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6. (15 points)

(a) **Answer:**

Let $\mathcal{S}$ and $\mathcal{U}$ denote the set of all suffix and UD first-order binary VLCs, respectively.

First, note that since $\mathcal{S} \subseteq \mathcal{U}$, then

$$\begin{aligned} \min_{\mathcal{C}\in\mathcal{U}} \bar{L} &\leq \min_{\mathcal{C}\in\mathcal{S}} \bar{L} \\ \Rightarrow \bar{L}_{\text{UD}} &\leq \bar{L}_{\text{S}} \end{aligned} \tag{1}$$

Now let $\mathcal{C}^*$, with codeword lengths $l_1^*, \ldots, l_n^*$, be the optimal UD code ($\mathcal{C}^* \in \mathcal{U}$):

$$\bar{L}(\mathcal{C}^*) = \bar{L}_{\text{UD}}$$

where $\bar{L}(\mathcal{C})$ is the expected cost function, $\bar{L}$, of $\mathcal{C}$.

Then the codeword lengths $\{l_i^*\}_{i=1}^M$ of $\mathcal{C}^*$ must satisfy the Kraft inequality (in base D) since $\mathcal{C}^*$ is UD.

Then by the Theorem in class on the "Kraft Inequality for Prefix codes", $\exists$ prefix code $\mathcal{C}'$ with the same codeword lengths as C^*: $l_i' = l_i^*,\ i = 1, \ldots, M$. We can reverse the codewords of $\mathcal{C}'$ to obtain a suffix code $\mathcal{C}'' \in \mathcal{S}$, with the same codeword lengths as C^*: $l_i'' = l_i^*,\ i = 1, \ldots, M$.

$$\begin{aligned} \therefore \bar{L}(\mathcal{C}'') &= \sum_{i=1}^{M} p_i c(l_i'') \\ &= \sum_{i=1}^{M} p_i c(l_i^*) \\ &= \bar{L}_{\text{UD}} \end{aligned}$$

But, $\bar{L}(\mathcal{C}'') \geq \bar{L}_{\text{S}}$ by the definition of $\bar{L}_{\text{S}}$.

$$\therefore \bar{L}_{\text{UD}} \geq \bar{L}_{\text{S}}. \tag{2}$$

Then by (1) and (2), we have $\boxed{\bar{L}_{\text{UD}} = \bar{L}_{\text{S}}}$.

(b) **Answer:**

Compression efficiency is one appropriate metric, where we want to choose a code alphabet size D and maximize order n so that the code's average code rate, $\overline{R}$, is as close as possible to the DMSs source entropy, $H_D(X)$.

Encoding delay is another metric, where we want to minimize order n and design a code with minimal variance in length to minimize the delay.

Storage and computational complexity is a third metric, where we want to minimize the size of the source alphabet by minimizing the order n.

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Due to these considerations, we need to balance the metrics and evluate the trade-offs between different solutions depending on the design application.