20053722
Student Number

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3. We can model the problem as a Multivariate Hypergeometric distribution. To use the second version of the distribution, let X_1 and X_2 be of type "other". Then the probability density function of X_3 is

$$\begin{aligned} p_{X_3}(x_3) &= \begin{cases} \frac{\binom{10}{x_3}\binom{14-10}{7-x_3}}{\binom{14}{7}}, & \text{if } x_1 \in \{x \in \mathbb{Z}_{\geq 0} | \max(0, 7-(14-10)) \leq x \leq 7\} \\ 0, & \text{otherwise} \end{cases} \\ &= \begin{cases} \frac{\binom{10}{x_3}\binom{4}{7-x_3}}{\binom{14}{7}}, & \text{if } x_1 \in \{3,4,5,6,7\} \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

Then

$$\begin{aligned} E[X_3] &= \sum_{x_3=3}^{7} x_3 \, p_{X_3}(x_3) \\ &= \sum_{x_3=3}^{7} x_3 \frac{\binom{10}{x_3}\binom{4}{7-x_3}}{\binom{14}{7}} \\ &\boxed{= 5} \quad \text{(using a calculator)} \end{aligned}$$

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