MEMORANDUM

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DATE: 29 October 2018
SUBJECT: Assignment 2 Submission: Trip Distribution Analysis: Modeling of Trip Interchanges using Gravity Models

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Introduction

The results from the four-step travel demand model provide insight for how people move around, with each component providing crucial analysis into trip-making behaviour. The four-step model consists of trip generation, trip distribution, mode choice, and trip assignment. The basic analysis quantity of the travel demand model is a “trip”: defined by a singular person’s movement on the transportation system from one “location” to another. The most common spatial unit used to represent “location” is a traffic analysis zone (TAZ).

Zonal trip productions and attractions were generated with trip generation, and now for trip distribution, analysis is considered to determine the various trip interchanges between TAZ, using the generated and balanced quantities of trip productions and attractions as a starting point to the analysis. For trips that are considered “non-home based” (neither end of the trip involves the traveler’s home), trip productions for a TAZ usually correspond with trip origins departing that TAZ, and conversely trip attractions for a TAZ correspond with trips destined for that TAZ.

Two categories of trip distribution models are often used: growth factor methods and gravity models. Growth factor methods apply a growth rate to the observed trips of a study area to determine trip distributions (either to a future horizon year scenario, or toward a study area with similar characteristics as the observed study area). Growth rates may be applied uniformly throughout (uniform growth rate) or applied to each TAZ’s productions and attractions separately (doubly constrained Furness Method) among other growth factor methods (i.e. Fratar Method).

Gravity models in transportation forecasting is analogous to the Newtonian law of universal gravitation, drawing parallels to a stronger force of attraction between bodies if the bodies are sufficiently large and within proximity. The transportation context can be rethought as the quantity of trips from one TAZ to another is higher if each TAZ has a proportionally high quantity of productions, attractions, and that the travel cost or deterrence to travel is low. Trip interchanges using a doubly constrained gravity model (the method used for the remainder of this memorandum) are generally found via the following equation:

$$\boldsymbol{T_{ij} = A_i O_i B_j D_j f(C_{ij})}$$

Where $\boldsymbol{i}$ represents the origin TAZ of a trip, $\boldsymbol{j}$ for the TAZ the trip is destined, $\boldsymbol{O_i}$ and $\boldsymbol{D_j}$ are the sums of total origins and destinations for the respect zones, and $\boldsymbol{f(C_{ij})}$ is the generalized cost/deterrence function that trip-makers are subject to under this model. $\boldsymbol{A}$ and $\boldsymbol{B}$ are balancing factors under the form:

$$A_i = \left(\sum_{i=1}^{N} B_j D_j f(C_{ij})\right)^{-1}, B_j = \left(\sum_{j=1}^{N} A_i O_i f(C_{ij})\right)^{-1}$$

A way to convey the results of trip distribution analysis is through the formulation of a trip interchange matrix, in the form below:

$$\begin{bmatrix} Zones & 1 & 2 & \dots & n & O_i \\ 1 & T_{11} & T_{12} & \dots & T_{1n} & O_1 \\ 2 & T_{21} & T_{22} & \dots & T_{2n} & O_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ n & Tn_1 & T_{n2} & \cdots & T_{nn} & O_n \\ D_j & D_1 & D_2 & \dots & D_n & \Sigma T_{ij} \end{bmatrix}$$

The first column represents origin TAZ that trips can depart from, zone i. The first row represents destination TAZ that trips can depart to, zone j. Trip patterns from origin i to destination j are represented in the form T_{ij}. O_i and D_j are the sums of total origins and destinations for the respective zones.

Scope/Objectives

This memorandum focuses on calibrating the gravity model for an observed dataset of non-home-based trip distributions. Since the zonal productions and attractions are fixed due to the observed dataset, the only parameter that requires calibration is that of the cost/deterrence function.

The three primary objectives are listed:

1. A generalized travel cost function will be determined for trips between TAZ of the dataset.
2. The doubly constrained gravity model will be determined for the determined travel cost function in conjunction with the observed trip productions and attractions. This will require calibration of the presently unknown dimensionless parameters of three proposed forms of the deterrence function.
3. The preferred deterrence function will be presented after quantitatively evaluating which model is preferred.

A dataset provided by Prof. Tae J. Kwon is provided for analysis. It consists of base year observed trip distributions between 100 TAZ, travel time between TAZ (for automobiles in minutes), shortest path travel distance between TAZ (for automobiles in kilometres), and automobile parking costs for a specific TAZ.

A doubly constrained gravity model is required because the base year data is provided, meaning the trip productions and attractions for each TAZ is fixed, unchanging, and the calibrated models should reflect that characteristic as closely as possible.

The generalized travel cost function will be determined by normalizing the provided data into monetary figures. This requires analysis into the typical value of time for commuters and typical vehicle travel costs.

The deterrence function can take three general forms: as a power function, as an exponential function, and as a top-exponential function:

$$f(C_{ij}) = C_{ij}^{-b}, or\ f(C_{ij}) = \exp(-b \times C_{ij}), or\ f(C_{ij}) = (C_{ij})^{a} \exp(-b \times C_{ij})$$

The parameters that require calibration will be the a and b-values of the above equations.

Procedure

To meet the first objective of finalizing a general travel cost function, research is required to determine how to normalize travel time cost and travel distance cost into monetary figures. Unit conversion factors for dollars per automobile travel minute and dollars per automobile travel kilometre are determined. Research avenues will focus on recent Canadian data where available. The monetary figures are obtained and summed together into a cost matrix (similar in structure to a trip interchange matrix) that can represent the monetary cost that travelers bear when travelling between TAZ. It is noteworthy to mention that the automobile parking cost only applies to the destination, so each cost is only considered in each TAZ's j column rather than TAZ in row i.

To calibrate the unknown parameters of the gravity model deterrence function, the procedure is computerized via a programming script. In MATLAB, a script is written that tests a range of a and b-values of the deterrence function by generating hundreds of trip interchange matrices and comparing them with the observed base year data. The scripts are included as attachments to this memorandum and important functions are commented for readability.

Each estimated trip interchange matrix is recorded and compared against the base-year trip interchange matrix to determine each estimation's respective root mean square error (RMSE). The a and b-value corresponding to the lowest RMSE is selected as the calibrated parameters for the gravity model. This is not a graphical method of calibrating with minimum RMSE.

To save on computational power in defense of inefficient code, multiple searches are performed for each search of the unknown parameter. For instance, in the search of the $\boldsymbol{b}$-value of the exponential deterrence function of the form $f(C_{ij}) = e^{-b \times C_{ij}}$, the initial search is performed for test b-values from [-10,10] with a step of 0.1. Once the b-value is found to one decimal place, the second search narrows around the b-value: [-1,0] in steps of 0.01. The parameters in this memorandum are found to three decimal places. This method involves more human input, however the script is able to accommodate an exhaustive search at a small step and large interval at the cost of computational power.

The preferred deterrence function is chosen by evaluating several factors:

- Lowest RMSE among other deterrence functions
- Observed and estimated trip length frequency distributions
- Engineering judgement

Comparing the estimated and observed frequency distributions require PivotTables and data cleanup. Each cell of the generalized cost function (representing cost of a traveller from zone i to j) is rounded down to create histogram bins. The frequency distribution is then created for power function, exponential function, top-exponential function estimated trips and observed trips.

Results

Although the dataset being analyzed is for non-home-based trips (which are necessarily work related trips), it may not seem intuitive at first to use labour figures to assign monetary values to travel time. However, since people often value their time based on their employment worth, using labour cost is a reasonable assumption to measure travel time.

For example, for a businessperson contemplating whether to clean their home, they could think:

"I could spend 3 hours cleaning, which is worth $XX of my time at work, or I could pay a maid to do it for near-equivalent or more inexpensive value."

Or a teenager using the wages from their first job to buy a desired item:

"This $350 gaming console costs XX hours of my time and labour to attain."

Another example is an hourly wage restaurant worker experiencing a train delay, they could think,

"If I am thirty minutes late to my shift, then I will lose $XX from my paycheque for today."

Or contemplating choosing between different modes:

"I could drive like I always do, or I could take a taxi, which is worth XX hours of my time and labour at my place of employment."

Since the data provided is only for automobile travel time, we cannot assume convenience factors that other modes provide for reducing the value of time. For example, a businessperson commuting via wifi-enabled commuter train could work on their laptop, therefore receiving productive value out of their chosen mode and reduce the cost of travel for them and their employment/employer.

According to Statistics Canada (2016) the average 2015 median total income for all Canadians is $70,336. Assuming 260 annual workdays (this includes paid statutory and civic holidays in Canada) at 8 working hours per day, there are 2080 annual working hours per year. The value per hour of the median working Canadian is thus $33.815 per hour. The full calculations are included in Appendix A.

A report from Barnes and Langworthy (2004) quantifies the costs of operating a vehicle. In 2003, the baseline cost for operating a vehicle was US$0.153 per mile. Assuming a rate of inflation of 2.13% (Inflation Calculator 2015) to 2015 value, an exchange rate of C$1.39/US$1 (MBH Media 2015), a 2003 average fuel cost of $1.50/gal (Barnes and Langworthy 2004), and a 2015 fuel cost of $2.4/gal (Green 2015), the 2015 operating cost of a vehicle is determined to be C$0.2245 per kilometre. The full calculations are included in Appendix B.

To calibrate the unknown parameters of the gravity model deterrence function, several searches are performed, as detailed in Table 1. The chosen parameters are bolded in red.

Table 1: Searching for the unknown parameters of the gravity model deterrence function, chosen values in bold red.

	Search Range	Step	a-/b-value, RMSE
Power Function $f(C_{ij}) = C_{ij}^{-b}$	[-3, 3]	0.01	b = 0.37 RMSE = 2.5961
	[0.33, 0.4]	0.001	b = [0.370,0.376] (take the average) **b = 0.373** **RMSE = 2.5961**
Exponential Function $f(C_{ij}) = e^{-b\times C_{ij}}$	[-3, 3]	0.01	b = 0.27 RMSE = 2.3741
	[0.25, 0.3]	0.001	b = [0.265, 0.268] (take the average) **b = 0.2665** **RMSE = 2.3741**
Top-exponential Function $f(C_{ij}) = (C_{ij})^a e^{-b\times C_{ij}}$	a: [-10, 10] b: [-10, 10]	0.5	a = 0 b = 0.5 RMSE = 2.3943
	a: [-0.5, 1] b: [0, 1.5]	0.1	a = 0.2 b = 0.4 RMSE = 2.3629
	a: [0.1, 0.3] b: [0.3, 0.5]	0.01	a = 0.15 b = 0.35 RMSE = 2.3560
	a: [0.13, 0.17] b: [0.33, 0.37]	0.001	**a = 0.154** **b = 0.354** **RMSE = 2.3559**

This search method is like the graphical method of plotting a-/b-values versus RMSE, this computational method is simply a brute force version. Comparing by sheer RMSE value alone, the gravity model that yields the lowest RMSE is one that includes the top-exponential function at RMSE = 2.3559, as shown below:

$$f(C_{ij}) = (C_{ij})^{0.154} \times \boldsymbol{e}^{-\mathbf{0.354}\times \boldsymbol{C_{ij}}}$$

The frequency distribution for travel cost among estimated and observed trips is shown below in Table 2 and Figure 1.

Table 2: Frequency Distribution for Estimated Trips and Observed Trips with respect to Travel Cost ($)

Cij ($)	powerf Count	expf Count	topexp Count	Observed Count
0	125	125	125	125
1	319	319	319	319
2	607	607	607	607
3	854	854	854	854
4	1124	1124	1124	1124
5	1157	1157	1157	1157
6	1061	1061	1061	1061
7	893	893	893	893
8	748	748	748	748
9	711	711	711	711
10	599	599	599	599
11	548	548	548	548
12	438	438	438	438
13	324	324	324	324
14	208	208	208	208
15	118	118	118	118
16	81	81	81	81
17	45	45	45	45
18	27	27	27	27
19	9	9	9	9
20	2	2	2	2
21	2	2	2	2
SUM	10000	10000	10000	10000

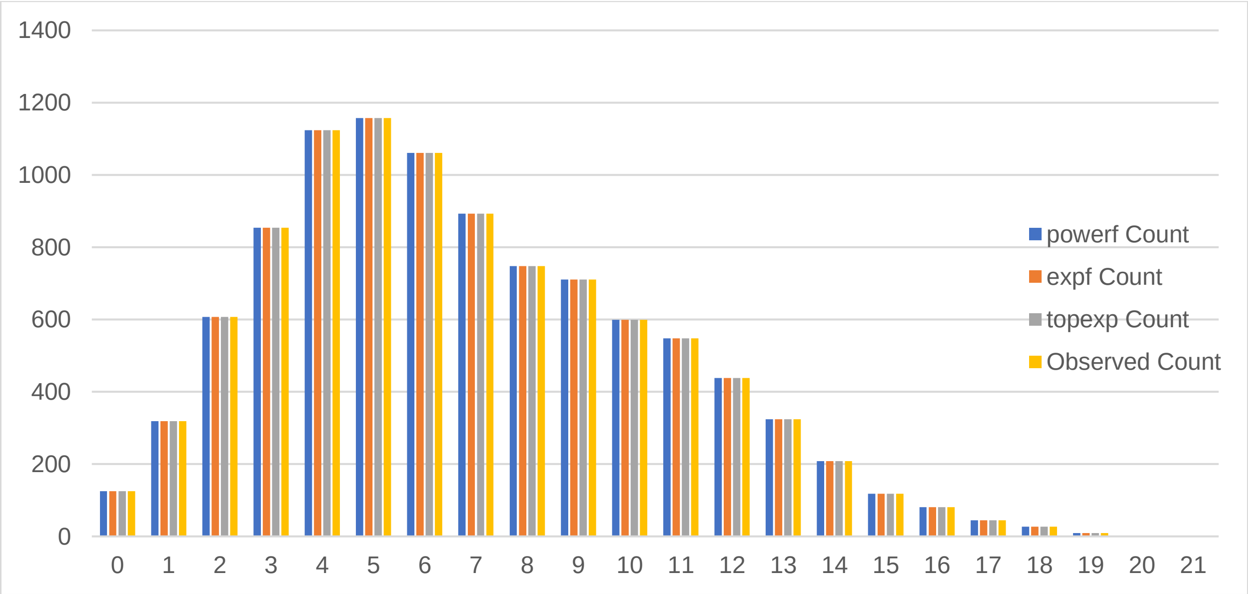

Figure 1: Graphical Presentation of the Frequency Distribution of Estimated Trips and Observed Trips

It is shown from the frequency distribution that no model differs from the observed trip distribution with respect to travel cost (when cost is rounded down to the nearest integer to create histogram bins).

The final trip distribution matrices are presented as attachments to this memorandum.

Conclusions/Recommendations

Research into an average working Canadian's value of time was determined to be $33.815 per hour, using a combination of census and calendar data. The average Canadian operating cost of an automobile per kilometre was determined to be C$0.2245.

Unknown a- and b-values were determined for the following forms of the gravity model deterrence function:

- Power function $f(C_{ij}) = C_{ij}^{-b}$, b = 0.373 for RMSE = 2.5961
- Exponential function $f(C_{ij}) = e^{-b \times C_{ij}}$, b = 0.373 for RMSE = 2.5961

Top-exponential function $f(C_{ij}) = (C_{ij})^{a} e^{-b \times C_{ij}}$, a = 0.154, b = 0.354 for RMSE = 2.3559

The preferred model is the top-exponential function in the form:

$$f(C_{ij}) = (C_{ij})^{0.154} \times \boldsymbol{e^{-0.354 \times C_{ij}}}$$

The metrics and measurements that determine this is the lowest RMSE value (2.3559) of the proposed calibration functions, as well as not differing from the trip cost and frequency distribution from the observed base year data.

The model presented is subject to some limitations, however. The most observable limitation present in this memorandum is that trip interchanges will inherently be created across TAZ that have no observed trips. Another limitation is that the generalized cost of travel is too simplistic in that it only considers the automobile mode of transport (i.e. travel times, travel costs, and out of pocket costs for transit or bicycle are not provided). Limitations in the data include the travel distance between each zone provided; only the distance of the shortest path was given, but in the case of inter-zonal transport offering multiple paths, other metrics than the shortest distance should be considered.

References

Barnes, G., and P. Langworthy. Per-mile Costs of Operating Automobiles and Trucks. *Transportation Research Record: Journal of the Transportation Research Board*, 2004. pp. 71-77.

MBH Media, Inc. *Canadian Dollars (CAD) to US Dollars (USD) exchange rate for December 31, 2015.* Exchange-Rates.org. **https://www.exchange-rates.org/Rate/CAD/USD/12-31-2015**. Accessed Oct 28, 2018.

Green, M. *AAA NewsRoom 2015 Gas Prices Second-Cheapest a Decade AAA Year-End Gas Price Report*. National AAA. **https://newsroom.aaa.com/2015/12/2015-gas-prices-second-cheapest-in-a-decade-aaa-year-end-gas-price-report/.** Accessed Oct 28, 2018.

Inflation Calculator *U.S. Inflation Rate, $100 in 2003 to 2015.* Official Data Foundation, Alioth LLC. **http://www.in2013dollars.com/2003-dollars-in-2015**. Accessed Oct 28, 2018

Minister of Natural Resources. *Fuel Focus, 2015 Annual Review.* Natural Resources Canada, Ottawa, Ontario. **https://www.nrcan.gc.ca/energy/fuel-prices/gasoline-reports/18031**. Accessed Oct 28, 2018

Statistics Canada. *Income of individuals by age group, sex and income source, Canada, provinces and selected census metropolitan areas.* Statistics Canada, Ottawa, Ontario. **https://www150.statcan.gc.ca/t1/tbl1/en/tv.action?pid=1110023901**. Accessed Oct 28, 2018

The Daily. *Household Income in Canada: Key results from the 2016 Census* Statistics Canada, Ottawa, Ontario. **https://www150.statcan.gc.ca/n1/daily-quotidien/170913/dq170913a-eng.htm.** Accessed Oct. 28, 2018

Appendix A: Travel Time Cost to an Individual

From Statistics Canada 2016 Census Report		Standard workday hours
Location	2015 Median TOTAL Income	
Canada	$ 70,336.00	8

Annual # of working days (Only Weekends Excluded; Assumes **PAID public holidays**)			
Annual Workdays	Total Annual Hours	Value/Hour	Value/Minute
260	2080	$ 33.82	$ 0.56

Appendix B: Operating Costs of a Vehicle

Assumes fuel costs at $1.50USD/gallon (3.785L) (2003 rates)			
	2003	Rates	
Baseline Cost (cents per mile)			
Cost Category	Automobile	Pickup/Van/ SUV	Commercial Truck
Total	15.3	19.5	43.4
Fuel	5.0	7.8	21.4
Maintenance/ Repair	3.2	3.7	10.5
Tires	0.9	1.0	3.5
Depreciation	6.2	7.0	8.0

	2015	Rates	
Baseline Cost (cents per mile)			
Cost Category	Automobile	Pickup/Van/ SUV	Commercial Truck
Total	23.6	31.1	72.4
Fuel	10.3	16.1	44.1
Maintenance/ Repair	4.1	4.8	13.5
Tires	1.2	1.3	4.5
Depreciation	8.0	9.0	10.3