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3. (25 points)

(a) **Answer:**
For the first-order linear predictor of Coder 1, let a be the prediction coefficient to be determined. Then the orthogonality principle states that

$$\begin{aligned}
E[(X_n - aX_{n-1})X_{n-1}] &= 0 \\
E[X_nX_{n-1}] - aE[X_{n-1}^2] &= 0 \\
a &= \frac{E[X_nX_{n-1}]}{E[X_{n-1}^2]} \\
&= \frac{r_1}{r_0} && \because \{X_n\} \text{ is wide sense stationary} \\
&\boxed{= \frac{1}{2}}.
\end{aligned}$$

For the second-order linear predictor of Coder 2, let a_1 and a_2 be the prediction coefficients to be determined. Then the orthogonality principle states that

$$E[(X_n - a_1X_{n-1} - a_2X_{n-2})X_{n-i}] = 0 \qquad \text{for } i = 1, 2$$

which gives the following system of linear equations:

$$\begin{aligned}
&\begin{cases} a_1r_0 + a_2r_1 = r_1, \\ a_1r_1 + a_2r_0 = r_2 \end{cases} \\
\Rightarrow &\begin{cases} a_1 \cdot 4 + a_2 \cdot 2 = 2, \\ a_1 \cdot 2 + a_2 \cdot 4 = 1 \end{cases} \\
\Rightarrow &\boxed{\begin{cases} a_1 = \frac{1}{2}, \\ a_2 = 0 \end{cases}}.
\end{aligned}$$

(b) **Answer:**
Let Δ_i denote the step size, N_i denote the quantization level, and Q_i denote the uniform quantizer of Coder i. Then since $\delta_i = \frac{2B}{N_i}$, we get

$$\begin{aligned}
E\left[(e_n - Q_i(e_n))^2\right] &\approx \frac{\Delta_i^2}{12} && \because \text{the overload distortion is negligible and the prediction error is uniform} \\
&= \frac{\left(\frac{2B}{N_i}\right)^2}{12} \\
&= \frac{B^2}{3N_i^2}.
\end{aligned}$$

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Then overall system gain for Coder i (System SNR, not in dB) is given by

$$\begin{aligned} G^i &= G^i_{clp} G_{Q_i} \\ &\approx G^i_{olp} G_{Q_i} \\ &\approx \frac{r_0}{r_0 - r_1 a_1} \cdot \frac{\sigma^2}{\frac{B^2}{3N_i^2}} \\ &= \frac{r_0}{r_0 - r_1 a_1} \cdot \frac{3N_i^2 \sigma^2}{B^2}. \end{aligned}$$

Therefore, the approximate SNR improvement is

$$\begin{aligned} 10 \log_{10} \frac{G^2}{G^1} &= 10 \log_{10} \frac{N_2^2}{N_1^2} \\ &= 10 \log_{10} \frac{256^2}{64^2} \\ &\boxed{\approx 12\,\mathrm{dB}}. \end{aligned}$$