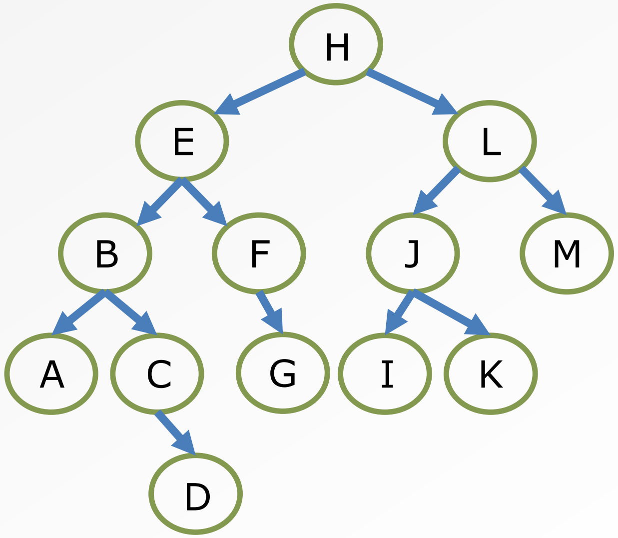
CE1107/CZ1107: DATA STRUCTURES AND ALGORITHMS

Binary Search Trees

College of Engineering
School of Computer Science and Engineering

OUTLINE

- **Binary Search Trees (BST)**
- BST Operations:
  - Traversal
  - Inserting a node
  - Removing a node

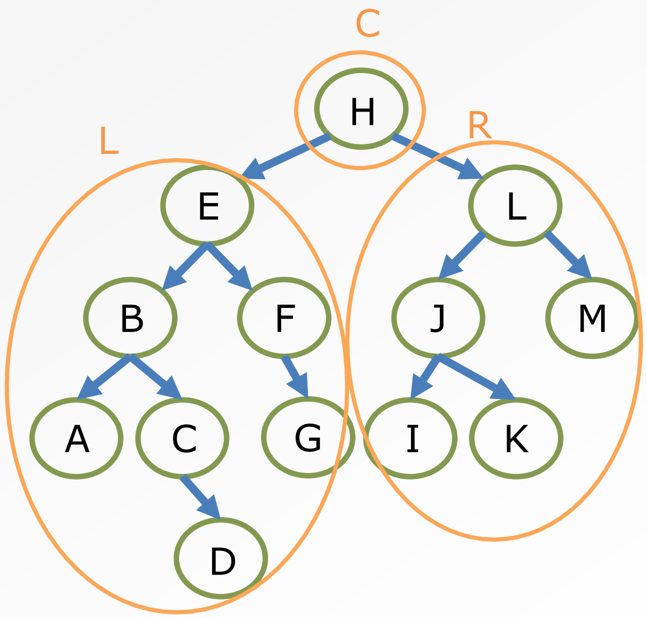


- BSTs are a special form of BT
- BST rule:

  At every node C,

  L < C < R, where

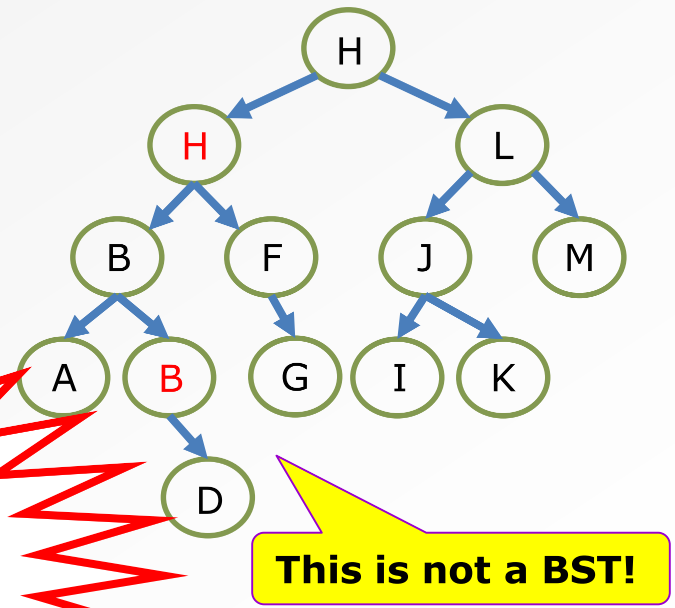
  - C is the data in the current node
  - L represents the data in any/ all nodes from C's left subtree
  - R represents the data in any/all nodes from C's right subtree



- BSTs are a special form of BT
- At every node C,

  L <= C <= R, where

  - C is the data in the current node
  - L represen[...]s t[...]e d[...]ta in an[...]



NO **=** in the BST!
There must be no duplicate nodes in BST!

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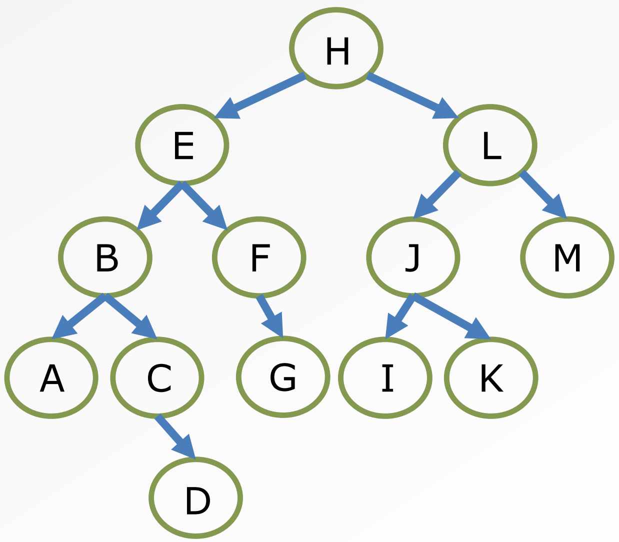

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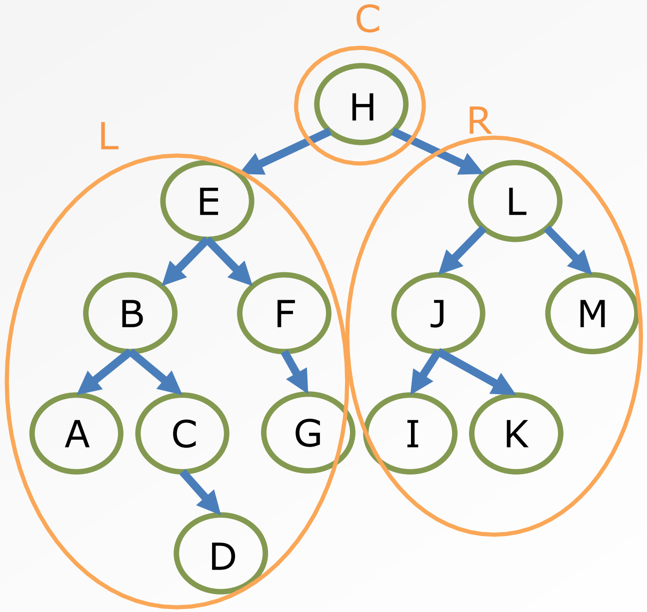
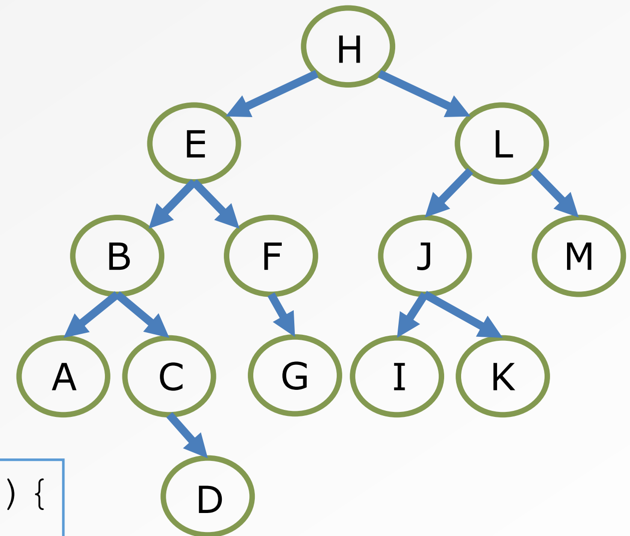


BST TRAVERSAL (BSTT)

- BSTT() traverses a BST to search for a node with a matching item
- Begin with TreeTraversal template




```
void BSTT(BTNode *cur, char c){

    if (cur == NULL)
       return;

    // Do something

    BSTT(cur->left);
    BSTT(cur->right);
}
```

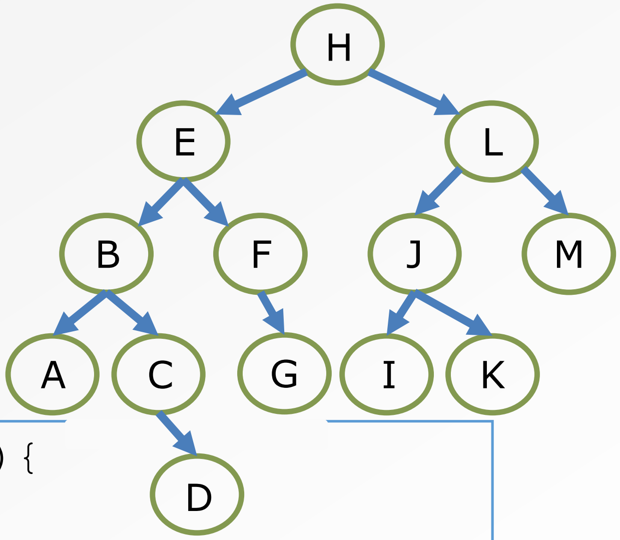
Do something with the current node's data

Visit the left child node

Visit the right child node

BST TRAVERSAL (BSTT)

- Now, at each node, we need to determine which subtree to keep visiting (and which subtree to ignore)




```
void BSTT(BTNode *cur, char c){
    if (cur == NULL) return;

      //do something
    if (c < cur->item)
        BSTT(cur->left,c);
    else
        BSTT(cur->right,c);
}
```

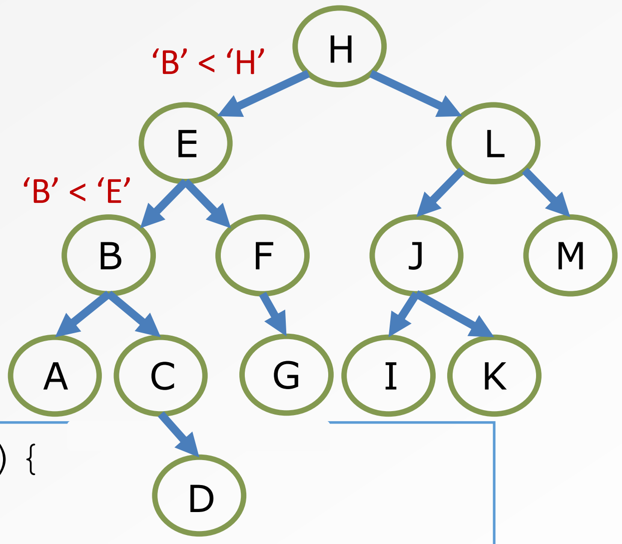
Do something with the current node's data

Visit the left child node

Visit the right child node

BST TRAVERSAL (BSTT)

- Check the traversal pattern for **BSTT(root, 'B')**



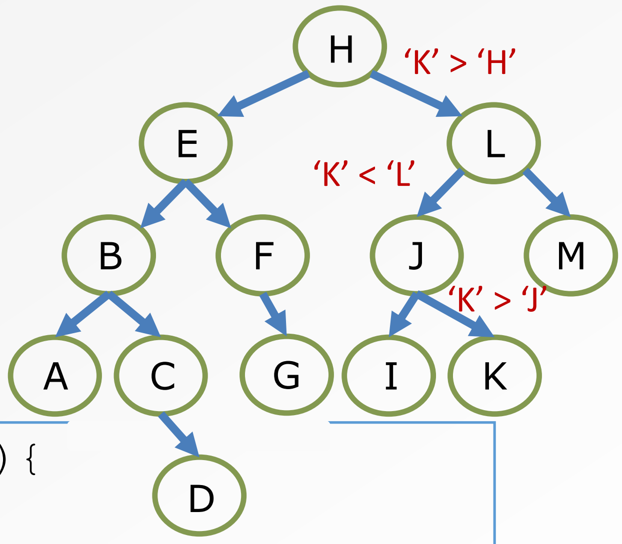

```
void BSTT(BTNode *cur, char c){
    if (cur == NULL) return;
    if (c==cur->item)
    { printf("found!\n"); return;}
    if (c < cur->item)
        BSTT(cur->left,c);
    else
        BSTT(cur->right,c);
}
```

BST TRAVERSAL (BSTT)

- Check the traversal pattern

  for

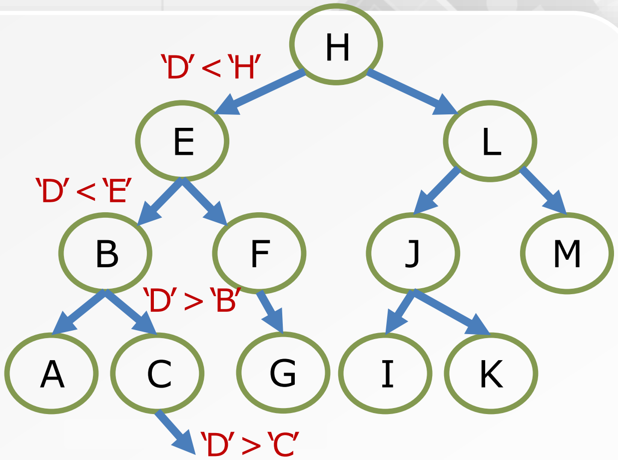
  BSTT(root, 'K')




```
void BSTT(BTNode *cur, char c){
    if (cur == NULL) return;
    if (c==cur->item)
    { printf("found!\n"); return;}
    if (c < cur->item)
        BSTT(cur->left,c);
    else
        BSTT(cur->right,c);
}
```

BST TRAVERSAL (BSTT)

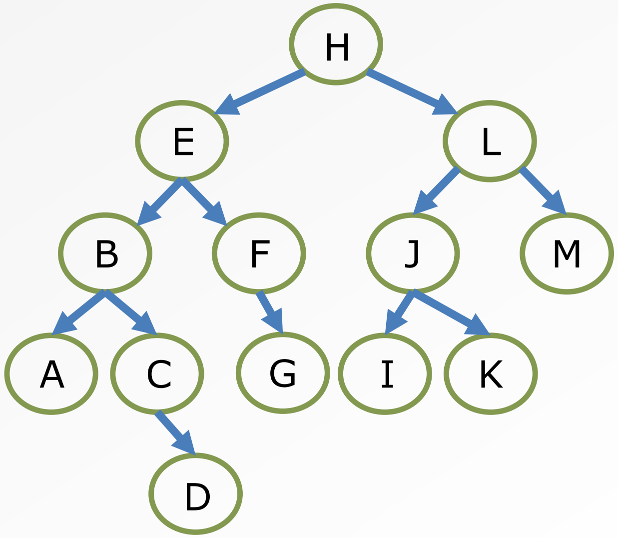
- What if the item doesn’t exist?
- If we remove node ‘D’, and then check the traversal pattern for **BSTT(root, ‘D’)**




```
void BSTT(BTNode *cur, char c){
     if (cur == NULL){
        printf(“can’t find!”);return;}
     if (c==cur->item){
        printf(“found!\n”); return;}
     if (c < cur->item)
         BSTT(cur->left,c);
     else
         BSTT(cur->right,c);
}
```

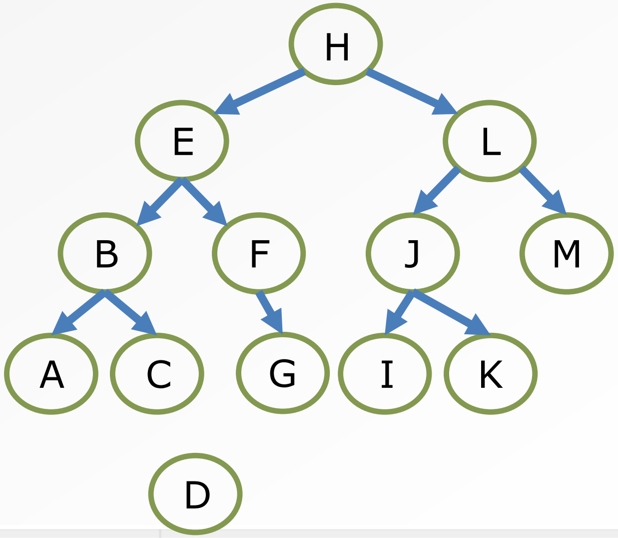
OUTLINE

- Binary Search Trees (BST)
- BST Operations:
  - Traversal
  - **Inserting a node**
  - Removing a node



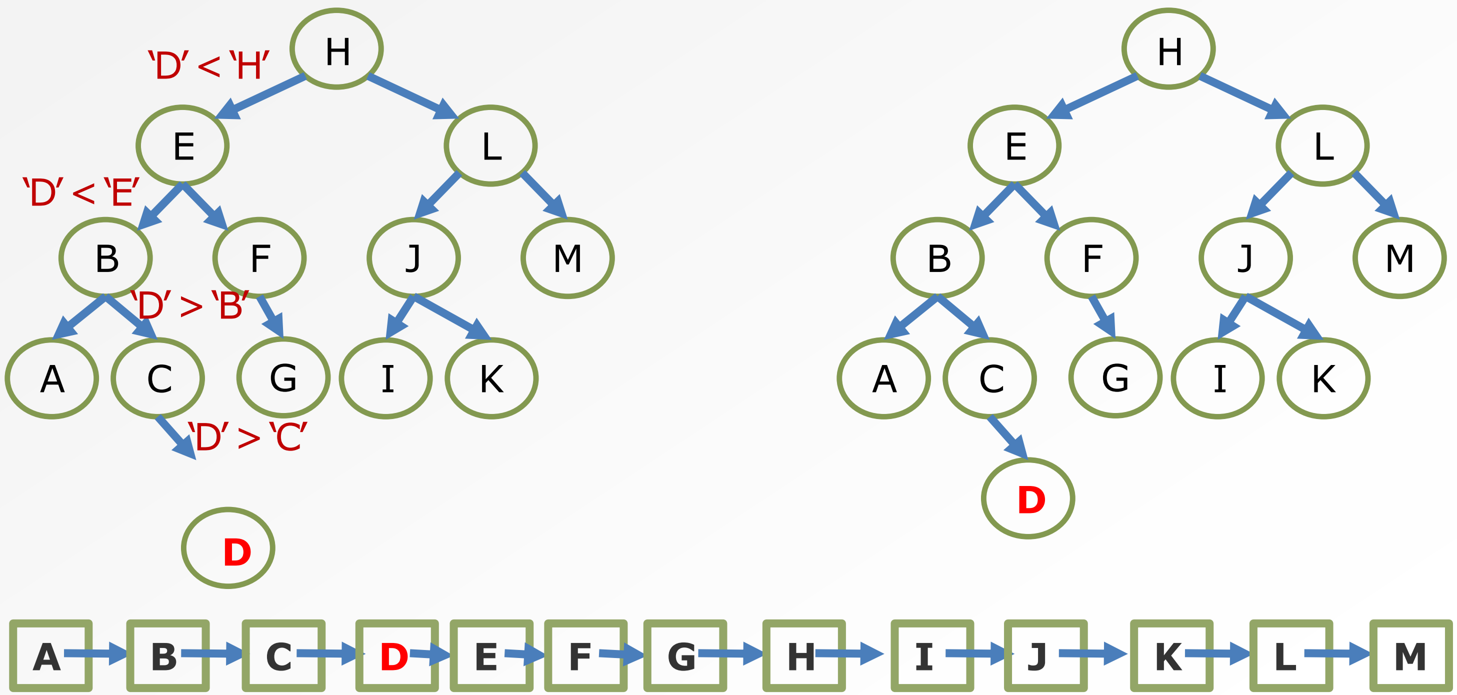
INSERTING A NODE INTO A BST

- Given an existing BST, an insertion operation must result in a BST
- How do we know where to place a new node 'D'?
- Given an existing BST and a new value to store, there is always a unique position for the new value



INSERTING A NODE INTO A BST

1. Use BSTT() to get to the correct empty location
2. Add the new node



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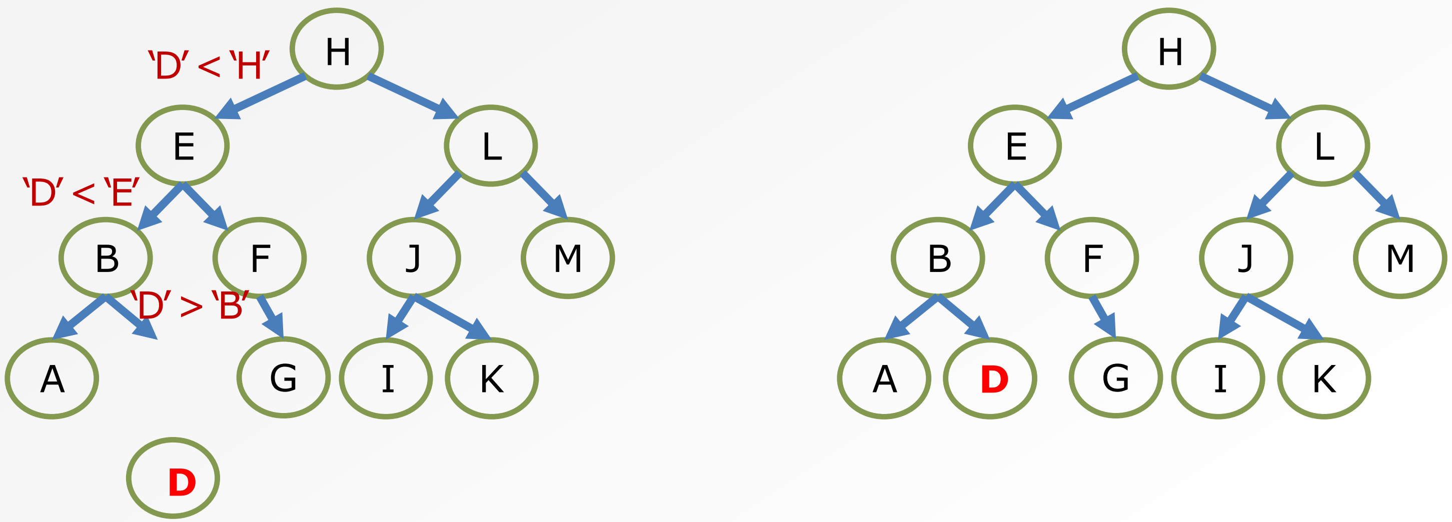
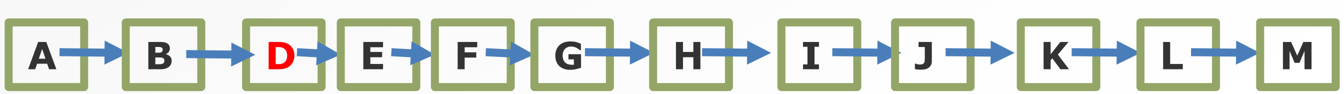
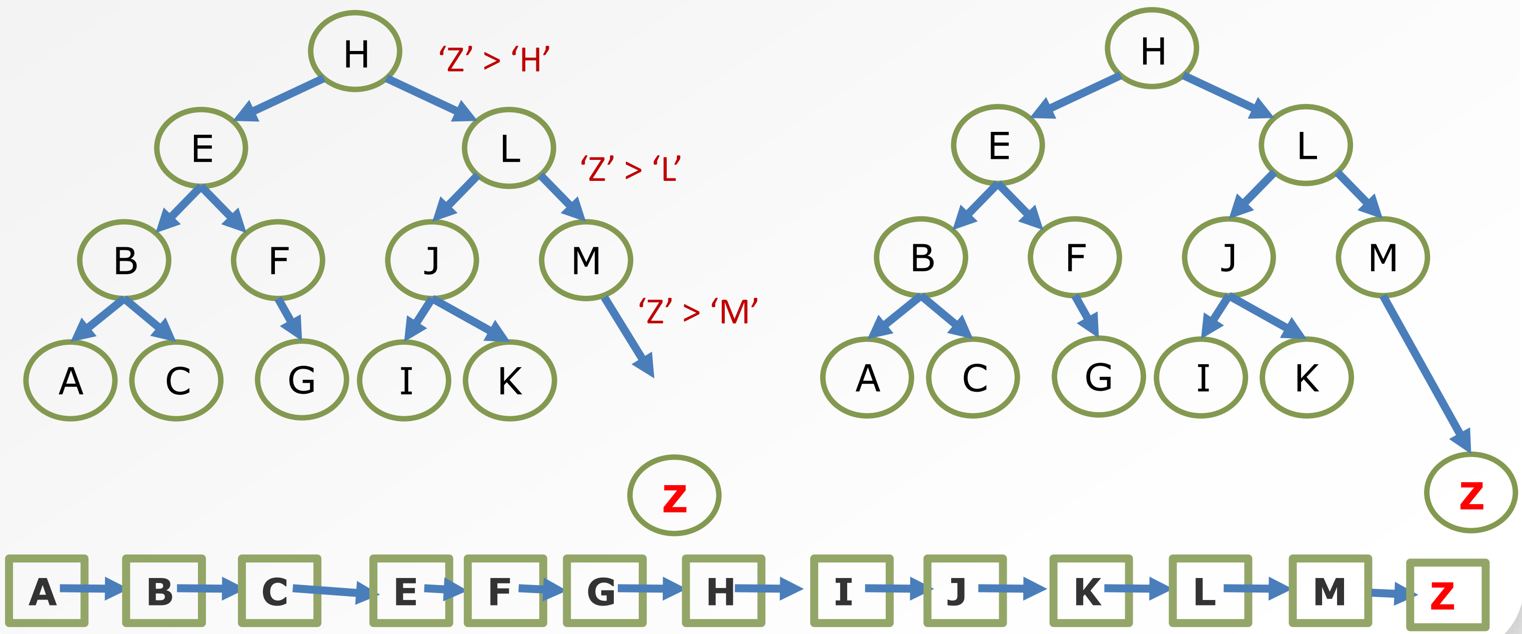


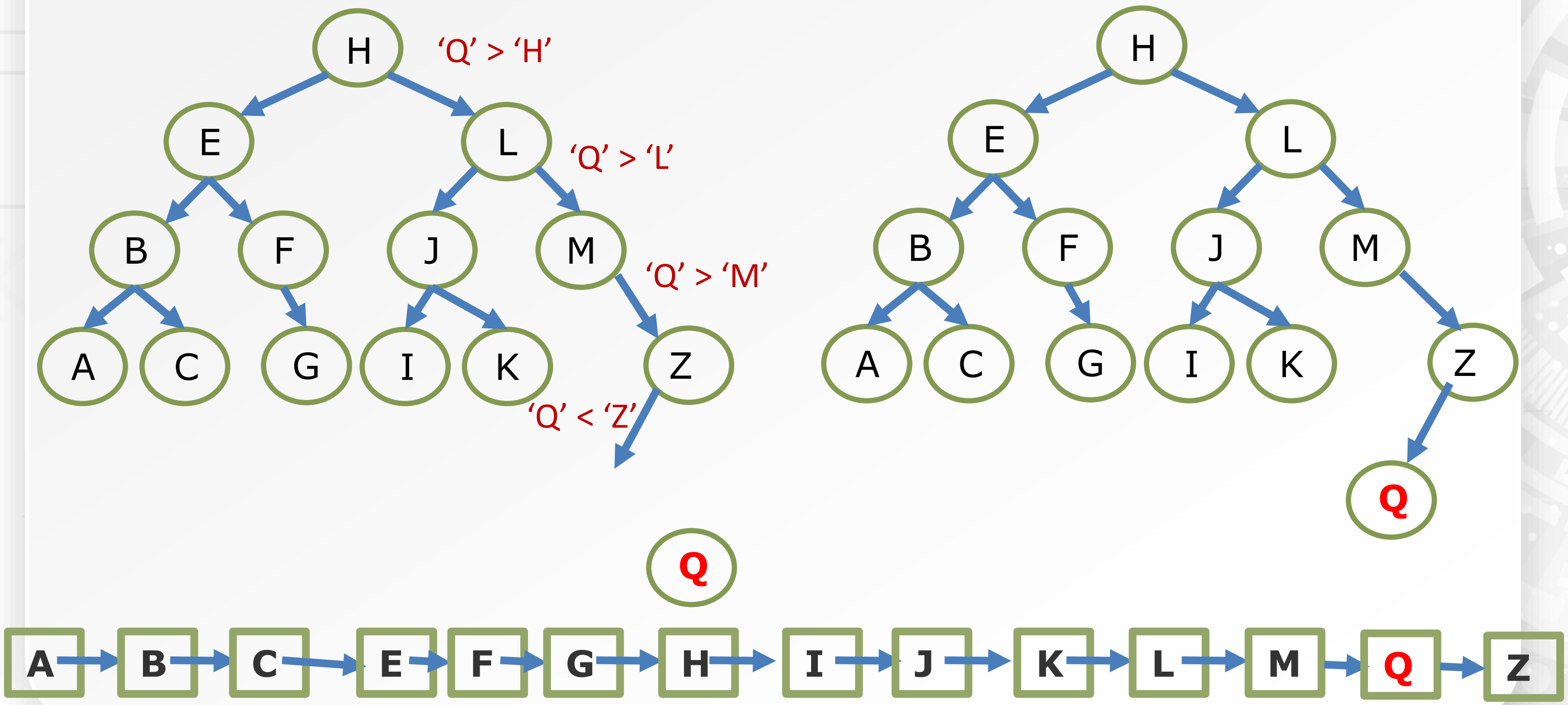


INSERTING A NODE INTO A BST

- Node insertion is relatively simple!
- Further exercise: Try Inserting ‘Z’



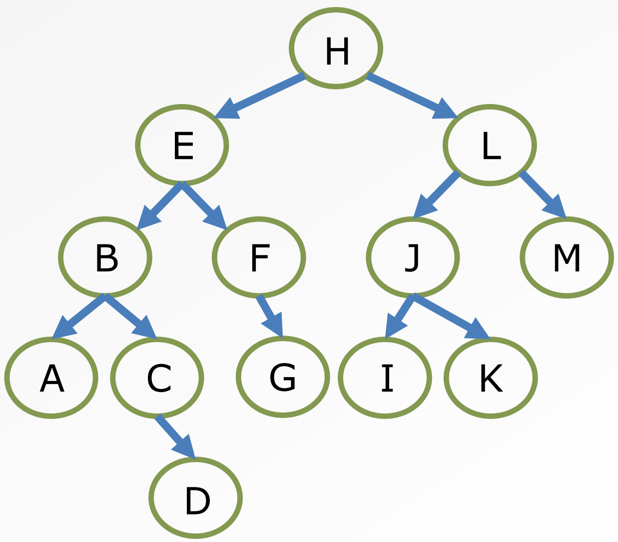
- Node insertion is relatively simple!
- Further exercise: Try Inserting ‘Q’



OUTLINE

- Binary Search Trees (BST)
- BST Operations:
  - Traversal
  - Inserting a node
  - **Removing a node**

After removal, the tree is still a BST

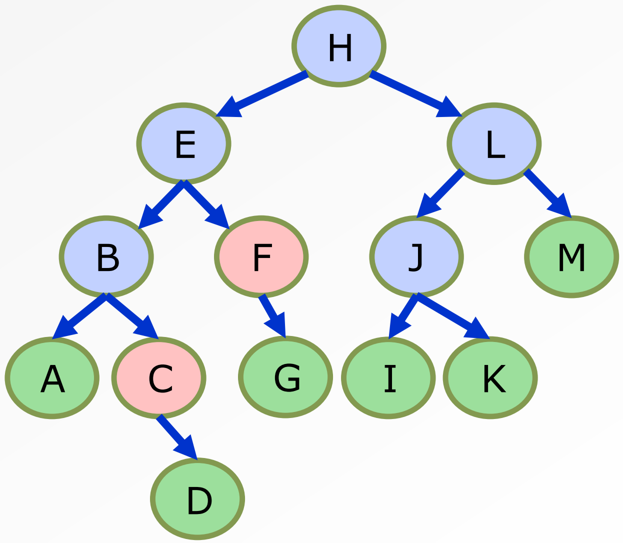


REMOVING A NODE FROM A BST

- Node removal is more complicated
- Beginning with a BST, the resulting tree after removing a node must still be a BST

Obey the BST rule: L < C < R

- Remove node X - a bit tricky
- 3 cases:
  1. x has no children:
     - Remove x
  2. x has one child y:
     - Replace x with y
  3. x has two children:
     - Swap x with successor
     - Perform case 1 or 2 to remove it



REMOVING A NODE FROM A BST

- Remove node X - a bit tricky
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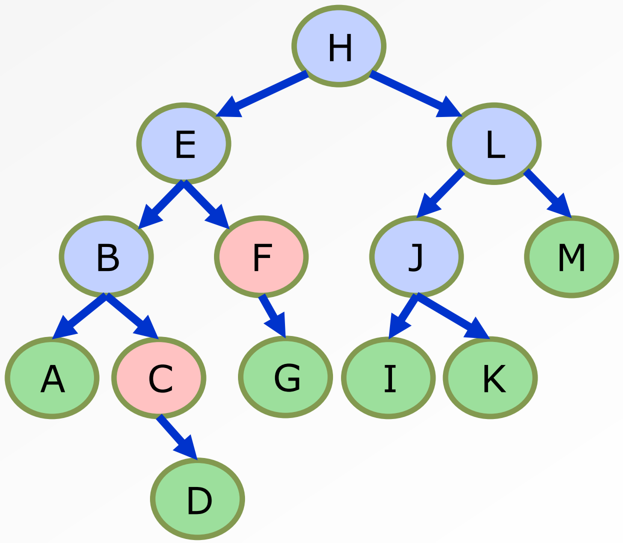

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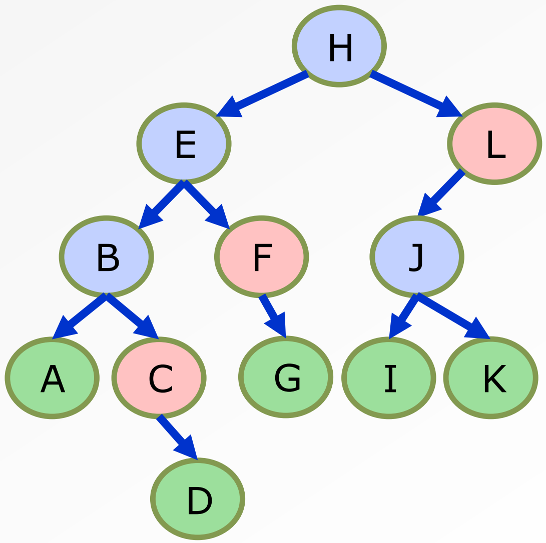
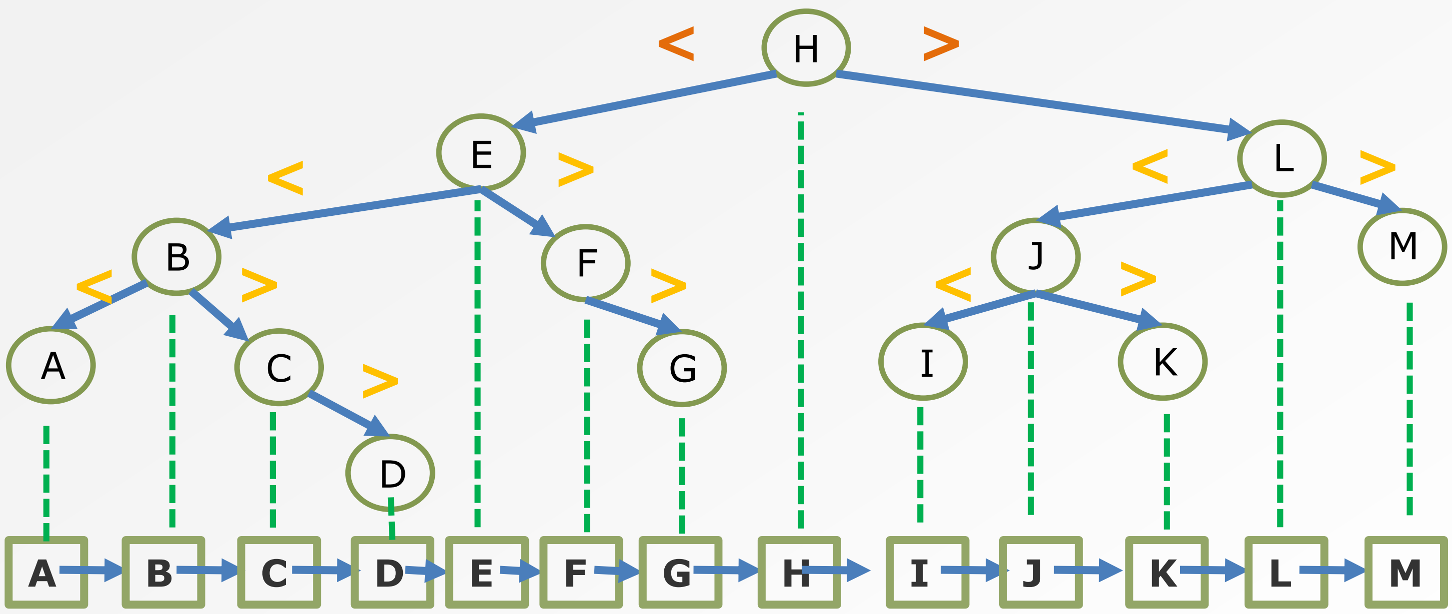


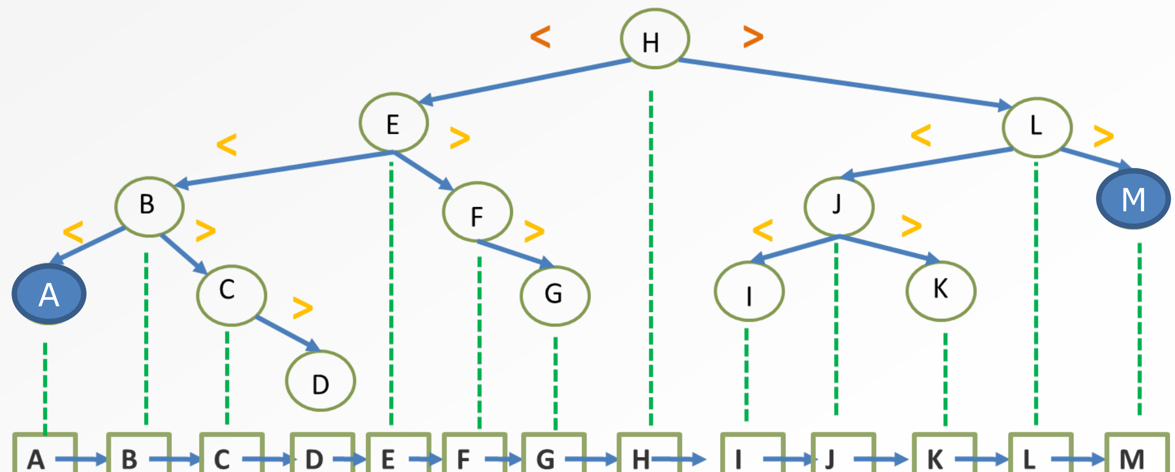
MAPPING: TREE(IN-ORDER) → LIST



- If we draw the BST carefully:
  - **Left subtree on the left side of the current node;**
  - **Right subtree on the right side of the current node;**
- Mapping to X-axis will produce **a sorted list.**

FEATURES

- BST's in-order traversal produces a sorted list!
  - **L < C < R** rule ensures sorted order
- The binary-search-tree property guarantees that:
  - The minimum is located at the left-most node
  - The maximum is located at the right-most node



REMOVING A NODE FROM A BST

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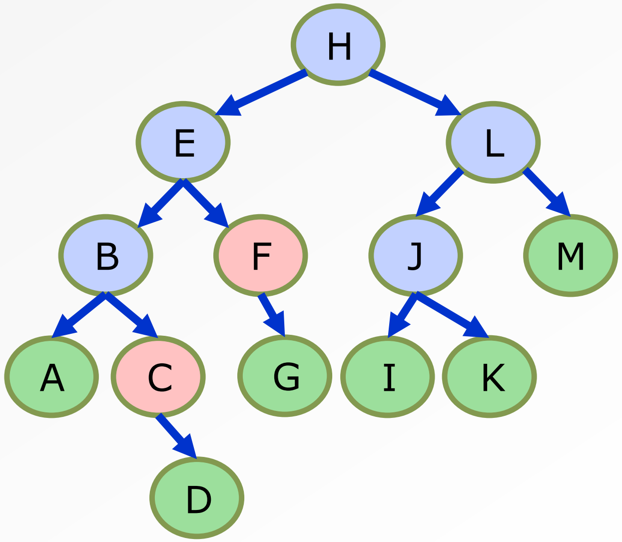


WHAT IS THE SUCCESSOR OF X?

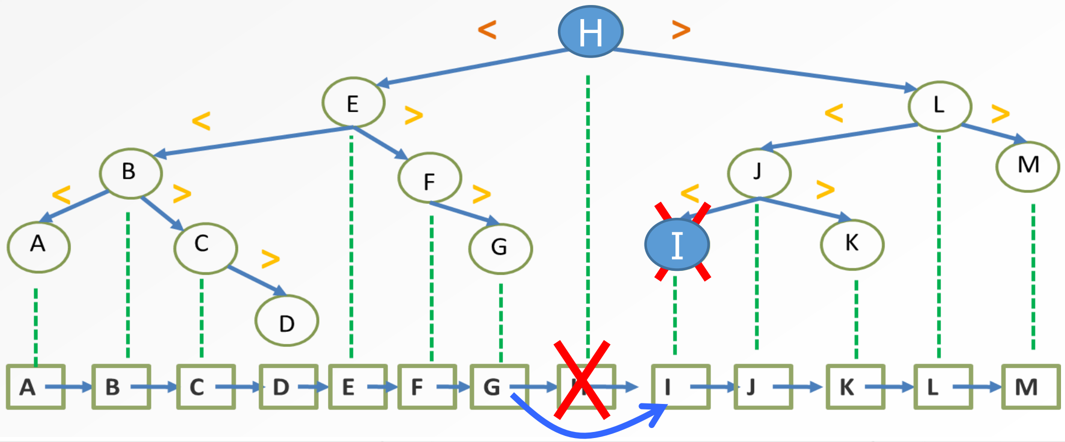
Replacing a node with its in-order successor ensures that the BST rule (L<C<R) is maintained

In-order traversal of a BST produce a sorted list (in ascending order)

Successor is:

- **The node immediately after it in the sorted list,** or
- **The next node visited using an in-order traversal**

X has two children, so X's successor is minimum node in its right subtree.
E.g.: H's successor is I, E's successor is F, J's successor is K.



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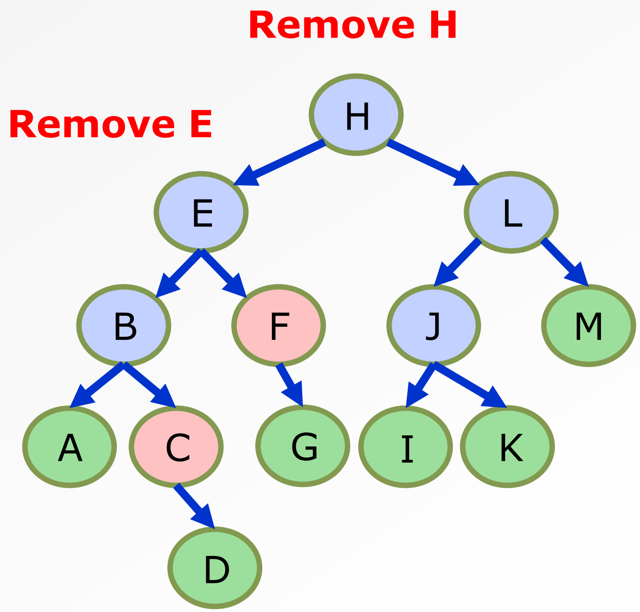

QUESTIONS

- **Why will case 3 always go to case 1 or case 2?**

  A: because when X has 2 children, its successor is the minimum in its right subtree, so the successor should not have left child.

  It might have no child(case 1) or one right child(case 2).

- **Could we swap x with predecessor instead of successor?**

  A: yes.

- Remove node X - a bit tricky
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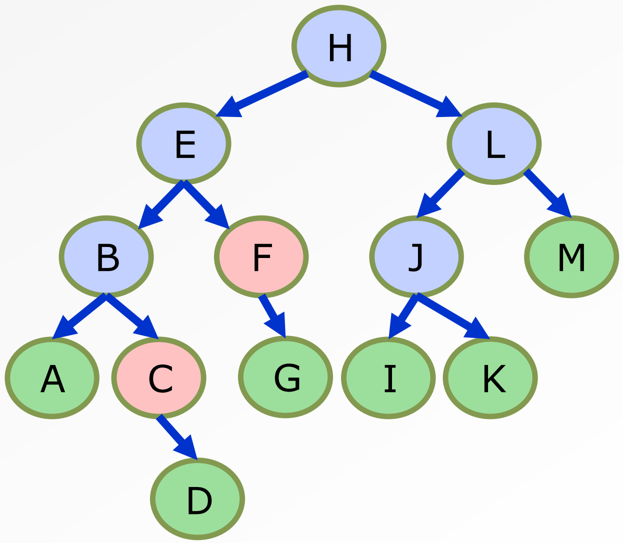


WHAT IS THE SUCCESSOR OF X?

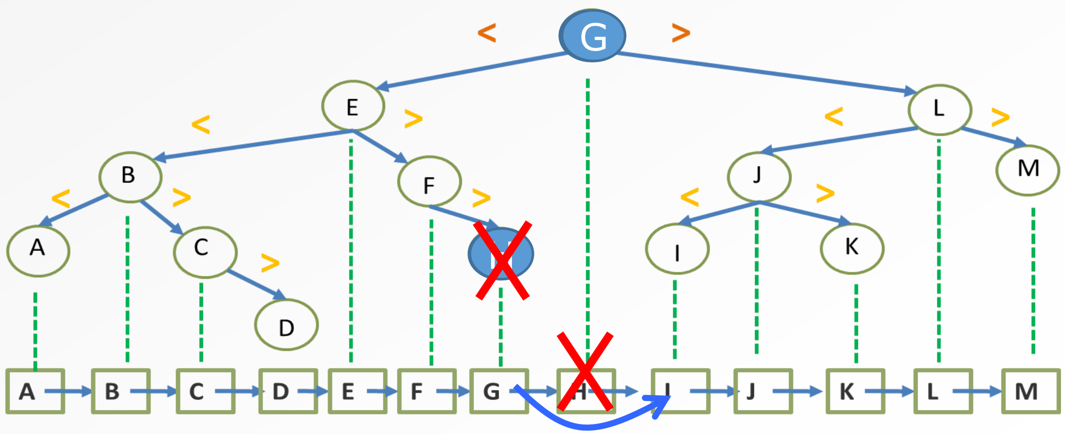
Replacing a node with its in-order **predecessor** ensures that the BST rule (L<C<R) is maintained

In-order traversal of a BST produce a sorted list (in ascending order)

Successor/predecessor:

- **The node immediately after/before it in the sorted list**
- **The next/previous node visited using an in-order traversal**

X has two children, so X's predecessor is maximum node in its left subtree.
E.g.: H's predecessor is G, E's predecessor is D, J's predecessor is I.



TODAY YOU SHOULD BE ABLE TO

- Define a Binary Search Tree
- From a list, how do we construct a Binary Search Tree? Is it efficient?
- How do we traverse a BST to search a item?
- How do we insert/remove a node from a BST?