Student Number: [redacted] Name: Bryan Hoang

5. (10 points)

(a) **Answer:**

$$\begin{aligned} \boldsymbol{w} &= \underbrace{6.22}_{t_1}\boldsymbol{v}_1 + \underbrace{133.98}_{t_2}\boldsymbol{v}_2 \\ &\Rightarrow \begin{cases} a_1 = \lfloor t_1 \rceil = 6 \\ a_2 = \lfloor t_2 \rceil = 134 \end{cases} \\ &\Rightarrow \boldsymbol{v} = 6\boldsymbol{v}_1 + 134\boldsymbol{v}_2 \\ &= \boxed{(43086, 11448)} \\ &\Rightarrow \boxed{\|\boldsymbol{v} - \boldsymbol{w}\| = 107.15}. \end{aligned}$$

(b) **Answer:**

$$\begin{aligned} \sqrt{\frac{\det(\boldsymbol{L})}{\|\boldsymbol{v}_1\|\|\boldsymbol{v}_2\|}} &\approx \sqrt{\frac{158709}{486.2 \cdot 329.2}} \\ &\boxed{\approx 0.9958} \\ &\approx 1. \end{aligned}$$

Since the ratio is close to 1, we can conclude that the basis $\{\boldsymbol{v}_1, \boldsymbol{v}_2\}$ is a "good" basis.

(c) **Answer:**

Proof.

$$\begin{aligned} &\boxed{\begin{cases} \boldsymbol{v}_1' = 5\boldsymbol{v}_1 + 6\boldsymbol{v}_2 \\ \boldsymbol{v}_2' = 19\boldsymbol{v}_1 + 23\boldsymbol{v}_2 \end{cases}} \\ \Rightarrow \det &\begin{pmatrix} 5 & 6 \\ 19 & 23 \end{pmatrix} = \boxed{1}. \end{aligned}$$

□

(d) **Answer:**

$$\begin{aligned} \boldsymbol{w} &= \underbrace{-2402.52}_{t_1}\boldsymbol{v}_1' + \underbrace{632.57}_{t_2}\boldsymbol{v}_2' \\ &\Rightarrow \begin{cases} a_1 = \lfloor t_1 \rceil = -2403 \\ a_2 = \lfloor t_2 \rceil = 633 \end{cases} \\ &\Rightarrow \boldsymbol{v}' = -2403\boldsymbol{v}_1' + 633\boldsymbol{v}_2' \\ &= \boxed{(46548, 9561)} \\ &\Rightarrow \boxed{\|\boldsymbol{v}' - \boldsymbol{w}\| = 3860.08}, \\ &\gg 107.15 \\ &= \|\boldsymbol{v} - \boldsymbol{w}\| \end{aligned}$$

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(e) **Answer:**

$$\sqrt{\frac{\det(\boldsymbol{L})}{\|\boldsymbol{v}_1\|\|\boldsymbol{v}_2\|}} \approx \sqrt{\frac{158709}{3323.2 \cdot 12673.8}}$$
$$\boxed{\approx 0.061}$$
$$\ll 1.$$

Since the ratio much smaller than 1, we can conclude that the basis $\{\boldsymbol{v}_1, \boldsymbol{v}_2\}$ is a "bad" basis.