Student Number: [redacted] Name: Bryan Hoang

8. (10 points)

(a) **Answer:**
Computing $P \oplus Q$ yields

$$\begin{aligned}
\lambda &= \frac{y_2 - y_1}{x_2 - x_1} \\
&= \frac{-5-2}{3-0} \\
&= -\frac{7}{3}, \\
x_3 &= \lambda^2 - x_1 - x_2 \\
&= \left(-\frac{7}{3}\right)^2 - 0 - 3 \\
&= \frac{22}{9}, \\
y_3 &= \lambda(x_1 - x_3) - y_1 \\
&= -\frac{7}{3}\left(0 - \frac{22}{9}\right) - 2 \\
&= \frac{100}{27}, \\
\Rightarrow \boxed{P \oplus Q} &= \boxed{\left(\frac{22}{9}, \frac{100}{27}\right)}.
\end{aligned}$$

(b) **Answer:**
Computing $P \oplus P$ yields

$$\begin{aligned}
\lambda &= \frac{3x_1^2 + A}{2y_1} \\
&= \frac{3 \cdot 0^2 - 2}{2 \cdot 2} \\
&= -\frac{1}{2}, \\
x_3 &= \lambda^2 - x_1 - x_2 \\
&= \left(-\frac{1}{2}\right)^2 - 0 - 0 \\
&= \frac{1}{4}, \\
y_3 &= \lambda(x_1 - x_3) - y_1 \\
&= -\frac{1}{2}\left(0 - \frac{1}{4}\right) - 2 \\
&= -\frac{15}{8}, \\
\Rightarrow \boxed{P \oplus P} &= \boxed{\left(\frac{1}{4}, -\frac{15}{8}\right)}.
\end{aligned}$$

Student Number: [redacted] Name: Bryan Hoang

Computing $Q \oplus Q$ yields

$$\begin{aligned}
\lambda &= \frac{3x_1^2 + A}{2y_1} \\
&= \frac{3 \cdot 3^2 - 2}{2 \cdot (-5)} \\
&= -\frac{5}{2}, \\
x_3 &= \lambda^2 - x_1 - x_2 \\
&= \left(-\frac{5}{2}\right)^2 - 3 - 3 \\
&= \frac{1}{4}, \\
y_3 &= \lambda(x_1 - x_3) - y_1 \\
&= -\frac{5}{2}\left(3 - \frac{1}{4}\right) - (-5) \\
&= -\frac{15}{8}, \\
\Rightarrow \boxed{Q \oplus Q} &= \left(\frac{1}{4}, -\frac{15}{8}\right).
\end{aligned}$$

(c) **Answer:**

Computing $P \oplus P \oplus P$ yields

$$\begin{aligned}
\lambda &= \frac{y_2 - y_1}{x_2 - x_1} \\
&= \frac{2 - (-\frac{15}{8})}{0 - \frac{1}{4}} \\
&= -\frac{31}{2}, \\
x_3 &= \lambda^2 - x_1 - x_2 \\
&= \left(-\frac{31}{2}\right)^2 - \frac{1}{4} - 0 \\
&= 240, \\
y_3 &= \lambda(x_1 - x_3) - y_1 \\
&= -\frac{31}{2}\left(\frac{1}{4} - 240\right) + \frac{15}{8} \\
&= 3718, \\
\Rightarrow \boxed{P \oplus P \oplus P} &= \left(240, 3718\right).
\end{aligned}$$

Student Number: [redacted] Name: Bryan Hoang

Computing $Q \oplus Q \oplus Q$ yields

$$\begin{aligned}
\lambda &= \frac{y_2 - y_1}{x_2 - x_1} \\
&= \frac{-5 - (-\frac{15}{8})}{3 - \frac{1}{4}} \\
&= -\frac{25}{22}, \\
x_3 &= \lambda^2 - x_1 - x_2 \\
&= \left(-\frac{25}{22}\right)^2 - \frac{1}{4} - 3 \\
&= -\frac{237}{121}, \\
y_3 &= \lambda(x_1 - x_3) - y_1 \\
&= -\frac{25}{22}\left(\frac{1}{4} + \frac{237}{121}\right) + \frac{15}{8} \\
&= -\frac{845}{1331}, \\
\Rightarrow \boxed{Q \oplus Q \oplus Q} &= \left(-\frac{237}{121}, -\frac{845}{1331}\right).
\end{aligned}$$