20053722
Student Number

Bryan Hoang
Name

1. **Solution:**

To find the distribution of X, let's find the mgf of X.

$$\begin{aligned} M_X(t) &= \mathrm{E}\left[e^{tX}\right] \\ &= \mathrm{E}\left[\sum_{n=0}^{\infty} \frac{(tX)^n}{n!}\right] \\ &= \sum_{n=0}^{\infty} \frac{\mathrm{E}[X^n]}{n!} t^n \end{aligned}$$

Assuming we can take expectation inside the infinite sum. Then

$$\begin{aligned} M_X(t) &= 1 + \sum_{n=1}^{\infty} \frac{\mathrm{E}[X^n]}{n!} t^n \\ &= 1 + \sum_{n=1}^{\infty} \frac{\left(\frac{n!}{\lambda^n}\right)}{n!} t^n \\ &= 1 + \sum_{n=1}^{\infty} \left(\frac{t}{\lambda}\right)^n \\ &= \sum_{n=0}^{\infty} \left(\frac{t}{\lambda}\right)^n \\ &= \frac{1}{1 - \frac{t}{\lambda}} && \text{if } |t| < \lambda \\ &= \left(\frac{\lambda}{\lambda - t}\right)^1 && \text{if } |t| < \lambda \end{aligned}$$

which is the mgf of a random variable with a Gamma$(1, \lambda) \sim$ Exponential(λ) distribution as seen in lecture. Thus, $\boxed{X \sim \text{Exponential}(\lambda)}$.