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1. (10 points)

(a) **Answer:**Computing $P \oplus Q$ yields

$$\begin{aligned}
 \lambda &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{5 - 4}{2 - (-1)} \\
 &= \frac{1}{3}, \\
 x_3 &= \lambda^2 - x_1 - x_2 \\
 &= \frac{1}{3}^2 - (-1) - 2 \\
 &= -\frac{8}{9}, \\
 y_3 &= \lambda(x_1 - x_3) - y_1 \\
 &= \frac{1}{3} \left(-1 + \frac{8}{9} \right) - 4 \\
 &= -\frac{109}{27}, \\
 \Rightarrow P \oplus Q &= \left(-\frac{8}{9}, -\frac{109}{27} \right).
 \end{aligned}$$

Computing $P \ominus Q = P \oplus (-Q) = (-1, 4) \oplus (2, -5)$ yields

$$\begin{aligned}
 \lambda &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{-5 - 4}{2 - (-1)} \\
 &= -3, \\
 x_3 &= \lambda^2 - x_1 - x_2 \\
 &= (-3)^2 - (-1) - 2 \\
 &= 8, \\
 y_3 &= \lambda(x_1 - x_3) - y_1 \\
 &= (-3)(-1 - 8) - 4 \\
 &= 23, \\
 \Rightarrow P \ominus Q &= (8, 23).
 \end{aligned}$$

Student Number: XXXXXXXXXXName: Bryan Hoang(b) **Answer:**Computing $2P = P \oplus P = (-1, 4) \oplus (-1, 4)$ yields

$$\begin{aligned}
 \lambda &= \frac{3x_1^2 + A}{2y_1} \\
 &= \frac{3(-1)^2 + 0}{2(4)} \\
 &= \frac{3}{8}, \\
 x_3 &= \lambda^2 - x_1 - x_2 \\
 &= \left(\frac{3}{8}\right)^2 - 2(-1) \\
 &= \frac{137}{64}, \\
 y_3 &= \lambda(x_1 - x_3) - y_1 \\
 &= \frac{3}{8}\left(-1 - \frac{137}{64}\right) - 4 \\
 &= -\frac{2651}{512}, \\
 \Rightarrow 2P &= \left(\frac{137}{64}, -\frac{2651}{512}\right).
 \end{aligned}$$

Computing $2Q = Q \oplus Q = (2, 5) \oplus (2, 5)$ yields

$$\begin{aligned}
 \lambda &= \frac{3x_1^2 + A}{2y_1} \\
 &= \frac{3(2)^2 + 0}{2(5)} \\
 &= \frac{6}{5}, \\
 x_3 &= \lambda^2 - x_1 - x_2 \\
 &= \left(\frac{6}{5}\right)^2 - 2(2) \\
 &= -\frac{64}{25}, \\
 y_3 &= \lambda(x_1 - x_3) - y_1 \\
 &= \frac{6}{5}\left(2 + \frac{64}{25}\right) - 5 \\
 &= \frac{59}{125}, \\
 \Rightarrow 2Q &= \left(-\frac{64}{25}, \frac{59}{125}\right).
 \end{aligned}$$