Student Number: [redacted] Name: Bryan Hoang

4. (15 points) **Answer:**

Proof. Suppose that $X \to Y \to (Z, W)$ form a Markov chain. Then we have

$$
\begin{aligned}
& I(X;Y) + I(Z;W) \\
\geq\ & I(X;Z,W) + I(Z;W) && \text{by the DPI} \\
=\ & I(X;Z,W) + I(Z;W) + I(X;Z) - I(X;Z) \\
=\ & I(X;Z) + I(Z;W) + I(X;W|Z) && \text{by the chain rule for MI} \\
=\ & I(X;Z) + I(X,Z;W) && \text{by the chain rule for MI} \\
\geq\ & I(X;Z) + I(X;W) && \text{by the chain rule for MI}
\end{aligned}
$$

□