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4. (20 points) **Answer:**

Proof. Suppose that C_n is the Shannon-Fano code for $p(x^n)$. Then for $i = 1, \dots, M$,

$$\begin{aligned}
\lim_{n\to\infty} \frac{1}{n} D(p_i \| p) &= \lim_{n\to\infty} \frac{1}{n} \sum_{x^n \in \mathcal{X}^n} p_i(x^n) \log \frac{p_i(x^n)}{p(x^n)} \\
&= \lim_{n\to\infty} \frac{1}{n} \sum_{x^n \in \mathcal{X}^n} p_i(x^n) \log \frac{p_i(x^n)}{\sum_{j=1}^{M} \alpha_j p_j(x^n)} \\
&\le \lim_{n\to\infty} \frac{1}{n} \sum_{x^n \in \mathcal{X}^n} p_i(x^n) \log \frac{p_i(x^n)}{\alpha_i p_i(x^n)} \\
&= \lim_{n\to\infty} \frac{1}{n} (-\log \alpha_i) \\
&= 0 && \because \alpha_i > 0 \quad \forall i = 1, \dots, M.
\end{aligned}$$

Thus, the code sequence $\{C_n\}$ is universal with respect to $\mathcal{P}$. $\square$