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4. (25 points) **Answer:**

From the textbook, the capacity of the q-ary symmetric DMC is

$$C = \log_2 q + \varepsilon \log_2 \frac{\varepsilon}{q-1} + (1-\varepsilon)\log_2(1-\varepsilon).$$

With $|\mathcal{Z}| = q$, the rate distortion function of the q-ary symmetric DMS is

$$R(D) = \begin{cases} \log_2 q - D\log_2(q-1) - H_b(D), & \text{if } D \in [0, \frac{q-1}{q}], \\ 0, & \text{if } D \geq \frac{q-1}{q}. \end{cases}$$

Then we want to see if the uncoded source-channel transmission scheme can achieve the Shannon limit of the communication system for it to be optiml. That is,

$$\begin{aligned} R(D_{SL}) &= C \\ \cancel{\log_2 q} - D_{SL}\log_2(q-1) - H_b(D_{SL}) &= \cancel{\log_2 q} + \varepsilon \log_2 \frac{\varepsilon}{q-1} + (1-\varepsilon)\log_2(1-\varepsilon) \\ D_{SL}\log_2(q-1) + H_b(D_{SL}) &= \varepsilon \log_2(q-1) + H_b(\varepsilon) \end{aligned}$$

$\because x\log_2(q-1) + H_b(x)$ is injective on $x \in [0, \frac{q-1}{q}]$,

$$D_{SL} = \varepsilon. \tag{1}$$

With that fact, let's see if the distortion constraint is satisfied for the scheme with $R_{sc} = 1$ source symbol/channel use. First, let W denote the additive noise of the channel (mod q). Then the expected distortion is

$$\begin{aligned} Ed(Z, \hat{Z}) &= P(Z \neq \hat{Z}) \\ &= P(W \neq 0) \\ &= 1 - (1-\varepsilon) \\ &= \varepsilon \\ &= D_{SL} && \text{by (1)}. \end{aligned}$$

Thus, the proposed scheme achieves the Shannon limit of the communication system, and thus $\boxed{\text{is optimal}}$ for the communication system.