Student Number: [redacted] Name: Bryan Hoang

4. (10 points)

(a) **Answer:**

$$\begin{aligned}\det(A_1) &= \det\begin{pmatrix}3 & 1\\ 2 & 2\end{pmatrix}\\ &= 4\\ &\neq \pm 1.\end{aligned}$$

Therefore, $\boxed{A_1 \notin \mathrm{GL}_2(\mathbb{Z})}$.

(b) **Answer:**

$$\begin{aligned}\det(A_2) &= \det\begin{pmatrix}3 & -2\\ 2 & -1\end{pmatrix}\\ &= 1.\end{aligned}$$

Therefore, $\boxed{A_2 \in \mathrm{GL}_2(\mathbb{Z})}$.

$$\boxed{A_2^{-1} = \begin{pmatrix}-1 & 2\\ -2 & 3\end{pmatrix}}$$

(c) **Answer:**

$$\begin{aligned}\det(A_3) &= \det\begin{pmatrix}3 & 2 & 2\\ 2 & 1 & 2\\ -1 & 3 & 1\end{pmatrix}\\ &= -9\\ &\neq \pm 1.\end{aligned}$$

Therefore, $\boxed{A_3 \notin \mathrm{GL}_3(\mathbb{Z})}$.

(d) **Answer:**

$$\begin{aligned}\det(A_4) &= \det\begin{pmatrix}-3 & -1 & 2\\ 1 & -3 & -1\\ 3 & 0 & -2\end{pmatrix}\\ &= 1.\end{aligned}$$

Therefore, $\boxed{A_4 \in \mathrm{GL}_3(\mathbb{Z})}$.

$$\boxed{A_4^{-1} = \begin{pmatrix}6 & -2 & 7\\ -1 & 0 & -1\\ 9 & -3 & 10\end{pmatrix}}$$