20053722
Student Number

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Name

1. For $i \in \{1, \ldots, 6\}$, let

$$X_i = \#\text{ of times the number i is rolled in ten dice rolls}$$

Then $(X_1, \ldots, X_6)$ have a multinomial distribution with parameters 10 and $(\frac{1}{6}, \ldots, \frac{1}{6})$.

(a) Finding the probability that one 1, two 2's, three 3's, and four 4's are rolled.

Solution:

Note: $10 - 1 - 2 - 3 - 4 = 0$

Using the joint marginal pmf of (X_1, X_2, X_3, X_4), we can calculate the probability as follows:

$$\begin{aligned} P(X_1 = 1, X_3 = 3, X_2 = 2, X_4 = 4) &= \frac{10!}{1!2!3!4!(0)!} \left(\frac{1}{6}\right)^{10} \left(1 - 2\left(\frac{1}{6}\right)\right)^{0} \\ &= \frac{10!}{1!2!3!4!(6)^{10}} \\ &= \frac{175}{839\,808} \\ &\boxed{\approx 0.00021} \end{aligned}$$

(b) Finding the probability that the number of 1's plus the number of 2's equals three and the number of 3's equals four.

Solution:

Let's calculate the probability of the above event using combinatorics.

$$\begin{aligned} P(X_1 + X_2 = 3, X_3 = 4) &= \overbrace{\binom{10}{3}\left(\frac{1+1}{6}\right)^3}^{\text{choosing 1's or 2's}} + \overbrace{\binom{7}{4}\left(\frac{1}{6}\right)^4}^{\text{choosing 3's}} + \overbrace{\binom{3}{3}\left(\frac{1}{2}\right)^3}^{\text{choosing other numbers}} \\ &= \frac{175}{11\,664} \\ &\boxed{\approx 0.015} \end{aligned}$$

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(c) Finding the probability that three 5's were rolled, given that exactly four of the ten rolls resulted in an outcome less than 4.

Solution:

Let's calculate the probability of the above event using combinatorics.

$$P\left(X_5 = 3 \mid \text{four dice} < 4\right) = \frac{P(X_5 = 3, \text{four dice} < 4)}{P(\text{four dice} < 4)}$$

$$= \frac{\overbrace{\binom{10}{4}\left(\frac{1+1+1}{6}\right)^4}^{\text{choosing four dice} < 4} \cdot \overbrace{\binom{6}{3}\left(\frac{1+1}{6}\right)^3}^{\text{choosing three 3's or 4's}} \cdot \overbrace{\binom{3}{3}\left(\frac{1}{6}\right)^3}^{\text{choosing three 5's}}}{\binom{10}{4}\left(\frac{1+1+1}{6}\right)^4 \underbrace{\binom{6}{6}\left(\frac{1}{2}\right)^6}_{\text{choosing six dice} \geq 4}}$$

$$= \frac{160}{729}$$

$$\boxed{\approx 0.219}$$