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2. (10 points)

(d) **Answer:**

$$
\begin{aligned}
\lim_{k\to\infty} \frac{(\ln k)^{375}}{k^{0.001}} &= \lim_{k\to\infty} \frac{375(\ln k)^{374}k^{-1}}{0.001k^{-0.999}} && \text{by L'Hopital's rule} \\
&= \lim_{k\to\infty} \frac{375(\ln k)^{374}}{0.001k^{0.001}}.
\end{aligned}
$$

Applying L'Hopital's rule over and over again yields

$$
\begin{aligned}
\lim_{k\to\infty} \frac{(\ln k)^{375}}{k^{0.001}} &= \lim_{k\to\infty} \frac{375!}{0.001^{375}k^{0.001}} \\
&= 0 \\
&< \infty
\end{aligned}
$$

$\therefore (\ln k)^{375} = \mathcal{O}(k^{0.001})$.

(e) **Answer:**

$$
\begin{aligned}
\lim_{k\to\infty} \frac{k^2 2^k}{e^{2k}} &< \lim_{k\to\infty} \frac{k^2 e^k}{e^{2k}} \\
&= \lim_{k\to\infty} \frac{k^2}{e^k} \\
&= 0 \\
&< \infty.
\end{aligned}
$$

$\therefore k^2 2^k = \mathcal{O}(e^{2k})$.

(f) **Answer:**

$$
\begin{aligned}
\lim_{N\to\infty} \frac{N^{10}2^N}{e^N} &= \lim_{N\to\infty} \frac{N^{10}}{\left(\frac{1}{2}\right)^N e^N} \\
&= \lim_{N\to\infty} \frac{N^{10}}{\left(\frac{e}{2}\right)^N} \\
&= 0 \\
&< \infty.
\end{aligned}
$$

$\therefore N^{10}2^N = \mathcal{O}(e^N)$.