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3. (15 points)

(a) **Answer:**

DMC 1 is a Z-channel (binary asymmetric channel). Then calculating the channel's capacity yields

$$\begin{aligned}
C &= \max_{p_x} I(X;Y) \\
&= \max_{p_x} \{H(Y) - H(Y|X)\} \\
&= \max_{p_x} \{H(Y)\} - \max_{p_x} \left\{ \sum_{a_i \in \mathcal{X}} p_X(a_i) H(Y|X=a_i) \right\} \\
&= \log_2 |\mathcal{Y}| - \max_{p_x} \left\{ p_X(a_1) H(Y|X=a_1) + p_X(a_2) H(Y|X=a_2) \right\} \\
&= \log_2 |2| - \max_{p_x} \left\{ p_X(a_2) h_b(\beta) \right\} \\
&= 1 - \max_{p_x} \left\{ p_X(a_2) h_b(\beta) \right\}
\end{aligned}$$

The capacity input distribution is the Bernoulli distribution with parameter $1-\alpha$. i.e.,

$$p_X(x) = \begin{cases} 1-\alpha & \text{if } x = a_2 \\ \alpha & \text{if } x = a_1, \end{cases}$$

where

$$\alpha = 1 - \frac{1}{(1-\beta)(1+2^{\frac{h_b(\beta)}{1-\beta}})}.$$

(b) **Answer:**

(c) **Answer:**