Student Number: Name: Bryan Hoang

- 8. (10 points)
- (a) Answer:

Computing  $P \oplus Q$  yields

$$\lambda = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-5 - 2}{3 - 0}$$

$$= -\frac{7}{3},$$

$$x_3 = \lambda^2 - x_1 - x_2$$

$$= \left(-\frac{7}{3}\right)^2 - 0 - 3$$

$$= \frac{22}{9},$$

$$y_3 = \lambda(x_1 - x_3) - y_1$$

$$= -\frac{7}{3}\left(0 - \frac{22}{9}\right) - 2$$

$$= \frac{100}{27},$$

$$\Rightarrow P \oplus Q = \left(\frac{22}{9}, \frac{100}{27}\right).$$

(b) **Answer:** 

Computing  $P \oplus P$  yields

$$\lambda = \frac{3x_1^2 + A}{2y_1}$$

$$= \frac{3 \cdot 0^2 - 2}{2 \cdot 2}$$

$$= -\frac{1}{2},$$

$$x_3 = \lambda^2 - x_1 - x_2$$

$$= \left(-\frac{1}{2}\right)^2 - 0 - 0$$

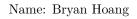
$$= \frac{1}{4},$$

$$y_3 = \lambda(x_1 - x_3) - y_1$$

$$= -\frac{1}{2}\left(0 - \frac{1}{4}\right) - 2$$

$$= -\frac{15}{8},$$

$$\Rightarrow P \oplus P = \left(\frac{1}{4}, -\frac{15}{8}\right).$$



Computing  $Q \oplus Q$  yields

$$\lambda = \frac{3x_1^2 + A}{2y_1}$$

$$= \frac{3 \cdot 3^2 - 2}{2 \cdot (-5)}$$

$$= -\frac{5}{2},$$

$$x_3 = \lambda^2 - x_1 - x_2$$

$$= \left(-\frac{5}{2}\right)^2 - 3 - 3$$

$$= \frac{1}{4},$$

$$y_3 = \lambda(x_1 - x_3) - y_1$$

$$= -\frac{5}{2}\left(3 - \frac{1}{4}\right) - (-5)$$

$$= -\frac{15}{8},$$

$$\Rightarrow Q \oplus Q = \left(\frac{1}{4}, -\frac{15}{8}\right).$$

## (c) **Answer:**

Computing  $P \oplus P \oplus P$  yields

$$\lambda = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2 - (-\frac{15}{8})}{0 - \frac{1}{4}}$$

$$= -\frac{31}{2},$$

$$x_3 = \lambda^2 - x_1 - x_2$$

$$= \left(-\frac{31}{2}\right)^2 - \frac{1}{4} - 0$$

$$= 240,$$

$$y_3 = \lambda(x_1 - x_3) - y_1$$

$$= -\frac{31}{2}\left(\frac{1}{4} - 240\right) + \frac{15}{8}$$

$$= 3718,$$

$$\Rightarrow P \oplus P \oplus P = \left(240, 3718\right).$$

Name: Bryan Hoang

Student Number:



Computing  $Q \oplus Q \oplus Q$  yields

$$\lambda = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-5 - (-\frac{15}{8})}{3 - \frac{1}{4}}$$

$$= -\frac{25}{22},$$

$$x_3 = \lambda^2 - x_1 - x_2$$

$$= \left(-\frac{25}{22}\right)^2 - \frac{1}{4} - 3$$

$$= -\frac{237}{121},$$

$$y_3 = \lambda(x_1 - x_3) - y_1$$

$$= -\frac{25}{22}\left(\frac{1}{4} + \frac{237}{121}\right) + \frac{15}{8}$$

$$= -\frac{845}{1331},$$

$$\Rightarrow Q \oplus Q \oplus Q = \left(-\frac{237}{121}, -\frac{845}{1331}\right).$$