

20053722  
Student Number

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5.

i. Finding the distribution function of  $Y$ .

$$\begin{aligned} F_Y(y) &= P(Y \leq y) \\ &= P(X_1^2 + X_2^2 + X_3^2 \leq y) \end{aligned}$$

Let  $S_y = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid : x_1^2 + x_2^2 + x_3^2 \leq y\}$ . Then

$$\begin{aligned} &= \iiint_{S_y} f_X(x_1, x_2, x_3) \, dx_1 \, dx_2 \, dx_3 \\ &= \frac{3}{4\pi r^3} \iiint_{S_y} dx_1 \, dx_2 \, dx_3 \\ &= \frac{3}{4\pi r^3} \frac{4\pi \sqrt{y}^3}{3} \end{aligned}$$

$$= \begin{cases} 0, & \text{if } y < 0 \\ \frac{y^{\frac{3}{2}}}{r^3}, & \text{if } y \in [0, r^2] \\ 1, & \text{if } y > r^2 \end{cases}$$

ii. Finding the pdf of  $Y$ ,  $f_Y(y)$ .

$$\begin{aligned} f_Y(y) &= \frac{d}{dy} F_Y(y) \\ &= \frac{d}{dy} \frac{y^{\frac{3}{2}}}{r^3} \\ &= \begin{cases} \frac{3\sqrt{y}}{2r^3}, & \text{if } y \in [0, r^2] \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

iii. Finding  $E[Y]$ .

$$\begin{aligned} E[Y] &= \int_0^{r^2} y f_Y(y) \, dy \\ &= \int_0^{r^2} y \frac{3\sqrt{y}}{2r^3} \, dy \\ &= \frac{3}{2r^3} \left[ \frac{2}{5} y^{\frac{5}{2}} \right]_0^{r^2} \\ &= \frac{3}{5} r^2 \end{aligned}$$