x_2) = \frac{1}{\sqrt{2\pi}^2 \sqrt{3}} e^{-\frac{2x_2^2-2x_1x_2+2x_1^2}{2(3)}}, \quad \forall x_1, x_2 \in (-\infty, \infty)$$

Then for the marginal pdf of X_1 ,

$$\begin{aligned}
 f_{X_1}(x_1) &= \int_{-\infty}^{\infty} f_{X_1, X_2}(x_1, x_2) dx_2 \\
 &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}^2 \sqrt{3}} e^{-\frac{2x_2^2-2x_1x_2+2x_1^2}{2(3)}} dx_2 \\
 &= \frac{1}{\sqrt{2\pi}^2 \sqrt{3}} \int_{-\infty}^{\infty} e^{-\frac{2x_2^2-2x_1x_2+2x_1^2}{2(3)}} dx_2
 \end{aligned} \tag{3}$$

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Let's do the same thing we did for (1) to (3). i.e. **add 0**.

$$\begin{aligned}
 & \frac{(-2x_2^2 + 2x_1x_2 - 2x_1^2 + \frac{3}{2}x_1^2) - \frac{3}{2}x_1^2}{2(3)} \\
 &= \frac{-2(x_2^2 + 2x_2(\frac{1}{2}x_1) + \frac{1}{4}x_1^2) - \frac{3}{2}x_1^2}{2(3)} \\
 &= \frac{-(x_2 - \frac{1}{2}x_1)^2}{2\sqrt{\frac{3}{2}}} - \frac{1}{4}x_1^2
 \end{aligned} \tag{4}$$

$$(4) \Rightarrow f_{X_1}(x_1) = \frac{1}{\sqrt{2\pi}^2\sqrt{3}} e^{-\frac{1}{4}x_1^2} \underbrace{\int_{-\infty}^{\infty} e^{\frac{-(x_2 - \frac{1}{2}x_1)^2}{2\sqrt{\frac{3}{2}}}} dx_2}_I$$

\Rightarrow The integrand of I is related to the p.d.f. of a Normal($\frac{1}{2}x_1, \frac{3}{2}$) r.v.

$$\Rightarrow I = \sqrt{2\pi} \sqrt{\frac{3}{2}}$$

$$\Rightarrow \boxed{f_{X_1}(x_1) = \frac{1}{\sqrt{2\pi}\sqrt{2}} e^{-\frac{1}{4}x_1^2}, \quad \forall x_1 \in (-\infty, \infty)}$$

□