CFD Homework 2 - BRYAN ACOSTA

Contents


```
clear
clc
close all
```

Calculating Work & Alpha

```
x1 = linspace(0,1,30);
x2 = linspace(0,1,32);
x3 = linspace(0,1,34);
x4 = linspace(0,1,36);
x5 = linspace(0,1,38);
x6 = linspace(0,1,40);

h = [32 , 34, 36, 40];
global counter

diffequation=@(x,y) -50*(y - cos(x));
```

EXACT ODE VS EXPLICIT EULER

```
countervec1 = (linspace(1,4,4)).*0;
error1 = (linspace(1,4,4)).*0;

[exactval,~] = exact_solution(diffequation,x2);
[estval,counter] = Explicit_Euler(x2);
input = abs(exactval(end) - estval(end));
countervec1(1) = counter;
error1(1) = input;

[exactval,~] = exact_solution(diffequation,x3);
[estval,counter] = Explicit_Euler(x3);
input = abs(exactval(end) - estval(end));
countervec1(2) = counter;
error1(2) = input;

[eactval,~] = exact_solution(diffequation,x4);
[estval,counter] = Explicit_Euler(x4);
input = abs(exactval(end) - estval(end));
countervec1(3) = counter;
error1(3) = input;

[exactval,~] = exact_solution(diffequation,x6);
[estval,counter] = Explicit_Euler(x6);
input = abs(exactval(end) - estval(end));
countervec1(4) = counter;
error1(4) = input;
```

EXACT ODE VS IMPLICIT EULER

```
countervec2 = (linspace(1,4,4)).*0;
error2 = (linspace(1,4,4)).*0;

[exactval,~] = exact_solution(diffequation,x2);
[estval,counter] = Implicit_Euler(x2);
input = abs(exactval(end) - estval(end));
countervec2(1) = counter;
error2(1) = input;

[exactval,~] = exact_solution(diffequation,x3);
[estval,counter] = Implicit_Euler(x3);
input = abs(exactval(end) - estval(end));
countervec2(2) = counter;
error2(2) = input;

[exactval,~] = exact_solution(diffequation,x4);
[estval,counter] = Implicit_Euler(x4);
input = abs(exactval(end) - estval(end));
countervec2(3) = counter;
error2(3) = input;

[exactval,~] = exact_solution(diffequation,x6);
[estval,counter] = Implicit_Euler(x6);
input = abs(exactval(end) - estval(end));
countervec2(4) = counter;
error2(4) = input;
```

```
% EXACT ODE VS MIDPOINT METHOD

countervec3 = (linspace(1,4,4)).*0;
error3 = (linspace(1,4,4)).*0;

[exactval,~] = exact_solution(diffequation,x2);
[estval,counter] = Midpoint(x2);
input = abs(exactval(end) - estval(end));
countervec3(1) = counter;
error3(1) = input;

[exactval,~] = exact_solution(diffequation,x3);
[estval,counter] = Midpoint(x3);
input = abs(exactval(end) - estval(end));
countervec3(2) = counter;
error3(2) = input;

[exactval,~] = exact_solution(diffequation,x4);
[estval,counter] = Midpoint(x4);
input = abs(exactval(end) - estval(end));
countervec3(3) = counter;
error3(3) = input;

[exactval,~] = exact_solution(diffequation,x6);
[estval,counter] = Midpoint(x6);
input = abs(exactval(end) - estval(end));
countervec3(4) = counter;
error3(4) = input;
```

EXACT ODE VS TRAPEZOIDAL METHOD

```
countervec4 = (linspace(1,4,4)).*0;
error4 = (linspace(1,4,4)).*0;

[exactval,~] = exact_solution(diffequation,x2);
[estval,counter] = trapezoidal(x2);
input = abs(exactval(end) - estval(end));
countervec4(1) = counter;
error4(1) = input;

[exactval,~] = exact_solution(diffequation,x3);
[estval,counter] = trapezoidal(x3);
input = abs(exactval(end) - estval(end));
countervec4(2) = counter;
error4(2) = input;

[exactval,~] = exact_solution(diffequation,x4);
[estval,counter] = trapezoidal(x4);
input = abs(exactval(end) - estval(end));
countervec4(3) = counter;
error4(3) = input;

[exactval,~] = exact_solution(diffequation,x6);
[estval,counter] = trapezoidal(x6);
input = abs(exactval(end) - estval(end));
countervec4(4) = counter;
error4(4) = input;
```

EXACT ODE VS ADAMS-BASHFORTH2 METHOD

```
countervec5 = (linspace(1,4,4)).*0;
error5 = (linspace(1,4,4)).*0;

[exactval,~] = exact_solution(diffequation,x2);
[estval,counter] = AdamsB2(x2);
input = abs(exactval(end) - estval(end));
countervec5(1) = counter;
error5(1) = input;

[exactval,~] = exact_solution(diffequation,x3);
[estval,counter] =AdamsB2(x3);
input = abs(exactval(end) - estval(end));
countervec5(2) = counter;
error5(2) = input;

[exactval,~] = exact_solution(diffequation,x4);
[estval,counter] = AdamsB2(x4);
input = abs(exactval(end) - estval(end));
countervec5(3) = counter;
error5(3) = input;

[exactval,~] = exact_solution(diffequation,x6);
[estval,counter] = AdamsB2(x6);
input = abs(exactval(end) - estval(end));
countervec5(4) = counter;
error5(4) = input;
```

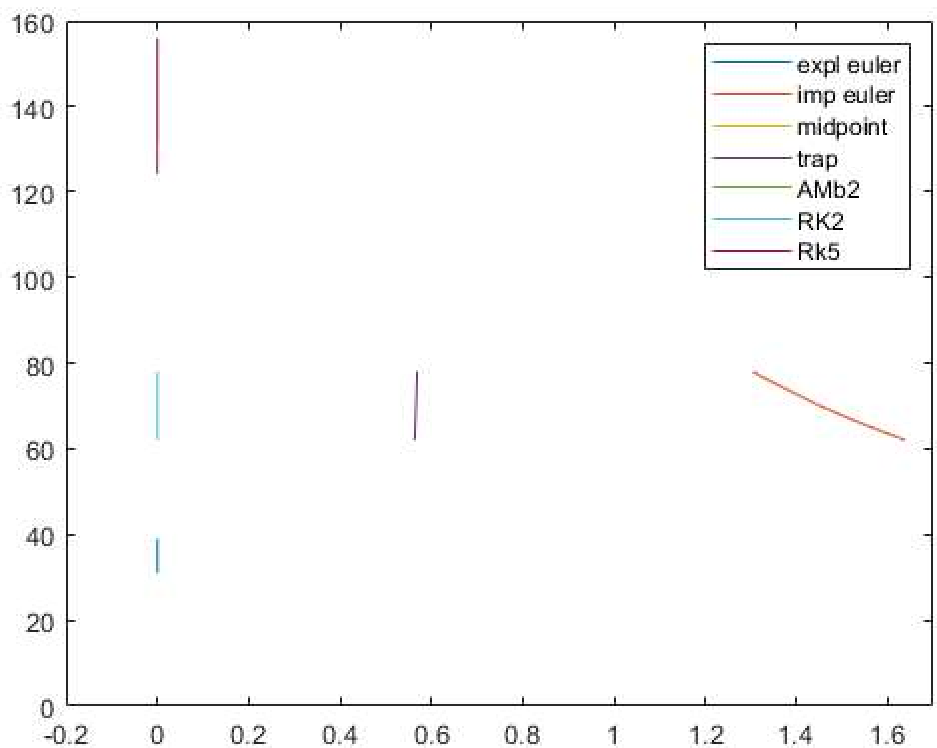
EXACT ODE VS RUNGE-KUTTA 2 METHOD

```
countervec6 = (linspace(1,4,4)).*0;
error6 = (linspace(1,4,4)).*0;

[exactval,~] = exact_solution(diffequation,x2);
[estval,counter] = RK2(x2);
input = abs(exactval(end) - estval(end));
countervec6(1) = counter;
error6(1) = input;

[exactval,~] = exact_solution(diffequation,x3);
[estval,counter] = RK2(x3);
input = abs(exactval(end) - estval(end));
countervec6(2) = counter;
error6(2) = input;

[exactval,~] = exact_solution(diffequation,x4);
[estval,counter] = RK2(x4);
input = abs(exactval(end) - estval(end));
countervec6(3) = counter;
error6(3) = input;

[exactval,~] = exact_solution(diffequation,x6);
[estval,counter] = RK2(x6);
input = abs(exactval(end) - estval(end));
countervec6(4) = counter;
error6(4) = input;
```

EXACT ODE VS RUNGE-KUTTA 4 METHOD

```
countervec7 = (linspace(1,4,4)).*0;
error7 = (linspace(1,4,4)).*0;

[exactval,~] = exact_solution(diffequation,x2);
[estval,counter] = RK4(x2);
input = abs(exactval(end) - estval(end));
countervec7(1) = counter;
error7(1) = input;

[exactval,~] = exact_solution(diffequation,x3);
[estval,counter] = RK4(x3);
input = abs(exactval(end) - estval(end));
countervec7(2) = counter;
error7(2) = input;

[exactval,~] = exact_solution(diffequation,x4);
[estval,counter] = RK4(x4);
input = abs(exactval(end) - estval(end));
countervec7(3) = counter;
error7(3) = input;

[exactval,~] = exact_solution(diffequation,x6);
[estval,counter] = RK4(x6);
input = abs(exactval(end) - estval(end));
countervec7(4) = counter;
error7(4) = input;
```

```
% counting power
figure(1)
plot(error1,countervec1) %blue
hold on
plot(error2,countervec2) %red
hold on
plot(error3,countervec3) %yellow
hold on
plot(error4,countervec4) %purple
hold on
plot(error5,countervec5) %green
hold on
plot(error6,countervec6) %blue
hold on
plot(error7,countervec7) %darkred
legend('expl euler','imp euler','midpoint','trap','AMb2','RK2','Rk5')
xlim([-0.2 1.7])
ylim([0 160])
```




```
%{
This graph conveys the idea that the higher the h, the faster the the graph
converges onto the exact solution. This shows that even though the solution
requires a lot more work to be done for a calculation, in the end it is
worth it because of how much faster it finishes. It also visualizes how
much faster the more advanced methods like RK4 are than Explicit Euler.
%}
```

```
num = 30;
for i = 2:5
```

```
    vec = linspace(0,1,num);
    powers = getpower(vec);

    fprintf('For size of num')
    disp(num)

    fprintf('Power with Eplicit Euler')
    disp(powers(i))
    fprintf('Power with Implicit Euler')
    disp(powers(i))
    fprintf('Power with Midpoint')
    disp(powers(i))
    fprintf('Power with Trapezoid Euler')
    disp(powers(i))
    fprintf('Power with Adams Bashforth Method')
    disp(powers(i))
    fprintf('Power with RK2')
    disp(powers(i))
    fprintf('Power with RK5')
    disp(powers(i))
    num = num + 2;
end
```

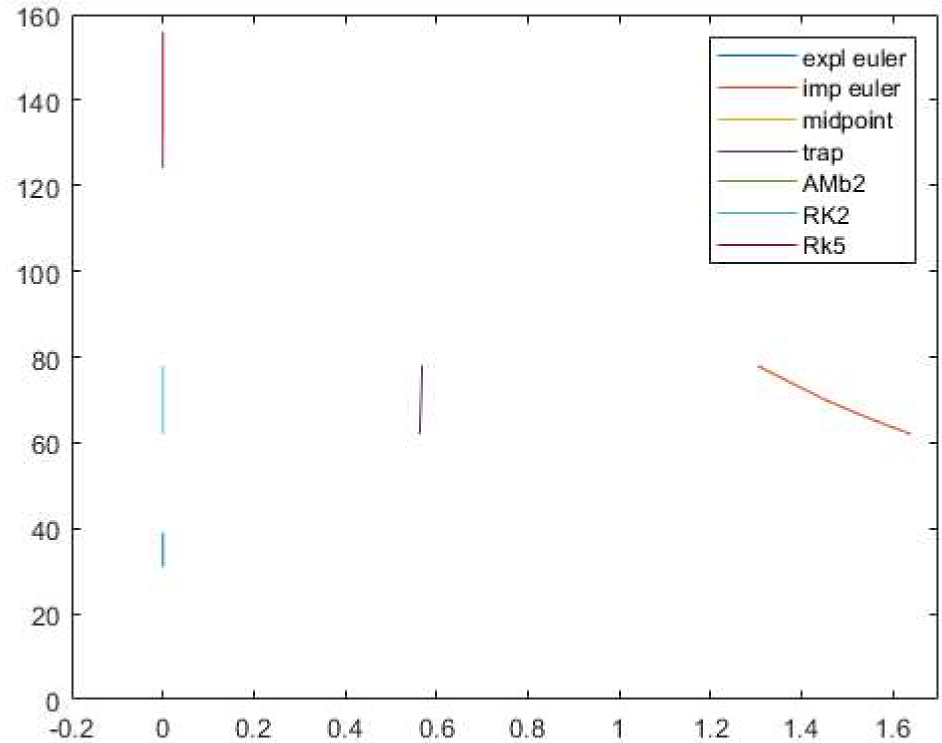
```
For size of num    30

Power with Eplicit Euler    29

Power with Implicit Euler    29

Power with Midpoint    29

Power with Trapezoid Euler    29

Power with Adams Bashforth Method    29

Power with RK2    29

Power with RK5    29

For size of num    32

Power with Eplicit Euler    62

Power with Implicit Euler    62

Power with Midpoint    62

Power with Trapezoid Euler    62

Power with Adams Bashforth Method    62

Power with RK2    62

Power with RK5    62

For size of num    34

Power with Eplicit Euler    66
```

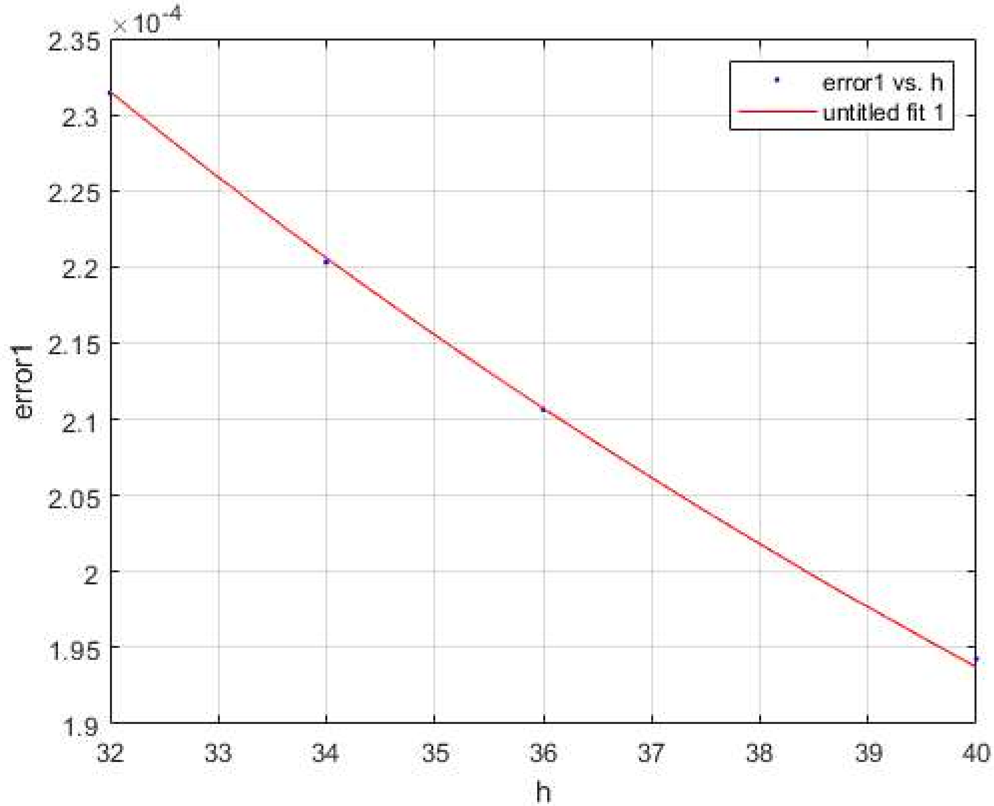
```
Power with Implicit Euler    66

Power with Midpoint    66

Power with Trapezoid Euler    66

Power with Adams Bashforth Method    66

Power with RK2    66

Power with RK5    66

For size of num    36

Power with Eplicit Euler    70

Power with Implicit Euler    70

Power with Midpoint    70

Power with Trapezoid Euler    70

Power with Adams Bashforth Method    70

Power with RK2    70

Power with RK5    70
```

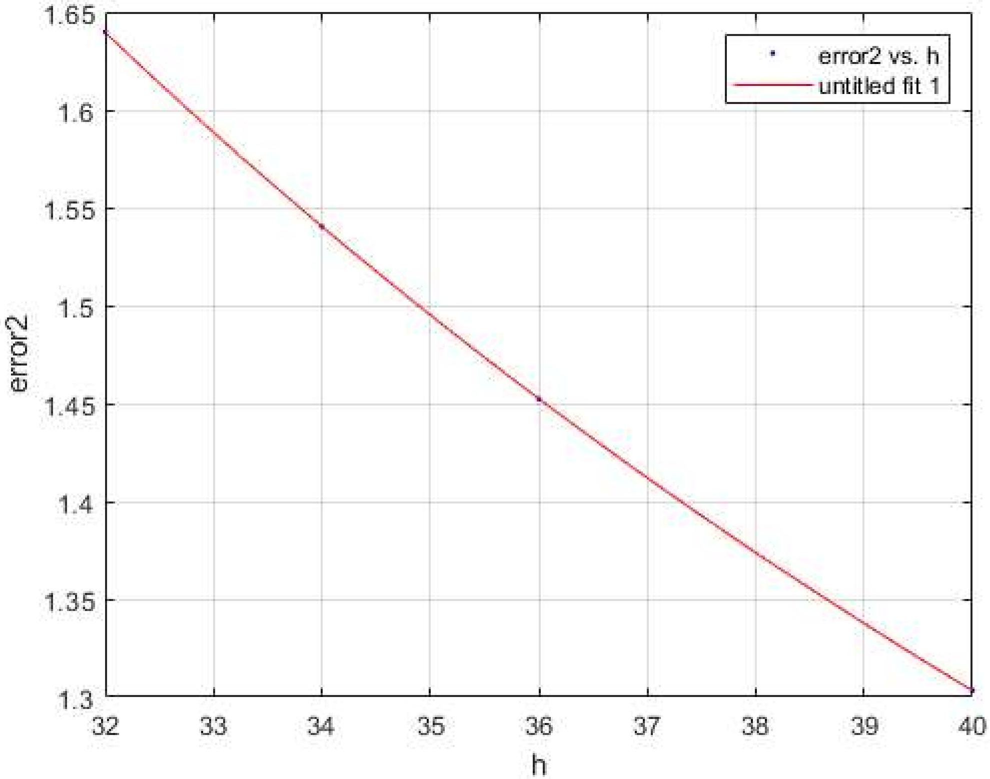
```
%For the Explicit Euler Method:
%Alpha = -0.7989
%For the Implicit Euler Method
%Alpha = -1.029
% For Midpoint.
% ALPHA = -7.047
% Trapezoid Method.
%ALPHA = 0.04238
%AdamsB2 Method
%ALPHA = 21.99
%RK2 Method
%ALPHA = -7.047
%RK4 Method
%Alpha =  -1.172
[xData, yData] = prepareCurveData( h, error1 );
ft = fittype( 'power1' );
opts = fitoptions( 'Method', 'NonlinearLeastSquares' );
opts.Display = 'Off';
opts.StartPoint = [0.0036911293190958 -0.798914698260166];
[fitresult, gof] = fit( xData, yData, ft, opts );
figure( 'Name', 'untitled fit 1' );
h_1 = plot( fitresult, xData, yData );
legend( h_1, 'error1 vs. h', 'untitled fit 1', 'Location', 'NorthEast', 'Interpreter', 'none' );
xlabel( 'h', 'Interpreter', 'none' );
ylabel( 'error1', 'Interpreter', 'none' );
grid on

%For the Implicit Euler Method
%Alpha = -1.029
[xData, yData] = prepareCurveData( h, error2 );
```

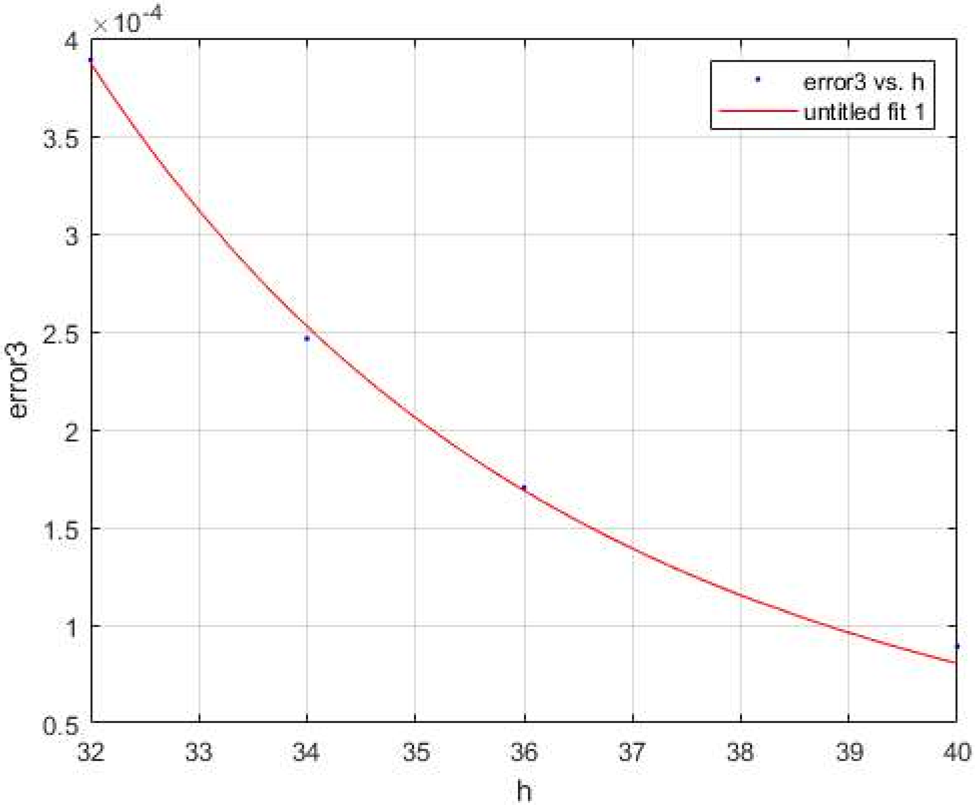
```
ft = fittype( 'power1' );
opts = fitoptions( 'Method', 'NonlinearLeastSquares' );
opts.Display = 'Off';
opts.StartPoint = [58.2972804846504 -1.03036117616061];
[fitresult, gof] = fit( xData, yData, ft, opts );
figure( 'Name', 'untitled fit 1' );
h_1 = plot( fitresult, xData, yData );
legend( h_1, 'error2 vs. h', 'untitled fit 1', 'Location', 'NorthEast', 'Interpreter', 'none' );
xlabel( 'h', 'Interpreter', 'none' );
ylabel( 'error2', 'Interpreter', 'none' );
grid on



% For Midpoint.
% ALPHA = -7.047
[xData, yData] = prepareCurveData( h, error3 );
ft = fittype( 'power1' );
opts = fitoptions( 'Method', 'NonlinearLeastSquares' );
opts.Display = 'Off';
opts.StartPoint = [16071999.4877197 -7.04748723927692];
[fitresult, gof] = fit( xData, yData, ft, opts );
figure( 'Name', 'untitled fit 1' );
h_1 = plot( fitresult, xData, yData );
legend( h_1, 'error3 vs. h', 'untitled fit 1', 'Location', 'NorthEast', 'Interpreter', 'none' );
xlabel( 'h', 'Interpreter', 'none' );
ylabel( 'error3', 'Interpreter', 'none' );
grid on



% Trapezoid Method.
%ALPHA = 0.04238
[xData, yData] = prepareCurveData( h, error4 );
ft = fittype( 'power1' );
opts = fitoptions( 'Method', 'NonlinearLeastSquares' );
opts.Display = 'Off';
opts.StartPoint = [0.48239321237515 0.0447063789106446];
[fitresult, gof] = fit( xData, yData, ft, opts );
figure( 'Name', 'untitled fit 1' );
h_1 = plot( fitresult, xData, yData );
legend( h_1, 'error4 vs. h', 'untitled fit 1', 'Location', 'NorthEast', 'Interpreter', 'none' );
xlabel( 'h', 'Interpreter', 'none' );
ylabel( 'error4', 'Interpreter', 'none' );
grid on

%AdamsB2 Method
%ALPHA = 21.99
[xData, yData] = prepareCurveData( h, error5 );
ft = fittype( 'power1' );
opts = fitoptions( 'Method', 'NonlinearLeastSquares' );
opts.Display = 'Off';
opts.StartPoint = [2.43119566493717e+44 -24.3901265610395];
[fitresult, gof] = fit( xData, yData, ft, opts );
figure( 'Name', 'untitled fit 1' );
h_1 = plot( fitresult, xData, yData );
legend( h_1, 'error5 vs. h', 'untitled fit 1', 'Location', 'NorthEast', 'Interpreter', 'none' );
xlabel( 'h', 'Interpreter', 'none' );
ylabel( 'error5', 'Interpreter', 'none' );
grid on
```

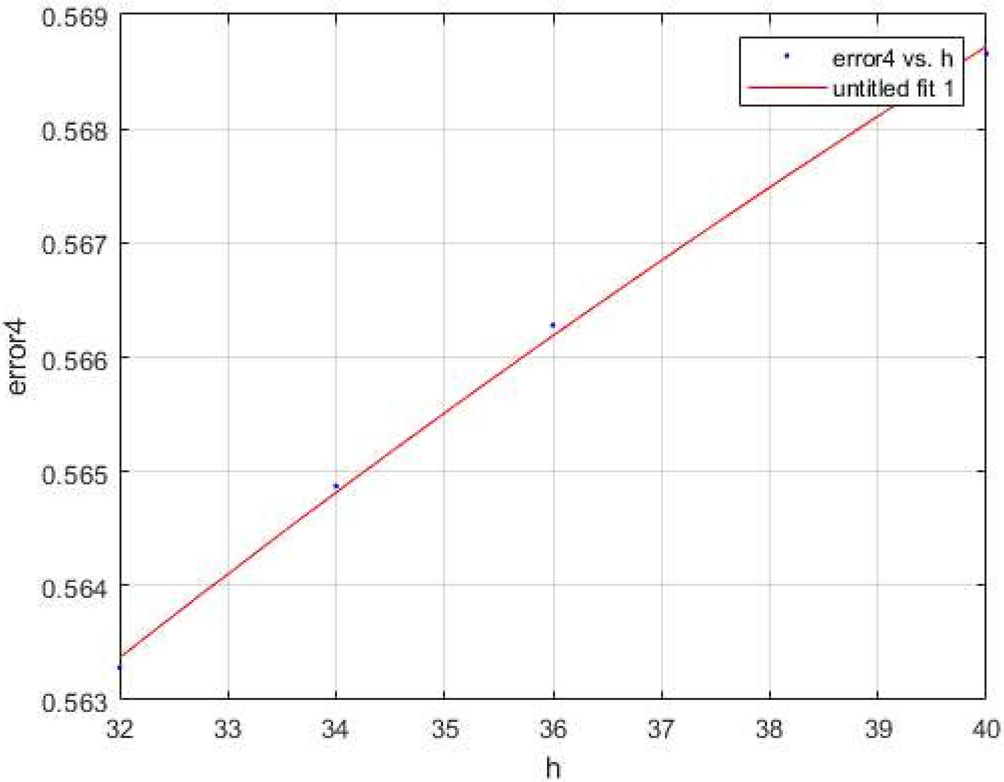
```
%RK2 Method
%ALPHA = -7.047
[xData, yData] = prepareCurveData( h, error6 );
ft = fittype( 'power1' );
opts = fitoptions( 'Method', 'NonlinearLeastSquares' );
opts.Display = 'Off';
opts.StartPoint = [16071999.4877197 -7.04748723927692];
[fitresult, gof] = fit( xData, yData, ft, opts );
figure( 'Name', 'untitled fit 1' );
h_1 = plot( fitresult, xData, yData );
legend( h_1, 'error6 vs. h', 'untitled fit 1', 'Location', 'NorthEast', 'Interpreter', 'none' );
xlabel( 'h', 'Interpreter', 'none' );
ylabel( 'error6', 'Interpreter', 'none' );
grid on

%RK4 Method
%Alpha =  -1.172
[xData, yData] = prepareCurveData( h, error7 );
ft = fittype( 'power1' );
opts = fitoptions( 'Method', 'NonlinearLeastSquares' );
opts.Display = 'Off';
opts.StartPoint = [1.10625849955209 -1.17162968027378];
[fitresult, gof] = fit( xData, yData, ft, opts );
figure( 'Name', 'untitled fit 1' );
h_1 = plot( fitresult, xData, yData );
legend( h_1, 'error7 vs. h', 'untitled fit 1', 'Location', 'NorthEast', 'Interpreter', 'none' );
xlabel( 'h', 'Interpreter', 'none' );
ylabel( 'error7', 'Interpreter', 'none' );
grid on

%{
For the Explicit Euler Method: Alpha = -0.7989
For the Implicit Euler Method: Alpha = -1.029
For Midpoint: ALPHA = -7.047
Trapezoid Method: ALPHA = 0.04238
AdamsB2 Method: ALPHA = 21.99
RK2 Method: ALPHA = -7.047
RK4 Method: Alpha =  -1.172
%}
```


expl euler
imp euler
midpoint
trap
AMb2
RK2
Rk5

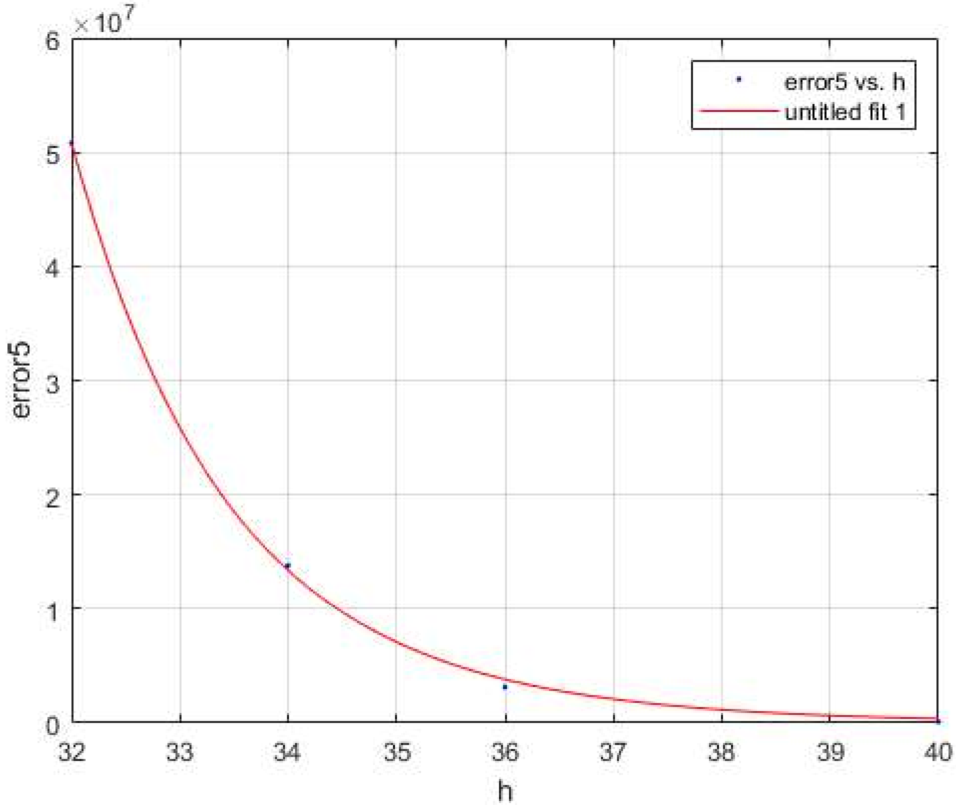

error1 vs. h
untitled fit 1
error1
h


error2 vs. h
untitled fit 1
error2
h

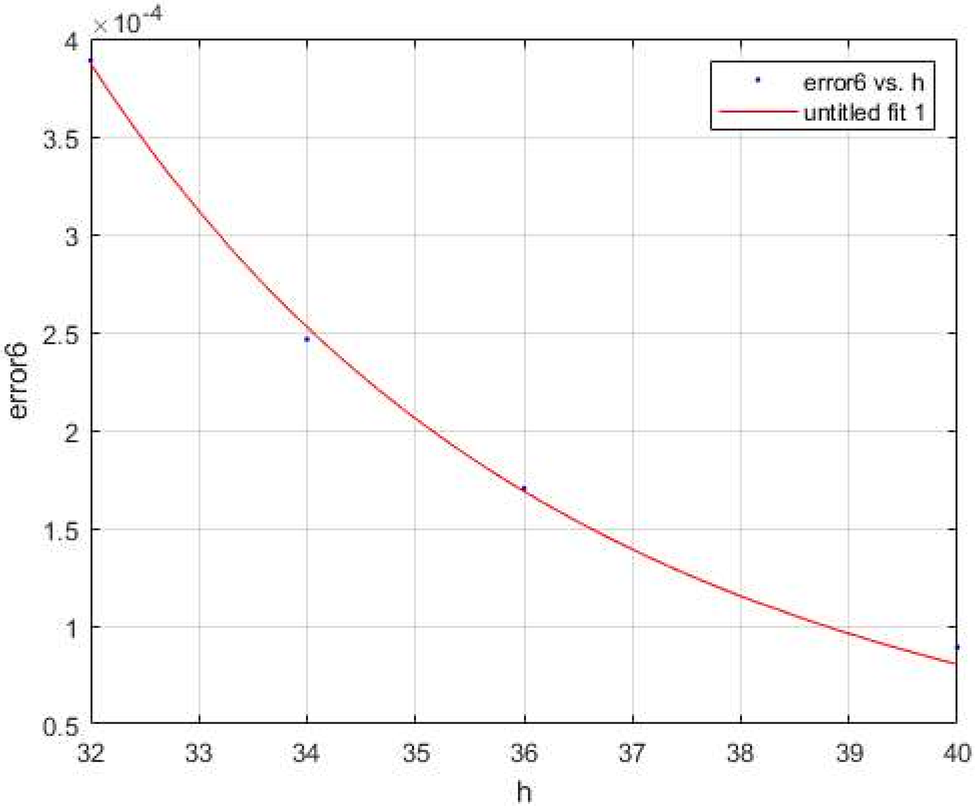

error3 vs. h
untitled fit 1
error3
h

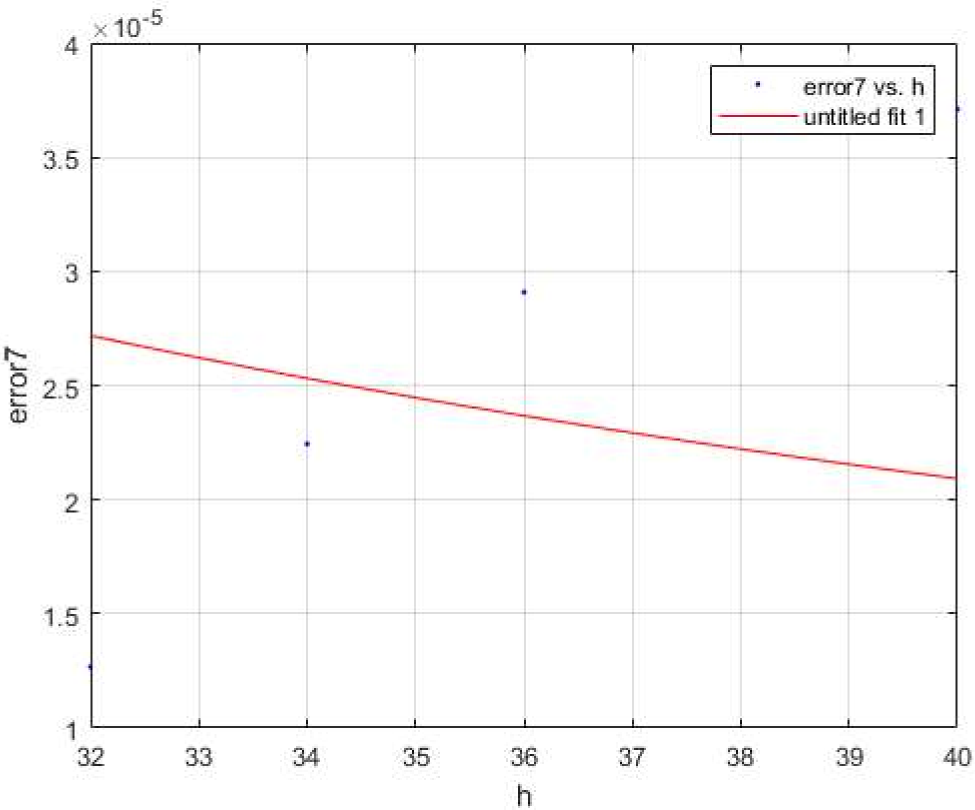


error4 vs. h
untitled fit 1
error4
h
0.569
0.568
0.567
0.566
0.565
0.564
0.563
32
33
34
35
36
37
38
39
40



error5 vs. h
untitled fit 1
$\times 10^7$
error5
h
6
5
4
3
2
1
0
32
33
34
35
36
37
38
39
40



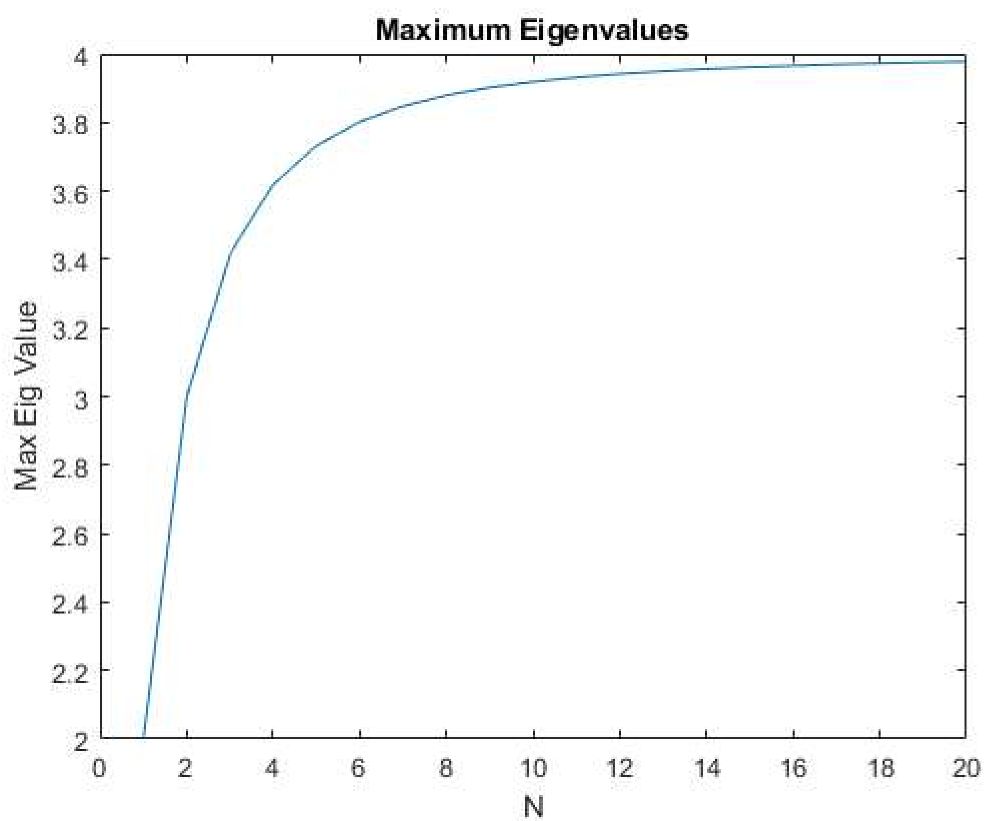


Question 2:

```
eig = zeros(length(i),1);
for i = 1:20
    eig(i) = max(abs(lambda((1:i)',i)));
```

```
end
plot(1:20, eig)
title('Maximum Eigenvalues')
ylabel('Max Eig Value')
xlabel('N')

eigen_values = (lambda((1:10)',10))';
disp(eigen_values)

%{
The amount of Eigen Values is linear to the amount of N the function has,
making this plot. This graph shows that how the eigenvalues approach 4 as N
goes to infinity.
%}
```

```
  Columns 1 through 7

   -0.0810   -0.3175   -0.6903   -1.1692   -1.7154   -2.2846   -2.8308

  Columns 8 through 10

   -3.3097   -3.6825   -3.9190
```




```
function [y,counter] = exact_solution(diffeq, xspan)
    [~, y] = ode45(diffeq,xspan, 0);
    counter = 1;

end
function [output,counter] = solve_equation(xval, yval, counter)
```

```
    counter = counter +1;
    diffequation=@(x,y) -50*(y - cos(x));
    output = diffequation(xval,yval);
end
function [output,counter] = Explicit_Euler(xspan)
    counter = 0;
    jump = xspan(2);
    output = xspan.*0;
    for i = 1: (length(xspan)-1)
           [soln, counter] = solve_equation(xspan(i), output(i),counter);
           output(i+1) = output(i) + jump*soln;
    end
end
function [output,counter] =Implicit_Euler(xspan)
    counter = 0;
    jump = xspan(2);
    misc = xspan.*0;
    output = xspan.*0;
    for i = 1: (length(xspan)-1)
           [soln, counter] =  solve_equation(xspan(i), misc(i),counter);
           misc(i+1) = misc(i) + soln*jump;
           [soln2, counter] = solve_equation(xspan(i+1), misc(i+1),counter);
           output(i+1) = output(i)+ jump*soln2 ;
    end
end
function [output,counter] =Midpoint(xspan)
    counter = 0;
    jump = xspan(2);
    halfjump = jump/2;
    output = xspan.*0;
    for i = 1: (length(xspan)-1)
        [soln, counter] =solve_equation(xspan(i),output(i),counter);
        [soln, counter] = solve_equation(xspan(i)+halfjump,output(i)+halfjump*soln,counter);
           output(i+1) = output(i) + jump*soln;
    end
end
function [output,counter] =trapezoidal(xspan)
    counter = 0;
    jump = xspan(2);
    halfjump = jump/2;
    output = xspan.*0;

    for i = 1: (length(xspan)-1)
            [soln, counter] = solve_equation(xspan(i), output(i),counter);
            [soln2, counter] = solve_equation(xspan(i+1), output(i+1),counter);
           output(i+1) = output(i)+ halfjump*(soln+soln2 );
    end
end
function [output,counter] =AdamsB2( xspan)
    counter = 0;
    jump = xspan(2);
    output = xspan.*0;
    for i = 1: (length(xspan)-2)
        [misc,counter] = solve_equation(xspan(i+1), output(i+1),counter);
        [misc2,counter]= solve_equation(xspan(i),output(i),counter);
           output(i+2) = output(i+1) + 0.5*jump*(3*misc-misc2);
    end
end

function [output,counter] = RK2( xspan)
    counter = 0;
```

```
    jump = xspan(2);
    output = xspan.*0;
    for i = 1: (length(xspan)-1)
            input1 = xspan(i)+ 0.5*jump;
            [misc,counter] = solve_equation(xspan(i),output(i),counter);
            input2 = output(i)+(0.5*jump*misc);
            [misc,counter] = solve_equation(input1,input2,counter);
            output(i+1) = output(i) + jump*misc;
    end
end
function [output,counter] = RK4(xspan)
    counter = 0;
    jump = xspan(2);
    halfjump = jump/2;
    output = xspan.*0;
    for i = 1: (length(xspan)-1)
            [input1,counter] = solve_equation(xspan(i),output(i),counter);
            [input2,counter] = solve_equation(xspan(i)+ halfjump,output(i)+halfjump*input1,counter);
            [input3,counter] = solve_equation(xspan(i)+ halfjump,output(i)+halfjump*input2,counter);
            [input4,counter] = solve_equation(xspan(i)+ jump,output(i)+jump*input3,counter);
            output(i+1) = output(i) + jump*((1/6)*input1+(2/6)*input2+(2/6)*input3+(1/6)*input4);
    end
end
function [countervec] = getpower(xspan)
    diffequation=@(x,y) -50*(y - cos(x));
    countervec = (linspace(1,8,8)).*0;
    [~,counter] = exact_solution(diffequation,xspan);
    countervec(1) = counter;
    [~,counter] = Explicit_Euler(xspan);
    countervec(2) = counter;
    [~,counter] = Implicit_Euler(xspan);
    countervec(3) = counter;
    [~,counter] = Midpoint(xspan);
    countervec(4) = counter;
    [~,counter] = trapezoidal(xspan);
    countervec(5) = counter;
    [~,counter] = AdamsB2(xspan);
    countervec(6) = counter;
    [~,counter] = RK2(xspan); %just need M
    countervec(7) = counter;
    [~,counter] = RK4(xspan);
    countervec(8) = counter;
end
function lambda = lambda(x,y)
    lambda  = -2*(1-cos(pi*x/(y+1)));
    return
end
```