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1. **Answer:**

Table 1: The joint distribution of X and Y

$X \backslash Y$	0	1	$p_X(x)$
0	$\frac{1}{4}$	0	$\frac{1}{4}$
1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$
$p_Y(y)$	$\frac{3}{4}$	$\frac{1}{4}$	

Based on Table 1, the joint pmf of X and Y is given by

$$p_{X,Y}(x,y) = \begin{cases} \frac{1}{2} & \text{if } x = 1 \text{ and } y = 0, \\ \frac{1}{4} & \text{if } x = y, \\ 0 & \text{if } x = 0 \text{ and } y = 1, \end{cases}$$

and the marginal pmfs are given by

$$p_X(x) = \begin{cases} \frac{1}{4} & \text{if } x = 0, \\ \frac{3}{4} & \text{if } x = 1, \end{cases} \qquad p_Y(y) = \begin{cases} \frac{3}{4} & \text{if } y = 0, \\ \frac{1}{4} & \text{if } y = 1, \end{cases}$$

Then the entropy of X is

$$\begin{aligned} H(X) &= -\sum_{x \in \mathcal{X}} p_X(x) \log_2 p_X(x) \\ &= -\big(p_X(0) \log_2 p_X(0) + p_X(1) \log_2 p_X(1)\big) \\ &= -\left(\frac{1}{4} \log_2 \frac{1}{4} + \frac{3}{4} \log_2 \frac{3}{4}\right) \\ &\approx 0.811 \text{ bits} \end{aligned}$$

and the entropy of Y is

$$\begin{aligned} H(Y) &= -\sum_{b \in \mathcal{Y}} p_Y(y) \log_2 p_Y(y) \\ &= -\big(p_Y(0) \log_2 p_Y(0) + p_Y(1) \log_2 p_Y(1)\big) \\ &= -\left(\frac{3}{4} \log_2 \frac{3}{4} + \frac{1}{4} \log_2 \frac{1}{4}\right) \\ &\approx 0.811 \text{ bits} \end{aligned}$$

which implies that the joint entropy of X and Y is

$$\begin{aligned} H(X,Y) &= -\sum_{x\in\mathcal{X}}\sum_{y\in\mathcal{Y}} p_{X,Y}(x,y)\log_2 p_{X,Y}(x,y) \\ &= -\left(\frac{1}{2}\log_2\frac{1}{2} + 0\log_2(0) + 2\cdot\frac{1}{4}\log_2\frac{1}{4}\right) \\ &\approx 1.5 \text{ bits} \end{aligned}$$

By the chain rule of conditional entropy, we then have

$$\begin{aligned} H(X|Y) &= H(X,Y) - H(Y) \\ &= 1.5 - 0.811 \\ &= 0.689 \text{ bits} \end{aligned} \qquad \begin{aligned} H(Y|X) &= H(X,Y) - H(X) \\ &= 1.5 - 0.811 \\ &= 0.689 \text{ bits} \end{aligned}$$

The mutual information between X and Y is

$$\begin{aligned} I(X;Y) &= H(X) + H(Y) - H(X,Y) \\ &= 0.811 + 0.811 - 1.5 \\ &= 0.123 \text{ bits} \end{aligned}$$

Therefore, the Venn diagram for the computed properties can be seen in Figure 1.

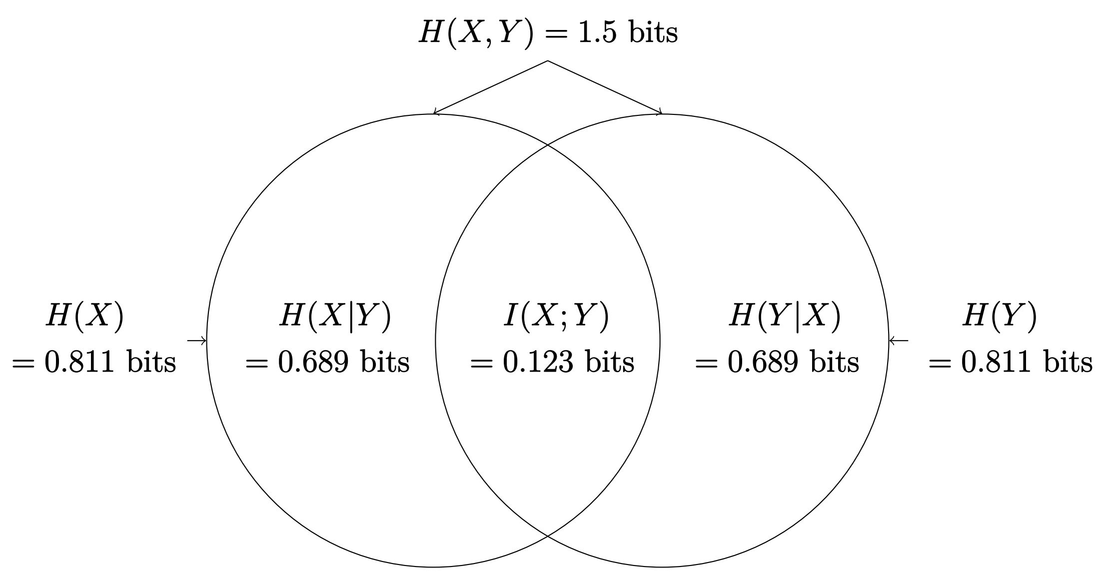

Figure 1: Venn diagram of entropy and mutual information for X and Y