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2. **Answer:**

To determine the joint entropy and mutual information of X and Y, we need to determine the joint and marginal pmfs.

Table 1: The joint distribution of X and Y

Y \ X	1	2	3	4	5	6	$p_Y(y)$
0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{5}{12}$
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{6}{12}$
2	0	0	0	0	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{12}$
$p_X(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	

From Table 1, we can see that the joint distribution of X and Y is

$$p_{X,Y}(x,y) = \begin{cases} \frac{1}{12} & \text{if } y = 2 \text{ or } x \in \{1,\dots,4\} \text{ and } y = 0, \\ \frac{1}{24} & \text{if } x \in \{5,6\} \text{ and } y \in \{0,2\}, \\ 0 & \text{otherwise,} \end{cases}$$

and that the marginal pmfs of X and Y are

$$p_X(x) = \frac{1}{6} \quad \forall\, x \in \{1,\dots,6\} \qquad p_Y(y) = \begin{cases} \frac{1}{2} & \text{if } y = 1, \\ \frac{5}{12} & \text{if } y = 0, \\ \frac{1}{12} & \text{if } y = 2, \end{cases}$$

Then the joint entropy of X and Y is

$$\begin{aligned} H(X,Y) &= -\sum_{x\in\mathcal{X}}\sum_{y\in\mathcal{Y}} p_{X,Y}(x,y)\log_2 p_{X,Y}(x,y) \\ &= -\left(10\cdot\frac{1}{12}\log_2\frac{1}{12} + 4\cdot\frac{1}{24}\log_2\frac{1}{24} + 4\cdot 0\log_2 0\right) \\ &\boxed{\approx 3.752 \text{ bits}} \end{aligned}$$

To make the calculation of the mutual information between X and Y easier, let's first

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determine the marginal entropies.

$$\begin{aligned} H(X) &= -\sum_{x\in\mathcal{X}} p_X(x)\log_2 p_X(x) \\ &= -\left(6\cdot\frac{1}{6}\log_2\frac{1}{6}\right) \\ &\approx 2.585 \text{ bits} \end{aligned} \qquad \begin{aligned} H(Y) &= -\sum_{y\in\mathcal{Y}} p_Y(y)\log_2 p_Y(y) \\ &= -\left(\frac{5}{12}\log_2\frac{5}{12}+\frac{1}{2}\log_2\frac{1}{2}+\frac{1}{12}\log_2\frac{1}{12}\right) \\ &\approx 1.325 \text{ bits} \end{aligned}$$

Then the mutual information between X and Y is

$$\begin{aligned} I(X;Y) &= H(X)+H(Y)-H(X,Y) \\ &= 2.585+1.325-3.752 \\ &\boxed{= 0.158 \text{ bits}} \end{aligned}$$