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8. (10 points)

(a) **Answer:**

$$\tau\sigma^2 = (\tau\sigma)\sigma = (\sigma^2\tau)\sigma = \sigma^2(\tau\sigma) = \sigma^2(\sigma^2\tau) = \sigma^4\tau = (\sigma^3)\sigma\tau = \boxed{\sigma\tau}.$$

(b) **Answer:**

$$\tau(\sigma\tau) = (\tau\sigma)\tau = (\sigma^2\tau)\tau = \sigma^2\tau^2 = \boxed{\sigma^2}.$$

(c) **Answer:**

$$(\sigma\tau)(\sigma\tau) = \sigma(\tau\sigma)\tau = \sigma(\sigma^2\tau)\tau = \sigma^3\tau^2 = \boxed{e}.$$

(d) **Answer:**

$$(\sigma\tau)(\sigma^2\tau) = \sigma(\tau\sigma)\sigma\tau = \sigma(\sigma^2\tau)\sigma\tau = \sigma^3(\tau\sigma)\tau = e(\sigma^2\tau)\tau = \sigma^2\tau^2 = \boxed{\sigma^2}.$$

Answer:

$\mathcal{S}_3$ is not a commutative group. For instance, the third rule of multiplication says that $\tau\sigma = \sigma^2\tau \neq \sigma\tau$.