$\min_{w} \frac{1}{2} w' \Sigma w$, 2 constraints: $r + (\mu - r\mathbb{1})'w = m$ & $\mathbb{1}'w = 1$

$L(w, \lambda, \gamma) = \frac{1}{2} w' \Sigma w - \lambda(m - r - (\mu - r\mathbb{1})'w) - \gamma(1 - \mathbb{1}'w)$.

$$\left.\begin{aligned} \frac{\partial L}{\partial w} &= w'\Sigma + \lambda(\mu - r\mathbb{1})' + \gamma \mathbb{1}' = 0 \\ \frac{\partial L}{\partial \lambda} &= m - r - (\mu - r\mathbb{1})'w = 0 \\ \frac{\partial L}{\partial \gamma} &= 1 - \mathbb{1}'w = 0 \end{aligned}\right\}$$

$w^* = \Sigma^{-1}(-\lambda(\mu - r\mathbb{1})' - \gamma \mathbb{1}')$

Plugging w^* to constraints, we get: $\mathbb{1}' w^* = 1 = \mathbb{1}'\Sigma^{-1}(-\lambda(\mu - r\mathbb{1})' - \gamma\mathbb{1}')$

$r + (\mu - r\mathbb{1})'\Sigma^{-1}(-\lambda(\mu - r\mathbb{1})' - \gamma \mathbb{1}') = m$

$r + (-\lambda)(\mu - r\mathbb{1})'\Sigma^{-1}(\mu - r\mathbb{1})' + (-\gamma)(\mu - r\mathbb{1})'\Sigma^{-1}\mathbb{1}' = m$

$(-\lambda)\mathbb{1}'\Sigma^{-1}(\mu - r\mathbb{1})' + (-\gamma)\mathbb{1}'\Sigma^{-1}\mathbb{1}' = 1$

$A = \mathbb{1}'\Sigma^{-1}(\mu - r\mathbb{1})' = (\mu - r\mathbb{1})\Sigma^{-1}\mathbb{1}$

$B = \mathbb{1}'\Sigma^{-1}\mathbb{1}' = \mathbb{1}\Sigma^{-1}\mathbb{1}$

$C = (\mu - r\mathbb{1})'\Sigma^{-1}(\mu - r\mathbb{1})' = (\mu - r\mathbb{1})\Sigma^{-1}(\mu - r\mathbb{1})$

$D = (\mu - r\mathbb{1})'\Sigma^{-1}\mathbb{1}' = \mathbb{1}\Sigma^{-1}(\mu - r\mathbb{1})$

$$\text{so} \quad \left.\begin{aligned} r - \lambda A - \gamma B &= 1 \\ r - \lambda C - \gamma D &= m. \end{aligned}\right\}$$

$\lambda A = -\gamma B - 1$, $\lambda = \frac{-\gamma B - 1}{A}$, plug in λ into second equation $(r - m - \gamma D)/C = \lambda$

$\frac{r - m - \gamma D}{C} = \frac{-\gamma B - 1}{A} \Rightarrow Ar - Am - \gamma AD = -\gamma CB - 1 \rightarrow \gamma CB - \gamma AD = -1 + Am - Ar$.

$\gamma = \frac{-1 + Am - Ar}{CB - AD}$

$\lambda = -\left(\frac{-1 + Am - Ar}{CB - AD}\right)B - 1 \,/A$

☆ Trouble evaluating matrices, it seems my calculation of A, B, C, D are incorrect as dimensions don't add up.

Question 2

Assets	μ	σ	w
1	0	0.3	50%
2	0	0.2	20%
3	0	0.5	30%

$$Corr = \begin{pmatrix} 1 & 0.8 & 0.5 \\ 0.8 & 1 & 0.3 \\ 0.5 & 0.3 & 1 \end{pmatrix}$$

$$Cov = \begin{pmatrix} 0.3 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.5 \end{pmatrix}\begin{pmatrix} 1 & 0.8 & 0.5 \\ 0.8 & 1 & 0.3 \\ 0.5 & 0.3 & 1 \end{pmatrix}\begin{pmatrix} 0.3 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.5 \end{pmatrix} = \begin{pmatrix} 0.09 & 0.048 & 0.075 \\ 0.048 & 0.04 & 0.03 \\ 0.075 & 0.03 & 0.25 \end{pmatrix}$$

$$Factor = \Phi^{-1}(1-c) = -2.33, \quad \phi(Factor) = \phi(-2.33) = 0.0267$$

$$\Sigma w = \begin{pmatrix} 0.0771 \\ 0.041 \\ 0.1185 \end{pmatrix} \qquad \frac{\partial VAR(w)}{\partial w_1} = 0 + (-2.33) \times \frac{0.0771}{0.287} = -0.626$$

$$w^T\Sigma w = 0.0823 \qquad \frac{\partial VAR(w)}{\partial w_2} = 0 + (-2.33) \times \frac{0.041}{0.287} = -0.333$$

$$\sqrt{w^T\Sigma w} = 0.287$$

$$\frac{\partial VAR(w)}{\partial w_3} = 0 + (-2.33) \times \frac{0.1185}{0.287} = -0.962$$

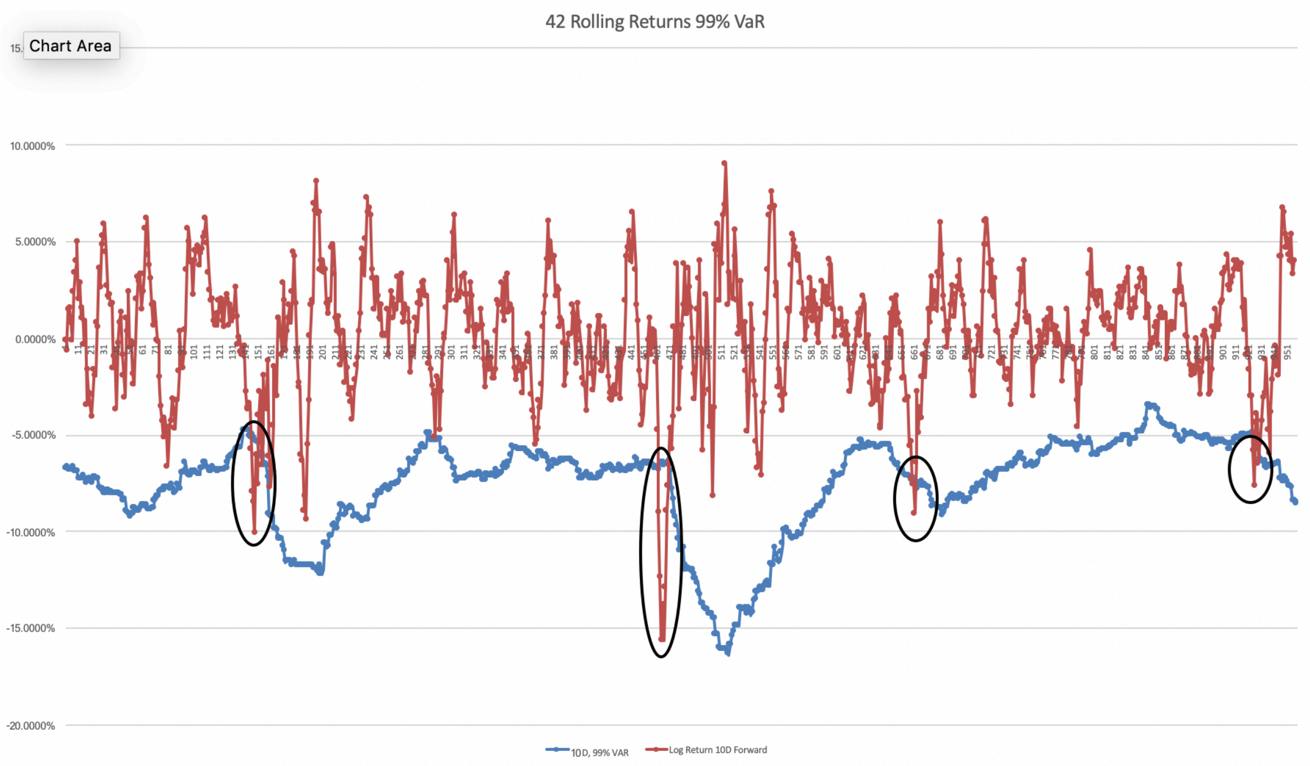
$$\frac{\partial ES(w)}{\partial w_1} = 0 + \frac{0.0267}{0.01} \times \frac{0.0771}{0.287} = 0.717$$

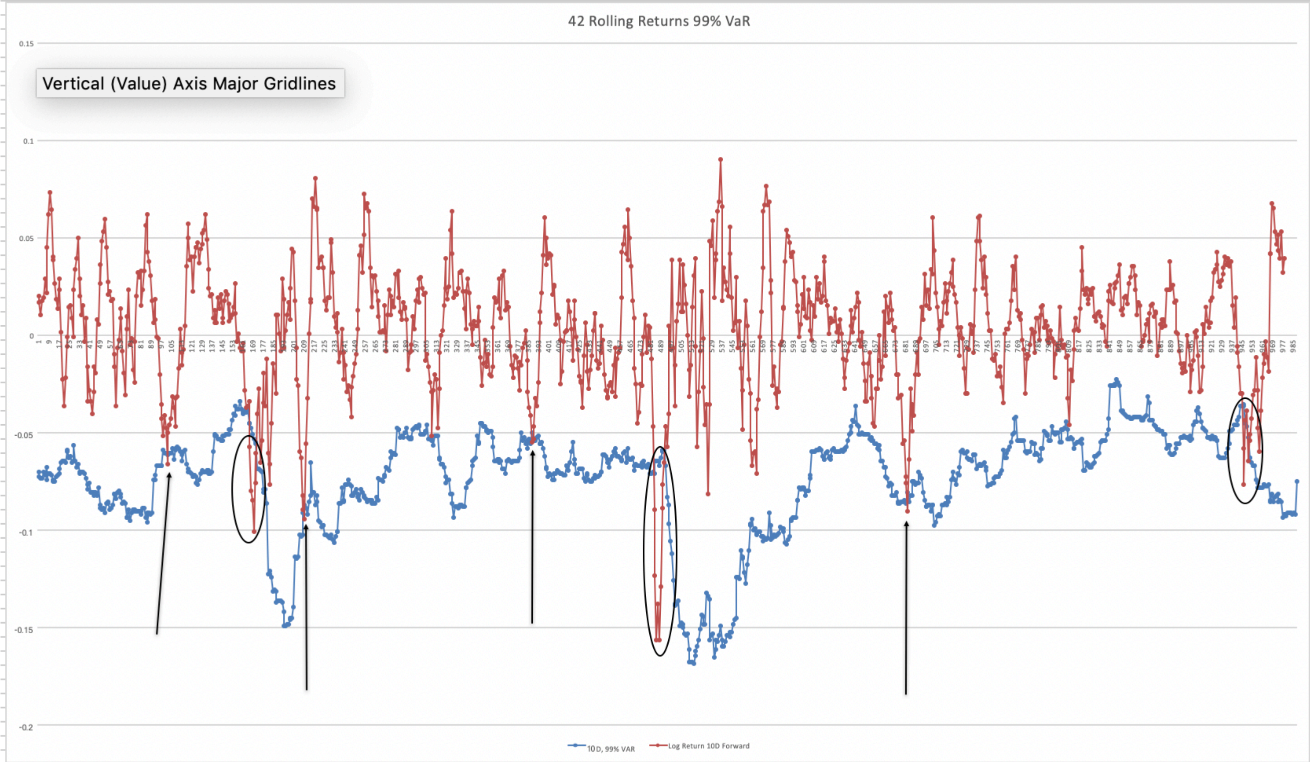
$$\frac{\partial ES(w)}{\partial w_2} = 0 + \frac{0.0267}{0.01} \times \frac{0.041}{0.287} = 0.381$$

$$\frac{\partial ES(w)}{\partial w_3} = 0 + \frac{0.0267}{0.01} \times \frac{0.1185}{0.287} = 1.10242$$

Question 3: VaR Backtesting

VaR Backtesting	Rolling Window of 21 Returns	Rolling Window of 42 Returns
% of VaR Breaches	2.25%	2.20%
# of Consecutive Breaches	14	15





Question 4: The Basel Framework

1. How many exceptions are allowed in a trading desk-level back testing at 99th percentile before it will be forced to exercise the standardised approach?
    a. Back testing requirements compare the Value at Risk (VaR) measure to a one-day holding period against each of the actual P&L and hypothetical P&L over a period of 12 months (250 trading days). Back testing of the risk model is based on VaR measure at 99th percentile confidence level.
    b. An exception is when the actual loss or hypothetical loss of the trading book in a day of back testing exceeds the daily VaR measure. The regulation separately counts exceptions for actual losses and exceptions for hypothetical losses. The overall number of exception is the greater of these two.
    c. If the trading desk has more than 12 exceptions for 99th percentile or more than 30 exceptions at 97.5th percentile in the most recent 12-month period, then the capital requirement for the trading desk must be standardised.
2. What are two key metrics to compare risk-theoretical P&L (RTPL) and hypothetical P&L (HPL)?
    a. The PLA test compares daily RTPL with daily HTP for each trading desk. There are two test metrics for the PLA test: Spearman correlation metric and Kolmogorov-Smirnov.
    b. Spearman correlation metric: this assess the correlation between RTPL and the HPL. A well-modelled trading desk would have a high correlation, which means both move in the same direction on a daily basis.
    c. Kolmogorov-Smirnov (KS) metric: assess similarity of distribution between RTPL and HPL. The closeness of distribution means that the bank's model is accurately capturing the range of losses of the trading desk.

Question 5: The EU Capital Requirements Regulation 2

1. Stable funding, together with a brief definition of Net Stable Funding Ratio:
    a. Stable funding: Article 427, Article 428
    b. Net Stable Funding Ratio: Article 510
2. Internal risk model for 'correlation trading', together with a list of main risks to be captured:
    a. Internal risk model: Article 377