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5. Showing that (1) is true.

$$\lim_{n\to\infty} P(X_{(n)} - \ln n \le x) = \exp\left(-e^{-x}\right) \tag{1}$$

Solution:

Proof. Since the **mean** of each X_i, $i \in \{1, \ldots, n\}$, is 1, that **means** $\lambda = 1$. Given the premise, the pdf of $X_{(n)}$ is

$$f_n(x) = nf(x)F(x)^{n-1}$$

Then the cdf of $X_{(n)}$ is

$$F_n(x) = \int_{\infty}^{x} nf(x)F(x)^{n-1} \, dx_n$$
$$= F(x)^n$$

where $F(x)$ is the common cdf of the X_i

Hence, the LHS of (1) is

$$\lim_{n\to\infty} P(X_{(n)} - \ln n \le x) = \lim_{n\to\infty} P(X_{(n)} \le x + \ln n)$$
$$= \lim_{n\to\infty} F_n(x + \ln n)$$
$$= \lim_{n\to\infty} F(x + \ln n)^n$$
$$= \lim_{n\to\infty} (1 - e^{-(x+\ln n)})^n$$
$$= \lim_{n\to\infty} \left(1 + \frac{(-e^{-x})}{n}\right)^n$$
$$= \exp\left(-e^{-x}\right) \qquad \square$$