Student Number: [redacted] Name: Bryan Hoang

5. (15 points)

(a) **Answer:**

Claim. The statement is **true**.

Proof.

$$\begin{aligned}
h(-4X-3Y-10) &= h(-4X-3Y) && \because \text{differential entropy doesn't change user translation}\\
&\geq h(-4X-3Y \mid Y) && \because \text{conditioning reduces differential entropy}\\
&= h{-4X}|Y && \because \text{differential entropy doesn't change user translation}\\
&= h{-4X} && \because X \perp\!\!\!\perp Y\\
&= hX + \log_2|-4| && \text{by the property of differential entropy acting on scaling}\\
&= hX + 2.
\end{aligned}$$

□

(b) **Answer:**

Claim. The statement is **false**.

Proof.

$$\begin{aligned}
I(X;Z) &\leq I(X;Y) && \text{by the DPI}\\
&= h(X) + h(Y) - \mathrm{H}(X,Y)\\
&= \log_2(2\pi e) - \left(\frac{1}{2}\log_2(1-\rho^2) + \log_2(2\pi e)\right)\\
&= \frac{1}{2}\log_2\left[\frac{1}{1-\rho^2}\right].
\end{aligned}$$

□

(c) **Answer:**

Claim. The statement is **true**.

Proof. □