CQF Exam One

June 2020 Cohort

Instructions

Answers to all questions **are required**. Requested mathematical and all computational workings must be provided. Please submit report in ONE PDF including all graphs. If your report is a Python notebook or Excel with handwritten inserts – please print out as ONE PDF without unnecessary output.

All supplementary code/Excel to be uploaded as ONE ZIP FILE. Name PDF and ZIP files to start with your own LASTNAME_.

If you submit multiple pdf or image files/absence of one clear report that provides workings and answers to all exam tasks, in format requested – will result in a request for resubmission (with Extensions) or deduction in marks.

Portal, upload and extension questions to Orinta.Juknaite@fitchlearning.com. Clarifying questions but not re-explanation are welcome to Richard.Diamond@fitchlearning.com.

All tasks are possible to solve in Excel using *MMULT(), MINV()* for portfolio computation and *IF()* for VaR backtesting. Computation in Python/Matlab/R encouraged.

Marking Scheme: Q1 26% **Q2** 20% **Q3** 30% **Q4** 12% **Q5** 12%

Optimal Portfolio Allocation

Consider an investment universe composed of the following risky assets with a dependence structure – the data below applies to all portfolio optimisation tasks as appropriate.

Assets	$\boldsymbol{\mu}$	$\boldsymbol{\sigma}$
A	0.04	0.07
B	0.08	0.12
C	0.12	0.18
D	0.15	0.26

$$\boldsymbol{Corr} = \begin{pmatrix} 1 & 0.2 & 0.5 & 0.3 \\ 0.2 & 1 & 0.7 & 0.4 \\ 0.5 & 0.7 & 1 & 0.9 \\ 0.3 & 0.4 & 0.9 & 1 \end{pmatrix}$$

Question 1. For all four assets, consider the optimization for a target return m, with the net of allocations invested (borrowed) in the risk-free asset:

$$\underset{\boldsymbol{w}}{\operatorname{argmin}} \frac{1}{2}\boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{\Sigma}\boldsymbol{w}, \qquad \text{s.t. } r + (\boldsymbol{\mu} - r\mathbf{1})^T\mathbf{w} = m$$

(a) Formulate the Lagrangian function and its partial derivatives. Derive the analytical solution for optimal allocations $\boldsymbol{w}^*$. Provide handwritten or typeset mathematical working.

(b) For the risk-free rate of 3% and a range of target return values $m = 5\%, 7.5\%, 10\%, 12.5\%$ compute optimal allocations $\boldsymbol{w}^*$ (each portfolio), portfolio risk $\sigma_\Pi = \sqrt{\boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{\Sigma}\boldsymbol{w}}$, and plot the Efficient Frontier.
Note: it is your task to determine the nature of μ_Π and shape of the frontier.

Question 2. For a given allocated portfolio compute VaR and ES sensitivities *wrt* each asset i.

Asset	$\boldsymbol{\mu}$	$\boldsymbol{\sigma}$	$\boldsymbol{w}$
1	0	0.30	50%
2	0	0.20	20%
3	0	0.15	30%

$$\boldsymbol{Corr} = \begin{pmatrix} 1 & 0.8 & 0.5 \\ 0.8 & 1 & 0.3 \\ 0.5 & 0.3 & 1 \end{pmatrix}$$

$$\frac{\partial \text{VaR}(w)}{\partial w_i} = \mu_i + \text{Factor} \times \frac{(\boldsymbol{\Sigma w})_i}{\sqrt{\boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{\Sigma w}}} \quad \text{and} \quad \frac{\partial \text{ES}(w)}{\partial w_i} = \mu_i - \frac{\phi(\text{Factor})}{1-c} \times \frac{(\boldsymbol{\Sigma w})_i}{\sqrt{\boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{\Sigma w}}}$$

where $()_i$ refers to i-th element, confidence $c = 99\%$, $1 - c = 0.01$, and Factor $= \Phi^{-1}(1-c)$ is *inverse Normal cdf*, while $\phi()$ is simply Normal *pdf*. Signs in formulae are for the left tail – lower percentile of 0.01. Tutor will not confirm Factor sign/matrix computation/output.

Regulation and Techniques

Question 3. As a market risk analyst, each day you calculate VaR from the available prior data. Then, you wait ten days to compare your prediction value VaR_{t-10} to the realised return and check if the prediction about the worst loss was breached. You are given a dataset with *Closing Prices* (FTSE 100).

- Implement VaR backtesting by computing 99%/10day Value at Risk using the rolling window of 21 returns to compute σ. The rolling window technique will give you the time series of $\text{VaR}_{10D,t}$.

$$\text{VaR}_{10D,t} = \text{Factor} \times \sigma_t \times \sqrt{10}$$

3.1 (a) Report the percentage of VaR breaches and (b) number of consecutive breaches. (c) Provide a plot which clearly identifies breaches. Further Instructions given below, use as needed.

3.2 Repeat backtesting for the sample of 42 returns to compute σ. Provide outputs (a), (b), (c) in one Table, together with 3.1.

Question 4. The Basel Framework, can now be referred to in interactive form: `https://www.bis.org/basel_framework/index.htm`. For regulation MAR32 Internal models approach, answer with definitions:

(a) How many exceptions are allowed in a trading desk-level backtesting at 99th percentile before it will be forced to exercise the standardised approach?

(b) What are two key metrics to compare risk-theoretical P&L (RTPL) and hypothetical P&L (HPL)?

Question 5. The EU Capital Requirements Regulation 2 (CRR2), can be found at European Banking Authority `https://eba.europa.eu/regulation-and-policy/single-rulebook/interactive-single-rulebook/100427`. Provide article numbers that specifically prescribe requirements for the following:

(a) Stable funding, together with a brief definition of Net Stable Funding Ratio.

(b) Internal risk model for 'correlation trading', together with a list of main risks to be captured.

END OF EXAM

Further Instructions

Please make good use of lecture exercises and problem-solving sessions. The tutor is unable to confirm numerical answers and methods of calculation/spreadsheets.

To compute the 99%/10day Value at Risk for an investment in the market index on the rolling basis. We drop the expected return (mean) from the VaR formula

$$\text{VaR}_{10D,t} = \text{Factor} \times \sigma_t \times \sqrt{10}$$

- Practical VaR calculation drops μ_{10D} for two reasons. First, 21-day sample average return (or alike) is not a robust quantity. Second, for a diversified portfolio/market index the quantity is negligible.
- Appropriate Factor value to be used (Standard Normal Percentile), the tutor will not confirm the numerical value. It is also your task to identify the eligible number of observations for which VaR is available and can be backtested: N_{obs} will not be confirmed.
- Compute a column of rolling standard deviation over log-returns for observations $1-21, 2-22, \ldots$. Compute VaR for each day t, after the initial period. This is your worst loss prediction.
- **Regardless** of how many observations there are in a sample (10, 21, 100, etc.), variance is *an average of squared daily differences* $\frac{\sum(r_t-\mu)^2}{(N-1)}$ and so, timescale remains 'daily'.
- VaR is fixed at time t and compared to the return realised from t to $t+10$. A breach occurs when that forward realised 10-day return $\ln(S_{t+10}/S_t)$ is below the VaR_t quantity.

  $$r_{10D,t+10} < \text{VaR}_{10D,t} \qquad \text{means breach, given both numbers are negative.}$$

  In Excel, you will have a column for VaR_t series, a column of $r_{10D,t+10}$ series, and indicator column $\{0,1\}$ for a breach using $IF()$ function.
- To obtain the conditional probability of breach $N_{conseq}/N_{breaches}$, identify consecutive breaches. For example, the sequence $1,1,1$ means two consecutive breaches occurred.

Extra Tasks

These tasks are not part of the exam and not graded. However, they provide paths for advanced modelling and your own further exploration.

Backtesting For comparison to naive sample std dev, backtesting can be done with EWMA on variance, with the same rolling window of 21 or 42 observations. EWMA estimated for each next period as follows:

$$\sigma^2_{t+1\,|\,t} = \lambda\,\sigma^2_{t\,|\,t-1} + (1-\lambda)\,r_t^2$$

advised $\lambda = 0.72$ is smaller than suggested by RiskMetrics but minimises out of sample forecasting errors.

The full GJR-GARCH model requires estimation of $(\omega, \alpha, \gamma, \beta)$ first (please see ARCH Lecture and its spreadsheet). GARCH model itself is biased towards the long-run average variance $\bar{\sigma}^2$ and not as responsive to the recent return information r_t^2. Therefore, we prefer EWMA.

Build Q-Q plots for 1D and 10D returns, and conclude if Normally distributed returns was a reasonable assumption. Log-returns being Normally distributed is the main assumption of Analytical VaR.
Without this assumption holding, the Normal Factor is not applicable.

As to the issue of independence of breaches in VaR, the applicable statistical test is for Christoffersen's exceedances independence.